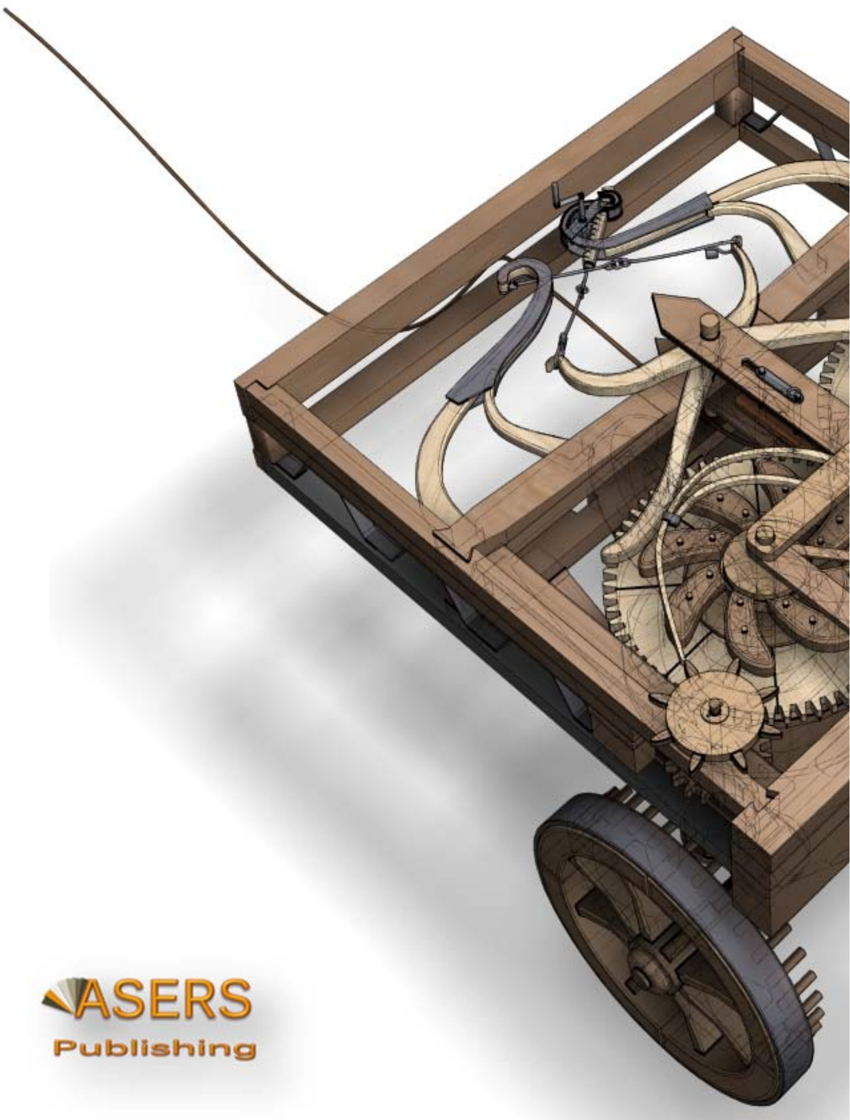
Theoretical and Practical Research in Economic Fields



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UNDERSTANDING SLL/US$ EXCHANGE RATE EXPECTATIONS IN SIERRA LEONE USING BOX-JENKINS ARIMA APPROACH

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Abstract: *This study was carried out with the purpose of producing twelve out-of-sample forecast for a univariate Exchange Rate variable as a way of addressing challenges faced around dollarization issues in the Sierra Leone economy. In pursuit of this, the ARIMA model was utilized, with the best model [1,4,7] indicating that the Sierra Leone - Leone [SLL] currency will continue to depreciate against the United States Dollar [US$] throughout the year 2020. This was done on the assumption of Ceteris Paribus condition, and most importantly on the view that past events of the univariate exchange rate variable are a determinant of future outcomes or performances. Robustness check was also carried out on within-sample forecast; the within-sample forecast was also utilized to produce out-of-sample forecast for a twelve months duration and this was then compared with the original out-of-sample result. Overall, results from the robustness results proved consistent with each other and only with infinitesimal level of error. In a bid to moving forward, recommendation for policy actions were highlighted, particularly in relation to the establishment of collaboration between relevant policy institutions like the Bank of Sierra Leone and the Ministry of Finance to address issues of concern, for example, a boost to the real sector and many more - in view of the current crisis of COVID-19, action of the central bank seem to be well in place to address abuse of the exchange rate system by unscrupulous traders / importers).*

Keywords: Box-Jenkins ARIMA; exchange rate; forecast; Sierra Leone.

JEL Classification: C52; C53; E47; F31; F47.

1. Introduction

Foreign exchange dynamics is a highly topical concern in the stabilization of the world economy, particularly some of the struggling economies around Sub-Saharan Africa (SSA), which are highly dependent on import transactions to support domestic consumption (Jackson *et al.* 2020; Bangura *et al.* 2013). Exchange rate modelling is very important for policy decisions (Jabbie and Jackson, forthcoming), hence it is very important that the dynamics of such variable is accurately predicted on a regular basis in a bid to address on-going concerns around macroeconomic fundamentals in a country like Sierra Leone.

Exchange rate measures a country's currency worth in comparison with others, which is closely related to the purchasing power parity [PPP] – an area that was recently addressed in Jabbie and Jackson's (forthcoming) study. Where it is such that the domestic currency cannot be exchanged at the same rate in comparison with international currencies, there is likelihood that the domestic currency is being devalued, which to some extent may impact on citizens' welfare and also, prices that people pay in exchange for goods and services (Nyoni 2019a; Jabbie and Jackson, forthcoming).

Exchange rate is very important in the international monetary market, particularly in areas concerned with offsetting payment on trade transactions, stabilisation of price dynamics and also, in support of basic structural development capacity in individual economies around the world (Jackson and Tamuke, 2018; Iyke and Odhiambo, 2017). Therefore, an intense study on exchange rate, and more so its prediction is very critical for macroeconomic stabilisation and planning. Fluctuations in exchange rate movement as witnessed in the Sierra Leone economy can stir up inertia of inflation expectations, which is not good for an economy (Jabbie and Jackson, forthcoming; Jackson and Tamuke, 2018; Bangura et al, 2012). This on many occasions can result in a build-up of dollarisation market, commonly used by people to hedge risks against collapse in investments connected with marketable securities and many more.

There is an expectation that with the continued devaluation of a country's currency, people will anticipate higher wages through collective bargaining agreements (Jackson, 2020b). Higher level wage and salary settlements can also result in increased cost to employers, and the pass-through of this is normally felt through high cost of goods purchased and also, services delivered. There is also a higher chance for money supply to increase, which also exert intense pressure on central bank to utilise available policy instruments to address high risks of '*too much money chasing too few goods*' in the system. Unlike in inflation targeting regimes adopted by many of the developed economies where central banks are perceived as totally autonomous, decisions taken to address exchange rate concerns in politically-meddled system as witnessed in many of the small open economies around Africa can be a very hard thing to achieve (Warburton and Jackson, 2020; Bleaney, Morozumi and Mumuni, 2018). In the absence of economic diversification, it seem very certain that exchange rate will continue to threaten many of the small open economies around the world – a typical example is Sierra Leone, which is highly dependent on importation of essential goods and services to support domestic consumption.

1.1. Rational for the Study and Emerging Knowledge Exploration

Exchange rate dynamics in Sierra Leone is a topical concern for economic prosperity - people's expectations about the economy, particularly prices of goods and services consumed have always being hinged on dynamic events in the exchange rate market, also highly influenced by parallel market street traders. Added to the highlighted expectations, the current state of low productivity in the real sector is also an attestation of the on-going crisis, with essential and basic consumable items like Rice and Onion continued to be the dominant imports, which also increase pressure on institutions like the central bank to settle high demand for foreign currencies to settle import bills.

On this note, this study has taken as its rationale by utilising data on Exchange Rate variable within the period 2001-2012 in observance of how past events of the series will impact on out-of-sample forecast for a twelve months duration. It is a general expectation that such outcome will induce exchange rate depreciation, on the backdrop of the country's low capacity to absorb shocks, attributable to an almost inactive real sector to support the production of basic goods and services needed for consumption in the domestic economy. The proposed methodology [ARIMA] will generate automated iteration of (likely) best models suitable for predicting out-of-sample forecast of exchange rate dynamics in the country.

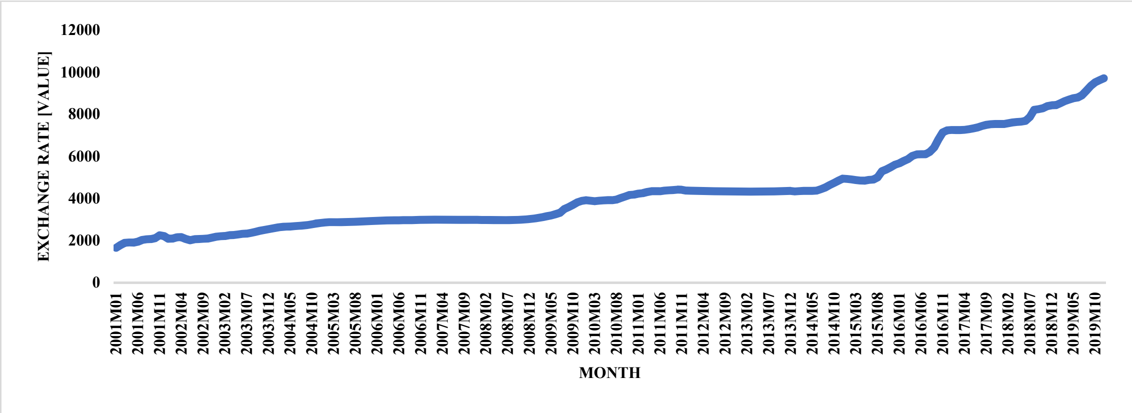
Despite the wide usage of ARIMA model to address exchange rate predictions, the uniqueness of this study is its automated approach to data utilization (as shown in Appendix 1). It is hoped the outcome will be useful for addressing knowledge exploration connected with the formulation of robust policy measures to address short-term concerns pertaining to exchange rate depreciation and scarcity in Sierra Leone. It is also important to note that, assumption built in this model is based on past events of the series, in which out-of-sample forecast is to be produced. Policy recommendations from the study will seek to address pressures in the exchange rate market and its pass-through effects to economic agents in the domestic economy.

1.2. Stylized Facts on Exchange Rate Dynamics in Sierra Leone

As already mentioned, exchange rate is an on-going challenge in the economy, attributable to an unproductive real sector, which has been heavily reliant on imports to support economic activities in the domestic economy. Figure 1 shows a steady increase in exchange rate in SLL/US$ over the period, which makes it very difficult for growth to be achieved in the economy. The country's susceptibility to shocks on account of its weak real sector is partly to be blamed for the situation as depicted in the steep rise in exchange rate between the SSL and the US$.

In addition to the weak real sector operation in the country, Sierra Leone has gone through series of traumas, some of which are man-made disasters, while others are occurrences attributable to natural disasters and events in the global economy (supply-side driven). The occupation of the country by rebel intruders is something that cannot be forgotten by people in the country – the legacy of this is still resonating in the fabrics of structural problems the country is still faced with, for example, destructive towns / villages and some urban towns, which were once utilized as means for activities relating to agriculture, mineral exploration, etc. The data as provided for exchange rate is indicative of recurring problems, which is making it easy for the SLL to be regularly devalued against international currencies like the US$. Equally, the experience of the twin shocks around 2014-15 is also an added trauma for a small open economy like Sierra Leone to absorb.

Figure 1. Exchange Rate Stylised Facts [2001M01 - 2019M12]



Source: BSL data warehouse

The lack of prudence on the part of people during the boom time of Iron Ore exploration also exposed the country's vulnerability to shocks (see Jackson, 2016). It was quite evident from the twin shock of Iron Ore price slump and the Ebola epidemic that proceeds from the boom time of mineral exploration were not utilised judiciously to cushion uncertainties revolving around natural disaster and supply-side driven shocks (Jackson and Jabbie, forthcoming; Jackson, 2016a).

1.3. Objectives and Organisation of the Study

In view of the above discussion, the main objectives of the paper are hereby stated as follows:

- Utilise automated ARIMA codes to produce the best model.
- Produce out-of-sample forecast covering twelve months period for the year 2020.
- Provide suggested policy recommendations in addressing exchange rate concerns.

In view of the above discourse, the remaining sections of the paper are divided as follows: Section two addresses the literature review, which is further sub-sectioned into theoretical and empirical literatures. Section three addresses the econometric model, which in this case is Autoregressive Integrated Moving Averages [ARIMA] and the data range, which is typically based on a univariate monthly usage of exchange rate variable sourced from the Bank of Sierra Leone data archive system. Section four covers the analysis of data and the model evaluation, which is based on an automated system produced from EVIEWS application. Section five covers the conclusion and relevant policy recommendations.

2. Literature Review

2.1. Theoretical

In order to address concerns around exchange rate dynamics and its forecast outcome, it is very important that attention is drawn to the theoretical connections of exchange rate determination – in this study, the researcher has identified three connections as explained below:

Purchasing Power Parity [PPP] – this typically suggest concerns relating to cause and effect between change in price level (which is the main cause) and exchange rate (which in this case, is the effect). As explained by Jabbie and Jackson (forthcoming), a simple mathematical expression can be used as outlined below in equation 1.

$$e = {}^{P}/_{P*} = \frac{\text{price of standard market prices of goods}}{\$\, price\ of\ the\ same\ standard\ basket} \qquad 2.1$$

As explained by Nyoni (2019b), the theory of PPP is based on the simple "*law of one price*". With reference to equation 2.1, it explains exchange rate movement between currency in two different countries, for example, Sierra Leone (SLL) and the United States of America (US$). Under a typical goods-market arbitrage condition, it is possible that such situation will make it possible for exchange rate to equalize prices in the two countries – in this case, where US imported goods are considered more expensive in the Sierra Leonean market, it is possible for economic agents in Sierra Leone to purchase more of what is produced in the domestic economy, which eventually may result in the appreciation of Sierra Leonean manufactured goods due to high demand over that of US traded goods in the local market.

Mint parity theory – this is based on the simple idea of two or more countries utilizing the same metallic standard of either gold or silver currency as medium of exchange for transaction. In this case, countries that uses gold for instance will be inclined to exchange based on the gold standard value in the respective countries. This essentially will make it possible for there to be a standard differential in prices of goods and services purchased in one country as against the other, given the exchange value of metallic currencies that are being utilized in the different countries. As pointed out by Aahana (Online) in an economics discussion portal, the mint parity of foreign exchange rate has highlighted two main concerns – the first is based on the actual rate of exchange, which can differ from the equilibrium rate of exchange. Secondly, under the gold standard, there is a limit specified beyond which fluctuations of exchange rate cannot take place. This thereby adds pressure in the determination of exchange rate parity, and thus price dynamics in the country in which the gold standard may not be of worthwhile value.

Uncovered Interest Rate Parity (UIP) – this is based on movement in exchange rate, which is hinged on holdings of assets in two different currencies, for example SLL and US$. The theory of UIP gives premium to arbitrage mechanism without much emphasis on transaction costs and liquidity constraints. On the condition that UIP is to hold, the resulting arbitrage relationship will produce the following (Nyoni 2019b, Warburton 2018):

$$E_t\ (Ine_{t+h} - Ine_t) = i_t\ - i_t^* \qquad 2.2$$

Market expectation of exchange rate return is defined based on the left-hand side of the equation from time t to time t+h, and i_t and i_t^* while i_t is the interest rate of the domestic and foreign currencies respectively.

2.2. Empirical Review

The relevance of forecasting exchange rate dynamics has been demonstrated lately in several studies produced by Nyoni (2019a; 2019b) – such studies captures forecast outcomes of Naira/US$ and the Indian Rupee/US$ exchange rate in India respectively. Each of the models made use of ARIMA with the best model outcomes indicating full robustness of the system in the underpinning policy recommendations. In the case with the Indian Rupee/US$ outcomes, there is an indication that the Rupee will appreciate, but also indicating a devaluation of the Rupee, while also encouraging local productivity and promoting capital inflows.

The practicality of ARIMA to forecast outcome has become very popular, given its rigor in providing high level of certainty; evidence of this in the area of forecasting inflation dynamics has been done more lately to address going forward situation with price stability and even the influence of exchange rate as an exogenous determinant of inflation dynamics in the Sierra Leone economy to be more specific (Jackson, Tamuke and Jabbie 2019; Jackson 2018; Jackson and Tamuke 20108; Jackson, Tamuke and Sillah 2018; Jackson, Tamuke, Jackson and Sillah 2018).

Babu and Reddy (2015) made an attempt to forecast exchange rate using ARIMA, Neutral Network and Fuzzy Neuron. The results proved contradictory, on the ground that ARIMA proved to be a better model in predicting exchange rate market in India than the other two counterpart models.

Olakorede *et al*. (2018) utilised univariate time series ARIMA model to determine exchange rate movement between Nigeria Naira and US$, with monthly data ranging from 1980M01 to 2015M12. The best fitted model (0,1,1) with minimal AIC and BIC indicated that the Naira will continue to depreciate against the US$ during the periods under investigation (2016M01 to 2018M12).

Driss and Fatima (2018) utilises Box-Jenkins ARIMA and Vasicek Stochastic Models to predict exchange rates in Morocco. Comparisons were then made from the two model outputs for the month of March 2018. The exchange rate outcome then made it possible for the best ARIMA (2,1,2) to be retained for EUR/MAD, while the Vasicek best model (3,1,2) was retained for the US$/MAD.

In pursuit of exploring means of forecast accuracy, Trafalis and Ince (2006) utilised hybrid model approach for predicting exchange rate performances. In this study, they proposed two stage forecasting that incorporated parametric techniques such as autoregressive integrated moving average (ARIMA), Vector

Autoregression (VAR) and co-integration techniques, and nonparametric techniques such as support vector regression (SVR) and Artificial Neutral Networks (ANN). It was particularly noted that input selection in the model estimation was very important in the final forecast output – in short, the SVR technique outperformed the ANN for two input selection models.

In pursuit of exploring the best model approach of forecasting exchange rate dynamics, Leung et al (2000) made an attempt by utilizing the General Regression Neutral Network (GRNN) – this study was intended to explore opportunities for international firms that are engaged in conducting substantial currency transfers in the course of business transactions, and the GRNN model was seen to be able to be a good approach in improving their overall prospects for profitability. The monthly data used was able to predict exchange rate for three currencies namely, the British Pound, Canadian Dollar and Japanese Yen. The overall outcome from the study proved that GRNN is better than other neutral network and econometrics techniques utilized in the study (namely, the Multi-Layered Feed-forward Neutral Network [MLFN]), which is a better means of solving financial forecasting problems.

West and Cho (1994) compared out-of-sample forecasting performance of univariate homoscedastic, GARCH, Autoregressive and Non-Parametric Models for conditional variances, using five bilateral weekly exchange rates for the dollar during the period 1973-1989. GARCH models were seen to demonstrate high level of forecast accuracy during a one-week horizon period. In the case of longer horizon, it was a difficult task of selecting the best model for forecasting out-of-sample performances. More confusing, none of the models were seen to perform better in a conventional test of forecast efficiency.

Canova (1993) also utilizes the Bayesian Time-Varying Coefficients (TVC) approach to model and forecast exchange rate data. In this, it was shown that leptokurtic behaviour disappears under condition of time aggregation. Overall, it was proved that the Bayesian TVC model improves over a random walk and such improvements are robust enough to effect changes in the forecasting environment.

Despite the relevance of the methodologies utilized in the aforementioned empirical reviews produced by researchers from selected countries, the uniqueness of this study is its ease in utilizing ARIMA automated system to model exchange rate for twelve months out-of-sample forecast period in Sierra Leone – as opposed to manual iteration, the automated system will generate easy means of diagnostically iterating model choices, while also selecting the best model for utilization in out-of-sample forecast for a specified period. It is hoped that such outcome will remain very important in supporting executive decisions that requires urgent attention in addressing exchange rate issues in the Sierra Leone economy.

3. Econometric Model and Data Usage

3.1. Autoregressive (AR) Models

$$SLL_t = \emptyset_1 SLL_{t-1} + \emptyset_2 SLL_{t-2} + \cdots + \emptyset_p SLL_{t-p} + \uparrow \varepsilon_t \qquad 3.1$$

The above series (SLL_t) as expressed in equation 3.1, epitomize as the Sierra Leone (Leone) currency (SLL) is assumed to be an autoregressive process of the order p, expressively denoted as AR(p). The above equation signify that the SLL exchange rate is explained by previous values of the SSL/US$ series (see Jackson 2018; Nyoni 2019a).

Operationalization of equation 3.1 makes it possible for ε_t to be made the subject, as shown in equation 3.2 below.

$$SLL_t \left(1 - \emptyset_1 B - \emptyset_2 B^2 - \cdots - \emptyset_p B^p\right) = \varepsilon_t \qquad 3.2$$

where: B in equation 4 is a backshift operator; $\emptyset_1$ ... $\emptyset_p$ are the parameters of the model; ε_t is a normally distributed random process with mean 0 and a constant variance σ_ε^2 which is assumed to be independent of all process values; SLL_{t-1}, SLL_{t-2}, are AR models, with stationary data, which means the data should be stationary prior to being fitted into the model (See Appendix 1 for automated codes, reflecting properties for satisfying stationary conditions).

3.2. Moving Average (MA) Models

White noise series properties with mean 0 and variance σ_ε^2 are normally referred to as moving average, with order q, normally expressed as MA(q). This can be expressed as a weighted linear sum of previous forecast errors:

$$SLL_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q} \qquad 3.3$$

Equation 3.3 implies that SLL/US$ exchange can be explained in terms of current and past period disturbances (in this case the random errors, or attributed to shocks, which is categorized on the basis of natural disasters or man-made events (see Jackson 2018a).

The backshift operator process of equation 3.3 can be rearranged to produce a new expression as indicated in equation 3.4 below.

$$SLL_t = \varepsilon_t(1 + \theta_1 B^1 + \theta_2 B^2 + \cdots + \theta_q B^q) \quad 3.4$$

where: $\theta_1, \theta_2, \ldots \theta_q$, are said to be coefficients, with lagged error terms; B is backshift operator; ε_t is normally distributed random process.

3.3. Autoregressive Moving Average (ARMA) Models

This is a combination of both the AR and MA models, which is also expressed as [ARMA(p,q)]. It is expressed thus:

$$SLL_t = \emptyset_1 SLL_{t-1} + \emptyset_2 SLL_{t-2} + .. + \emptyset_p SLL_{t-p} + \varepsilon_t + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q} \quad 3.5$$

A rearrangement of equation 3.5 will then produce a new expression as shown in equation 3.6 below:

$$SLL_t - \emptyset_1 SLL_{t-1} + \emptyset_2 SLL_{t-2} + .. + \emptyset_p SLL_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q} \quad 3.6$$

The backshift operator expression then results in a new equation as expressed in equation 3.7 below:

$$SLL_t\left(1 - \emptyset_1 B^1 - \emptyset_2 B^2 - \cdots - \emptyset_p B^p\right) = \varepsilon_t(1 + \emptyset_1 B^1 + \emptyset_2 B^2 + \cdots + \emptyset_q B^q) \quad 3.7$$

Simplifying equation 3.7 produces a new expression as shown below:

$$\emptyset(B) SLL_t = \emptyset(B)\varepsilon_t \quad 3.8$$

This is a simple case of parsimony ARMA(p,q), which gives a better representation of the data used.

3.4. Autoregressive Integrated Moving Average (ARIMA) Models

This is applicable to data that is representative of stationary series, which are thereby differenced on several occasions to make sure the date at use is fit-for-purpose. In this case, series can be differenced once or twice, depending on the properties of the series as shown in the equation below.

$$(1 - B)^d SLL_t \quad 3.9$$

where d is the number of times the process is differenced to ensure stationarity is achieved.

Hence, drawing on from equation 3.7, the expression with differencing can be written as shown in equation 3.10 below:

$$SLL_t\left(1 - \emptyset_1 B^1 - \emptyset_2 B^2 - \cdots - \emptyset_p B^p\right)(1 - B)^d = \varepsilon_t(1 + \emptyset_1 B^1 + \emptyset_2 B^2 + \cdots + \emptyset_q B^q) \quad 3.10$$

This can now be simplified to form equation 3.11 as indicated below:

$$\emptyset(B)(1 - B)^d SLL_t = \emptyset(B)\varepsilon_t \quad 3.11$$

Equation 3.10 in particular is referred to as the ARIMA process, with the following characteristics:

- White noise [ARIMA(0,0,0)]
- Random walk [ARIMA(0,1,0)]
- Autoregressive process [ARIMA(0,0,q)]
- Autoregressive moving average [ARIMA(p,0,q)]

Random Walk process as mentioned above can be expressed as indicated in equation 3.12 below:

$$SLL_t = SLL_{t-1} + \varepsilon_t \quad 3.12$$

The first difference of the expression in equation 3.12 will now result in a stationary series as expressed here; $SLL_t - SLL_{t-1}$

3.5. Data and Application Usage

Data utilised ranged between 2001M01-2019M12 and this was sourced from the Bank of Sierra Leone data warehouse system, with a total of 227 data points. The automated coding system developed in EVIEWS application made it such that all data were seasonally adjusted in a bid to smoothen their pattern (See Appendix

1). Relevant diagnostic checks are also incorporated in the automated coding system to enable quick outcome of results, and more so with precision in the iteration process compared to manual iteration.

4. The Box Jenkins ARIMA Process, Analysis and Evaluation

In this section, the process of model analysis and evaluation can be determined by applying the Box-Jenkins ARIMA methodology. This is simplified in Appendix 1, with relevant codes to illustrate each stage of the process – the model selection process commenced by differencing the series in a bid to achieve stationarity. The model selection as specified in the codes have been given maximum AR and MA properties of 11, and then seasonally adjusted to smoothen the series. Identification of the ARMA is done by assigning optimal AR and MA orders, which is based on the AIC, SIC and Hannan-Quinn criterion. This will also help to generate set of residuals, while also performing automated diagnostic tests to ascertain conformity with the characteristics of a white noise property (Nyoni 2019). The process can be repeated, which addresses the order of integration until the best models are sourced out for use in forecasting out-of-sample exchange rate (See Appendix 1).

The automated system for this study produced 426, with 227 observed data, which is more than sufficient to run the model. In order to minimise repetition of all the model iterations, the best two model properties are illustrated in Appendix 2 and 3 ([1,4,7] and [1,9,9] respectively) to enable effective evaluation of the best model, which can be used to forecast out-of-sample values for exchange rate in the identified period of 2020M01 to 2020M12.

