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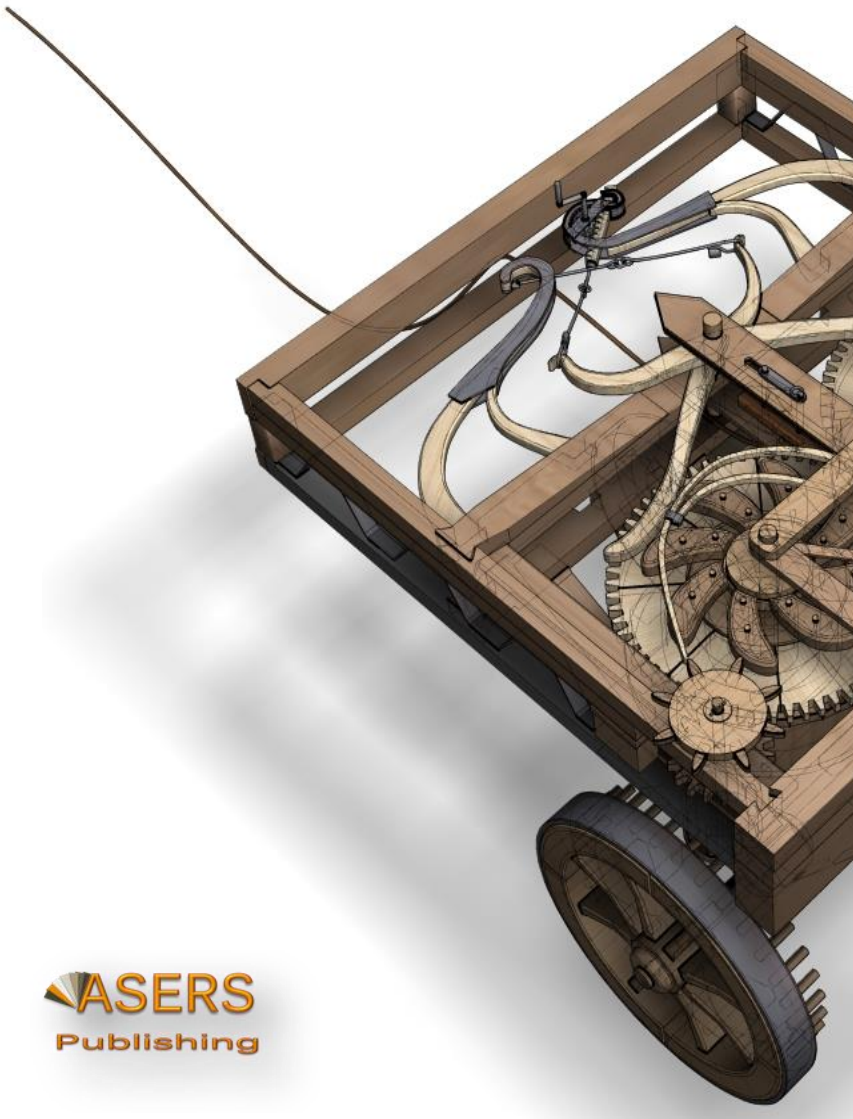

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OIL PRICE SHOCK AND ECONOMIC GROWTH: EXPERIENCE OF CEMAC COUNTRIES

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Abstract:

The objective of this paper is to evaluate the impact of oil shocks on the growth rate of Growth Domestic Product (GDP) in CEMAC countries. We use a panel VAR model approach to the variation of the real GDP growth rate, oil price inflation rate and money supply between 2000 and 2015. Our main results show that CEMAC countries mostly depend on oil pension. Consequently, the analysis of impulsion response functions and the decomposition of variance show that, the shock on oil price negatively affects the growth rate of the GDP. We then suggest CEMAC countries to diversify their production, the destination of their exports and the sources of budgetary income or takings.

Keywords: oil shock; GDP; Panel VAR

JEL Classification : C23; F2; F43; F45.