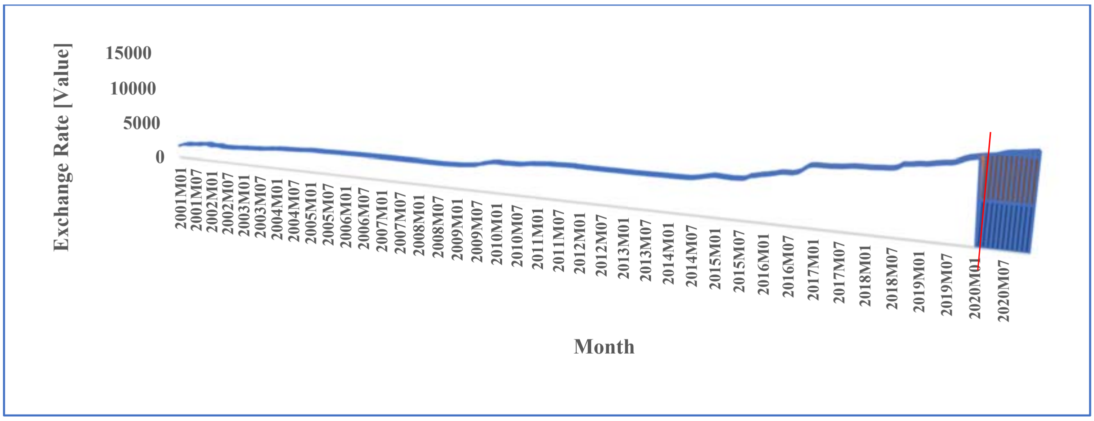
Table 1. Summary of Two Best Model Selected out of 426 Iterations

Component	Best Model	R-square	AIC	Durbin Watson	Root [AR and MA]	Prob (F-Stat)
Exchange Rate	(1,4,7)	52.7%	-6.39	1.94	AR <1 MA<1	0.00
	(1,9,9)	51.9%	-6.25	1.96	AR<1 MA=1	0.00

Source: EVIEWS Output

Out of the 426 iterated models, the two models specified in Table 1 were judged to be the best for forecasting exchange rate. Amongst the two as shown above in Table 1, Model [1,4,7] is the preferred choice given its R^2 value, which is slightly higher than that of model [1,9,9]. Equally, values relating to AIC, Durbin Watson and both the AR and MA roots, which addresses stability condition for forecasting are also comparatively smaller.

Figure 2. Exchange Rate Actual and Forecast



Source: EVIEWS generated Out-of-Sample Forecast

As shown in Table 1, the automated model [1,4,7] was selected to generate out-of-sample forecast for Exchange rate, and it shows that exchange rate dynamics will continue to move above Le10,000 (Ten thousand Leone mark, which is approximately equivalent to $1) as shown in Figure 2 in the shaded portion – this is an indication that the SLL will continue to depreciate against the US$ throughout most part of the year 2020 (Reference to Figure 2 and Appendix 4). The model being univariate in nature only took into consideration past events of the series, which commenced from 2001M01 to 2019M12.

If the condition of ceteris paribus is to be held constant, that is, with no policy intervention, then SLL/US$ exchange rate will certainly become a concern in the domestic economy. This will have wide scale implications, for example, goods in the domestic economy will become expensive to purchase using the SSL currency in comparison to the same item sold in the USA. In this regard, purchasing power will reduce on account of continued devaluation to the SSL currency against the US$. Given the fact that the country is highly dependent on imports to support domestic consumption, this will likely result into serious problems in areas relating to some of the undermentioned points:

- Prices of basic goods and services will continue rise due to the inertia of expectations manifested by citizens in relation to issues concerned with exchange rate problems.
- There is also a chance for unscrupulous individuals, particularly parallel market street traders to continue their habit of hoarding the dollar currency, which eventually will make it difficult for the central bank to meet requests submitted by importers in settlement of import bills.
- There is also a chance that citizens, particularly those in employment would channel request for collective bargaining agreements with employers in a bid to negotiate wage / salary increase, given the fact that inflation would have undermine the value of consumers' purchasing power, and in particular their sustained means of livelihoods (Jackson, 2020a; Jackson, 2020b).
- Equally, there is a chance for money supply to increase due to possible devaluation of the SLL currency, with citizens also being inclined to hoard money due to expectations of occurrences pertaining to price increase in domestically consumed items.

5. Model Robustness Check

5.1. 12-Months within - Sample Forecast (2019M1-2019M12)

In order to ascertain the robustness of the model estimation technique, the author carried a 12-months within the given sample data utilised (2001M1-2019M2); in that vein, the forecast considered the forecast from 2019M01-2019M12 and the results in their log-transformation are provided as shown below in Table 4:

Table 2: Comparing Sample Results: Normal NEXR Data with Within Forecast Sample Data

Year / Time	NEXR Normal Data (Log)	Within Forecast Estimate (Log) 2019M1-2019M12
2019M1	9.04041	9.04041
2019M2	9.050905	9.050905
2019M3	9.062303	9.062303
2019M4	9.071559	9.071559
2019M5	9.078563	9.078563
2019M6	9.085552	9.085552
2019M7	9.100071	9.100071
2019M8	9.118193	9.118193
2019M9	9.09971	9.09971
2019M10	9.161757	9.161757
2019M11	9.172177	9.172177
2019M12	9.181047	9.181047

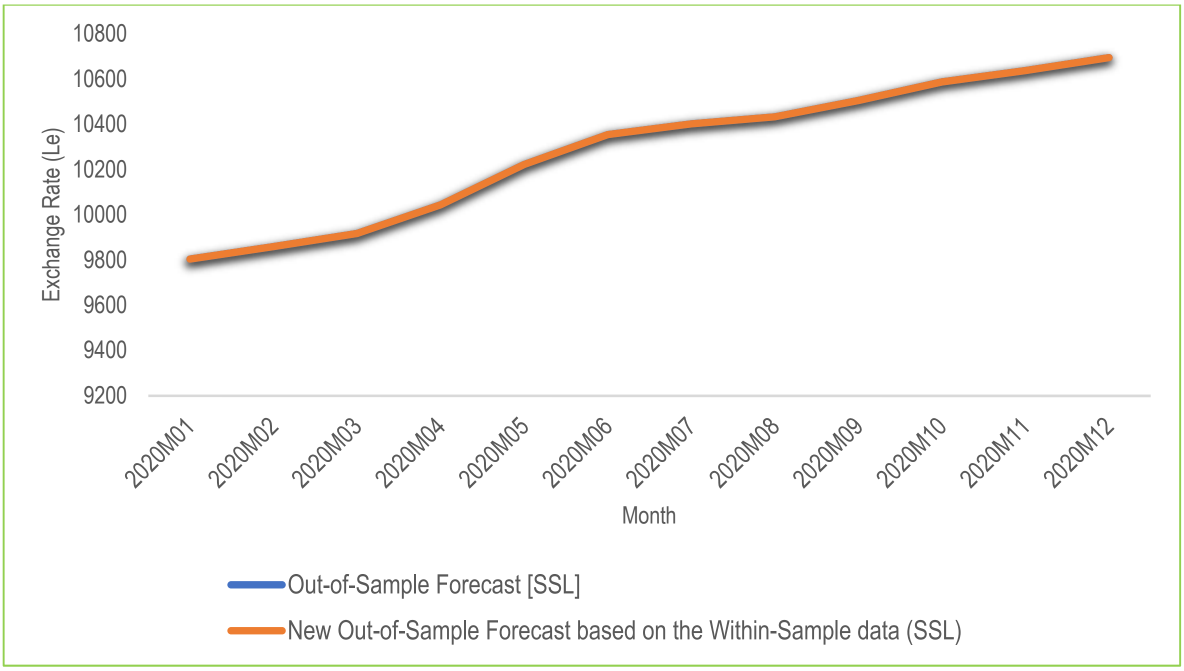
Source: EVIEW Estimation

Margin of error in the above estimation is almost ZERO, which attest to the consistency and efficiency of the model estimation to support within sample forecast of 12-months period, technically considered to be a short term period forecast as the strength of the Box-Jenkins model is designed to not so efficient in producing medium to long term forecast estimates.

5.2. 12-Months New Out-of-Sample Forecast (2020M1-2020M12)

Consistent with the forecast result produced in Appendix 3, we also ran a twelve months out-of-sample forecast from 2020M1-2020M12 using the within sample forecast as shown in column 3 of Table 4 to see whether similar results will be outputted. The table below provide result outcome for the new out-of-sample forecast taken from the within-sample forecast data (see Appendix 5). Figure 3 below provide a visual plot of the comparative outcome of the normal out-of-sample forecast and that which was taken from the within-sample forecast. The margin of error is infinitesimal, which indicate that model is sufficiently robust to address Exchange Rate dynamics in the country. Such outcome was produced without much consideration given to the escalating impact of Corona Virus 2019 (COVID-19) on the global economy (Jackson, 2020a). On that note, given the country's weak state of real sector operations, there is possibility that the currency may depreciate further than the Le10,000.00 forecast range. On the whole, the given the wide-scale impact of COVID-19 on the global economy, and in particularly on commodity prices like Crude Oil, there is a high possibility for Exchange Rate depreciate in Sierra Leone to be subdued on account of the country's pricing formulae (%% + or – band) used to work out Petrol Pump price (in Litre).

Figure 3: Comparative Chart of Outcome of Sample Forecast



Source: *Conversion Estimation from MS Excel*

Conclusions and Recommendations

Exchange rate dynamics is quite a topical concern for the global economy, particularly those in the global south that seem to be struggling with limited scope of improving their productive capacity in addressing citizens' basic needs. Equally as in the case with inflation forecasting, exchange rate forecasting is also a growing interest to policy makers and particularly for those in the business community. Given the volatility of events in the global economy, it is always such that exchange rates can be intrinsically noisy, non-stationary and produces level of confusion for those who are policy makers (Nyoni, 2019a, 2019b and Box and Jenkins, 1994).

Out-of-sample forecast from this study shows that exchange rate is set to rise in the year 2020 (reference to Figure 2 and Appendix 3) on the back drop of expectation inertia, attributable to the country's unproductive real sector in addressing consumption needs of essential food and non-food items. Outcome from this study is critical for policy makers, whose decisions are considered very important in addressing exchange rate concerns in the domestic economy. In reality, the situation in Sierra Leone is worrying, given the high level of unproductive capacity in the real sector and also, the inertia of expectations from economic agents in the country, which is generally connected with price increase and continued devaluation of the SSL currency against the dollar in this case. Regardless of authorities' decisions (in the case, the Central Bank and Ministry of Finance) to affirmatively address issues, people's mindsets have been attuned to thinking in a negative way that makes it harder for the situation to normalize in the country.

The way forward in this difficult condition is for both the monetary and fiscal authorities to work collaboratively in addressing issues of practical concern. In this situation, investments can be selectively diverted in the short-run to address basic commodity needs in the country – this could involve '*hand-picking*' of trustworthy importers earmarked to receive foreign exchange from the central bank in support of its core mandate in stabilizing prices, while also monitoring dynamics in the exchange rate market.

The prevalence of corruption that has gone unchecked for decades in the country is also making it very difficult for institutions like the central bank to meet its core objective of stabilising prices (see Jackson, 2018a; Jackson, 2017). Medium to long term strategies should be directed at addressing ways by which the country's real sector can be strengthened through diversion of investments in core areas connected with food and other essential non-food production, which seem to be the most influential in the country's Consumer Price Index [CPI] basket (see recent studies produced by Jackson, Tamuke and Jabbie, 2019).

In view of the empirical output and analysis of the study done so far, it seems more reasonable for authorities to develop affirmative measures in a bid to address issues around exchange rate dynamics, which is crippling the fabrics of developmental and growth prospects in the country. Continued depreciation of the SLL against the US$ is a deterrent to economic prosperity, particularly when the real sector is not productively capable to support domestic activities and also, the lack of prudence on the part of citizens in supporting sustained developmental agendas (Jackson and Jabbie, 2019; Jackson, 2016). There is a high risk of Exchange Rate surfacing given the current global crisis of COVID-19 – equally as mentioned, the pass-through effect of subdued global commodity prices like Crude Oil, will make it less likely for prices to be inflated in the domestic market. Equally, action taken by the central bank through its announced Monetary Policy Statement to support importation of essential commodities will also help to keep exchange rate at a reasonable rate for the time being.

In recommendation of the way forward on things, particularly in the short term, measures should be set in place to monitor revenue generating sources, particularly earnings from overseas trade (e.g., proceeds or royalties from mining companies, etc.). In addition, there should be transparent means of ensuring that those in receipt of foreign currencies to settle payments for goods imported in the country are monitored to avoid abuse of potential reserves that can be utilized by the central bank for judicious purposes, and also in ensuring minimum international requirements relating to three months of import cover is regularly maintained.

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APPENDICES

Appendix 1: 'EVIEWS ARMA selection of optimal lag lengths'

```
smpl @all 'Set sample period
scalar n1=@obs(nexr) 'Number of observations of NEXR data
scalar components = 1 'Number of NEXR components, including aggregate index
scalar maxar = 11
scalar maxma = 11

'Rename series
series nexr_1 = nexr

'Seasonally adjust data
for !i = 1 to components
nexr_!i.x12(mode=m) nexr_!i
next

'For each component produce ARMA(a,m) with varying orders
for !i = 1 to components
    for !a = 1 to maxar '12
        for !m = 1 to maxma '12

        smpl 2001m1 2001m1+n1-1
        equation arma_!i_!a_!m.ls d(nexr_!i_sa) c ar(1 to !a) ma(1 to !m)

        next
    next
next

'Identify the ARMA for each component with the optimal AR and MA orders according to the Akaike Information Criterion.
Change to @schwarz or @hq for Schwarz and Hannan-Quinn criteria.
for !i = 1 to components
!mininfocrit = 9999
    for !a = 1 to  maxar '12
        for !m = 1 to maxma '12

        if arma_!i_!a_!m.@aic<!mininfocrit then
        !besta = !a
        !bestm = !m
        !mininfocrit = arma_!i_!a_!m.@aic
        endif
        next
    next

'Save the equation with the best order structure
smpl 2001m1 2007m1+n1-1
equation arma_best_!i.ls d(nexr_!i) c ar(1 to !besta) ma(1 to !bestm)

smpl 2001m1+n1 2020
arma_best_!i.forecast nexr_forecast_!i
next

'Show best ARMA models for selected components
for !i = 1 to 1
show arma_best_!i
next

show exp(nexr_forecast_1)/exp(nexr_forecast_1(-12))*100-100
'*
```

Source: *EVIEWS Output*

Appendix 2: Best Model

Dependent Variable: D(NEXR_1_SA) – [1,9,9]
Method: ARMA Maximum Likelihood (BFGS)
Date: 12/27/19 Time: 23:37
Sample: 2001M02 2019M12
Included observations: 227
Convergence achieved after 320 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.008223	0.002687	3.060118	0.0025
AR(1)	2.130098	0.209186	10.18282	0.0000
AR(2)	-2.045876	0.435270	-4.700246	0.0000
AR(3)	0.351086	0.580099	0.605217	0.5457
AR(4)	1.187092	0.608753	1.950037	0.0525
AR(5)	-1.139257	0.524579	-2.171755	0.0310
AR(6)	-0.442927	0.457537	-0.968068	0.3341
AR(7)	1.698625	0.426458	3.983103	0.0001
AR(8)	-1.510659	0.317392	-4.759602	0.0000
AR(9)	0.517494	0.139128	3.719545	0.0003
MA(1)	-1.570731	47.38617	-0.033147	0.9736
MA(2)	0.927326	56.39745	0.016443	0.9869
MA(3)	0.784794	74.75416	0.010498	0.9916
MA(4)	-1.234469	154.8589	-0.007972	0.9936
MA(5)	0.509392	78.66760	0.006475	0.9948
MA(6)	0.867417	164.2629	0.005281	0.9958
MA(7)	-0.980542	216.3296	-0.004533	0.9964
MA(8)	0.520619	130.5464	0.003988	0.9968
MA(9)	0.032411	9.146762	0.003543	0.9972
SIGMASQ	7.90E-05	0.000239	0.330926	0.7410

R-squared	0.527244	Mean dependent var	0.007770
Adjusted R-squared	0.483851	S.D. dependent var	0.012956
S.E. of regression	0.009308	Akaike info criterion	-6.395626
Sum squared resid	0.017935	Schwarz criterion	-6.093869
Log likelihood	745.9036	Hannan-Quinn criter.	-6.273863
F-statistic	12.15042	Durbin-Watson stat	1.945309
Prob(F-statistic)	0.000000		

Inverted AR Roots	.82-.45i	.82+.45i	.75	.59+.74i
	.59-.74i	.15+.97i	.15-.97i	-.87-.39i
	-.87+.39i			
Inverted MA Roots	.87-.50i	.87+.50i	.59-.66i	.59+.66i
	.22-.89i	.22+.89i	-.06	-.87-.35i
	-.87+.35i			

Source: *EVIEWS Output*

Appendix 3: Alternative Best Model Selected

Dependent Variable: D(NEXR_1) – [1,4,7]
Method: ARMA Maximum Likelihood (BFGS)
Date: 12/27/19 Time: 23:37
Sample: 2001M02 2019M12
Included observations: 227
Convergence achieved after 426 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.007970	0.002302	3.462829	0.0006
AR(1)	1.684868	0.063894	26.36983	0.0000
AR(2)	-2.177746	0.081889	-26.59390	0.0000
AR(3)	1.580565	0.083919	18.83440	0.0000
AR(4)	-0.807957	0.064109	-12.60278	0.0000
MA(1)	-1.002605	4.439207	-0.225852	0.8215
MA(2)	1.207362	2.537261	0.475853	0.6347
MA(3)	-0.221457	0.539772	-0.410279	0.6820
MA(4)	0.151773	1.730492	0.087705	0.9302
MA(5)	0.446037	2.341793	0.190468	0.8491
MA(6)	0.027203	1.932681	0.014075	0.9888
MA(7)	0.280735	3.922650	0.071568	0.9430
SIGMASQ	9.76E-05	0.000983	0.099234	0.9210

R-squared	0.518713	Mean dependent var	0.007797
Adjusted R-squared	0.491725	S.D. dependent var	0.014269
S.E. of regression	0.010173	Akaike info criterion	-6.248394
Sum squared resid	0.022146	Schwarz criterion	-6.052252
Log likelihood	722.1927	Hannan-Quinn criter.	-6.169248
F-statistic	19.22011	Durbin-Watson stat	1.964563
Prob(F-statistic)	0.000000		

Inverted AR Roots	.71-.58i	.71+.58i	.13+.97i	.13-.97i
Inverted MA Roots	.78-.63i	.78+.63i	.25-.90i	.25+.90i
	-.18+.66i	-.18-.66i	-.69	

Source: *EVIEWS Output*

Appendix 4: Actual Forecast of SLL/US$ Exchange Rate in 2020

Month	Out-of-Sample Forecast [SSL]
2020M01	9804.666
2020M02	9860.084
2020M03	9917.775
2020M04	10044.49
2020M05	10222.86
2020M06	10355.31
2020M07	10402.77
2020M08	10433.78
2020M09	10506.23
2020M10	10587.92
2020M11	10638.13
2020M12	10695.46
Source: *EVIEWS Empirical Output*	

Appendix 5: Comparative Outcome of Actual Forecast of SLL/US$ Exchange Rate in 2020 and the New Out-of-Sample Forecast Based on Within-Sample Results

Month	Out-of-Sample Forecast [SSL]	New Out-of-Sample Forecast based on the Within-Sample data (SSL)
2020M01	9804.666	9804.666
2020M02	9860.084	9860.084
2020M03	9917.775	9917.775
2020M04	10044.49	10044.49
2020M05	10222.86	10222.86
2020M06	10355.31	10355.31
2020M07	10402.77	10402.77
2020M08	10433.78	10433.78
2020M09	10506.23	10506.35
2020M10	10587.92	10587.97
2020M11	10638.13	10638.58
2020M12	10695.46	10695.55
Source: *EVIEWS Empirical Output*		

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THE ARMEY CURVE IN BULGARIA (2000-18) – THEORETICAL CONSIDERATIONS AND EMPIRICAL RESULTS

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Abstract: *In this paper we provide a theoretical basis for the so-called" Armey curve," the inverted U-shape relationship between the level of government purchases and GDP growth, named after Armey (1995). We use an otherwise standard Keynesian model, augmented with a quadratic relationship between investment and lagged government expenditure, which was documented empirically. This modelling approach is a useful shortcut that aims to capture the common link shared by both variables, namely their dependence on the real interest rate, as suggested also by the extended static IS-LM model. This resulting dynamic relationship is a newly documented stylized fact, at least in Bulgarian data for the period 2000-2018, and the source in the extended Keynesian model that generates an Armey curve for Bulgaria.*

Keywords: armey curve; GDP growth; government purchases; Bulgaria.

JEL Classification: E12; E22.

Introduction

One of the major postulates and policy recommendations of the standard Keynesian theory is that governments could affect economic activity through the use of fiscal policy. In particular, when the economy is in a recession, the government can stimulate aggregate demand by increasing government purchases, by decreasing taxes, or both. Therefore, the theory predicts that a higher level of government purchases can increase the gross domestic product (GDP) of the economy.

Some authors take those recommendations one step further: In addition to the level effect on output, they argue that there is also a systematic growth effect on output. In other words, a higher level of government purchases could affect the growth rate through more public investment in education, healthcare, infrastructure, etc. However, according to Armey (1995), this growth effect is non-linear, and thus not necessarily a positive one. More precisely, Armey (1995) argues that after some level government spending is harmful for economic growth. The existence of a threshold level of government purchases is then a critical issue as it represents an important constraint for policy and public finance consolidation and austerity plans. Such effects deserve a rigorous treatment in order to be understood in depth.

This non-linear relationship between the level of government purchases and GDP growth was a relatively new stylized fact, named "the Armey curve", even though it had been empirically documented earlier in Engen and Skinner (1992), and later in Sheehey (1993). This finding came into stark contrast with a much older empirical relationship, known as the Wagner's law (1883), which postulated that there is only a positive relationship between government spending and economic growth. The finding at that time was due to the fact that the share of government spending in output was very small compared to current, or post-World-War-II levels.

Despite being linked to the Keynesian theory, the Armey curve was never explicitly derived in a formal manner. Most of the studies in the literature, *e.g.*, Sheehey (1993) and later studies, are empirical and are all based on *ad hoc* assumptions. The interested reader is referred to Afonso and Furceri (2010), Arpaia and Turrini (2007), Dar and Amirkhalkhali (2002), Folster and Henrekson (2001), Gwartney *et al.* (1998), Lin (1994), Sattar (1993), Engen and Skinner (1992), and the references therein. Armey (1995) himself argues verbally why the curve is hump shaped. We aim to bridge that gap by providing a relevant theoretical basis for the Armey curve. The reason is that the Keynesian model is static in nature, while growth is a dynamic concept. Without a dynamic extension of the model, no Armey curve can arise.

We thus start from an otherwise standard Keynesian framework, and extend it another novel stylized fact: the existence of a quadratic relationship between the level of current private investment, and the level of lagged government purchases. The dynamic inter-relationship introduces dynamics in the model in a simple way. We take this underlying dynamic relationship as an empirical regularity and incorporate it in the model. As a suitable testing case we use Bulgaria over the period 2000-18: a country that is both an EU member state, but also still developing. We can argue that the link captures the common inter-dependence on the main interest rate, which might be rigid due to the fact that the monetary authority, or the central bank, is focused on price stability, and not on full-employment considerations. Another possible explanation could be the dynamic negative effects of high debt levels. For now, we leave this interesting interest rate channel for future study.

With this new mechanism in place, the Keynesian framework is now made dynamic, and we can now generate an inverted U-shape relationship between the level of government purchased and output growth. Yet another advantage of the framework is that we can find the approximate threshold-, or congestion level of public spending, *i.e*., the level of expenditure that maximizes economic growth.

The rest of the paper is organized as follows: In the next section, we present the model setup calibrated to Bulgarian data, and derive the theoretical Armey curve; the optimal-, or the growth-maximizing level of government purchases is solved determined from both the calibrated theoretical model, as well as from data directly, using the empirical Armey curve. The two cases are compared and contrasted. The paper then concludes.

1. Model Setup

This section describes a two-period open economy standard Keynesian model with the simple extension, as outlined in the introduction. We need at least 2 periods in the timing of the model in order for the setup to have a dynamic dimension. After all, the Armey curve features economic growth. The results obtained using a 2-period model can be then easily extended to any number of periods.

We begin with the national income accounting identity

$$Y_t = C_t + I_t + G_t + X_t - M_t, \qquad 1.1$$

where Y_t denotes *GDP* in period *t*, C_t is private final consumption spending, I_t denotes investment, G_t are government purchases, X_t are exports, and M_t are imports. In other words, output equals the sum of its uses.

Next, aggregate consumption behavior is assumed to be characterized by a standard Keynesian consumption function:

$$C_t = \bar{C} + b(Y_t - T_t), \qquad 1.2$$

where $\bar{C} > 0$ denotes the autonomous consumption spending, $0 < b < 1$ is the marginal propensity to consume out of disposable (after-tax) income, and T_t denote lump-sum taxes in period *t*. Such a relationship has been also documented for Bulgaria in Vasilev (2015b).

Next, in an open economy context, imports are proportional to disposable income, with the degree of proportionality $0 < m < 1$, also referred to as the marginal propensity to imports:

$$M_t = m(Y_t - T_t) \qquad 1.3$$

In contrast, since exports depend on foreign countries' demand, which is taken to be exogenous in the model, we will set $X_t = \bar{X}$, and keep it unchanged.

For the sake of realism, we set total tax revenue to be proportional to income, or:

$T_t = tY_t$, 1.4

where $0 < t < 1$ is the average (effective) tax rate. As in Vasilev (2015a), that corresponds to a proportional tax system, where all forms of income - labor, capital, and profit (corporate) income are all taxed at the same rate of 10 percent. Given that total revenues are now endogenous, the government will use spending as an instrument to achieve a balanced budget, which is what is observed in Bulgarian data over most of the period 2000-18. With an Armey curve a higher spending feeds into higher future output, and thus higher future tax revenue.

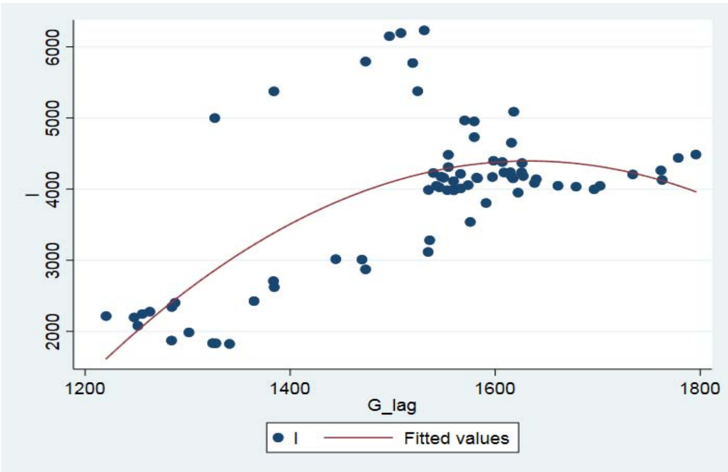
Lastly, the novelty in this paper is that the model will try to capture the (partial) crowding out effect of government purchases, and the fact that more public spending discourages private saving and leaves less resources for private investment tomorrow. (As mentioned earlier, the lagged effect could be driven by some stickiness exhibited in the behavior of the interest rate.) The intertemporal price of those resources is the real interest rate, hence there is a direct link between the two variables, which can be represented after some simple algebra as:

$I_t = f(G_{t-1})$, 1.5

where $f'(.) > 0$, $f''(.) < 0$.

The assumptions imposed on this function are easily verified using data on Bulgaria over the period 2000-18, where all data is from NSI (2019). As documented in Figure 1 (where G_{lag} denotes G_{t-1}), a non-linear relationship was established when a quadratic regression specification was fitted through the scatterplot. In addition, the formal regression estimation output is presented in Figure 2. R^2 is 48 percent, which means that the model explains half of the variation in investment, and all variables are statistically significant.

Figure 1. The facts



Source: National Statistical Institute, Bulgaria (2019)

Figure 2. Regression Output: Investment Function

Source	SS	df	MS			
				Number of obs	=	74
				F(2, 71)	=	33.33
Model	41454837.5	2	20727418.7	Prob > F	=	0.0000
Residual	44151447.4	71	621851.372	R-squared	=	0.4842
				Adj R-squared	=	0.4697
Total	85606284.9	73	1172688.83	Root MSE	=	788.58

I	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
G_lag	53.39736	12.18918	4.38	0.000	29.09281	77.7019
G_lagsq	-.0163497	.0040864	-4.00	0.000	-.0244977	-.0082018
_cons	-39202.68	9035.525	-4.34	0.000	-57219	-21186.35

Now that the assumed relationship between investment and lagged government purchases has been empirically verified, we proceed and introduce it into the model framework. In addition, we also need to fix initial investment. Therefore, in the first period, we will assume that private investment is pre-determined, and set to some exogenous level, or $I_1 = \bar{I}$. Therefore, in period 1,

$$Y_1 = \bar{C} + b(Y_1 - tY_1) + \bar{I} + G_1 + \bar{X} - m((Y_1 - tY_1). \quad 1.6$$

Similarly, in perioiod 2.

$$Y_2 = \bar{C} + b(Y_2 - tY_2) + f(G_1) + G_2 + \bar{X} - m(Y_2 - tY_2). \quad 1.7$$

Differencing output produces the following expression:

$$\Delta Y = Y_2 - Y_1 = b(1 - t)\Delta Y + f(G_1) - \bar{I} + G_2 - G_1 - m(1 - t)\Delta Y, \quad 1.8$$

Or

$$\Delta Y = \frac{1}{1-(b-m)(1-t)}[f(G_1) - \bar{I} + G_2 - G_1] \quad 1.9$$

Divide now by Y to transform the expression into output growth rate to obtain

$$\Delta Y/Y = \frac{1}{1-(b-m)(1-t)}[f(G_1) - \bar{I} + G_2 - G_1] \quad 1.10$$

Thus, the effect of government purchases on economic growth equals

$$\partial(\frac{\Delta Y}{Y})/\partial G_1 = \frac{1}{1-(b-m)(1-t)}[f'(G_1) - 1] \quad 1.11$$

In other words, depending on the level of government purchases G_1, the effect of government spending on output growth can be either positive, zero, or negative. For low levels of spending, $f'(.) > 1$, *i.e.*, the demand effect is very large (like it was during the Great Depression), and the effect is positive. In contrast, for large levels of spending, the effect is negative $f'(.) < 1$. There could be also some value for intermediate G_1 for which $f'(G_1) = 1$, so there is zero effect on growth.

We now use the mean level of government spending over the period 2000-18, G_{avg} = 1526.466 (in BGN mln.), in Bulgaria, as well as the estimated functional form for f (.) in order to make some computational experiments. In particular

$$f'(G_{avg}) - 1 = 53.397 - 0.016 * 2G_{avg} - 1 = 3.551 > 0 \quad 1.12$$

or the effect on growth at the average level of public expenditure on growth has been positive. This result shows that spending can be increased further in order to speed up economic growth. In particular, we can obtain the model-predicted threshold level of government purchases, denoted by $\hat{G}$, that maximizes economic growth by setting $f'(\hat{G}) = 1$ *i.e.*, $\hat{G}$ = 1637.4. In Bulgarian data, we observe such values (in BGN mln.) and above from 2016 onwards, which is an indication that the economy is now operating beyond the peak of the Armey curve, and the government needs to lower the level of government spending.

Figure 3. Armey curve regression

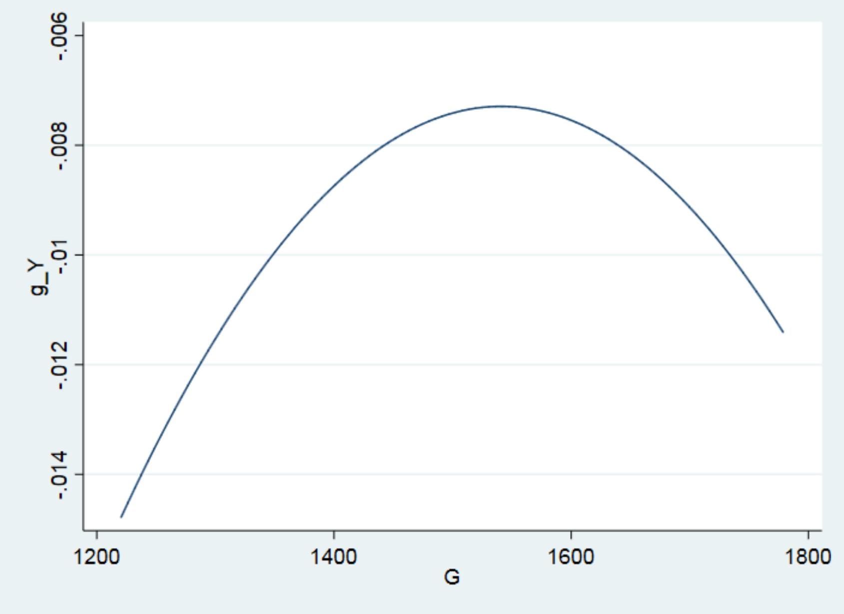
Source	SS	df	MS		
				Number of obs =	74
				F(2, 71) =	1.39
Model	.000269636	2	.000134818	Prob > F =	0.2551
Residual	.006873452	71	.000096809	R-squared =	0.0377
				Adj R-squared =	0.0106
Total	.007143087	73	.000097851	Root MSE =	.00984

g_Y	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
G	.0002247	.0001645	1.37	0.176	-.0001033	.0005528
Gsq	-7.29e-08	5.54e-08	-1.32	0.193	-1.83e-07	3.76e-08
_cons	-.180432	.1213136	-1.49	0.141	-.4223245	.0614605

Alternatively, we can estimate the Armey curve empirically, and obtain the growth-maximizing level of government spending by running a quadratic regression of growth on government purchases The results are presented in the Figure 3, where g_Y denotes output growth rate, G is the level of government purchases, and Gsq is the square of government purchases. According to the OLS estimates produced, the growth of the economy is maximized at G^* = 1572.43 (in BGN mln.), which is lower than the level predicted by the calibrated Keynesian model above. This value corresponds to the level observed in 2012.

The qualitative conclusion - that the economy is now operating beyond the peak of the Armey curve, and the government needs to lower the level of government spending - continues to hold. In addition, given the low R^2, any inference based on this regression is to be taken with some caution. This result can be also seen from the fitted Armey curve presented in Figure 4. Thus, the presence of a peaking relationship between the level of government spending and economic growth has been established both theoretically and empirically in Bulgaria over the period 2000-18.

Figure 4. Empirical Armey curve in Bulgaria, 2000-18



Source: Author's calculations

Conclusions

In this paper we provide a theoretical basis for the so-called "Armey curve," the inverted U- shaped relationship between the level of government purchases and GDP growth, named after Armey (1995). We use an otherwise standard Keynesian model, extended with a quadratic relationship between investment and government expenditure, which is a new documented stylized fact in Bulgarian data for the period 2000-2018. The link is through the dependence of both on the interest rate. The model is able to generate a realistic Armey curve for Bulgaria through this new transmission channel alone.

As a future extension, we may consider next a dynamic IS-LM model, in order to provide more detail on the interest rate link outlined above. The ambition is eventually to construct a micro-founded New Keynesian general equilibrium model with physical capital, maybe along the lines of Barro (1990) and Easterly and Rebelo (1993), and augmented with sticky prices, in order to understand better, the quantitative effect of this new propagation mechanism.

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SHADOW EFFECT FROM LAFFER TAX ALLERGY: NEW TAX POLICY TOOL TO FIGHT TAX EVASION

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Abstract: *This study is inspired by the Laffer curve to develop and formalize a concept around optimal tax policy considering asymmetric information. This is the "Shadow effect". This theory states that when the tax burden is high, producers tend to inflate their fictitious expenses to reduce their declared profit (to avoid paying a high tax). The theoretical developments show that the propensity of producers to the Shadow effect is positively related to the square of the tax rate. The relationship is non-linear. They also show that there is an inverse and non-linear relationship between the tax rate and the level of production. Also, producers' sensitivity to the Shadow Effect can be influenced by fluctuating the tax burden. This study provides governments a new fiscal policy tool. For instance, a numerical application has shown that if the Cameroonian government wants to encourage production in such a way that it could reach 50% more, it should reduce the corporate tax rate down ceteris paribus, to 16.19%.*

Keywords: tax evasion; tax burden; Laffer curve; Shadow effect.

JEL Classification: H21; H26.

Introduction

"We have a system that increasingly taxes work and subsidizes nonwork." – Milton Friedman

Can markets regulate without state intervention? There is not a lot of differences between this question and the following: Should we pay the tax? Indeed, tax is one of the main government's sources of funding. Without the tax that gives to the State the means of implementing its politics (Fauvelle-Aymar 1999), the latter wouldn't have any power on markets, or even conversely (Quinn et Shapiro 1991, Best 1976). This brings tax at the centre of the liberalism – interventionism debate. But, even if the major school of thought agree on the necessity of the presence of the State as an institution in the economy (at least for the Friedrich Hayek's Rule of law and property right), the problem of the " degree " of intervention will still persists (Friedman et Friedman 1998). This research raises the issue of tax management: what to tax? who to tax? where to tax? and above all, how to tax?

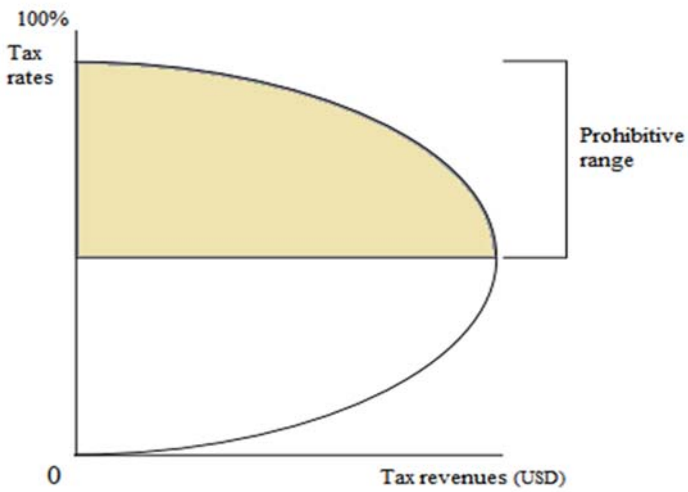
The washouts of capitalism during 1929 and 2008's crises (J. Stiglitz 2010), the failure of the Marxist's socialism and communism in Ex-URSS have shown the limits of economic radicalism. By extrapolating, it's obvious that not taxing is as dangerous as overtaxing. It is, therefore, necessary to find the right balance.

But, economic literature around that issue is abundant and even proposes some solutions. One of them is the theory of optimal taxation. Indeed, the optimal tax theory aims to design and implement a tax that maximises a social welfare function that is subjected to economic constraints (Mankiw et Weinzierl 2009). From this theory, the majority of taxes distort individual behaviour and consequently reduces the individual incentive to the taxed activity (Keane 2011). Although useful, optimal tax theory is often criticized to not considering administrative costs of tax systems (Burgess et Stern 1993). And the practice of ignoring the full set of tax instruments under uncertainty leads to misleading results (Dhami et al-Nowaihi 2006). Moreover, in its current state, optimal tax theory is incomplete as a guide to action for critical issues in tax policy. It is incomplete because it has not yet come to terms with taxation as a system of coercively collecting revenues from individuals who will tend to resist (Slemrod 1990).

Another attempt to build an appropriate tax policy lays in the famous Laffer Curve. Much closer to a political concept than an economic theory, the Laffer curve also aims to provide a solution to the optimal taxation issues. Indeed, an anecdote reported by Jude Wanniski in *The Public Interest* says that during dinner, Arthur Laffer grabbed his napkin and a pen and sketched a curve on the napkin illustrating the trade-off between tax rates and tax revenues (Wanniski 1978, Laffer 2004).

Laffer's curve is particularly interesting and seductive as it is simple to explain. About that, Wanniski (1978) summarises it into that Laffer's statement: "There are always two tax rates that yield the same revenues." And the famous Curve has the following shape:

Figure 1. The Laffer Curve



Source: (Laffer 2004)

As exposed on the figure, the Laffer curve is a hump-shaped curve showing tax revenue as a function of the tax rate. Revenue initially increases with the tax rate but then it can decrease (prohibitive range) if taxpayers reduce market labour supply and investments, if they switch compensation into non-taxable forms, and engage in tax evasion (D. Fullerton 2008). From the Curve, the revenue-maximizing tax rate can be calculated from an estimate of the elasticity of taxable income concerning the after-tax share. Fullerton D. (2008) explains that the mid-range for this elasticity is around 0.4, with a revenue peak of around 70%.

According to Laffer (2004), lower tax rates change economic behaviour and stimulate growth, which causes tax revenues to exceed static estimates. Furthermore, Wanniski (1978) argue that when the tax rate is 100%, all production ceases in the money economy (as distinct from the barter economy); Indeed, people will not work in the money economy if all the fruits of their labours are confiscated by the government, leading government revenues to zero (Wanniski 1978).

The particularity of this study is that it focuses on the tax evasion alternative, meaning that instead of ceasing production when the tax burden is high, producers increase their informal activities. This behaviour is considered as the "Shadow effect". Indeed, Mirowski (1982) was already arguing that the derivation of the Laffer Curve has nothing to do with tax evasion. Thus, Mirowski (1982) places a particular emphasis on this study "Shadow effect". Moving in the same direction, the optimal tax problem is a game of imperfect information between taxpayers and the social planner (Mirrlees 1971). Indeed, imperfect information is due to asymmetric information that enables producers to produce fraudulent financial statements by inflating their expenses. Their goal is to avoid high tax burden. An issue that has not been formally considered by the optimal tax theory (Slemrod 1990) and the so-called Laffer Curve (Mirowski 1982).

However, this did not taint the Laffer Curve glamour. Maybe because Laffer only reinvented the wheel. Indeed, Laffer himself said that the first studies on the relationship between the tax rate and the economic growth date back to the fourteenth century with the writings of Ibn Khaldoun[1] (Laffer 2004). Moreover, in the

[1] Ibn Kahaldun wrote in the 14th century that: at the beginning of the dynasty, taxation yields a large revenue from small assessments. At the end of the dynasty, taxation yields a small revenue from large assessments (Giertz 2008).

Wealth of Nations, Adam Smith yet observed that "…the economic incomes of private people are of three main types: rent, profit and wages. Ordinary taxpayers will ultimately pay their taxes from at least one of these revenue sources". Also, Jean-Baptiste Say in *Treaty of Political Economy* concluded that an excessive tax destroys the basis that carries it. The curve named the Laffer curve was formally presented by French economist Jules Dupuit in the early 1840s (Giertz 2008).

The tax is so important as it seems to go back to the time when humankind started living in community. In this connection, Marshall Sahlins (1976) explained that primitive societies already used their surplus of production as offerings to deities for their protection. A little later, the tax will rather serve to constitute the mode of the social organisation of the sedentary population (Sahlins 1976). Nowadays, in addition to ensuring the state's sovereign functions, taxes are supposed to contribute to the reduction of inequality and poverty by redistributing the wealth created (Lambert 1993). Goals that are very far from being achieved (Leigh 2008) and whose can even have negative consequences on financial markets[2].

Indeed, the tax has not always been considered by taxpayers as a contribution or participation. They consider it more like a penalty, a punishment; Although, a famous quote from an unknown author says, "A fine is a tax for doing something wrong. A tax is a fine for doing something right." Indeed, too much tax kills tax; and this can lead to tax evasion. Facing a high tax burden, entrepreneurs will choose to inflate their fictitious expenses to declare a low profit and therefore maintain their living condition. This is what this study considers as being the "Shadow effect". In such a situation, tax authorities may need to have clear information on how low they have to reduce the tax rates. Such policy, as an incentive, may encourage production and therefore economic development.

This is the framework in which this study fits. The objective of this paper is to develop and formalise the Shadow effect theory. Then, the study will provide policymaker with an effective and reliable tool for deriving tax policy considering tax evasion.

As recounted by Mirowski (1982), when Laffer was called before the US Congress to testify on the then-proposed Kemp-Roth tax cut. Senator Packwood questioned Laffer point-blank on his method of empirically determining the peak of the Laffer Curve; Laffer answers: "I cannot measure it frankly, but I can describe to you what the characteristics of it are (…)".

This study aims to measure the shadow effect propensity in addition to describing its characteristics.

1. Background of the Study

There is a plethora of work on taxation. These works cover many aspects; they are related to the nature of taxation as well as to its purpose. In this regard, Ramsey's contribution has had a significant impact on tax theory as well as other fields such as public goods pricing and regulation. Indeed, Ramsey (1927) proposes to tax only goods and services so that goods with the most inelastic demand are the most heavily taxed. The latter explains that, when taxes focus on goods whose demand varies little with price, it is more likely that the consumer will not change his consumption behaviour too much (Ramsey 1927). Kaldor (1965) connected the taxation issue with economic development from two points of view: the point of view of incentives and the point of view of resources. According to him, improving the tax system from an incentive point of view is made thru the granting of additional concessions of various kind, with less regard to the unfavourable effects on the public revenue. On the other side, additional taxation is made at the expense of worsening its disincentive effects (Kaldor 1965). Afterwards, Mirrlees's work shed new light on optimal taxation. Indeed, the author focused on the management of inequality at the centre of taxation by suggesting a way to formalize the policymaker's problem that deals explicitly with unobserved heterogeneity among taxpayers. According to him, if the policymaker taxes income in an attempt to tax those of high capacities, individuals will be discouraged from exerting as much effort to earn that income (Mirrlees 1971). A pioneering thought close to the Laffer curve. Moreover, his thought already considers the asymmetry of information. Indeed, the optimal tax problem turns out to be a game of imperfect information between taxpayers and the social planner.

It will not be long before the concept of "Laffer Curve" emerges. Wanniski was one of the first economists to speak of it in a scientific paper. According to him, all around the Laffer curve, there is this simple but powerful statement by Arthur Laffer: "There are always two tax rates that yield the same revenues". Indeed, when the tax rate is 100 per cent, all production stops in the money economy. Wanniski (1978) from the Laffer curve, argues that people will not work in the money economy if all the fruits of their labours are confiscated by

[2] Financing after bailout is costly because increased taxation reduces the non-financial sector's incentives to invest (Acharya, Drechsler et Schnabl 2014).

the government. Thus, as production ceases, there is nothing for the 100 % rate to confiscate, leading government revenues to zero. On another side, when the tax rate is zero, people can keep 100 % of what they produce in the money economy (Wanniski 1978). There could, therefore, be a critical point or an equilibrium from which the tax becomes heavy. Any further increase in the tax rate from this point would result in a reduction in tax revenues. But, at political equilibrium, both governmental decision and makers and taxpayers, as a group find themselves in a dilemma (Buchanan et Lee 1982). In such a situation, Buchanan and Lee (1982) explains that both would be better off if rates could be reduced and revenues increased. But this will not be the end of the dilemma because taxpayers will not respond to the reduction in rates as they predict a return to the equilibrium rate. Therefore, the government cannot increase tax revenues by moving down the long-run Laffer curve unless it can convince taxpayers that the rate cuts are permanents (Buchanan et Lee 1982).

When Arthur Laffer plotted total tax revenue as a function of a particular tax rate, he drew an upward-sloping segment called the normal range, followed by a downward-sloping segment called the prohibitive range (D. Fullerton 1982). From this, Fullerton (1982) indicates that tax rates on the prohibitive range in theoretical and empirical models have been caused by high tax rates, high elasticity parameters, or both.

But the Laffer Curve does not make unanimity among researchers. Indeed, into ''What's wrong with the Laffer Curve?'', Mirowski (1982) raises some of the main criticisms of the Laffer Curve. The author groups them in 4 points: the first one lays in questions about the magnitudes of elasticities of incentives that are not formally determined; the second lays into the problems of empiricism; the third one is the omission of some potentially relevant variables; the last one is the subsidiary controversy about the size of the underground economy. The most acute criticism is that the procedure of empirical attempts to formalize the Laffer curve lacks both theoretical and statistical rationale (Mirowski 1982). For a scientific theory, it is undeniably an important issue.

But yet, critics have not reduced economists' craze for the Laffer Curve. In the same wake, Feige and McGee (1983) focus on the public finance implications of the Laffer curve. They developed a simple macro-model from which it is possible to derive a Laffer curve. Their model reveals that the shape and position of Laffer curve depend upon the strength of supply-side effects, the progressivity of the tax system and the size of the unobserved economy (Feige et McGee 1983). But in a general equilibrium model with one private good, one public good, labour and an income tax, Malcomson (1986) explains that certain widely assumed properties of the Laffer curve do not necessarily hold. Indeed, for well-behaved functional forms, it may not be continuous and may not have an interior maximum (Malcomson 1986). But, one of the major contributions of Malcomson (1986) lies in the fact that the slope of the Laffer curve depends on technology as well as on the tax elasticity of labour supply.

Stiglitz (1987) comes back on the asymmetry of information and the inequality issues around taxation. According to him, the new Welfare Economics is distinguished by two features: first, it does not assume that the government has at its disposal the information required to make lump-sum redistributions and second, it identifies who can pay higher taxes (J. E. Stiglitz 1987). But it exists so many factors influencing tax revenue. About that, the existence of a negatively-sloped section on the tax revenue – tax rate relationship is shown to crucially depend on the nature of government expenditures (Gahvari 1989).

According to Slemrod (1990), the optimal tax theory also named the theory of optimal taxation is the study of designing and implementing a tax that reduces inefficiency and distortion in the market under given economic constraints. But in its current state, optimal tax theory is incomplete as a guide to action for serious issues in tax policy. It is incomplete because it has not yet come to terms with taxation as a system of coercively collecting revenues from individuals who will tend to resist (Slemrod 1990). This contribution allows considering willful tax evasion. According to the latter, the differences in the ease of administering various taxes are a critical determinant of appropriate tax policy. But appropriate tax policy may be reached thru tax equilibrium. About that, Guesnerie & Jerison (1991) investigate the form of the tax equilibrium set in simple Diamond-Mirrlees models and characterizes the corresponding Laffer curves They argue that the curves do not necessarily slope downward and can have multiple local maxima. Thus, local information about them is thus not sufficient to place restrictions on the optimal choice among tax systems (Guesnerie et Jerison 1991).

Some empirical studies exist around the taxpayers' sensitivity of taxable income to changes in tax rates. This is the case of Feldstein (1995). The latter argues that changes in marginal tax rates induce taxpayers to alter their behaviour in ways that affect taxable income and therefore tax revenue. The magnitude of this response is of critical importance in the formulation of appropriate tax and budget policies (Feldstein 1995). These empirical studies also focus on Laffer curves. This is the case of Hsing (1996) whose examines the Laffer curve for the U.S., based on time-series data during 1959–1991. His results show that the bell-shaped Laffer curve is statistically significant and that the revenue-maximizing tax rate is between 32.67% and 35.21%

(Hsing 1996). But all economists do not agree with the critical level of the maximum tax rate, maybe because of the existence of tax evasion. Indeed, in the presence of differing abilities to evade taxes, markets select producers for their evasive skills and their abilities to keep costs of production low (Palda 1998). According to Palda (1998), when the least efficient firms are the best tax evaders, adverse selection is severe and output comes entirely from the high-cost end of the supply curve. Inefficiency can have different causes. About that, Goolsbee, Hall and Katz (1999) argue that basic theory suggests that high marginal rates cause an inefficiency that rises with the square of the tax rate. The greater the behavioural response, the less revenue is raised by the higher rates (Goolsbee, Hall et Katz 1999). In the same vein of empirical work, the elasticity of taxable income has received much attention. About that, Gruber and Saez (2002) show that the overall elasticity of taxable income is approximately 0.4, and the elasticity of real income is much lower (Gruber et Saez 2002).

Since income can have different sources, the same is true for tax revenues. Indeed, Mirrlees pioneer contribution paves the way for several works around optimal taxation (Salanie 2003, Kaplow 2008). These studies suggest that taxation of income can be based on capital, environmental, credits for low-income families, and consumption tax (Salanie 2003). Thus, much progress has been made in this area. The progress that has shadowed the Laffer curve drowned in its criticisms.

The premises of response to critics on of the Laffer curve's lacks in theoretical and statistical rationale lies into Laffer (2004). In his paper, Laffer (2004) argues that lower tax rates change economic behaviour and stimulate growth, which causes tax revenues to exceed static estimates. Because tax cuts create an incentive to increase output, employment, and production, they help balance the budget by reducing means-tested government expenditures (Laffer 2004). This contribution by Laffer brings some theoretical elements but still suffers from the same empiricism and theoretical demonstration limits. Indeed, the custom is to speculate on the shape of the curve, with or without empirical elements. This is the case of Fullerton (2008). According to him, the Laffer curve is a hump-shaped curve showing tax revenue as a function of the tax rate. Revenue initially increases with the tax rate but then can decrease if taxpayers reduce market labour supply and investments, switch compensation into non-taxable forms, and engage in tax evasion (D. Fullerton 2008).

Meanwhile, the theory of the optimal tax has made significant progress; especially through the social aspect. Indeed, from the theory of optimal taxation, a tax system should be chosen to maximize a social welfare function subject to a set of constraints (Mankiw et Weinzierl 2009). Here, the social planner is considered as a utilitarian; meaning that the social welfare function is based on the utilities of individuals in the society. According to Mankiw and Weinzierl (2009), that welfare function is a nonlinear function of individual utilities. Summarizing the theory of the optimal tax, Mankiw and Weinzierl (2009) highlights eight general lessons suggested by optimal tax theory. The first one suggests that the optimal marginal tax rate schedules depend on the distribution of ability; the second is that the optimal marginal tax schedule could decline at high incomes; the third would like that a flat tax, with a universal lump-sum transfer, could be close to optimal; the fourth is that optimal extent of redistribution rises with wage inequality; the fifth explains that taxes should depend on personal characteristics as well as income; the sixth lesson is that only final goods ought to be taxed, and typically they ought to be taxed uniformly; the seventh is that capital income ought to be untaxed, at least in expectation; and the last one is that, in stochastic dynamic economies, optimal tax policy requires increased complexity (Mankiw et Weinzierl 2009).

These theoretical developments in the field of the optimal tax will be followed by empirical contributions attempting to make the Laffer curve useful. For instance, revisiting Laffer curve, Trabandt and Uhlig (2011) show that the United States can maximally increase tax revenues by 30% with labour taxes and 6% with capital taxes. They obtain 8% and 1% for the European Union. According to them, the consumption tax Laffer curve does not peak. Moreover, endogenous growth and human capital accumulation affect taxes quantitatively (Trabandt et Uhlig 2011). Similarly, other macroeconomic aggregates are also tied to the tax. Indeed, economic theory suggests that inconsistent tax handling of investments distorts investment decisions and drives disfavoured investments high at the expense of good investments (Simkovic 2015). According to Simkovic (2015), differences in the tax treatment of higher education relative to other forms of investment could create an undersupply of educated labour relative to physical or financial capital. Therefore, such distortions would reduce economic growth and social welfare.

2. The "Shadow Effect's" Theoretical Presentation

2.1. Theoretical Assumptions and Definitions

The shadow effect is the propensity of producers to inflate their fictitious expenses. They do it to avoid the tax burden.

The study will define the other terms as they are introduced into the theoretical developments.