1. Introduction

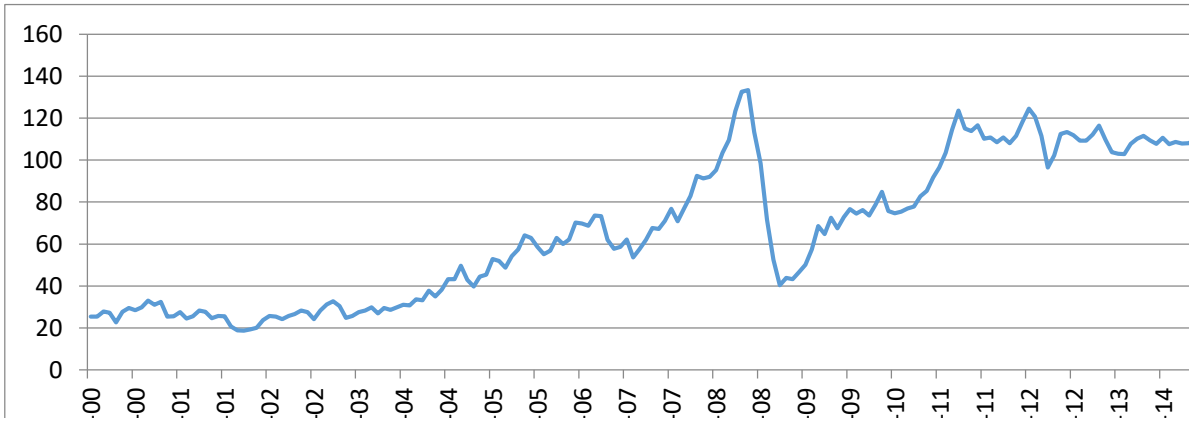
Economic activity in sub-Saharan Africa has slowed considerably for more than 20 years. It should be noted that this global analysis is marked by strong heterogeneity which appears from one country to another. Economic growth in the region fell to 3½ per cent in 2015, the lowest level in a decade.

Since 2014, the Central African Economic and Monetary Community (CEMAC) faces at the same time several types of shocks: securities (terrorist threats), politics (political crisis) and falling prices of natural resources. Concerning the persistent fall in the cost of natural resources, principally oil, started since June 2014, the cost of barrel passed from more than 100 US $, to less than 50 US $. Consequently, the economic growth curve in the zone follows a fall starting from 4.8 % in 2014 to 2.4 % in 2015, and in 2016, it is forecast at 1%.

2. Impact of falling oil prices on the activity of the CEMAC countries

The relation between oil price variables and the principal macroeconomic indicators was already the subject of numerous theoretical and empirical studies (Hamilton 1983, 1988, 1996, 2003, Rasche and Tatom 1981, Mork 1989, Hooker 1996). This dynamic interest at the same time academicians, policy decision-makers, the actors of finance and of the civil society since the first crisis of oil triggered in 1970. So, a series of dramatic events in the 1970s sent the price of crude oil over $40 a barrel by the end of that decade, which would be over

$100 a barrel at current prices. The price remained very volatile after the collapse in the 1980s but was still as low as $20 a barrel at the end of 2001 (Hamilton 2009). After 2005, the barrel price remained above $60 despite the strong volatility. But since the fall in August 2014, the barrel price dropped below $60 in March 2015 and is maintained until now. The consequences of this strong decrease are dynamic for the exporting countries, in particular those of the CEMAC zone.



Source: INSEE

Figure 1. Evolution of barrel price of January 1990

This real decrease in barrel price influence the decisions of budgetary and monetary policies in function of the weight of the oil returns in the gross domestic product and the budgetary returns. It is in this light that Copinschi (2015) brings out the weight of the oil rent in the Gross domestic product and the budgetary returns of CEMAC countries. Thus, in Cameroon, it is observed that the returns from oil represents 10% of Gross Domestic Product, 20% of the budget and represent 50% of export returns for a production of 75 000 barrel/d. In Congo, oil returns represents 50% of Gross Domestic Product, 75% of budgetary returns and 80% of export returns for a production of 281 000 barrel/d. In Gabon, oil returns represent 45% of Gross Domestic Product, 50% of budgetary returns and 70% of export returns for a production of 236 000 barrel/d. In Equatorial Guinea, oil returns represent 85% of Gross National Product, 85% of budgetary returns and 90% of export returns for a production of 281 000 barrel/d.

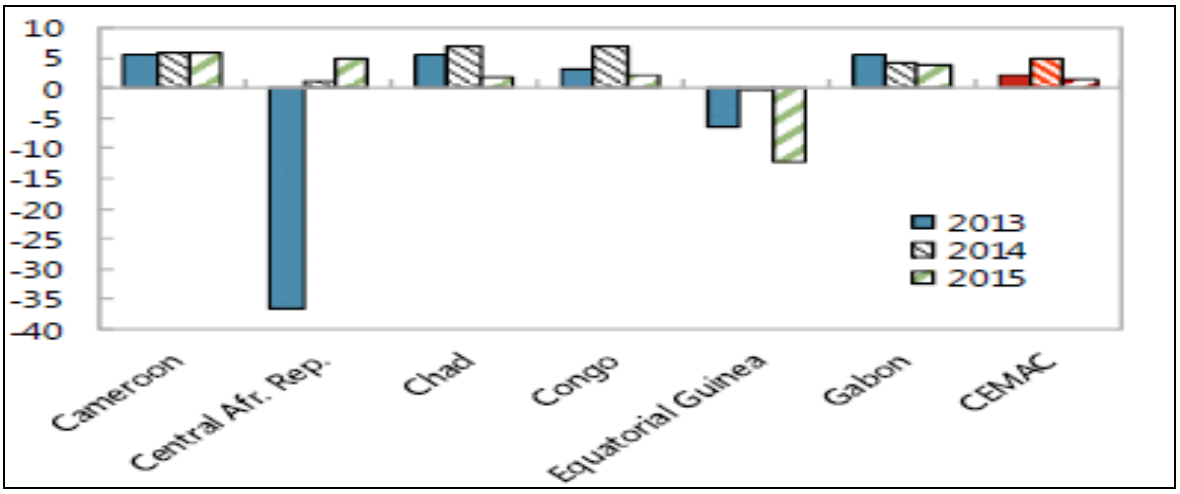
Table 1. Relative size of economies and importance of oil sector, 2015