The theoretical assumptions are:

A1: The study assumes that entrepreneurs (producers) are sensitive to the Shadow effect;

A2: The fictitious expenses are determined ex-post based on the real value of the tax rate. The study name those fictitious expenses "input-tax", as they are used as input in the profit declaration to tax authorities[3];

A3: The production function is a Cobb-Douglas with 3 production factors and constant returns to scale;

A4: The real profit of entrepreneurs is strictly non-negative.

Indeed, the profit is as: $\pi \geq 0$; moreover, if $\pi = 0$ entrepreneurs have no reason to create fictitious expenses. This is because the tax authorities do not tax zero profits.

The non-negative condition is also because the official corporate tax rate can't be negative: $t_r \geq 0$.

In addition to the assumptions made by Cobb and Douglas, the study stated that:

1. If either tax-input, labour or capital vanishes, then so will production.

2. The marginal productivity of tax-input is proportional to the amount of production per unit of tax-input.

2.2. Model Description

As mentioned above, this study considers a Cobb-Douglas production function (Cobb et Douglas 1928). In fact, in most analyses on tax issues, authors generally use a social welfare function that is the nonlinear function of individual utilities (Mankiw et Weinzierl 2009).

Therefore, the production function without fictitious cost is as follows:

$$\mathrm{Y} = \mathrm{A}.\mathrm{K}^{\alpha}.\mathrm{L}^{\beta} \tag{1.1}$$

With $\alpha + \beta = 1$;

Y is the level of production;

K is the capital input;

L is the labour input;

A is the level of the total factor productivity;

α and β are respectively, the output elasticities of labour and capital. These values are constants and they are determined by available technology used by the producer.

The study assumes that the entrepreneur inflates his fictitious expenses to reduce the amount of the corporate tax he will have to pay. This is the "Shadow effect". There are several reasons for such behaviour; one of them is to avoid the high tax burden.

To measure the propensity of producers to blow up their expenses, the study will first estimate the maximum amount of fictitious expenses that entrepreneurs can add to their real expenses. This fictitious amount is named: the input-tax ($\overline{\mathrm{T}}$).

$\overline{\mathrm{T}}$ is as: $\mathrm{TC}_{\mathrm{fictitious}} = \mathrm{K} + \mathrm{L} + \overline{\mathrm{T}}$,

With $\mathrm{TC}_{\mathrm{fictitious}}$ the Total fictitious Costs (the amount entrepreneurs will declare to tax authorities).

This is consistent with the Mirrlees framework. The latter states that the optimal tax problem becomes a game of imperfect information between taxpayers and the social planner (Mirrlees 1971).

Indeed, the real expenses (Total Cost) are: $\mathrm{TC}_{\mathrm{real}} = \mathrm{K} + \mathrm{L}$, with $\overline{\mathrm{T}} > 0$.

As $\overline{\mathrm{T}} > 0, \Rightarrow \mathrm{TC}_{\mathrm{fictitious}} > \mathrm{TC}_{\mathrm{real}}$; $\Rightarrow \pi_{\mathrm{fictitious}} > \pi_{\mathrm{real}}$

When tax burden increases, entrepreneurs do not reduce their production output (in this case, because they can produce fraudulent financial statements), they prefer to increase their fictitious expenses. This is why in theoretical developments, (for the first economic problem) the level of production is a constant.

From this, the production function becomes:

$$\mathrm{Y} = \mathrm{A}.\mathrm{K}^{\alpha}.\mathrm{L}^{\beta}.\overline{\mathrm{T}}^{\gamma} \tag{1.2}$$

[3] Producers create fictitious costs and maintain their level of production; this practice allows them to reduce the profit declared to the tax authorities. Thus, they increase their informal income.

2.2.1. Ex-Post Determination of the Input-Tax

Since the input tax is determined ex-post, entrepreneurs calculate it to maintain their level of production. The formula is as follow:

$$\overline{\mathrm{T}} = \mathrm{t_r}.\pi = \mathrm{t_r}.(\mathrm{Y} - \mathrm{TC_{real}}) = \mathrm{t_r}.(\mathrm{Y} - \mathrm{K} - \mathrm{L})$$

The value of the input-tax is computed ex-post using the value of K and L from the real production function that is (1.1).

Once the value of $\overline{\mathrm{T}}$ is obtained, the Total Cost becomes $\mathrm{TC} = \mathrm{K} + \mathrm{L} + \overline{\mathrm{T}}$.

From this, the production function becomes as it follows: $\mathrm{Y} = \mathrm{A}.\mathrm{K}^{\alpha}.\mathrm{L}^{\beta}.\overline{\mathrm{T}}^{\gamma}$ (1.2)

2.2.2. The Entrepreneur's Economic Problem

The standard theory of optimal taxation posits that a tax system should be chosen to maximize a social welfare function subject to a set of constraints (Mankiw et Weinzierl 2009). But this study considers minimising the cost function of entrepreneurs.

Since total production remains constant, but not the total cost, the economic problem of the producer is to minimize the costs subjected to a constant amount of production.

In other words:

$$\begin{cases} Min\ K + L + \overline{\mathrm{T}} \\ \quad Subject\ to \\ \mathrm{A}.\mathrm{K}^{\alpha}.\mathrm{L}^{\beta}.\overline{\mathrm{T}}^{\gamma} = \mathrm{Y} \end{cases} \qquad (1.3)$$

The Lagrangian is written as follows:

$$\mathcal{L}(\mathrm{K}, \mathrm{L}, \overline{\mathrm{T}}, \lambda) = \mathrm{K} + \mathrm{L} + \overline{\mathrm{T}} - \lambda(\mathrm{A}.\mathrm{K}^{\alpha}.\mathrm{L}^{\beta}.\overline{\mathrm{T}}^{\gamma} - \mathrm{Y}) \qquad (1.4)$$

The first-order conditions are written as follows:

a) $\frac{\partial \mathcal{L}}{\partial K} = 0 \Rightarrow 1 = \lambda.\alpha.\mathrm{A}.\mathrm{K}^{\alpha-1}.\mathrm{L}^{\beta}.\overline{\mathrm{T}}^{\gamma}$

b) $\frac{\partial \mathcal{L}}{\partial L} = 0 \Rightarrow 1 = \lambda.\beta.\mathrm{A}.\mathrm{K}^{\alpha}.\mathrm{L}^{\beta-1}.\overline{\mathrm{T}}^{\gamma}$

c) $\frac{\partial \mathcal{L}}{\partial \overline{\mathrm{T}}} = 0 \Rightarrow 1 = \lambda.\gamma.\mathrm{A}.\mathrm{K}^{\alpha}.\mathrm{L}^{\beta}.\overline{\mathrm{T}}^{\gamma-1}$

d) $\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow \mathrm{Y} = \mathrm{A}.\mathrm{K}^{\alpha}.\mathrm{L}^{\beta}.\overline{\mathrm{T}}^{\gamma}$

The resolution of the above system of equations will give the values of K and L which enables to keep the level of production constant while inflating the input-cost costs at $\overline{\mathrm{T}}$.

$\frac{(a)}{(b)}: \Rightarrow \frac{\alpha}{\beta}.\mathrm{K}^{\alpha-1-\alpha}.\mathrm{L}^{\beta-\beta+1} = 1 \Rightarrow L = \frac{\beta}{\alpha}.K$

$\frac{(a)}{(c)}: \Rightarrow \frac{\alpha}{\gamma}.\mathrm{K}^{\alpha-1-\alpha}.\overline{\mathrm{T}}^{\gamma-\gamma+1} = 1 \Rightarrow K = \frac{\beta}{\alpha}.\overline{\mathrm{T}}$

$\frac{(a)}{(c)}: \Rightarrow \frac{\beta}{\alpha}.\mathrm{L}^{\beta-1-\beta}.\overline{\mathrm{T}}^{\gamma-\gamma+1} = 1 \Rightarrow L = \frac{\beta}{\gamma}.\overline{\mathrm{T}}$

By replacing K and L by their values from $\frac{(a)}{(c)}$ and $\frac{(a)}{(c)}$ into d), we have:

d) $\mathrm{Y} = \mathrm{A}.(\frac{\beta}{\alpha}.\overline{\mathrm{T}})^{\alpha}.(\frac{\beta}{\gamma}.\overline{\mathrm{T}})^{\beta}.\overline{\mathrm{T}}^{\gamma}$

The logarithmic transformation thru the natural logarithm gives:

(d) $\Rightarrow$ $ln\frac{Y}{A} = \alpha.ln\frac{\alpha}{\gamma} + \alpha.ln\overline{\mathrm{T}} + \beta.ln\frac{\beta}{\gamma} + \beta.ln\overline{\mathrm{T}} + \gamma.ln\overline{\mathrm{T}}$

$$\Rightarrow \quad \overline{\mathrm{T}} = \frac{\mathrm{Y}}{\mathrm{A}}.\frac{\gamma^{\alpha+\beta}}{\alpha^{\alpha}.\beta^{\beta}} \qquad (1.5)$$

(1.5) gives the value of the optimal tax-input. The value of K and L are the following:

$\frac{(a)}{(c)}: \Rightarrow \quad \mathrm{K} = \frac{\beta}{\alpha}.\frac{\mathrm{Y}}{\mathrm{A}}.\frac{\gamma^{\alpha+\beta}}{\alpha^{\alpha}.\beta^{\beta}} \quad \Rightarrow \quad \mathrm{K} = \frac{\mathrm{Y}}{\mathrm{A}}.\frac{\gamma^{\alpha+\beta-1}}{\alpha^{\alpha}.\beta^{\beta-1}}$

$\frac{(a)}{(c)}: \Rightarrow \quad \mathrm{L} = \frac{\beta}{\gamma}.\frac{\mathrm{Y}}{\mathrm{A}}.\frac{\gamma^{\alpha+\beta}}{\alpha^{\alpha}.\beta^{\beta}} \quad \Rightarrow \quad \mathrm{L} = \frac{\mathrm{Y}}{\mathrm{A}}.\frac{\gamma^{\alpha+\beta-1}}{\alpha^{\alpha}.\beta^{\beta-1}}$

2.2.3. The Shadow Effect Propensity

As shown above, (1.5) gives the amount of the optimal input-tax that allows the producer to reduce the taxable profit.

The study focuses on the value of γ: the responsiveness of the output to a change in levels of the input-tax factor. From tax officials, γ is interpreted as the propensity of entrepreneurs to Shadow effect.

Therefore, the objective of the tax authorities (or government) is to bring γ closer to 0. This is opposite to the producer's goal. This statement means that there is a positive link between the propensity of entrepreneurs to Shadow effect (γ) and the level of tax rate (t_r).

From the value of the input-tax into (1.5), the value of γ can be derived:

$$(1.5) \quad \Rightarrow \overline{T} = \frac{Y}{A}.\frac{\gamma^{\alpha+\beta}}{\alpha^{\alpha}.\beta^{\beta}}; \Rightarrow \gamma = \exp\left[\frac{1}{\alpha+\beta}\ln\left(\frac{\alpha^{\alpha}.\beta^{\beta}}{Y}A.\overline{T}\right)\right] \quad (1.6)$$

From there, the study can express the tax rate (t_r) as a function of the shadow effect propensity (γ).

Indeed, $\overline{T} = t_r.\pi$ and $\pi = Y - TC = Y - K - L$

$$\Rightarrow \quad \overline{T} = t_r.\pi = t_r.(Y - K - L)$$

(1.6) becomes:

$$\gamma = \exp\left[\frac{1}{\alpha+\beta}\ln\left(\frac{\alpha^{\alpha}.\beta^{\beta}}{Y}A.t_r.(Y-K-L)\right)\right]$$

$$\Rightarrow \quad \ln t_r = (\alpha+\beta).\ln\gamma - \ln\left[\left(1-\frac{K+L}{Y}\right)\alpha^{\alpha}.\beta^{\beta}.A\right]$$

⇒

The tax rate is as:

$$t_r(\gamma) = \frac{\gamma^{\alpha+\beta}}{\left(1-\frac{K+L}{Y}\right)\alpha^{\alpha}.\beta^{\beta}.A} \quad (1.7)$$

With $0 \le t_r(\gamma) \le 1$;

When the tax function reaches its minimum, $t_r(\gamma = 0) = 0$; This is consistent with the study's assumptions.

Proof:

According to the above function, the only way to get γ to equal zero is to set t_r to zero. Indeed, the function $t_r(\gamma)$ exists if and only if: $\left(1-\frac{K+L}{Y}\right)\alpha^{\alpha}.\beta^{\beta}.A \neq 0$

Since $\alpha^{\alpha}.\beta^{\beta}.A \neq 0; \Rightarrow 1 \neq \frac{K+L}{Y}$;

Indeed, $\alpha + \beta \neq 0$ because $\alpha + \beta = 1 - \gamma$ (due to the constant return to scale assumption -A3-).

Now, $K + L = TC$ cannot be greater than Y because of assumption A4. Indeed, A4 states that the real profit of entrepreneurs is strictly non-negative. Meaning that $\frac{K+L}{Y} \ge 1$.

But, since $1 \neq \frac{K+L}{Y}$,

$t_r(\gamma)$ exists if and only if $\frac{K+L}{Y} > 1$.

Meaning that the only way to bring γ to zero is to set t_r to zero.

The relationship between the shadow effect propensity and the tax rate can also be presented in the other direction. That is the expression of γ as a function of t_r.

From (1.7), $\quad \gamma^{\alpha+\beta} = \left(1-\frac{K+L}{Y}\right)\alpha^{\alpha}.\beta^{\beta}.A.t_r$

$$\Rightarrow \quad \ln\gamma = \frac{1}{\alpha+\beta}.\ln\left[A.\left(1-\frac{K+L}{Y}\right).\alpha^{\alpha}.\beta^{\beta}.t_r\right]$$

$$\Rightarrow \quad \gamma(t_r) = \left[A.\left(1-\frac{K+L}{Y}\right).\alpha^{\alpha}.\beta^{\beta}\right]^{\frac{1}{\alpha+\beta}}.t_r^{\frac{1}{\alpha+\beta}} \quad (1.8)$$

This function (1.8) summarizes the relationship between the shadow effect and the tax rate. This function summarizes the relationship between the shadow effect and the tax rate. From here, raises the first instrument to influence entrepreneurs' propensity to shadow effect. This is the tax rate.

2.2.4. The Slope of the Curve and Some Properties

The slope of γ enables to represent the graph of the function and determine the elasticities during variations.

- The extremums of the curve are:

$\gamma(t_r = 0) = 0$;

$\gamma(t_r = 1) = \left[A.\left(1 - \frac{K+L}{Y}\right).\alpha^{\alpha}.\beta^{\beta}\right]^{\frac{1}{\alpha+\beta}}$;

$\Rightarrow \gamma(t_r = 1) \neq 1$ if $\left[A.\left(1 - \frac{K+L}{Y}\right).\alpha^{\alpha}.\beta^{\beta}\right]^{\frac{1}{\alpha+\beta}} \neq 1$

$\Rightarrow \gamma(t_r = 1) = 1$ if $\left[A.\left(1 - \frac{K+L}{Y}\right).\alpha^{\alpha}.\beta^{\beta}\right]^{\frac{1}{\alpha+\beta}} = 1$

The curve on the graph below in shape (a) gives the shape of the function for $\gamma(t_r = 1) = 1$. In such a situation:

$ln\left[A.\left(1 - \frac{K+L}{Y}\right).\alpha^{\alpha}.\beta^{\beta}\right]^{\frac{1}{\alpha+\beta}} = ln1 = 0$; $\Rightarrow \frac{1}{\alpha+\beta}\ln\left[A.\left(1 - \frac{K+L}{Y}\right).\alpha^{\alpha}.\beta^{\beta}\right] = 0$;

$\Rightarrow \left(1 - \frac{K+L}{Y}\right) = \frac{1}{\alpha^{\alpha}.\beta^{\beta}.A}$.

- The slope of the curve is: $\frac{\partial\gamma(t_r)}{\partial t_r} = \frac{\partial\gamma}{\partial t_r}\left[A.\left(1 - \frac{K+L}{Y}\right).\alpha^{\alpha}.\beta^{\beta}\right]^{\frac{1}{\alpha+\beta}}.t_r^{\frac{1}{\alpha+\beta}}$

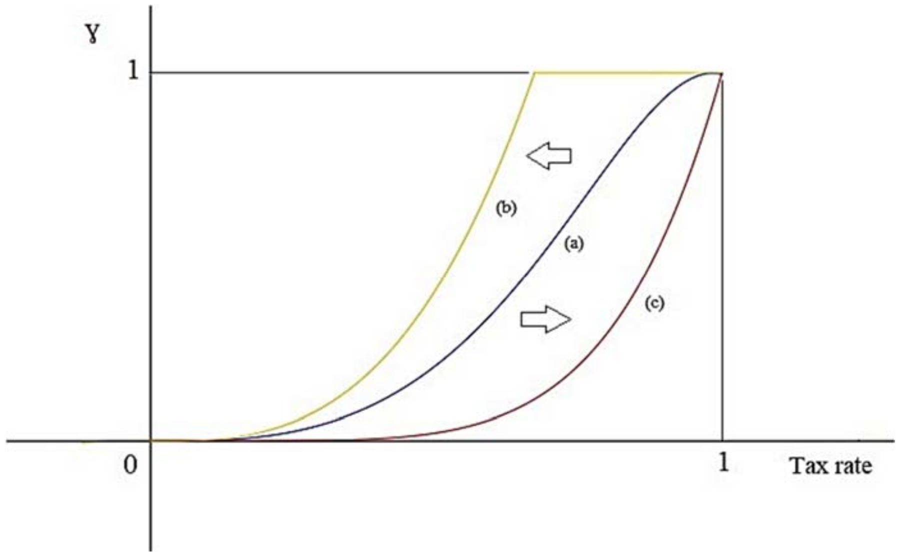
$\Rightarrow \quad \frac{\partial\gamma(t_r)}{\partial t_r} = \frac{1}{\alpha+\beta}.\left[A.\left(1 - \frac{K+L}{Y}\right).\alpha^{\alpha}.\beta^{\beta}\right]^{\frac{1}{\alpha+\beta}}.t_r^{\frac{1-\alpha-\beta}{\alpha+\beta}}$;

$\Rightarrow \quad \frac{\partial\gamma(t_r)}{\partial t_r} > 0$, meaning that the curve has a positive slope.

- From $t_r^{\frac{1}{\alpha+\beta}}$, as $\frac{1}{\alpha+\beta} \neq 1$, the curve is non-linear.
- Another property of the function is already given above as $\gamma(t_r)$ exists if and only if $\frac{K+L}{Y} > 1$. Meaning that the only way to bring γ to zero is to set t_r to zero.

The curve of shadow effect propensity as a function of the tax rate will, therefore, have the following graphical representation:

Figure 2. Shadow effect propensity as a function of tax rate



All along the line of each curve, we have the different combinations of γ and t_r. There is a positive but non-linear relationship between the entrepreneur propensity to shadow effect and the level of the tax rate.

The curve will shift to the left from position (a) to the position (b) due, *ceteris paribus*, to a decrease into the value of the slope.

From $\frac{\partial\gamma(t_r)}{\partial t_r}$, when $\alpha + \beta$ increase, the slope tends to decrease. Therefore, the curve can shift to the right, from (a) to (c).

This is an important finding. It suggests that the technology used to produce goods has a negative influence on the producer's sensitivity to shadow effect (the slope of the curve).

Indeed,

- $\lim_{\alpha+\beta\to 1}\left[\frac{\partial\gamma(t_r)}{\partial t_r}\right] = A.\left(1 - \frac{K+L}{Y}\right) > 0$;

- $\lim_{\alpha+\beta\to 0}\left[\frac{\partial\gamma(t_r)}{\partial t_r}\right] = 0$.

2.3. The Use of Tax Rate Leverage on Shadow Effect Propensity

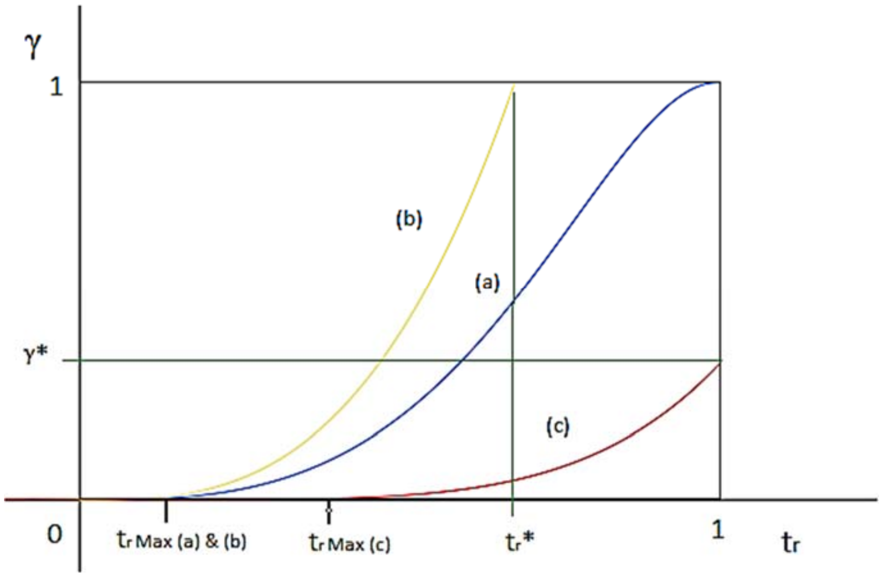
From the objective of this paper, the study aims to provide tools to reduce the shadow effect. The first instrument the study proposes is the corporate tax rate. Theoretical development suggests that an increase in the level of the tax rate will lead to an increase in the producers' propensity to shadow effect. But with different proportions.

As shown above, the slope of $\gamma(t_r)$ is not constant and depends on α and β. That slope is considered as the sensitivity of producers to the shadow effect when there is a change in the tax rate level. Meaning that the curve of $\gamma(t_r)$ can have a lot of shapes.

Indeed, when the shape of the curve (slope of the curve that also is the sensitivity of producers to the shadow effect) is known, the government can move along the $\gamma(t_r)$ curve by changing the value of the tax rate.

According to the producer's sensitivity to shadow effect (the slope of the curve), the curve can take the following shapes:

Figure 3. Movements of the curve due to changes in shadow effect sensitivity



Let's consider the 3 following shapes of the curve:

- Entrepreneurs shadow propensity is highly sensitives to tax burden (b). This situation is closed to the case of developing countries, informal enterprises, very small and small businesses;
- Entrepreneurs shadow propensity is proportionally sensitive to tax burden (a). This situation is closed to the case of emerging countries, medium enterprises;
- Entrepreneurs shadow propensity is poorly sensitives to tax burden (c). This situation is closed to the case of developed countries, large enterprises.

Explanations:

The same increase in the level of the tax rate will lead to a higher increase in the producer's propensity to shadow effect in (b) and a lower increase in (c). This means that government in the situation (c) can have a maximum optimal tax rate (tr Max) greater than the one of government in the situation (b). The maximum optimal tax rate is the maximum tax rate above which producers start creating fictitious expenses.

When producers are highly sensitives to shadow effect (b), up to a certain level of the tax burden (tr*), they declare zero profit because $\gamma(t_r) = 1$. Due to the burden of tax being too high for them, they increase their fictitious expenses to their maximum level. In such a situation (tr*), any additional increase in tax rate will result in the same level of shadow effect propensity: $\gamma(t_r) = 1$. An explanation is since entrepreneurship is subsistence entrepreneurship (informal entrepreneurship in sub-Saharan countries for instance). In such a situation there is any industrial production and small businesses don't have a legal form, but they pay their tax to the town hall. Another explanation leads in the fact that institutions are weak (high level of corruption for example).

When producers are poorly sensitive to shadow effect (c), it may happen that they never reach the maximum level of shadow effect. This can be due because of their scale or the technology they use. Another explanation can be the strength of institutions (tax authorities, credits registries, shareholders, stock markets, etc.) that constrain producers to provide real information about the health of the company.

2.4. Reducing the Tax Rate Level to Improve the Tax Revenue

The study still considers that entrepreneurs inflate their fictitious expenses to avoid the tax burden. Above, the study has shown that shadow effect propensity increases with the tax rate. This link enables to build up another useful tool: the level of production.

Indeed, the government's main objective, instead of reducing shadow effect can be: increase the country's Gross Domestic Products. In such a situation, the government will put in place policies to increase the country's production, therefore, the entrepreneurs' output. One of these policies can be the reduction of the corporate tax rate. Once entrepreneurs' production will increase, the government tax revenue will also increase. Indeed, the real profit of entrepreneurs will increase because the tax burden has decreased (shadow effect).

The economic problem will, therefore, be stated as (Mankiw et Weinzierl 2009):

$$\begin{cases} Max\ \mathrm{A.K^{\alpha}.L^{\beta}.\overline{T}^{\gamma}} \\ \quad Subject\ to \\ \mathrm{K + L + \overline{T} = TC} \end{cases} \tag{2.1}$$

The Lagrangian is written as follows:

$$\mathcal{L}(\mathrm{K, L, \overline{T}}) = \mathrm{A.K^{\alpha}.L^{\beta}.\overline{T}^{\gamma}} - \lambda(\mathrm{K + L + \overline{T} - TC}) \tag{2.2}$$

The first-order conditions are written as follow:

a) $\frac{\partial \mathcal{L}}{\partial K} = 0 \Rightarrow \alpha.\mathrm{A.K^{\alpha-1}.L^{\beta}.\overline{T}^{\gamma}} = \lambda$

b) $\frac{\partial \mathcal{L}}{\partial L} = 0 \Rightarrow \beta.\mathrm{A.K^{\alpha}.L^{\beta-1}.\overline{T}^{\gamma}} = \lambda$

c) $\frac{\partial \mathcal{L}}{\partial \overline{T}} = 0 \Rightarrow \gamma.\mathrm{A.K^{\alpha}.L^{\beta}.\overline{T}^{\gamma-1}} = \lambda$

d) $\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow \mathrm{K + L + \overline{T} = TC}$

By solving the above system of equations, the following results are found:

- $\mathrm{\overline{T} = TC.\gamma}$ (2.3)
- $\mathrm{K = TC.\alpha}$ (2.4)
- $\mathrm{L = TC.\beta}$ (2.5)

In this situation, the study considers that the government doesn't consider the reduction of shadow effect as a priority, but the increase of tax revenues. Therefore, the propensity of shadow effect will be influenced indirectly from government policies.

(2.3) gives: $\mathrm{\overline{T} = TC.\gamma}$ $\Rightarrow (\mathrm{Y - TC}).\mathrm{t_r} = \mathrm{TC.\gamma}$

$\Rightarrow \mathrm{Y} = \frac{\mathrm{TC.\gamma}}{\mathrm{t_r}} + \mathrm{TC}$

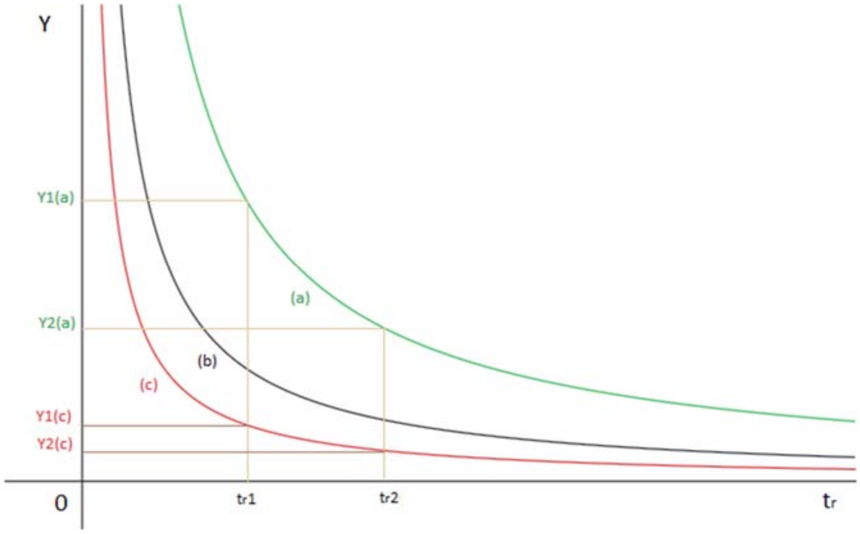
$$\Rightarrow \quad \mathrm{Y(t_r)} = (1+\gamma).\mathrm{TC.t_r}^{-1} \tag{2.6}$$

This suggests that there is an inverse relationship between the level of production and the tax rate. Indeed, the slope of the curve of $\mathrm{Y(t_r)}$ is negative:

$$\frac{\partial \mathrm{Y}}{\partial \mathrm{t_r}} = -\gamma.\mathrm{TC.t_r}^{-2} < 0 \tag{2.7}$$

Since $\frac{\partial^2 \mathrm{Y}}{\partial \mathrm{t_r}} = 2\gamma.\mathrm{TC.t_r}^{-3}$ and $\mathrm{Y(t_r)}$ is non-linear, the shape of the curve is as:

Figure 4. Changes in production due to tax rate changes



The above graph shows that from the same variation in the tax rate (from t_r2 to t_r1), the variation in production changes depending on the curve slope and shape.

Indeed, the shape of the curve in (a) is explained by a higher level of γ. When γ increases, the curve shifts to the right; and when γ decreases, the curve shifts to the left (c). Meaning that γ in (a) is greater than γ in (c).

A higher level of shadow propensity leads to a higher change in production when the tax rate changes. Indeed, from the graph, the same reduction in the tax rate (from t_r2 to t_r1), leads to a higher level of increase in production (From $Y2(a)$ to $Y1(a)$ in (a) than (From $Y2(c)$ to $Y1(c)$ in (c).

From (2.7), the formula for computing the value of the change is the following:

$$\Delta t_r = -\frac{t_r^{-2}}{\gamma.TC}.\Delta Y(t_r) \qquad (2.8)$$

This formula helps the government to determine how low they have to reduce their tax rate to increase their production.

For instance, if the government wants to increase production by 50%, it must reduce the level of taxation by: $\Delta t_r = -\frac{t_r^{-2}}{\gamma.TC}.50\%$.

3. Numerical Application: The Case of Cameroon

A numerical application of the above theoretical developments is made choosing Cameroon's case. Cameroon is amongst the Sub-Saharan African countries with higher tax burden levels. About that, the Paying Taxes 2018 report from PricewaterhouseCoopers (PwC) ranks the Cameroonian tax system 183rd at the world level. Indeed, from World Bank enterprise survey 2016, data shows that 76.45% of Cameroonian entrepreneurs point out tax rate as an obstacle for the growth of their activities. Moreover, 36.56% of those entrepreneurs even describes the tax rate as a major and very severe obstacle. A worrying situation for the government. Besides, tax reforms (Institution of a single tax interlocutor, modernization of the Fiscal Investigations Brigade) in 2004 has contributed to the improvement of domestic tax revenues. Indeed, tax revenues from corporate tax rose to 1.6% of GDP in 2010 compared to 0.6% of GDP in 1995 (Assobo 2011). But confronted with the needs of the state, this tax revenue increase is not enough. This is one of the reasons why the government has set itself the next challenges: to improve the level of tax revenues; and above all, to consider the possibility of lowering the corporate tax rate (Assobo 2011).

3.1. Data and Determination of Input-Tax

The data used in the numerical application come from the world bank enterprise survey, last update 2016. The database provides information and opinions about the business environment in Cameroon.

The study uses the following variables from the database:

- n5a: Total annual expenditure for purchases of equipment in the last fiscal year, the variable is labelled K;

▪ n2a: Total labour cost (including wages, salaries, bonuses, etc) in the last fiscal year, the variable is labelled L;

▪ d2: Establishment's total annual sales in the last fiscal year, the variable is labelled Y.

To estimate the Cobb-Douglas production function, the study uses the natural logarithm of the data. The description of the available and non-negative data is presented in the table below:

Table 1. Data description of Cameroonian producers

Variable	Obs.	Mean	Std. Dev.	Min.	Max.
lnY	144	17.83422	2.452546	13.12236	26.02158
lnK	144	15.01748	2.361146	9.903488	22.10956
lnL	144	15.7905	2.195856	11.69525	23.719

Source: Author

The sample for the 2016 Cameroon Enterprise Survey is selected using stratified random sampling to obtain unbiased estimates for the whole population. Despite that the sampling consists of 363 firms, including small, medium and large firms, only 144 observation was complete in the database for the study selected variables.

From this database, the study derived the value of the input-tax as:

$$\overline{\mathrm{T}} = \mathrm{t_r}.\pi = \mathrm{t_r}.(\mathrm{Y} - \mathrm{TC_{real}}) = \mathrm{t_r}.(\mathrm{Y} - \mathrm{K} - \mathrm{L}) = 35\%.(\mathrm{Y} - \mathrm{K} - \mathrm{L})$$

Indeed, the official corporate tax rate in Cameroon is 35%.

Using the ex-post value of the input-tax, the study can proceed to the estimation of the Cobb-Douglas production function of Cameroonian entrepreneurs.

3.2. Production Function Estimation

The second step is estimating the production function. The purpose is to find the elasticities of production factors. From the same above sample, the study uses Bootstrapped[4] Ordinary Least Square to estimate the output that is total sales. The results are presented in the table below:

Table 2. OLS estimation result for the production function

lnY	Observed Coef.	Bootstrap Std. Err.	z	P>z
lnK	0.1418157***	0.0232532	6.10	0.000
lnL	0.1564977***	0.0314389	4.98	0.000
lnT	0.719882***	0.0360549	19.97	0.000
_cons.	1.482066***	0.1327167	11.17	0.000
Number of Obs. =	144			
Replications =	10000			
Wald chi2(3) =	15190.58			
Prob > chi2 =	0.0000			
R-squared =	0.9902			
Adj R-squared =	0.9900			
Root MSE =	0.2452			

Source: Author from Stata. With *** representing significance at 1% level.

From the above table, the study derives the Cameroonian entrepreneur's production function as:

$$\widehat{\mathrm{Y}} = (4.40).\widehat{\mathrm{K}}^{0.14}.\widehat{\mathrm{L}}^{0.15}.\widehat{\overline{\mathrm{T}}}^{0.71}$$

This is a Cobb-Douglas with constant returns to scale. Indeed, $\alpha + \beta + \gamma = 1$. And, all the coefficient values are significant at 1% level of significance. The exponential value of the constant-coefficient is replaced into the Cobb-Douglas as the value of $A = 4.40$.

The next step is to determine the shadow effect propensity.

3.3. Shadow Effect Propensity Derivation

As presented above, the economic problem of producer facing tax burden is to reduce its total costs subjected to its production. The problem to optimise can be stated as:

$$\begin{cases} Min\ K + L + \overline{\mathrm{T}} \\ \quad Subject\ to \\ \mathrm{A}.\mathrm{K}^{\alpha}.\mathrm{L}^{\beta}.\overline{\mathrm{T}}^{\gamma} = \mathrm{Y} \end{cases} \qquad (3.1)$$

[4] The bootstrapping consists of 10000 replications.

The Cameroonian entrepreneurs' economic problem can be stated as:

$$\begin{cases} Min\ K + L + \overline{T} \\ Subject\ to \\ (4.40).\widehat{K}^{0.14}.\widehat{L}^{0.15}.\widehat{\overline{T}}^{0.71} = \widehat{Y} \end{cases}$$

The Lagrangean is written as:

$$\mathcal{L}(K, L, \overline{T}, \lambda) = K + L + \overline{T} - \lambda(4.40).\widehat{K}^{0.14}.\widehat{L}^{0.15}.\widehat{\overline{T}}^{0.71} = \widehat{Y})$$

In a concern for accuracy, the value of $\widehat{Y}$ is the value of the efficient producer. Therefore, efficiency scores are determined using DEA (Data Envelopment analysis). The DEA is output-oriented and the study computes technical efficiency scores of Cameroonians producers.

When technical efficiency score is equal to zero, it means that producer (i) is not technically efficient. On the other hand, when the technical efficiency score is equal to one, it means that the firm (i) has reached the maximum technical efficiency (Coelli 1996).

The DEA model is the following:

$$\begin{cases} & Max\ (\frac{\beta_1.Y_i}{\sum(\alpha_1.K_i + \alpha_2.L_i + \alpha_3.\overline{T}_i)}) \\ Subject\ to: & \\ & \frac{\beta_1.Y_i}{\sum(\alpha_1.K_i + \alpha_2.L_i + \alpha_3.\overline{T}_i)} \leq 1: \end{cases}$$

With $i \in [1; 144]$ for the Cameroonian producers from the World Bank Enterprise Survey with complete data availability into the database;

With $\sum_{k=1}^{K}\alpha_k = 1$: input weight and $\sum_{k=1}^{K}\beta_k = 1$: output weigh;

The results are presented in the annexe.

From DEA results, the average efficient DMU (decision-maker unit), the shadow propensity function is as follows:

$$\gamma(t_r) = \left[A.\left(1 - \frac{K+L}{Y}\right).\alpha^{\alpha}.\beta^{\beta}\right]^{\frac{1}{\alpha+\beta}}.t_r^{\frac{1}{\alpha+\beta}} = 21{,}68.t_r^{3{,}45}$$

The shadow effect propensity functions for the efficient Cameroonian entrepreneurs are presented into the following table:

Table 3. Shadow effect propensity functions for the efficient Cameroonian entrepreneurs

DMU	DMU Label	$\left[A.\left(1 - \frac{K+L}{Y}\right).\alpha^{\alpha}.\beta^{\beta}\right]^{\frac{1}{\alpha+\beta}}$	$\gamma(t_r)$
339	A	11,8358908	$11{,}84.t_r^{3{,}45}$
189	B	12,7985868	$12{,}80.t_r^{3{,}45}$
319	C	16,3406329	$16{,}34.t_r^{3{,}45}$
256	D	12,2664263	$12{,}27.t_r^{3{,}45}$
238	E	13,6977065	$13{,}70.t_r^{3{,}45}$
17	F	16,4748069	$16{,}47.t_r^{3{,}45}$
114	G	20,9565672	$20{,}96.t_r^{3{,}45}$
346	H	17,9267581	$17{,}93.t_r^{3{,}45}$
314	I	18,8686879	$18{,}87.t_r^{3{,}45}$
Average DMU		21,6776137	$21{,}68.t_r^{3{,}45}$

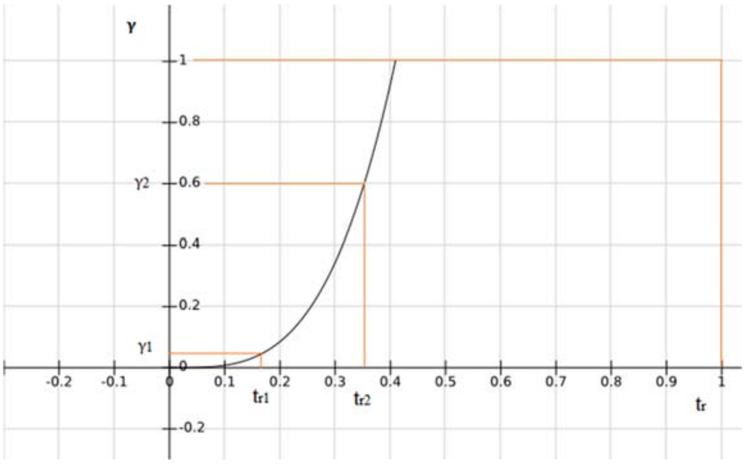
Source: Author

The study considers the average DMU data of Cameroonian producers in plotting the shadow effect propensity function.

The shape of the curve is shown in the Figure 5.

The average DMU situation is closed to the overall Cameroonian producers' situation from the estimations. Indeed, the official corporate tax rate in Cameroon is 35%. That could be t_{r2} in the graph leading to γ_2.

Figure 5. Shape of Shadow effect propensity for Cameroonian producers



The above graph shows that a reduction in corporate tax rate from 35% to approximatively 16% brings producers shadow effect propensity from approximatively 60% to approximatively 4%. The graph also shows that up to approximatively 41% of tax rate burden, Cameroonian entrepreneurs reaches their maximum shadow effect propensity.

3.4. Optimal Tax Rate Reduction in Cameroon

The instrument built in this study enables to determine the optimal tax rate according to the needs of the government.

From (2.8), the necessary tax rate change is: $\Delta \mathbf{t_r} = -\frac{\mathbf{t_r}^{-2}}{\gamma.\mathbf{TC}}.\Delta \mathbf{Y}(\mathbf{t_r})$

As shown above, this formula helps the government to determine how low they have to reduce their tax rate to increase their production. In the case of Cameroon, the considers a positive change of 50% in production. Indeed, "If SMEs were contributing to 50% of GDP, we would already be an emerging country." This statement is from the Cameroonian Minister for Small and Medium-sized enterprises.

The table below gives the optimal reductions in corporate tax rate considering a 50% increase in efficient Cameroonian producers' outputs.

Table 4. Optimal reduction in the corporate tax rate for Cameroonian producers

DMU	DMU Label	$\Delta \mathbf{t_r}$	New t_r
339	A	-23,16	11,84%
189	B	-22,20	12,80%
319	C	-18,66	16,34%
256	D	-22,73	12,27%
238	E	-21,30	13,70%
17	F	-18,53	16,47%
114	G	-14,04	20,96%
346	H	-17,07	17,93%
314	I	-16,13	18,87%
Average DMU		-18,81	16,19%

Source: Author

The results show that considering shadow effect and, to reach 50% more in term of entrepreneur's production, the Cameroonian government should reduce the corporate tax rate from 35% to 16.19%. The study considers the value of the average DMU.

Conclusion

This study develops a new concept to explain the behaviour of entrepreneurs that is to create fictitious expenses to avoid a tax burden. The study names that concept the: "Shadow effect". The idea behind the concept is that, when the tax burden increases, entrepreneurs falsify their financial statements to reduce the tax base and maintain their profits. The idea is closed to the concept of the Laffer Curve. But Laffer curves supposes that when tax burden is high, people reduce their work supply or increase it to maintain their living condition. This study adds another alternative linked to tax evasion: the shadow effect. In that alternative, people keep their real level of production constant but inflate their expenses. In order words, they increase their informal activities to evade tax. Therefore, the objective of the paper is to provide to policymakers a tool to reduce the shadow effect. A tool to drive tax policy considering tax evasion. The instrument is based on the usage of the corporate tax rate.

To measure the propensity of producers to inflate their expenses, the study first estimates the maximum amount of fictitious expenses that entrepreneurs can use.

This fictitious amount is named: the input-tax $(\overline{\mathrm{T}})$. The input-tax is determined ex-post of production. This is because the study supposes that producers inflate their expenses based on their real level of production.

The study made 4 theoretical assumptions:

A1: The study assumes that entrepreneurs (producers) are sensitive to the Shadow effect;

A2: The fictitious expenses are determined ex-post based on the real value of the tax rate;

A3: The production function is a Cobb-Douglas with 3 production factors and constant returns to scale;

A4: The real profit of entrepreneurs is strictly non-negative.

From this, the producer's economic problem is to reduce his expenses subjected to a constant level of production. A Lagrangian is derived from the producer economic problem and solved.

Working into this framework, the theoretical developments have shown that there is a positive non-linear relationship between entrepreneurs' sensitivity to the shadow effect and the evolution of the tax burden. Thus, an increase in the level of the tax rate will lead to an increase in the producers' propensity to shadow effect. But with different proportions. Three interesting cases are highlighted:

- When entrepreneurs' shadow propensity is highly sensitives to tax burden (depending on the slope of the curve). This situation is closed to the case of developing countries, informal enterprises, very small and small businesses;
- When entrepreneurs' shadow propensity is proportionally sensitive to tax burden (depending on the slope of the curve). This situation is closed to the case of emerging countries, medium enterprises;
- When entrepreneurs' shadow propensity is poorly sensitives to tax burden (depending on the slope of the curve). This situation is closed to the case of developed countries, large enterprises.

Indeed, when producers are highly sensitives to a shadow effect, up to a certain level of the tax burden (tr*), they declare zero profit because the shadow effect propensity is $\gamma(\mathrm{t_r}) = 1$. Because the burden of the tax is too high for them, they increase their fictitious expenses to their maximum level. In such a situation (tr*), any additional increase in tax rate will result in the same level of shadow effect propensity: $\gamma(\mathrm{t_r}) = 1$.

In the other hand, when producers are poorly sensitive to a shadow effect, it may happen that they never reach the maximum level of shadow effect. This can be due because of their scale or the technology they use. Another explanation can be the strength of institutions (tax authorities, credit bureau registries, shareholders, stock markets, etc.) that constraint the producers to provide real information about the health of the company.

The study also provides a simple instrument to the government based on the slope of the curve.

The above formula helps the government to determine how low their need to reduce their tax rate to increase their GDP. Indeed, when the government needs to spur economic growth while avoiding tax evasion (thru shadow effect), the above formula provides the needed change in the tax burden. This tool is of critical importance because if economic growth is not fairly distributed, poverty and inequality will increase.

An empirical application on the case of Cameroon (a high tax burden Sub-Saharan African country) has shown that considering shadow effect and, to reach 50% more (as expected by the government) in term of entrepreneur's production, the Cameroonian government should reduce the corporate tax rate from 35% to 16.19%.

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Annexe

dmu	lnK	lnL	lnY	lnT	Efficiency Scores	dmu	lnK	lnL	lnY	lnT	Efficiency Scores
339	9,90	14,91	16,12	14,71	1,00	53	14,91	16,30	17,03	15,07	0,93
359	10,13	14,51	15,61	14,15	0,99	84	14,91	14,29	17,07	15,83	0,94
174	10,31	12,61	13,12	10,99	1,00	133	14,91	16,81	17,22	14,71	0,96
278	10,82	13,59	14,91	13,53	0,98	176	14,91	14,51	17,37	16,17	0,94
189	10,92	14,98	17,82	16,71	1,00	225	14,91	15,42	17,62	16,38	0,94
129	11,00	14,00	15,42	14,08	0,98	302	14,91	14,91	16,30	14,56	0,93
265	11,51	13,12	14,91	13,64	0,97	316	14,91	15,95	16,76	14,79	0,93
290	11,51	13,46	14,91	13,55	0,96	238	15,07	11,92	16,07	14,53	1,00
327	11,51	14,22	16,12	14,89	0,97	16	15,20	14,51	17,20	15,92	0,94
167	11,92	13,12	14,91	13,62	0,96	105	15,20	16,91	17,91	16,29	0,93
244	11,92	13,59	15,20	13,88	0,96	236	15,20	14,91	16,45	14,71	0,93
194	12,21	12,39	14,51	13,21	0,96	274	15,20	14,69	16,21	14,29	0,93
259	12,21	13,82	16,12	14,94	0,96	143	15,42	16,81	19,19	18,01	0,95
322	12,21	13,82	14,91	13,35	0,95	182	15,42	14,40	16,81	15,35	0,93
342	12,21	14,22	15,89	14,61	0,96	198	15,42	14,22	16,52	14,91	0,93
232	12,43	13,82	14,51	12,48	0,96	307	15,42	16,81	18,83	17,59	0,94
92	12,55	17,50	19,52	18,33	0,99	310	15,42	14,22	16,52	14,91	0,93
70	12,61	13,18	15,61	14,41	0,96	323	15,42	14,85	16,45	14,59	0,93
99	12,61	14,29	15,42	13,90	0,94	332	15,42	15,96	17,11	15,37	0,93
279	12,61	13,82	15,76	14,51	0,95	173	15,61	15,32	17,03	15,44	0,92
281	12,61	14,22	15,52	14,07	0,95	226	15,76	16,12	16,81	13,86	0,95
286	12,61	13,82	15,07	13,55	0,94	18	15,89	15,68	17,37	15,79	0,92
319	12,61	18,20	20,72	19,59	1,00	26	15,89	18,83	19,64	17,96	0,95
354	12,61	14,73	15,76	14,20	0,95	28	15,89	14,91	17,73	16,43	0,94
179	12,77	14,51	16,12	14,80	0,95	29	15,89	14,73	16,52	14,27	0,94
234	12,79	13,12	15,07	13,74	0,95	30	15,89	14,09	16,81	15,13	0,94
191	12,90	11,92	15,20	14,00	0,98	48	15,89	14,40	17,22	15,77	0,94
43	13,12	14,00	16,35	15,15	0,95	184	15,89	15,15	18,20	16,99	0,94
69	13,12	14,91	15,89	14,27	0,94	178	16,12	16,52	19,28	18,11	0,95
74	13,12	12,61	16,52	15,42	0,98	262	16,12	18,83	20,72	19,50	0,96
101	13,12	15,07	15,76	13,86	0,95	336	16,12	16,12	20,72	19,65	0,98
130	13,12	14,29	15,61	14,13	0,94	347	16,12	15,42	17,91	16,57	0,93
243	13,12	15,76	16,52	14,78	0,95	135	16,21	13,82	17,03	15,33	0,94
258	13,12	13,82	15,42	14,02	0,94	201	16,24	16,00	17,37	15,45	0,93
71	13,30	14,40	15,42	13,72	0,94	11	16,30	16,81	17,62	15,33	0,94
117	13,30	15,61	17,55	16,33	0,95	17	16,30	14,73	16,52	12,07	1,00
147	13,30	16,38	18,08	16,82	0,96	31	16,30	16,76	19,40	18,22	0,95
222	13,46	14,40	15,42	13,68	0,93	273	16,38	18,42	20,50	19,30	0,95
266	13,46	14,69	16,52	15,24	0,94	353	16,45	16,01	18,34	17,00	0,93
313	13,46	14,51	16,12	14,75	0,94	86	16,52	17,15	20,08	18,94	0,95
344	13,46	14,22	14,91	12,54	0,95	102	16,52	16,30	18,17	16,70	0,93
152	13,59	15,10	16,81	15,51	0,94	116	16,52	19,81	22,11	20,95	0,97
256	13,59	11,70	17,22	16,14	1,00	299	16,76	17,22	18,13	16,02	0,93
34	13,82	16,30	18,60	17,44	0,96	249	16,81	16,71	19,34	18,12	0,94
140	13,82	15,94	18,06	16,87	0,95	338	16,81	18,70	20,66	19,43	0,95
148	13,82	14,51	17,73	16,62	0,96	109	17,03	15,52	17,71	15,68	0,93
163	13,82	14,91	16,52	15,16	0,93	224	17,03	18,42	20,72	19,54	0,95
220	13,82	17,50	19,67	18,50	0,97	39	17,22	17,15	19,81	18,60	0,94
268	13,82	12,21	15,42	14,10	0,97	40	17,22	15,79	18,89	17,58	0,94
289	13,82	13,82	14,91	12,77	0,94	114	17,22	23,72	26,02	24,87	1,00
295	13,82	14,51	15,42	13,46	0,93	137	17,22	15,42	17,73	15,47	0,94
326	13,82	15,42	16,93	15,57	0,94	346	17,22	16,45	22,64	21,58	1,00
68	14,00	15,83	17,37	16,04	0,94	89	17,50	19,67	22,11	20,96	0,96
205	14,00	15,25	16,43	14,88	0,93	126	17,73	18,47	19,92	18,45	0,94
23	14,22	16,08	18,20	17,00	0,95	231	17,94	17,50	18,62	15,76	0,97
64	14,22	15,15	16,43	14,88	0,93	352	17,97	20,22	22,39	21,21	0,96

dmu	lnK	lnL	lnY	lnT	Efficiency Scores	dmu	lnK	lnL	lnY	lnT	Efficiency Scores
73	14,22	16,30	17,22	15,57	0,94	107	18,06	20,03	21,64	20,33	0,96
110	14,22	15,91	17,91	16,68	0,95	146	18,13	18,06	20,42	19,15	0,94
190	14,22	15,89	17,22	15,79	0,94	112	18,20	17,62	20,03	18,69	0,94
141	14,51	15,20	16,81	15,40	0,93	260	18,42	14,51	18,86	16,74	0,96
151	14,51	15,42	16,71	15,16	0,93	207	18,83	20,72	23,03	21,85	0,97
165	14,51	15,42	16,52	14,85	0,93	122	19,19	17,71	19,86	17,82	0,96
175	14,51	15,14	16,52	14,99	0,93	6	19,52	18,83	21,13	19,72	0,96
237	14,51	14,40	17,50	16,35	0,95	103	19,52	21,82	24,64	23,52	0,99
197	14,63	15,66	16,95	15,43	0,93	351	19,52	20,56	23,43	22,30	0,98
283	14,69	13,12	17,73	16,62	0,97	12	19,54	19,08	24,31	23,24	0,99
21	14,73	15,42	16,81	15,29	0,93	59	19,67	19,11	20,44	18,06	0,97
214	14,73	15,20	16,52	14,91	0,93	7	19,82	20,30	22,67	21,46	0,97
227	14,73	15,42	16,12	13,68	0,94	136	20,03	22,33	23,36	21,80	0,98
14	14,91	19,34	21,75	20,61	0,98	216	20,72	19,61	21,82	20,19	0,97
25	14,91	15,42	17,03	15,60	0,93	314	21,42	14,22	23,03	21,75	1,00
52	14,91	16,52	17,34	15,54	0,93	56	22,11	19,81	23,72	22,42	0,99

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LEAD-LAG AND RELATIONSHIP BETWEEN MONEY GROWTH AND INFLATION IN TURKEY: NEW EVIDENCE FROM A WAVELET ANALYSIS

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Abstract: *The study investigates the relationship between money supply and inflation and Turkey by employing wavelet analysis, mainly continuous wavelet analysis, cross wavelet transforms and wavelet coherence and phase-difference, for the period from 1987 to 2019. Our main finding confirms the modern quantity theory of money about the existence of a relationship between inflation and money supply in the short-run and long-run, and also confirms the traditional quantity theory of money about the existence of a relationship in the long run. The phase difference confirms the existence of a bidirectional relationship between money supply and inflation. The result is consistent with both the traditional quantity theory of money in the long run and the modern quantity theory of money in the short-run and long-run in terms of the existence of a relationship between money supply and inflation.*

Keywords: money supply; inflation; wavelet analysis; Turkey; the quantity theory of money.

JEL Classification: C13; C15; E37; E41.

Introduction

In Turkey, policymakers have employed monetary and fiscal policy instruments as navigators to guide the country's economic policies in response to the economic situation and public finance. However, their effects were limited, and this is evident from the successive crises that impacted the country, which prompted the signing of agreements with the International Monetary Fund (IMF) aimed at containing these crises. However, the economic crisis suffocated the country due to of the significant amount of external debt and the rising inflation as the Turkish government was close to bankruptcy at the beginning of the millennium (Tursoy 2019). Monetary policy-makers were quick to devise new programs and strategies to target inflation and contain these crisis along with the IMF and in line with these reforms, in 2005, 6 zeros were removed from the local Turkish currency thus creating the "New Turkish Lira" (Bankası 2005).