	Country GDP/CEMAC's GDP	Country oil GDP/CEMAC's GDP	Country oil GDP/ country GDP	Country fiscal oil revenue/ country fiscal revenue	Country oil exports/country merchandise exports	Country external trade balance/ country GDP
Cameroon	38.7	1.8	4.7	14.1	38.5	-3.8
Central Africa Republic	2.2	0.0	0.0	0.0	0.0	-22.3
Chad	14.8	3.0	20.0	34.5	78.0	-15.5
Congo, Republic of	12.2	4.9	40.5	37.8	74.4	-14.6
Equatorial Guinea	12.8	3.8	30.0	81.6	81.4	41.7
Gabon	19.4	6.2	31.8	33.6	76.5	5.5
CEMAC	100	19.6	19.6	39.0	70.6	2.1

Source: IMF Country Report N 16/290

Thus, in the IMF Country Report N 16//277, it is shown that CEMAC growth was subdued in 2015. It slowed to 1.6 percent, from 4.9 percent in 2014, because of reduced public investment and lower oil production. Growth is projected to be 1.9 percent in 2016, as oil production and investment remain sluggish. From 2017 onward, growth is expected to reach 3½ percent a year, as oil prices gradually recover, some one percentage point below the average growth level of the past decade of high oil prices. Growth of money and credit to the economy turned negative in 2015 for the first time in a decade, contributing to keeping inflation low. The regional fiscal and current account deficits grew to 6 and 9 percent of GDP in 2015, respectively, as oil export

proceeds fell by 32 percent. Continued low oil prices and high public expenditure will contribute to maintaining both deficits at about 6 and 8 percent of GDP in 2016, respectively.



Source: IMF country Report n 16/277

Figure 2. Real GDP Growth, 2013-2015

Faced with the fall in oil returns, all countries of the region have, in the course of the year 2015, strongly reduce their public expenditures on investment, what aggravates the slowing effect of the economy by impacting the non-oil activity sectors but of which the financing greatly depends on oil returns (construction, etc.). Gabon and Congo has announced the important adjustments in the public expenditures and Cameroon has to follow. But it is in Equatorial Guinea that the recadrage is most severe: the amount of public investments for the year 2015 will experience a fall of close to 60% with respect to the previous year. Besides, the fall in foreign investments in the oil sector of these countries will equally have a negative impact on the growth of this year.

In the same context, Besso and Chameni (2016) show that the CEMAC countries are highly exposed to exogenous trade shocks. The consequences of this situation are noted in the report of the International Monetary Fund on the regional economic outlook on Africa (October 2016). The report notes that commodity-exporting countries face serious economic tensions because of the fall in prices of these commodities on international markets. Thus, according to Table 2, the analysis of the instability of individual countries reveals several discrepancies. In the case of Cameroon, there was a reduction in instability from 7.53 between 1980 and 1990 to 0.82 between 2001 and 2010. The poor less performance was achieved by Chad and the Central African Republic

Table 1. Indicator of instability in the economies of the CEMAC zone.

	GDP Growth			Agricultural production			Export		
	1981 to 1990	1991 to 2000	2001 to 2010	1981 to 1990	1991 to 2000	2001 to 2010	1981 to 1990	1991 to 2000	2001 to 2010
Cameroon	7.53	3.90	0.82	0.09	0.10		0.17	0.09	0.15
Central Africa Republic	5.45	4.43	4.02	0.13	0.10	0.14	0.15	0.08	0.03
Chad	9.33	7.64	9.81	0.19	0.22	0.35	0.33	0.22	0.35
Congo, Republic of	9.40	3.72	3.08	0.06	0.07	0.04	0.27	0.17	0.19
Gabon	8.01	4.59	3.14	0.06	0.15	0.05	0.17	0.17	0.26
CEMAC	6,66	3,65	2,26	0,07	0,10	0,05	0,17	0,11	0,16

Source: Besso and Chameni (2016)

Regarding the instability of agricultural production, Congo obtained the best indicators. Gabon is performing well in the last decade, despite the high risks in previous decades. As for Chad and the Central

Africa republic, they consolidate their places in the most unstable countries of the zone.