Turkish policy in terms of targeting inflation has experienced many deviations, where these deviation were not necessarily the result of mistaken economic policies, where Global crises, as global production and demand decreased, and the rise in oil prices affected Turkey being an oil importer (Durmuş 2018). Turkish economic policymakers attempted to deal with these challenges by directing the monetary and fiscal policy tools in the country and established new monetary policies aimed at reducing Turkey's vulnerability to external shocks and that also took into account inflation stability along with financial stability (Kayıkçı and Kaplan 2019).

The monetary authority can make a significant contribution to fostering economic stability by establishing a steady course and ensuring it follows this course. By making the direction one of steady but modest increase in the amount of money, this can make a significant contribution to preventing any inflation or price deflation. However, the government's policies of increasing spending and trying to reach full employment as well as subsequent mistakes made by the Central Bank are the factors that led to the increase in the money supply that caused inflation (Friedman 1968). It is of paramount importance for policymakers to ensure that policies to combat inflation are enforced in a stable way to ensure the financial stability needed for low inflation and sustainable economic growth.(Koyuncu 2014)

The importance of inflation as a macroeconomic factor in the literature is derived from its capacity to represent monetary policy's efficiency and effectiveness in affecting the macroeconomy. Specifically, the degree to which inflation influences production growth or economic conditions has proven to be a subject of significant importance to any Central Bank regarding price stability. Generally, the empirical evidence largely appears to lend support to the idea that inflation is counterproductive to economic growth and that price stability, described as a low and stable inflation rate, is an essential prerequisite for achieving economic growth (Mavikela *et al.* 2019).

Reacting to economic conditions is considered as a task of monetary policy, and an interesting tool that is normally employed by monetary policy is the money supply. The detection of the presence of a causal relationship between money supply and inflation is a popular area of economic research in a growing number of developing and developed countries. Generally, the causality between money supply and inflation is believed to be unidirectional from money supply to inflation (Göçmen 2016). The nature of the relationship between money supply and inflation in Turkey is not entirely clear due to the limited amount of studies that have dealt with this topic. The majority of researchers have neglected the relationship between money supply and inflation in Turkey, and most of them have also not taken into account time-frequency, which could provide important information about this relationship and how they interact in the frequency domain. In this study, we investigate the causality between money supply and inflation rate in Turkey and attempt to provide new insights regarding the direction of this relationship by employing wavelet analysis; wavelet analysis has a feature that reveals information in time-frequency that cannot obtained by normal analysis. We also employ cross wavelet analysis and phase deference to determine the lead lag relationship and to recognize the causality direction.

Since it is not always possible to obtain significant information from normal analysis of a data series, where the data's frequency contain also involves significant data information can't appear in time domain, wavelet analysis has become a popular tool for analysing time series due to its ability to facilitate the understanding about data in both time domain and frequency domains.

Our main finding confirms the modern quantity theory of money about the existence of a relationship between inflation and money supply in the short run and long run, and also confirms the traditional quantity theory of money about the existence of a relationship in the long run. The phase difference confirms the existence of a bidirectional relationship between money supply and inflation.

This paper is structured as follows. Sections 2 provides a literature review and examines previous studies on the relationship between money supply and inflation. Section 3 explains the methodology used in the study. In Section 4, the data are described, and the results discussed, while the conclusion is presented in Section 5.

1. Literature Review

Inflation refers to the situation where more money is being paid for the same goods or services. Although we have the ability to determine whether an economy is suffering from inflation, there is currently no consensus regarding what triggers it. Keynesian economists explain the relationship between money supply and inflation via demand-pull and cost-push and Demand-pull-inflation. Monetarists indicate that inflation occurs as a result of an increase in the money supply (Abdullah *et al.* 2020). As central banks are primarily focused on maintaining price stability, money flows have been assigned less importance. However, it seems that there less interest in concerning money supply go hands in hands with inflation to keeping inflation down and steady (King 2001).

Researchers who are interested in studying the nexus between money supply and inflation have generally used the most famous version of the quantity equation of exchange formulated by Fisher (1911). The Fisher and Brown (1911) equation is expressed as:

$$M(V_T) \equiv P_T(T) \tag{1.1}$$

Where M is money in circulation, V_T is the transaction velocity of circulation, P_T is the average price level of transaction and T is the total transaction per period. The left-hand side symbolizes money supply, while the right side symbolizes money demand. A higher supply of money leads to a higher price and vice versa. Consequently, a shift in the supply of money will lead to a price change. In other words, if a country faces high inflation rates, decreasing the amount of money supply will reduce inflation and vice versa when there is disinflation and deflation (Ditimi *et al.* 2017). The relationship between money supply and inflation is linked with money demand and money supply, where the expenditure surplus that is not met by increasing output will lead to an increase in prices, since the raise in expenditure changes the prices rather the quantity (Strano 2003).

Fisher and Brown (1911) assumed that as monetary authorities increase the level of currency (money supply) in the market, deposits must increase by the same ratio, while the velocity and quantity of goods remain constant. This is expressed as:

$$M(\bar{V}) = P(\bar{Y}) \quad 1.2$$

Changing the supply of money given the constancy of velocity and output will result in equivalent proportionate inflation rate changes (Su *et al.* 2016). However, many economists argue that monetary theory does not hold in the short-run and it focuses on the long-run properties of the economy rather than the short-run dynamics. Monetarists are sceptical about the potential of using monetary policy to achieve short-term stability (Meyer 2001).

Money growth in the long-term has an impact on inflation, as promoted by the quantity theory of money, where a growth in price level precedes an increase in money supply. However, empirical evidence on the connection between money growth and inflation and the effectiveness of money in terms of forecasting inflation is controversial (Vladova and Yanchev 2015).

The modern monetary theory claims that inflation is driven by an increase in the money supply that is greater than money demand, whereas changes in income and price are driven by money. Milton Friedman argued that inflation is monetary and monetary policy should be focused on reducing inflation. Consequently, money increase is generally believed to have a significant long-term influence on any country's economic activities (Ditimi *et al.* 2017).

In order to explain the nature of the relation between money and inflation, many researchers have studied it using different methods. Differences in their empirical results are due to different countries, methodologies and methods of analysis. Next, we will briefly review some studies about the nexus between money supply and inflation.

Abdullah *et al.* (2020) attempted to identify the determinants of inflation in Kuwait during the period from 1979 to 2015 and examined whether inflation is affected by exchange rate, interest rate, taxation, current account, unemployment, GDP and money supply using multiple regression analysis. The study results showed that a change in the inflation rate is positively significant with money supply.

Ditimi *et al.* (2017) investigated the impact of a rise in money supply on inflation in Nigeria using the co-integration autoregressive dynamic error correction model approach. Their findings showed that the money supply does not have a considerable effect on inflation in either the short run or long run while the Granger causality test indicated that there was no causality between money supply and inflation in either direction during the study period, which was 1970 to 2016. In contrast, Sasongko and Huruta (2018) found a causality between money supply and inflation rate by using the Granger causality method, but only a one-way causality whereby money supply affected inflation and not vice versa. The study aimed to determine the casual relationship between money supply and inflation in Indonesia during the period from 2007 to 2017.

Jiang *et al.* (2015) investigated the relationship between money supply and inflation rate in China for the period from the mid-1990s to the early 2000s using novel wavelet analysis. The study results support the modern quantity theory of money that money growth and inflation rate are positively related, although in the short run, this positive relationship deviated due to temporary shocks. Also in China, Su *et al.* (2016) examine the relationship between money supply and inflation rate by using bootstrap Granger full sample causality and sub sample rolling causality. The study aimed to verify whether the Chinese economy supported the quantity theory of money. The varying rolling-window approach applied in this research showed that the money supply had a positive and negative impact on inflation in several sub-periods and vice versa. The findings of the study were consistent with the modern quantity theory of money. By using wavelet analysis, Jiang *et al.* (2015) found that money growth has a positive effect on inflation in China with the existence of some deviations. The study results confirmed the modern quantity theory of money.

Vladova and Yanchev (2015) examined the relationship between dynamic money supply (currency in circulation, M1, M2) and prices in Bulgaria during the period 1998 to 2012 using three econometric methods, namely dynamic cross correlations, Granger causality test in the framework unrestricted VAR models and Johansen cointegration. The study concluded that there was a two-way relationship between money supply and price dynamics.

Nguyen (2015) investigated the effect of fiscal deficit and broad money supply M2 on inflation in nine Asian countries over the period 1985 to 2012 using a pooled mean group estimation-based error correction model and the general method of moments. The main finding was that the money supply M2 had a positive impact on inflation according to the pooled mean group method.

Regarding Turkey, Koyuncu (2014) investigated the impact of budget deficit and Money supply (M2) on inflation using Johansen cointegration and Granger causality during the period from 1987 to 2013. The Granger causality test showed a one-way relationship from Money supply to inflation but there was no evidence indicating that the direct cause of inflation was the money supply. Also, the Johansen test revealed that there was no cointegration between the series.

Rua (2012) investigated the relationship between money growth and inflation over the period 1970 to 2007 in the euro area using wavelet analysis. The study results showed a strong association between money supply and inflation in the short-term (low frequency) and this relation become stronger at long-term development than short-term fluctuation of business cycle.

2. Methodology

This section presents the methodology used in the study, namely wavelet analysis including continuous wavelet transform, cross wavelet transforms and wavelet coherence. Wavelet analysis has become a popular tool in analysing time series due to its ability to provide a better understanding for data in both time domain analysis and frequency domain analysis.

It is not always possible to obtain significant information from normal analysis of a data series. Sometimes the data's frequency also contains significant information. The Fourier transformation (FT) measures the frequency-amplitude of the data in the time domain, but it does not indicate when this frequency occurred in time. All frequency components take place at any point in time in the case of stationary data, but this is not true for non-stationary data. FT, therefore, is not suitable for non-stationary data, so wavelet analysis is considered as an alternative to the Fourier transformation. Wavelet analysis can handle non-stationary data and has the ability to provide information simultaneously in both time and frequency (Jeet and Vats 2017).

Continuous wavelet transforms (CWT) is considered a useful tool for extracting information from the time series and data self-similarity detection. CWT produces many wavelet coefficients C, which can represent a function of two factors, namely scale and position $C(scale, position)$. A specific wavelet ψ is obtained by projection onto the tested time series x(t):

$$C(scale, position) = \int_{-\infty}^{\infty} x\, t\psi(scale, position, t)dt \qquad 2.1$$

Any value consonant with the region of the time series x(t) could be taken in scale and position. Multiplying each coefficient by the fittingly scaled (dilated) and shifted wavelet produces the constituent wavelets of the authentic signal. There are several types of wavelets, which have various properties for different applications. In the analysis of both the amplitude and phase information. since we are interested in the most popular wavelet which is Morlet wavelet which is define as:

$$\psi_\eta(t) = \pi^{-\frac{1}{4}} e^{iw0\eta} - e^{-\frac{\eta^2}{2}} \qquad 2.2$$

where is w the frequency of the angle (rotation rate per time unit) which is assigned value to 6 as indicated in the literature, the normalization term is -1/4 is the dimensionless time parameter represented by, $\eta = t/\lambda$, t is the time parameter, λ is the scale of the wavelet. The process of scaling and shifting is described in equation xxx (Gençay *et al.* 2001; In and Kim 2013; Loh 2013).

Cross wavelet analysis and wavelet coherence are considered as powerful techniques for testing suggested linkages between two-time series. The continuous wavelet transform can be extended to include to time series and construct a cross wavelet transformation, which identifies areas with high joint influence and provides more detail on the phase association (Grinsted *et al.* 2004). When the two wavelet transformations being performed are in reference to the same mother wavelet, then the transformation of the cross wavelet shows the degree of the commonality between the two initial wavelet transforms or signal. To deduce the existence of a mutual signal or to locate the source of these common signals can be used by the magnitude' of the cross-wavelet transform. Similarity may occur for various reasons and each implementation may give different justifications for this commonality. The magnitude of the cross wavelet transform will have a peak showing this similarity (Randy and Young 1993).

In this paper, to avoid biased results and the production of any incorrect distortions or deviation by giving more weight to the large-scale linking phenomena than to the small-scale phenomena, the study uses the corrected wavelet transformation by normalizing the scale as follows:

$$wxy\ (t,s) = \frac{1}{s}.wx(t,s).\overline{w}y(t,s) \tag{2.3}$$

For more information about the suggested cross-wavelet bias corrected by normalizing scales, see Veleda *et al.* (2012)

For the phase differences, we follow Rösch and Schmidbauer (2016) and write:

$$Angle(\mathcal{T},\mathcal{S}) = Arg(Wave.xy(\mathcal{T},\mathcal{S})) \tag{2.4}$$

This so-called x-over-y phase difference at each time and scale is equal to the difference between the individual local phase separations when transformed into an interval angle$[-\pi,\pi]$. The two series are considered in phase (anti-phase) if the absolute value is less or greater than $\pi/2$ respectively at the scale. Appendix A presents an interpretation of phase differences.

Wavelet coherence is another useful measure used to determine how coherent the cross wavelet transform is in time-frequency space (Grinsted, Moore and Jevrejeva 2004, 564) and demonstrates how powerful the co-movement between two series is over time and frequency and it locate between zero and one, as we closer to one as we have more powerful association. In addition to the degree of the relation, the wavelet coherency plot indicates the frequencies at which the association is located (Özmen and Yılmaz 2017). To analyze the wavelet coherence between two time series, we follow Torrence and Webster (1998) and write:

$$R_n^2(s) = \frac{\left|s(s^{-1}w_n^{xy}(s))\right|^2}{s(s^{-1}|w_n^x(s)|^2)\, s(s^{-1}\left|w_n^Y(s)\right|^2)} \tag{2.5}$$

The smoothing operator is S a suitable smoothing operator for the Morlet wavelet and is given by:

$$s_{time}(w)|_s = (w_n(s) * c_1^{\frac{-t2}{s^2}})| \tag{2.6}$$

$$s_{time}(w)|_s = (w_n(s) * c_2\prod(0.6s))| \tag{2.7}$$

Where the normalization constants are c1 and c2 and $\prod$ is the rectangle function. The factor of 0.6 is the empirically determined scale decorrelation length for the Morlet wavelet.

For the phase differences, we follow Rösch and Schmidbauer (2016) and write:

$$Angle(\mathcal{T},\mathcal{S}) = Arg(Wave.xy(\mathcal{T},\mathcal{S})) \tag{2.8}$$

This so-called x-over-y phase difference at each time and scale is equal to the difference between individual local phase separations when transformed into an interval angle$[-\pi,\pi]$. The two series are considered in phase (anti-phase) if the absolute value is less or greater than $\pi/2$ respectively at the scale. Appendix A presents an interpretation of the phase differences.

3. Data and Results

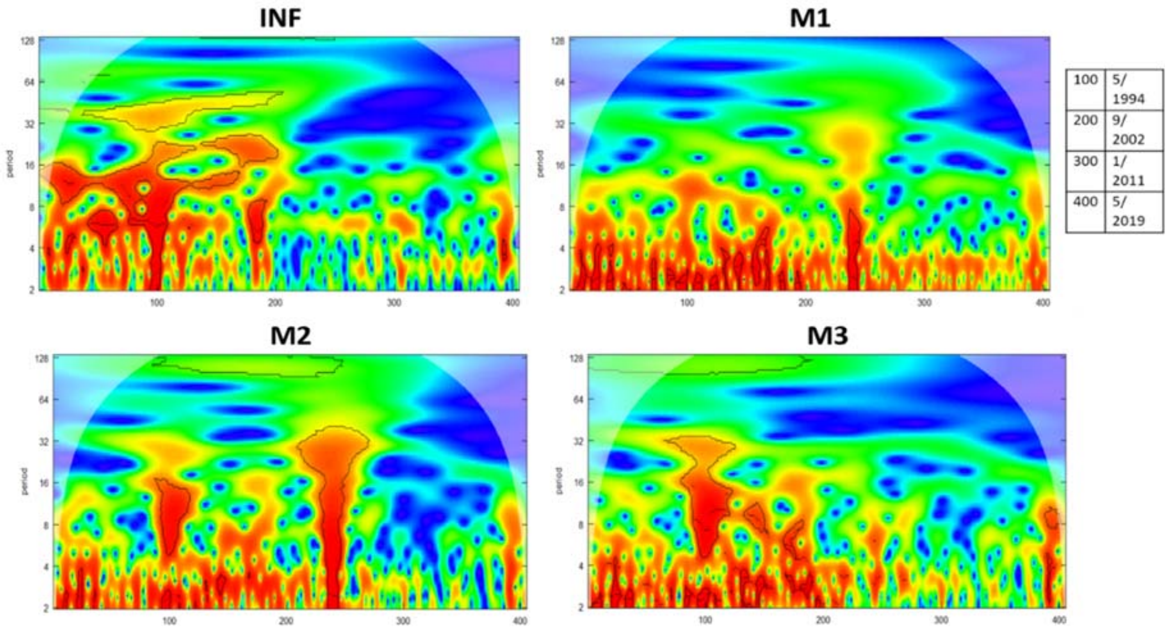
For the purpose of analysing the nexus and lead lag relationship between growth inflation (IN) and money growth (Narrow Measure (M1), Intermediate Measure (M2), Broad Measure (M3)), monthly data was obtained from the Turkish Central Bank. The data covered the period between February 1987 till October 2019, the data monthly percent change. No adjustment was made to the original data since the study data is growth and the wavelet transform can handle nonstationary data, so it is not necessary to check for a unit root or take any difference. Appendix B shows the plots for all variables A large deviation in inflation can be observed in 1994, which was caused by the currency crisis in Turkey, while another large deviation occurred in 2005 in M1 and M2, which was the year that the Turkish currency converted to the New Turkish Lira.

Appendix C shows the descriptive statistics of the variables. It can be observed the inflation fluctuated between -1% and 23% with a mean of 0.03 and a standard deviation of 0.03. The growth in money supply rates also showed a very wide range of fluctuation. For instance, M1 had a minimum of -20.19% and a maximum of

68.2%, with a mean of 3.39 and standard deviation of 7.98. These fluctuations can be explained by the major successive crises that impacted Turkey during the period of the study.

Before analysing the correlation relationship between money inflation and money supply, we start by conducting the continuous wavelet transform to capture the important information that occurred in frequency. Figure 1 shows the results of continuous wavelet transform for both inflation and money supply at different time horizons. The INF plot has many large significant plots at different times and frequencies. One interesting significant region is located around 1994, which is the year of the Turkish currency crisis. We can see the effect of the crisis in high frequency and extended to reach more than 32 periods.

Figure 1. Continuous wavelet transforms for the study variables



Note: 5% significance level against red noise designates by the contour, the lighter shade shows the cone of influence. Power color code ranges from blue (low power) to red (high power).

As a result of the currency crisis that occurred in 1994, the output fell by 6 percent, inflation increased to three-digit levels, the central bank lost half of its reserves, and in the first quarter of the year, the exchange rate (against the US dollar) depreciated by more than half (Celasun 1999). Another significant plot region can be observed around 2001, which was the year that the Turkish government was close to announcing bankruptcy as a result of its deficit (Tursoy 2019)

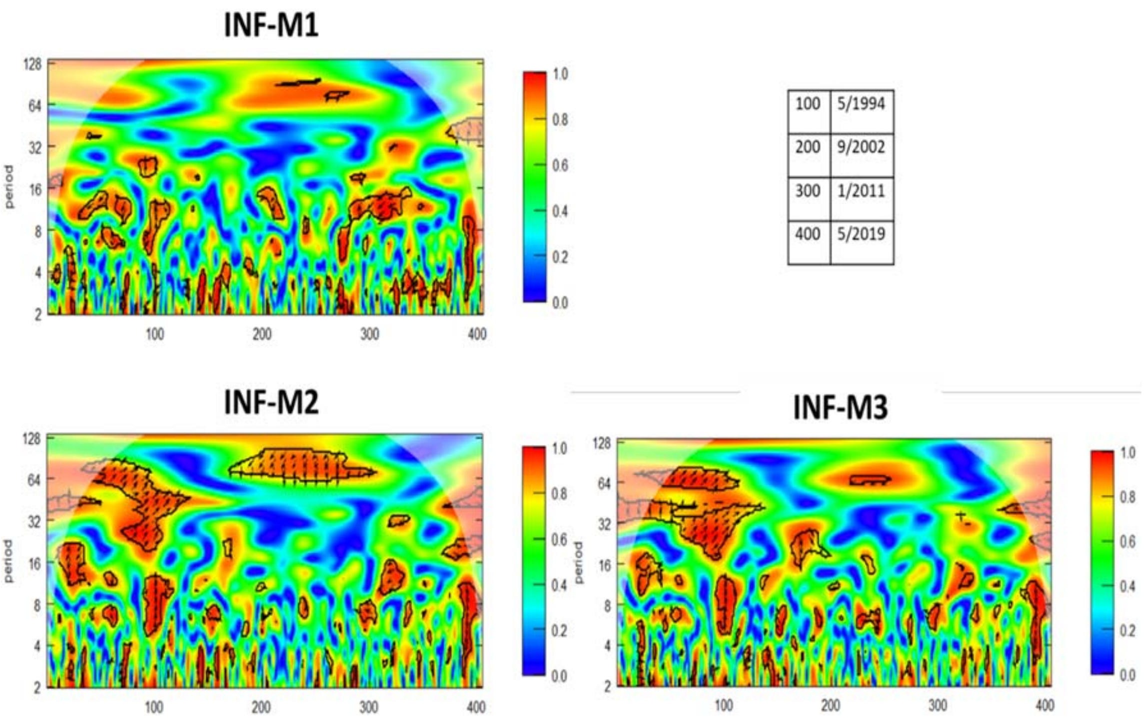
The effect of the crisis of 2001 was not as large as the crisis of 1994, which clear from the plot xx that extended for years at low frequency (around 16 month and 32 month). The last significant region located at high frequency is around 2018, which represents the fall of the Turkish lira against US dollar by 40% from the start of the currency crisis in 2018.

Regarding money supply, it is clear from Figure 1 that M3 was the most affected by the currency crisis in 1994, the effect of which appears from month 4 to month 32, while M2 was the most affected by the mortgage crisis in 2008. We can see a significant plot region extending from high frequency (2 month) and extended to low frequency (32 month). M1 shows many significant regions from 1987 to around 2008, where the greatest effect on money supply was caused by the global crisis in 2008.

For the correlation analysis between the study variables, we employ wavelet coherence with phase difference to capture the nature of the relationship between inflation and money supply. Figure 2 presents the wavelet coherence for the variable of the study. In terms of the relationship between the IN-M1 pair, we can see in Figure 2 many plot regions for the significant relationship between inflation and money supply. From 1987 until 1994 (up to frequency 7), there is generally a positive relationship between inflation and money supply, while from cycle 8 to cycle 6, there is largely a negative relationship (the arrows left). From 1995 to 2019, it can be seen from Figure 2 that there is a negative relationship between inflation and M1 at either high or low frequency.

It is clear from the IN-M1 pair results that it agrees with the modern quantity theory of existing the existence of a relationship between money supply and inflation. These results (from 1987 until 1994) confirm those of Abdullah et al. (2020) and Jiang et al. (2015) who found a positive relationship between money supply and inflation in the short run while, but conflict with them after 1995, where the change in the positive relationship may reflect the different policy that was followed after the currency crisis in 1994.

Figure 2. Wavelet coherence between money growth and selected inflation



Note: 5% significance level against red noise is designated by the contour, the lighter shade shows the cone of influence, by using phase randomized surrogate series is estimated from Monte Carlo simulations. Power colour code ranges from blue (low power) to red (high power). Arrows represent phase differences.

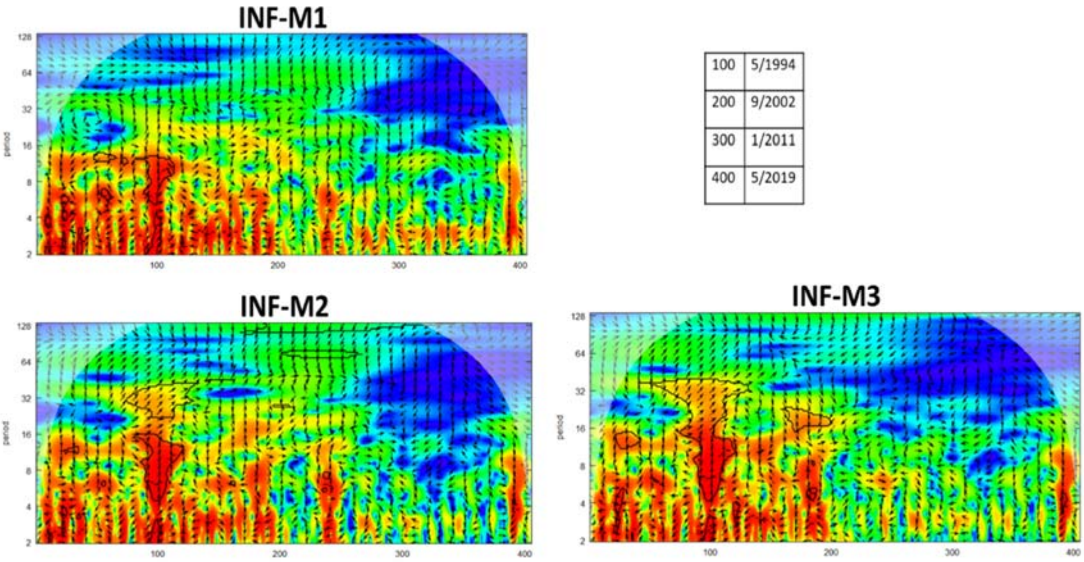
For the IN-M2 and IN-M3 pairs, it can be observed that the relationship quite similar together and almost similar to the IN-M1 pair, although the association between IN-M2 and IN-M3 is stronger at low frequency (around month 25 to month 120). The correlation between money supply growth and inflation almost up to 1, and there are two large significant region plots, where the first region plot shows a positive relationship while the second one shows a negative relationship.

In order to determine the similarity of change between two time series signals together as well as the causality, we employ cross wavelet analysis phase differences. Figure 3 shows cross the wavelet analysis, for the pair of IN-M1. We can observe significant plots from 1987 until around 1994 at different frequencies, around before 1990s we can notice significant plots for the pairs at high frequency (up to around 5 month), for the IN-M2 pair and the IN-M3 pair, as shown in Figure 3 it almost similar to the IN-M1 pair with minor differences. There is a significant plot region around 1994 at low frequency (around 25 to around 50) that did not appear for the IN-M1 pair. it's appears in this pair (IN-M1) that the leading inflation while M2, M3 lagging after neutral significant plot region in 1994 which it's not clear which variable was leading and which one lagging. The significant region plots in 1994 is larger for the IN-M3 pair than IN-M2. We can see another region plot during the study period, but it is not clear which is leading, and which is lagging.

In some frequencies and times, we notice that the inflation is leading while money supply growth is lagging. The explanation for this may come from Demand pull theory, which suggests that an increase in aggregate demand more than an economy's productive capacity leads to an increase in prices and wages and this pushes the central bank to increase the money supply.

The phase differences show that the causality is bidirectional and changeable between money supply and inflation. Although some researchers argue that the relationship between money supply and inflation is bidirectional, there is no exclusive result across countries about these relationships which different regarding country and methodology.

Figure 3. Cross wavelet transforms between money growth and selected inflation



Note: 5% significance level against red noise designated by the contour, the lighter shade shows the cone of influence, by using phase randomized surrogate series is estimated from Monte Carlo simulations. power colour code ranges from blue (low power) to red (high power). Arrows represent phase differences.

The monetary policy Procedures have a primary role in controlling and directing this direction. The result confirms both the traditional and modern quantity of theory they agree on existing relationship on long run. Although there is no consensus regarding the relationship in the short-run, there is agreement between the traditional quantity theory of money and the modern quantity theory of money about the existence of a relationship between money supply and inflation in the long-run (Jiang *et al.* 2015, 250). The results from the IN-M1 and IN-M2 pairs until the mid-1990s agree with both Abdullah *et al.* (2020) and Nguyen (2015) regarding the existence of a relationship in the short run. Our results also agree with Rua (2012), who argued that the relationship becomes stronger in the long run (low frequency) but the sign of the relationship differs in the long run compared to the short run. Also, after the mid-1990s our results still agree with Rua (2012) that the relationship becomes stronger in the long term. The results concur with both Vladova and Yanchev (2015) and Su *et al.* (2016) that there is a two-way relationship between inflation and money supply.

Conclusion

This paper has investigated the relationship between inflation and growth in money supply (M1, M2, M3) during the period from 1987 to 2019 and attempt to provide an insight regarding the nature and direction of this relation. The study employed continuous wavelet transform to capture important information in frequency and also used cross wavelet and different phases analysis to determine the lead lag relationship, while wavelet coherence was also adopted to describe the association between the study variables.

Generally, the results confirm the modern quantity theory of money about the existence of a relationship between inflation and money supply in the short run and long run, and also confirm the traditional quantity theory of money about the existence of a relationship in the long run. Also, the results are in complete agreement with Su *et al.* (2016) that money supply has a positive and negative impact on inflation in several sub-periods and vice versa.

The phase difference confirms the existence of a bidirectional relationship between money supply and inflation. Since the relation between money supply and inflation is changeable, the study shows that the lead lag is changeable between money supply and inflation. The changeable impact and changeable lead lag relationship between the variables appear to be connected with different policies implemented after each crisis that impacted Turkey.

The study recommends that sub-period analysis should be conducted to investigate money supply and inflation for each crisis and the polices subsequently implemented by the authorities to handle them. Also, we suggest that a review study should be performed focused on monetary and fiscal policies that followed these periods in order to obtain a better understanding on the nature of this relationship.

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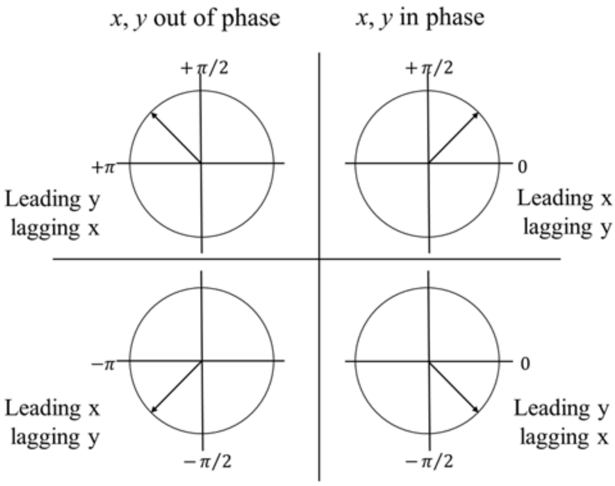
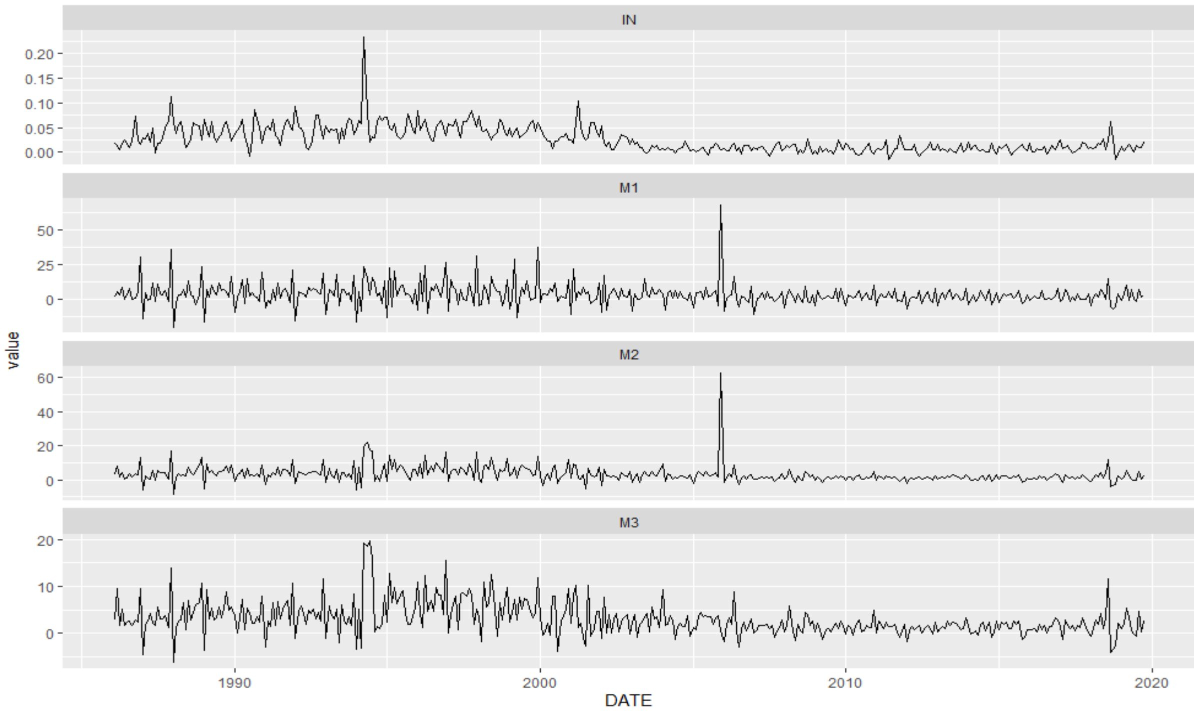
Appendices

Appendix A. Graph based on Rösch and Schmidbauer (2016).



Appendix B. Graphical distribution of time-series.



Appendix C. Statistical Distributions.

variable	mean	sd	median	min	max	range	skew	Kurtosis
INF	0.03	0.03	0.02	-0.01	0.23	0.25	1.93	9.94
M1	3.39	7.98	2.38	-20.19	68.2	88.39	2.03	12.38
M2	3.26	4.88	2.39	-8.75	62.94	71.69	5.14	54.68
M3	3.08	3.55	2.28	-624	19.79	26.3	1.34	3.29

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PUBLIC SPENDING AND ECONOMIC WELFARE IN ECOWAS COUNTRIES: DOES LEVEL OF DEVELOPMENT MATTER?

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Abstract: *Conflicting views on the sign of the relationship between government size and economic development have resulted into the testing of non-monotonic relationship in the literature. Therefore, the total effect of growing public spending on economic development is ambiguous. This study investigated how government size affect economic development and determine the optimal government size that promotes economic development in ECOWAS countries. The study employed secondary data covering the period 1986 to 2018. Data on Gross Domestic Product per capita, government size, population growth rate, inflation rate, gross fixed capital formation and financial development variables were sourced from World Development indicator database. The study constructed social welfare function as development indicator. Data were analysed using Least Absolute Deviation (LAD) regression and quantile regression (QR). The findings showed that quantile regression estimates are negative and significant ($p < 0.05$) in low quantiles, thus suggesting that deleterious effect of government size is more pronounced among countries with low level of economic development.*

Keywords: public spending, economic growth, GDP; ECOWAS countries.

Jel classification: O40; O55; E60; C21.

Introduction

The issue of relationship between government size and economic development is currently of burning importance to most economies across the world, especially in the United States and European Union because most countries have been confronted with an increasing public debt and a drop in their economic growth since global financial crisis of 2007. Faced with this crisis, countries like United State of America, chose to support economic activity with reflationary policies *i.e.* public spending, thus increasing public deficit and public debt. This choice seems to have been justified by the Keynesian paradigm, based on a vicious cycle of public spending through the multiplier effect.

This observation appears to hold across most countries regardless of the level of development. For the last 20 years, expansion in the share of government as a percentage of GDP appears to have been the norm in both developing and developed countries. In comparing developing and developed nations, the current levels, growth rates, composition, and determinants of government expenditures exhibit significant differences. Therefore, the total effect of growing public spending on economic welfare and development is ambiguous, especially with the realities of exposure to international trade and domestic factor such as institutions (Thorbecke 2013).

In West Africa, most of the countries are members of Economic Community of West African States (ECOWAS) economic group. Out of this group, countries such as Burkina Faso, Cote d'Ivoire, Guinea-Bissau and Togo experienced political turmoil during the period of analysis, while Mali, Niger and Nigeria remain vulnerable to security issues, which have contributed to the fragility of the group (AEO 2017). From the fiscal revenue viewpoint, none are considered predominantly natural resource-rich, perhaps except Nigeria; however, many generate fiscal revenues from natural resources (mining, oil) with increasing economic and fiscal potential. ECOWAS countries have among the lowest GDP per capita levels in the world and exhibit relatively low and irregular GDP per capita growth rates, mainly because their economies are not well diversified, and they have relatively high population growth rates.

Findings from the empirical literature on government size and economic development relationship are mixed (Folster and Henrekson 2001). In recent years, there is some convergence in term of the importance of public expenditure on economic development. But the result still changes across countries, economic regions or from one data sample to another. For instance, some studies are of opinion that government size promotes economic development (Komain and Brahmasrene 2007, Alexiou 2009). Other studies posit that the effect of government size on economic development is deleterious (Martins and Velga 2013; Churchill, Yew and Ugur 2015)

The debate on sign of the relationship between government size and economic development is still on. Attempt to resolve these conflicting views have led to the consideration of a non-linear relationship between the government size and economic development (Barro 1990). Ample evidence indicates that linear or monotonic relationship exist between government size and economic development in ECOWAS countries. For example, Ansari, Gordon and Akuamoach (1997), Enang (2010) and Mudaki & Masaviru, (2012) reported in their studies that large government is a drag on economic development, whereas Yasin (2003), Oriakhi and Arodoye (2013) and Gisore, Kiprop, Kalio, Ochieng & Kibet (2014) asserted that government is a spur to growth and economic development. Given that empirical literature supply conflicting views on the impact of government size on economic development, it indeed becomes plausible to consider the possibility of a non-linear relationship for ECOWAS countries.

Furthermore, the studies that have investigated the link between government size and economic development for developing economies, have discussed economic development from income-based perspective of development (studies such as, Iyare, Lorde and Francis 2005; Oteng-Abayie and Frimpong 2009; Moreno-Dodson and Bayraktar 2015). Recent development in macroeconomics has showed that income-based indicator (GDP growth) is not a good measure of economic development and well-being (Stiglitz, Sen and Fitoussi 2009; Stiglitz 2016). This study, therefore, looked beyond GDP measure by constructing social welfare function (SWF) as development indicator. This development indicator considers the spread of benefit that economic growth brings among the citizenry in terms of access to health care, education, infrastructures, improved quality of life e.t.c.

Upon the foregoing, this study tested the relationship between government size and economic development in ECOWAS countries in a non-monotonic framework as theoretically characterized by Pevcin (2004) and Davies (2009) using quantile regression.

1. Data and Methods

Beyond the standard linear regression model framework, the study applied quartile regression model. Quantile regression as introduced in Koenker and Bassett (1978) is an extension of classical least squares estimation of conditional mean models to the estimation of the whole conditional distribution of response variable (see Koenker 2005).

Given the data $(y_t, x_t^{'})'$ for t = 1, . . . , T, where x_t is k × 1, consider the following linear specification for the conditional quantiles of y:

$$y_t = x_t^{'}\beta + e_t \qquad 1.1$$

where y_t is the dependent variable – development indicator and x_t is a vector of explanatory variables – government spending and control variables. The primary objective is to estimate β for different conditional generic quantile functions given in equation 1.1.

As described by Koenker and Bassett (1978), the estimation of β is done by minimizing equation (2);

$$\hat{\beta}_\tau = min_{\beta\epsilon\Re}k\,[\tau \textstyle\sum_{yt\geq xt\beta} |\, y_t - x_t^{'}\beta \,| \; + \; (1-\tau) \textstyle\sum_{yt<xt\beta} |\, y_t - x_t^{'}\beta \,| \qquad 1.2$$

With equation 1.2 specification, the study was able to depict the conditional distribution in detail when more quantile regressions are estimated. Moreover, the conditional distribution would be skewed to the left if the upper quantile lines are close to each other, relative to the lower quantile lines. It has been found in many applications that the estimated quantile regressions are quite different across quantiles (Katrin 2009). This suggests that regressors may have distinct impacts on the dependent variable at different locations of the conditional distribution (Kuan 2007).

While the formulation of the quantile regression model is analogous to the conventional mean regression model, important differences arise in model estimation. The essential feature of a regression analysis is to examine the manner in which a set of explanatory variables affects the conditional distribution of a dependent variable. In the classical econometric techniques (Ordinary Least Squares, Instrumental Variable and Generalized Least Squares), the component around which the dependent variable randomly fluctuates is the conditional mean $E[y/x, \beta]$. However, unlike the classical approach, which amounts to estimating the conditional mean of the conditional distribution of y, the quantile estimator is employed on different quantiles of the conditional distribution.

The quantile function is a weighted sum of the absolute values of the residuals. Where the weights are symmetric for the median regression case in $\tau = ½$, the minimization problem stated in equation (2) reduces to $min_{\beta \epsilon \Re^k} \sum_{t=1}^{T} |(y_t - x_t^{'}\beta)|$ and asymmetric otherwise. It thus can be observed that varying the parameter τ on the [0,1] interval will generate the entire conditional distribution of economic development and government size series. The coefficient $\beta_i(\tau)$ can then be interpreted as the marginal impact on the τ^{th} conditional quantile due to a marginal change in the i^{th} policy variable.

The quantile regression approach makes it possible to identify the effects of the covariates at different points on the conditional distribution of the dependent variable. With economic development as dependent variable, suppose τ =0.05, i.e countries that are in the left tail of the conditional distribution of economic development (less developed countries) and τ =.95, that is, countries that are in the upper tail of the conditional distribution (most developed countries). Under traditional mean regression methods, the slope coefficient is constrained to be the same for all quantiles, as such there is insufficient information on how policy variables affect countries differently. Mello and Novo (2002) construed that the ability to distinguish the effects of policy variables among different quantiles is important empirically.

Hence, the study estimated equation 1.3 specify as;

$$dev_i = \rho_\tau + \delta_\tau govexp_i + \epsilon_\tau Z_i + \varepsilon_{\tau i} \quad 1.3$$

where *dev* represents economic development, *govexp* represents government size and *Z* captures the control variables, ρ_τ, δ_τ, and ϵ_τ are parameters to be estimated for different values of τ and, $\varepsilon_{\tau i}$ is the random error term. By varying τ from 0 to 1, the study can trace the entire distribution of economic development variable conditional on government size variable.

2. Definitions and Measurements of Variables

The dependent variable in Social Welfare Function (swf). Looking beyond GDP, the measure of economic development employed in this study is the Social Welfare Function developed by Sen (1973), using an individual's average income for a country, allowing it to be weighted by the inequality of distribution of income within a country and is calculated as:

$$swf_{it} = GDP\ per\ capita_{it} * (1 - GINI_{it}) \quad 2.1$$

GDP per capita is the average level of income in a given country *i* at time *t*. GINI is the most commonly used measurement of income inequality for country *i* at time *t*. The higher the value of *swf* index, the higher the level of social welfare. Using equation 2.1, the study constructed social welfare function for the sampled countries.

Four control variables in the models are; Inflation rate (*inf*) measure as the percentage change of consumer price index, population growth rate (*pop*) in percentage, domestic investment (*inv*), proxy by gross fixed capital formation as percentage of GDP captured the share of investment to output and financial deepening (*findev*) measure as ratio of credit to private sector to GDP .

As widely used in the growth literature (Islam 1995; Caselli *et al.* 1996; Levine *et al.* 2000; Hung 2011) averaging data over fixed intervals has the potential for eliminating business cycle fluctuations. Thus, allowing the focus to be on the medium – and long – term trend in the data. Therefore, all values of variables are five-year averages in order to eliminate short – term fluctuations and reduces potential impacts of single year abnormalities.

Thus, with inclusion of the control variables described above, equations 1.3 and 2.1 beget the estimated model as;

$$swf_{it} = \beta_0 + \beta_1 govexp_{it} + \beta_2 inf_{it} + \beta_3 inv_{it} + \beta_4 pop_{it} + \beta_5 fid_{it} + v_{it} \quad 2.2$$

Secondary data was the major source of data for this study. Data covering the period 1986 to 2018 are sourced as discussed below: government size (government expenditure to GDP), population growth rate, inflation rate, gross fixed capital formation and financial development, were sourced from World Development indicator (WDI) database.

3. Results and Discusions

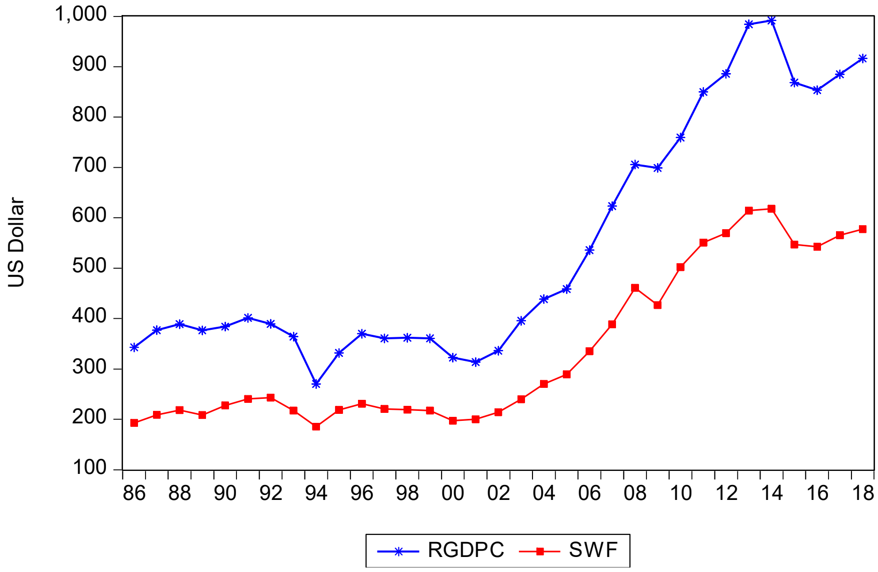
Table 1. Development Indicators for ECOWAS Countries, 1986-2018 Averages

Country	*rgdpc*	*swf*	*Remark*
Burkina Faso	411 [L]	417	low
Cote d'Ivoire	1016 [H]	906 [H]	very high
Gambia	495	300 [L]	low
Ghana	712 [H]	884 [H]	very high
Guinea	487	507	medium
Guinea Bissau	374 [L]	321 [L]	very low
Mali	482	536	medium
Niger	279 [L]	251 [L]	very low
Nigeria	992 [H]	1388 [H]	Very high
Senegal	767 [H]	592 [H]	very high
Sierra Leone	287 [L]	370	low
Togo	412	336 [L]	low

[L] – low, [H] – high

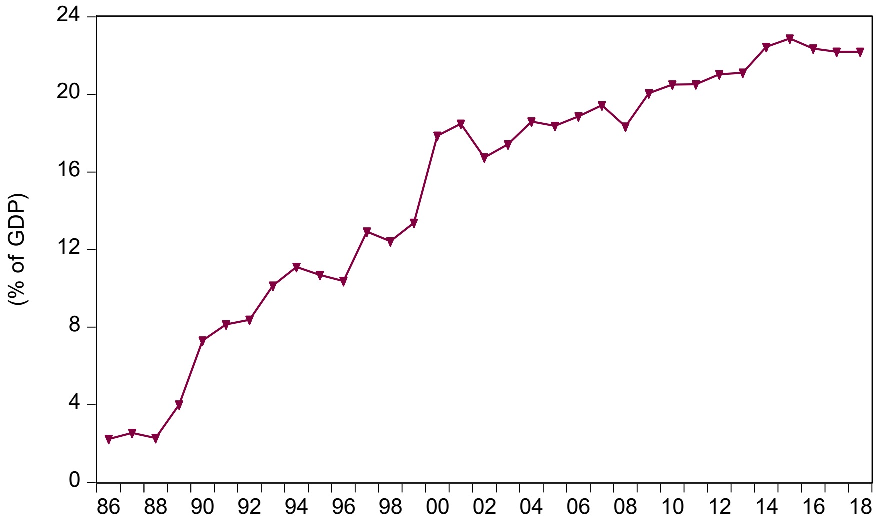
Table 1 presents averages of two development Indicators for ECOWAS Countries for the period 1986 to 2018 and it suggest relatively close correlation between real gross domestic product per capital and social welfare function. For examples, 8 out of 12 sampled countries show similar development status using the two indicators. This correlation is more explicit in figure 1, where the patterns of real gross domestic product per capital (*rgdpc*) and social welfare function (*swf*) are similar, although swf is lower throughout all the sample period.

Figure 1. Trend of Real GDP per capita and Social Welfare Function of ECOWAS countries (1986-2018)



Over the sample period, figure 2 shows that the share of government to GDP for ECOWAS countries has been rising since 1988. The growth of government size in the countries might be justified by need of government to finance public investment in building infrastructures, healthcare, education, improvement of labor force, and Research & Development.

Figure 2. Trend of Government Size of ECOWAS countries (1986-2018)



The panel unit root tests are first applied based on three different panel unit-root tests; Levin, Lin & Chu (LLC), Im, Pesaran and Shin (IPS) and Maddala – Wu (PP-Fisher) tests. The various tests are reported at level in Table 2 and result of tests after first difference in Table 3. As reported in Table 2, the panel unit root tests (at least two of the tests) show that the following series are stationary at levels at least at 5% significance level; economic development (*dev*), inflation (*infl*) and population (*pop*).

Table 2. Panel Unit Root (At level)

Variables	Levin, Lin & Chu t*	Im, Persaran & Shin W-stat	PP-Fisher Chi-sq	Decision
dev	-6.4276***	-1.5480*	53.8033***	S
govexp	-2.8772***	-0.1776	28.9204	NS
findev	-3.5400***	-0.3277	30.4909	NS
infl	-7.5572***	-3.5036***	75.5070***	S
invt	-4.8193***	0.8476	31.7029	NS
pop	-6.3192***	0.5912	42.0590**	S
*** (1%), ** (5%) & *(10%) level of significance				

Table 3 shows that the series that are non-stationary at levels, achieved stationarity after taking the first difference. Hence, we conclude that these variables are integrated of order one I(1), it therefore necessary to determine whether there is at least one linear combination of the variables that is I(0).

Table 3. Panel Unit Root (At first difference)

Variables	Levin, Lin & Chu t*	Im, Persaran & Shin W-stat	PP-Fisher Chi-sq	Decision
swf	-	-1.2675	-	I(0)
govexp	-24.2001***	-6.9245***	95.5057***	I(1)
findev	-10.2541***	-2.8408***	70.5942***	I(1)
infl	-	-	-	I(0)
Invt	-4.8562***	-0.4798	33.4226*	I(1)
Pop	-	0.9962	-	I(0)
rgdpc	-4.5106***	-0.6037	38.4059**	I(1)
*** (1%), ** (5%) & *(10%) level of significance				

The result of the Kao (1999) cointegration test, which is a residual-based cointegration technique, is presented in Table 4. Based on the results, the null hypothesis of no cointegration was rejected at 5%

significance level. Therefore, the Kao cointegration test supports the evidence of long-run equilibrium relationship among the variables.

Table 4. Kao Residual Cointegration Test Result

Test	*swf* model
t-statistic	-10.1846
p-value	0.0000***
Note: *** (1%), ** (5%), * (10%)	

Investigating how government size affect economic development in a non-linear framework, the result of quantile regression estimates is presented in Table 5. The least absolute deviation (LAD) regression generates negative and significant coefficient of government size at the 5% level. This shows that a unit percentage point increase in government size will bring about 3.76 units reduction in economic development and social welfare all things being equal. The results of quantile regression estimates show that out of the five quantile estimates of government size conditional on economic development, three (τ = 0.05, 0.25, 0.50) prove to be negative and significant at the 5% level.

Table 5. Quantile Estimates (Dependent variables are *swf*)

Variables	Tau (τ)	Coefficient	t-ratio
Constant	LAD	-2.78053	-0.0191
	0.050	-7.7574	-0.7127
	0.250	-103.758	-1.0047
	0.500	-2.7805	-0.0198
	0.750	-572.138***	-3.3313
	0.950	-1578.21***	-3.3584
Govexp	LAD	-3.7614*	-1.473
	0.050	-5.1738***	-3.6973
	0.250	-2.6704*	-1.5471
	0.500	-3.7614*	-1.4647
	0.750	4.6941**	1.9206
	0.950	-0.1418	-0.0188
Infl	LAD	-1.6044**	-2.710
	0.050	-1.0941**	-1.8366
	0.250	-0.5899**	-2.1099
	0.500	-1.6044**	-2.4566
	0.750	-1.3404*	-1.3849
	0.950	1.9404	0.9098
Invt	LAD	-0.5712	-0.3864
	0.050	2.4131***	3.0864
	0.250	3.0521***	3.2893
	0.500	-0.5713	-0.3938
	0.750	-2.9493**	-1.9209
	0.950	-7.8601**	-1.9762
logpop	LAD	40.5232**	1.833
	0.050	22.5751	1.2778
	0.250	30.7814**	2.1661
	0.500	40.5232**	1.8567
	0.750	114.302***	4.2538
	0.950	304.698***	3.6528
findev	LAD	6.9218**	2.644
	0.050	3.4433***	3.7146
	0.250	5.9181***	4.1950
	0.500	6.9218**	2.7792
	0.750	11.7890***	5.5024
	0.950	11.6973**	2.37995

Moreso, the quantile regression results illustrate that the marginal effect of government size on economic development in ECOWAS countries reduces as one move from 0.05 quantile to 0.25 quantile after which it rises to middle (0.5) quantile of government size variability. For instance, the marginal effect of a unit percentage point rise in government size brings about decrease of 5.17 units at 0.05 quantile, 2.67 units at 0.25 quantile and 3.76 units at 0.5 quantile. This implies that, at lower economic development quantiles, government size exerts a negative effect on economic development and welfare in countries such as Gambia, Guinea Bissau and Sierra Leone. At **τ** = 0.75, the marginal effect of government spending on economic welfare is positive and significant at 5% level in countries such as Ghana and Cote d'Ivoire while its negative and insignificant at **τ** = 0.95 (i.e upper quantile).

This evidence suggests that potential information gains associate with the estimation of the entire conditional distribution of level of economic development of sample countries or group, as opposed to the conditional mean only (such as given by LAD estimates).

Both the traditional literature on structural barriers to development and in the more recent debate on the proper role of government in a market-oriented development strategy, support exists for the contention that increases in government expenditure may spur growth and economic welfare in less developed countries, such as ECOWAS countries. This position is drawn out of growth-limiting characteristics specific to developing countries among which are structural inflexibilities, instance of market failure, and inability to hedge against risks of doing business. The theoretical supposition that follows is that increasing government size will have more positive (or less negative) impact on economic development in poor countries or developing countries. In testing the hypothesis of this study, the quantile regression results show that the effect of government size on economic welfare is significantly different across level of development.

Countries at the lower development distribution of the sample used such as Gambia, Guinea Bissau and Sierra Leone are expected to increase government spending especially in areas of education, health, the environment and infrastructure where markets alone are insufficient. It suffices to add that the need to increase government expenditures in these areas has not been met in all the ECOWAS countries. For instance, health expenditure remains below the 15% of government spending threshold prescribed under the 2001 Abuja Agreement (AEO 2017). Statistics shows that Nigeria spend less than 1% of GDP on health (WEO 2017).

In consonance with some previous studies, this study found the coefficient of the share of government expenditures to be significant, its sign consistently negative (Mudaki and Masaviru 2012; Martins and Velga 2013; Churchill, Yew and Ugur 2015) at low (0.050) to upper-middle (0.750) quantiles. At the upper quantiles of 0.950, the study also found that the coefficient of the share of government expenditures is positive and significant, which is consistent with findings of Komain and Brahmasrene (2007), Alexiou (2009) and Ochieng & Kibet (2014) among others.

This study therefore surmised that the hypothesis that level of economic development does not matter is not supported by these findings, rather it posited that effect of government size on economic development is conditioned on the level of development in ECOWAS countries. Furthermore, the results of the quantiles estimate of the control variables suggest that their relationships with economic development is indeed linear. For instance, financial development and population growth rate have positive effect on economic development across almost all the quantiles of development distributions while inflation rate and domestic investment are consistently negative and statistically significant at 5% level in near all the quantiles.

Conclusions

This study tested the hypothesis that the effect of government size on economic welfare is not significantly different irrespective of the level of development in ECOWAS countries. Using quantile regression, regression estimates showed that the marginal effect of government size on economic welfare varies across level of development. It Therefore, suggests that the relationship between government size and economic development is not linear rather ambiguous, that is it could be positive or/and negative depending on countries' development level. This study concluded that harmful effect (growth benefit) of increasing government spending will be more (less) pronounced among countries with low level of economic development than countries with high level of development. Adopting government spending as policy variable targeted at improving social welfare should be implemented with caution and selectiveness because of efficiency issue arising from weak institutions, especially in developing countries.

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THE THEORETICAL ASPECT OF DELPHI TECHNIQUES AND AHP METHOD

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Abstract: *Various researchers in Iran, Germany, Central Taiwan, Brazil and U.S have selected to apply the combined Delphi technique and AHP method in their studies. These two techniques have been used to rank and identify factors, elements, importance measurement, expert perceptions of a topic, etc.*

The areas where these studies, and we are based on our analysis, are tourism industry and beverage industry but the focus falls on marketing, social media, content marketing, e-commerce and e-business.

The general purpose of this paper is to address in general terms the Delphi technique and AHP method.

Keywords: Delphi technique; AHP method; factors; experts.

JEL Classification: B40; D78; D81.

Introduction

The Delphi technique functions as a process whereby through this technique to the expert, individually, is required to answer the questions asked for the data and subsequently to be submitted to the Principal Administrator which then will be processed and another questionnaire will be created to respondents and this process continues until the intended consensus is reached. This technique has a characteristic in itself, because the selected experts do not know who the other selected experts are (Grisham 2009). The characteristic of the AHP method is that the extraction of the priority scales is based on the judgments of the experts, a judgment which derives by comparison in pairwise (Saaty 2008).

1. Delphi Method

This technique originated in the early 1950s, which Rand Corporation had used as a predictive tool for the army. The main purpose of this technique is to get answers from a group of trusted experts on a particular dilemma or problem (Erffmeyer et al. 1969; Stitt-gohdes and Crews 2004).

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The Delphi method is characterized by four main features:

▪ Anonymity - the research participants are anonymous to each other. They are known only to researchers. Participants have no pressure from other participants regarding their opinion and the responses they give;

▪ Repeatability - provides the experts with the opportunity to change their views based on the findings of the group at a later stage;

▪ Controlled feedback - informs participants about the views of other experts and provides an opportunity to clarify and change their opinion;

▪ Statistical processing / aggregation of group responses allows for quantitative analysis and interpretation of data (Rowe and Wright 2001).

This method is accomplished by three consecutive operations, such as:

- Selecting a group of experts;
- Preparation of the questionnaire,
- Method administration

Individuals who have a global view of the problem and who usually combine multiple sources of information should always be given priority. Functional safety is thought to increase with group size and average errors decrease with decreasing group size. The questionnaires contain a mix of open-ended and closed-ended questions, so they are extensive and require deep reflection and personal judgment by experts. What makes this method more original is the fact that experts review their judgments many times, so we are dealing with an iterative process. It is important to maintain the anonymity of individual judgments: no expert knows exactly how the other responded (Ceku and Kola 2011).

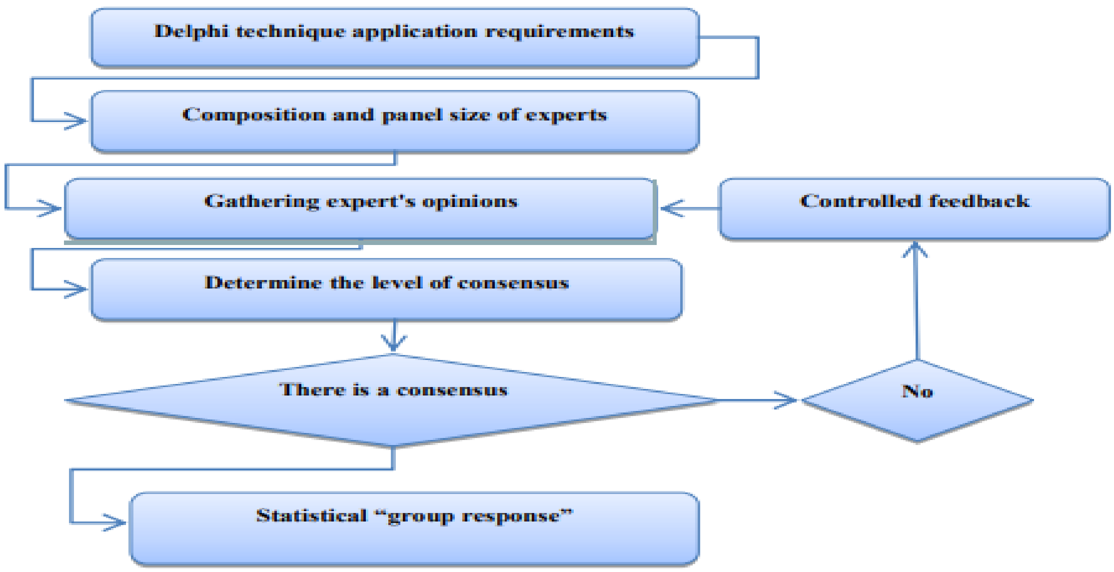
The success of a research applying the Delphi technique is on the careful selection of the expert panel Stitt-gohdes and Crews (2004), it is also thought that judgments made by multiple experts are more accurate than judgments by individual experts (Rowe and Wright, 2001). Qualitative research applies this technique, as we mentioned earlier, which is used to get expert opinion when there is not much information on a particular problem or field. This technique is considered simple in application because it also allows for interaction, but the fact is that this technique takes a long time to apply (Giannarou and Zervas, 2014).

According to Jolson and Rossow (1971) the validity and reliability of the subjective judgment method should be evaluated compared to the available alternatives.

The disadvantage of this technique is the lack of a theoretical framework, but in general a large number of scholars acknowledge that the methodology of this technique is not uniform in application, given the uncertainty about panel size, panel selection. and Delphi stages (Habibi *et al.* 2014).

Also, in his study, Habibi *et al.* (2014) presented a framework for applying the Delphi technique to qualitative decision making, which will presented in Figure 1:

Figure 1 Theoretical framework of Delphi technique in qualitative research

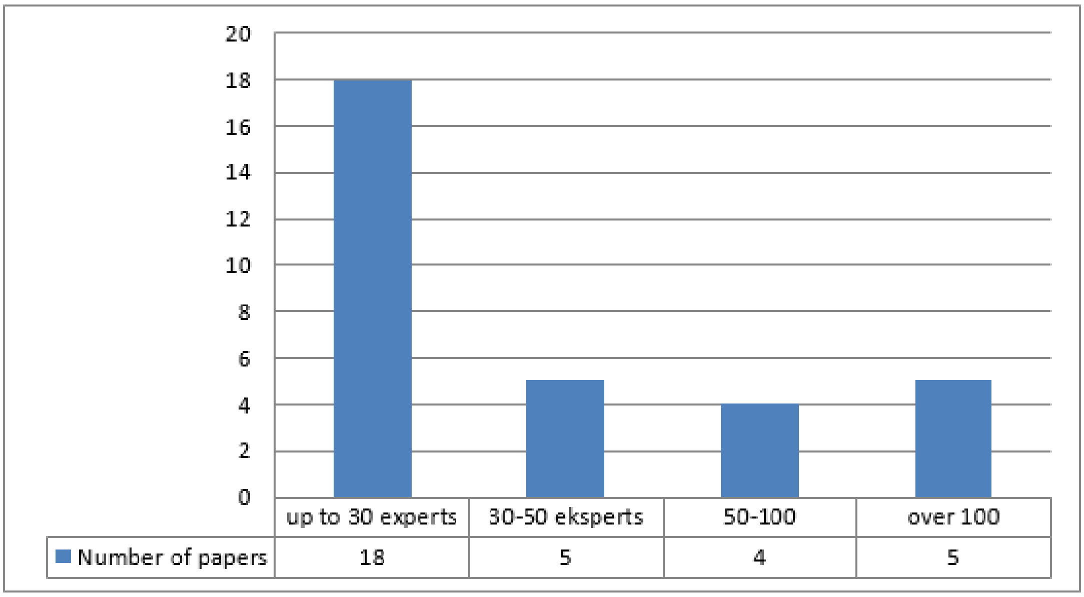


Source: Habibi *et al*. 2014

This framework clearly explains the application and principles of this technique to qualitative research.

In their research Giannarou and Zervas (2014) analyzed the previous 32 studies that applied the Delphi method to elaborate the differences between the number of participants, the number of rounds, and the Likert scale. The following is a summary of the general statistics in Figures 2, 3 and 4, which we derived from this empirical study, with the very fact that there is uncertainty as to panel size, panel selection, and stages in the Delphi technique.

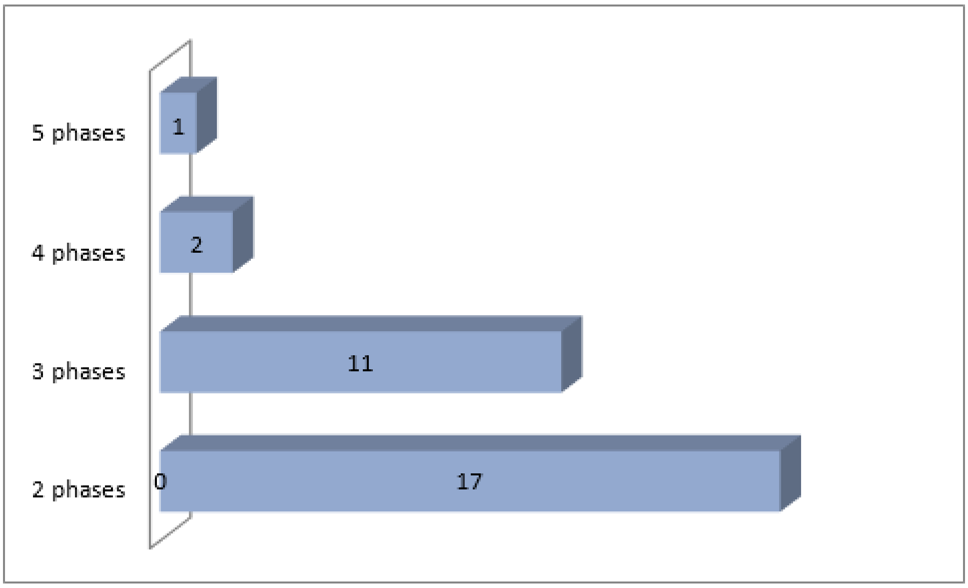
Figure 2. Results of empirical research with Delphi technique for number of experts



Source: Adopted according to the author based to (Giannarou & Zervas, 2014)

From this summary of 32 researches it can be seen that 18 papers have selected no more than 30 experts, 5 papers 30-50 experts, 4 papers 50-100 experts and 5 others have made part of the study of over 100 experts.

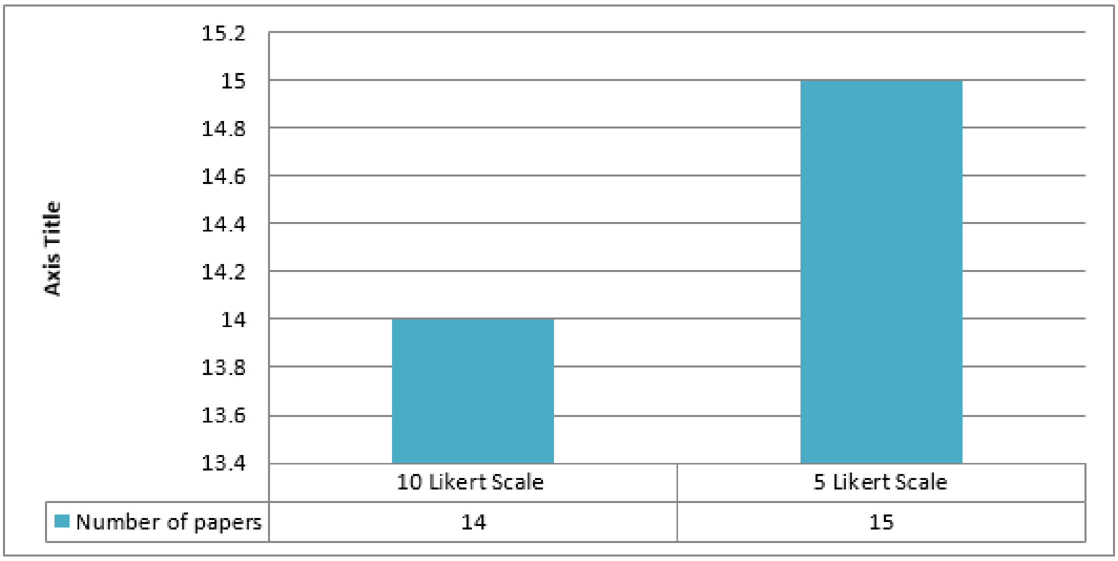
Figure 3. Results of empirical research with Delphi technique for number of research stages



Source: Adopted according to the author based to Giannarou and Zervas (2014)

Regarding the phases of the research, a large percentage have done the research in 2 and 3 phases, and a smaller percentage in the phases 4 and 5.

Figure 4. Results from empirical research with the Delphi technique for Likert scales



Source: Adopted according to the author based to Giannarou and Zervas (2014)

In all the researched works, following the results from the first phase, the questionnaires for the other stages were formulated according to the Likert scale, where 10 and 5 scales were used. The Delphi method functions as a process and is composed of several stages of research, a fact that increases the importance of this technique to be applied by many researchers (researchers in different fields).

The following figure illustrates the process of the research stages according to the Delphi method:

Figure 5. The Delphi method process in the general context

Source: Adopted according to the author and based to Hsu and Sandford (2012)

Researchers, in the first phase of creating the questionnaires, may use an alternative method by directly administering a structured questionnaire. The design of the questionnaire may be based on individual interviews from research conducted or extensive literature review of the topic of interest. In the second phase, subjects are required to review, analyze, and rank information extracted from the first phase of the research. Also, in the third phase they receive a third questionnaire where they are asked to review their previous evaluations to give justifications and rank the statement points (Hsu and Sandford 2012).

2. AHP Method

According to Saaty and Vargas, (2012) AHP is composed by seven pillars:

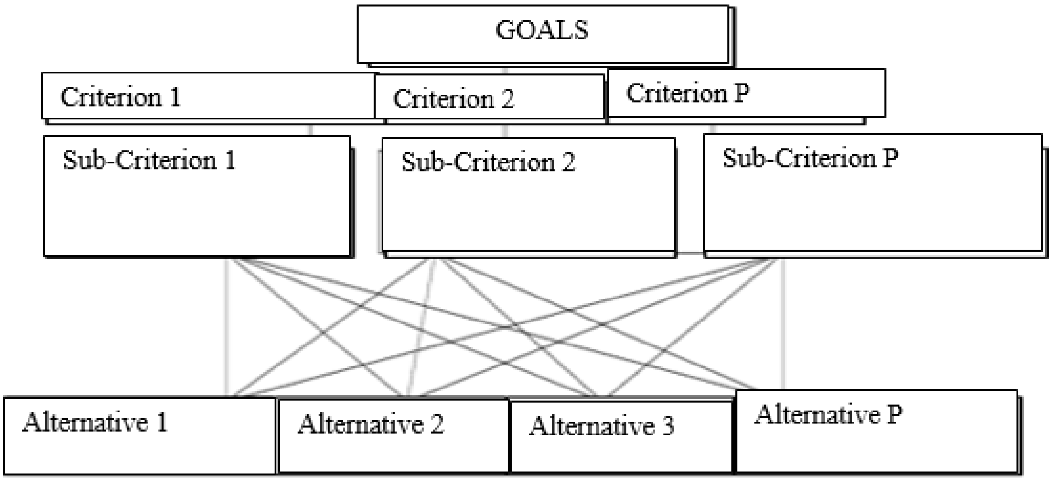
1.Calculation of the report, proportionality and scale normalized ratio
2.Mutual paired comparisons
3.Direct self-vector sensitivity
4.Homogeneity and aggregation,
5.Dependent synthesis and reactions,
6.Maintaining and changing rank,
7.Group decisions (T. Saaty and Vargas 2012) .

The following explains the AHP methodology in six steps:

The first step of this method - The problem is analyzed into a hierarchical structure where at the top level the main goals are set, at the second level the criteria are set and at the third level the sub-criteria, if any, and alternatives at the lower level. This is the most creative and important part of decision making (Bhushan and Rai 2004).

This general hierarchical structure is illustrated in Figure 6 by the AHP method.

Figure 6. Overall hierarchical structure of the AHP method



Sources: Bhushan and Rai (2004)

Describes the relationship between the elements of one level and the elements of another level which is below it. So each element is indirectly related to the other (Bhushan and Rai 2004).

The second step of this method is to determine the relative importance of different attributes / criteria in relation to goals. The basic scale of the AHP method is shown in the table 1.

A scale that determines how many times one element is more important or dominant over another than the criterion or property with which they are compared. This scale has the properties of the ratio scale which is composed of 9 points.

If attribute A is evaluated as absolutely more important than the other attribute B where it is evaluated with 9, it will turn out that attribute B will be absolutely less important than attribute A where it will be evaluated with 1 (Mousavi 2012).

Step Three builds a set of two-sided comparative matrices as ratios (Shrestha *et al.* 2004) .

In the step four the relative weight of the various criteria and sub criteria that are also compared to the evaluations of the alternatives is determined, an importance which is defined through the main eigenvalue (Bhushan and Rai 2004; Hamidi 2016) .

While in the fifth step assess the order of the order matrix n. In this method the comparisons made are subjective so AHP tolerates discrepancies through the amount of redundancy in access (Bhushan and Rai 2004; Hamidi 2016).

Table 1. Rate report

Intensity of Importance	Definition	Explanation
1	Equal Importance	Two activities contribute equally to the objective
2	Weak	
3	Moderate importance	Experience and judgment slightly favor one activity over another
4	Moderate plus	
5	Strong importance	Experience and judgment strongly favor one activity over another
6	Strong plus	
7	Very strong or demonstrated importance	An activity is favored very strongly over another; its dominance demonstrated in practice
8	Very, very strong	
9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation
Reciprocals of above	If activity *i* has one of the above nonzero numbers assigned to it when compared with activity *j*, then *j* has the reciprocal value when compared with *i*	A reasonable assumption
Rationals	Ratios arising from the scale	If consistency were to be forced by obtaining *n* numerical values to span the matrix

Source: Saaty and Vargas (2000)

In step six the evaluation of each alternative is multiplied by the weights of the sub-criteria and collected to obtain local estimates for each criterion. Then, local estimates are multiplied by the weight of the criteria and collected to get global estimates (Bhushan and Rai 2004, Hamidi 2016).

3. Review of the Literature on the Application of the Delphi Technique and the AHP Method

In his research Mousavi (2012), applied a combination of the Delphi technique and the AHP method, selecting 30 experts to identify the influencing elements that influence the e-marketing strategy in tourism, which extracted many variables which were then summarized into three factors, in third stage, to make the ranking.

The AHP method and the Delphi method have been used to evaluate the strategy formulation for the nine recreational areas in central Taiwan. This research provides information on competitiveness assessment in different recreational areas (Lee and Liu 2011). In their studies Chen and Wang (2010), have subsequently selected AHP and Delphi for proposing a comprehensive framework, with specific business elements, highlighting the six performance indicators in adapting the business strategy.

The Delphi method has been selected by researchers (Palazzo and Vollero 2015), in order to create an overall picture of content marketing.

A triple-blind study, based on the Delphi method, was conducted to identify and evaluate factors related to the use and interventions in internet (Brouwer *et al.* 2008).

In his study Pagani (2009) included 40 leaders in the US and Europe, aimed at tackling the uncertainties arising in the wireless 3G industries. The findings have identified the underlying trends and useful uncertainties in the development of corporate or business strategy. Also, in order to identify and evaluate the factors that influence the skills gap in the field of digital marketing, in their study Ghotbifar *et al.* (2017), as the most appropriate, have selected the Delphi method. In their research Bakhshimazdeh and Alikhasi (2015), have selected the Delphi and AHP method to identify and rank strategic and organizational factors. Also Varini K., (2006), to see how experts perceive the impact of ICT on hotels to maximize the contribution achieved, specifically, from the sale of their rooms have applied the Delphi method. Their study, since it was among the most important motivational factors that influence the selection of commercial sites by consumers (users) to buy online, Delphi method has been used as most appropriate for this nature of the study (Klaiber *et al.* 2015).

To reach consensus on the proposed objectives, strategies and key performance indicators in their research (Helmink 2013), they conducted the research in two rounds using the Delhpi method. Whereas, Safa *et al*. (2013), to identify the main problems faced by horticultural products during the implementation of e-commerce, conducted the study in four rounds according to the Delphi method by panel members.

Likewise, to design an e-marketing framework in the beverage industry, Maniyan *et al.* (2015) have used the Delphi method, selecting populations from faculty members and managers working in the beverage industry. beverages in Iran. From this study it emerges that the main criteria in implementing e-marketing are business strategy, e-marketing planning, and consumer interpretation.

In their study Serra *et al.* (2013) using the Delphi method, interviewed 12 specialists to understand how firms in Brazil in 2015 will use social media to achieve their goals. Also, in their paper Mejía Trejo (2017), applied the Delphi and AHP method over a 4-month period and included 200 business experts to determine the key variables of e-Business Innovation (eBusiness) for SMEs, since 2016.

Conclusion

There are four features of Delphi method characteristic: participant anonymity, repeatability, controlled responses and statistical processing / collection. This method is also carried out through three operations by selecting a group of experts, preparing the questionnaire and administering the method. Experts have the opportunity to review their opinions many times through the iterative process.

The AHP method is considered to be one of the most important decision-making methods by which the report rates from paired comparisons are derived. The ratio scale is derived from the main Eigen vector and the sustainability index is also derived from this vector. This scale is made up of 9 points by which it is more important to determine an element than another element in relation to the criterion to which it is compared.

Based on empirical research we conclude that Delphi technique, together with AHP method, is applied in combination when researchers aim at: identifying specific elements and strategic factors, measuring the importance of factors, evaluating strategy formulation, observing expert perceptions of a topic and reaching consensus on objectives, strategies and performance indicators, etc.

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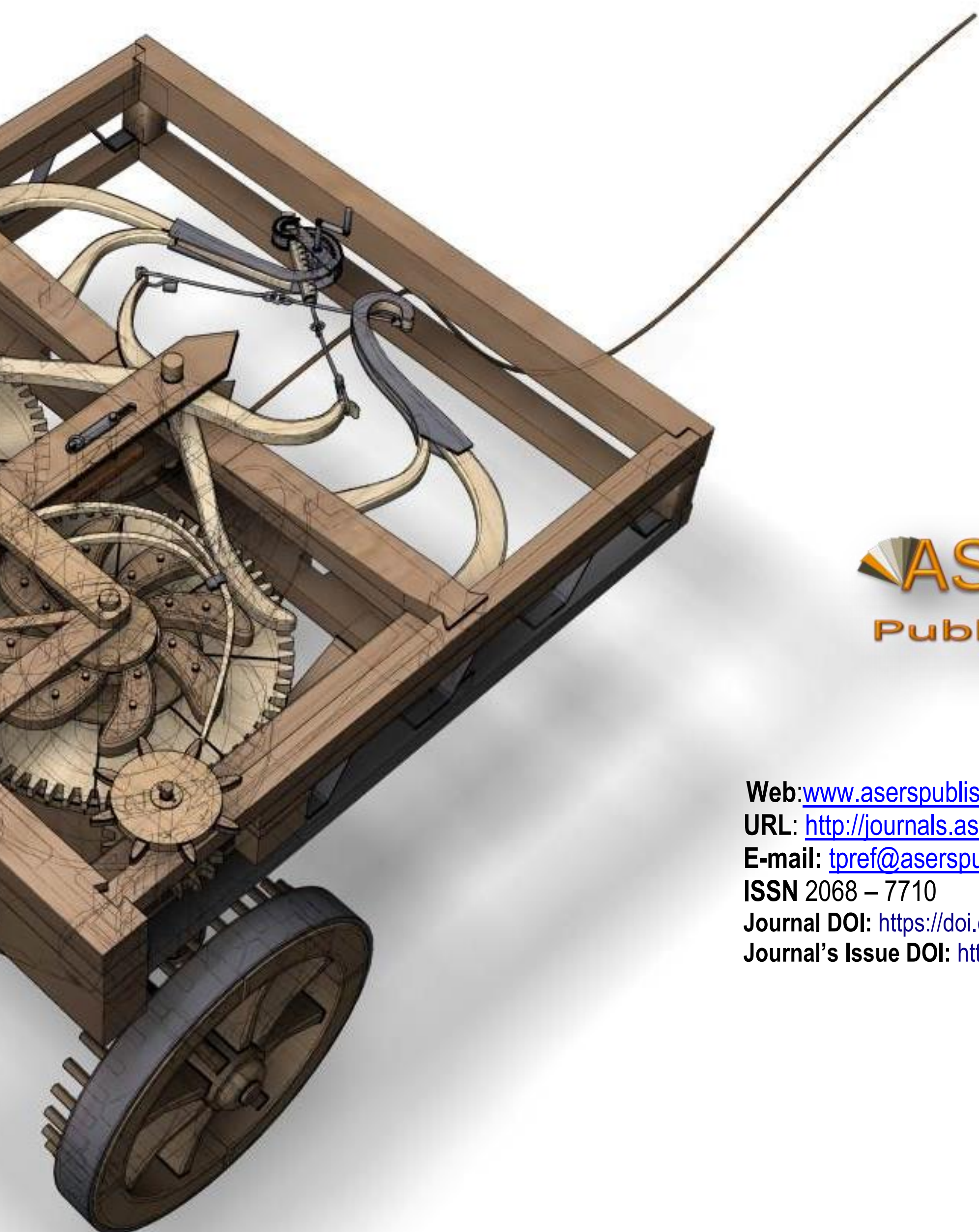

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