The last indicator in this table is closely related to international trade, which is the instability of exports. At this level, while the Central Africa Republic, achieves the best performance in terms of stability, Chad consolidates its position as the most unstable country in the FCFA zone. Nevertheless, it is important to note that for this indicator, the countries of the CEMAC zone represented in this study present the signs of high instability.

Considering this background, we study the macroeconomic dynamics between economic output growth, domestic price level, money supply and oil price over a set of CEMAC countries. To evaluate the relative importance of these variables in the movements of other variables in both short and long run, Impulse Response Functions (IRFs) and Forecast Error Variance Decompositions (FEVDs) are used.

3. Literature review

3.1. World

Many authors concentrated on analyzing the oil price-macroeconomic relationship (Hamilton 1983, 1988, 1996, 2003, Rasche and Tatom 1981, Mork 1989, Hooker 1996). The main results of the paper may be summarized as follows. Firstly, the linear (symmetric) oil price specification reveals that changes in oil price stimulate GDP growth in the short term, but cause GDP to decline in the long term. Secondly, for non-linear (asymmetric) specifications, positive oil price shocks cause GDP to decline in the long term without experiencing growth in the short term. Another interesting finding is the response of output growth to negative (decreasing) oil price changes. Using negative oil price shock measures, GDP responds negatively in the short term, but eventually recovers although responses in the long term are not statistically significant (Aziz and Dahala 2015, Basnet and Upadhyaya 2015).

Ozturk (2015) analyzes the impact of oil price shocks on the selected macroeconomic variables in Turkey for the period of 1990Q1-2011Q4. Vector Auto regression (VAR) models and bivariate Granger causality tests are applied to determine the oil price shocks - macroeconomic relationship. The empirical findings shows that both symmetric and positive oil price shocks decrease industrial production, money supply, and imports while the negative oil price shocks increase imports.

Baumeister and Peersman (2013). Using time-varying BVARs, we find a substantial decline in the short run price elasticity of oil demand since the mid-1980s. This finding helps explain why an oil production shortfall of the same magnitude is associated with a stronger response of oil prices and more severe macroeconomic consequences over time, while a similar oil price increase is associated with smaller output effects. Oil supply shocks also account for a smaller fraction of real oil price variability in more recent periods, in contrast to oil demand shocks. The overall effects of oil supply disruptions on the US economy have, however, been modest.

Lutz (2008). A comparison of the effects of exogenous shocks to global crude oil production on seven major industrialized economies suggests a fair degree of similarity in the real growth responses. An exogenous oil supply disruption typically causes a temporary reduction in real GDP growth that is concentrated in the second year after the shock. Inflation responses are more varied. The median CPI inflation response peaks after three to four quarters. Exogenous oil supply disruptions need not generate sustained inflation or stagflation. Typical responses include a fall in the real wage, higher short-term interest rates, and a depreciating currency with respect to the dollar. Despite many qualitative similarities, there is strong statistical evidence that the responses to exogenous oil supply disruptions differ across G7 countries.

Aziz and Dahalan (2015) investigates the asymmetric effects of oil price shocks on real economic activities in ASEAN-5 from 1991 to 2014 using an unrestricted panel Vector Auto Regressive (VAR) method. Results from the impulse response function (IRFs) shows evidence of an asymmetric relationship between oil prices and economic activities. Specifically, positive oil price shock measures negatively affect output growth both in the short term and in the long term. For oil price decrease specifications, real output responds negatively in the short term before recovering to its pre-shock level in the long term. The variance decomposition analysis (VDCs) also exhibit differences between the effects of positive and negative oil price shocks on economic activities, supporting the evidence of asymmetric relationship obtained in the IRFs simulations.

Brémond *et al.* (2014) study the relations between the price of oil and a large dataset of commodity prices, relying on panel data settings. Using second generation panel co-integration tests, our findings show that the WTI and commodity prices are not linked in the long term. Nevertheless, considering our results in causality tests, they show that short-run relations exist, mainly from the price of crude oil to commodity prices. We thus implement a Panel VAR estimation with an impulse response function analysis. Two main conclusions emerge: (i) fast co-movements are highlighted, while (ii) market efficiency is emphasized.

Blanchard and Gali (2007) characterize the macroeconomic performance of a set of industrialized economies in the aftermath of the oil price shocks of the 1970s and of the last decade, focusing on the differences across episodes. We examine four different hypotheses for the mild effects on inflation and economic activity of the recent increase in the price of oil: (a) good luck (i.e. lack of concurrent adverse shocks), (b) smaller share of oil in production, (c) more flexible labour markets, and (d) improvements in monetary policy. We conclude that all four have played an important role.

Cologni and Manera (2009) using a Markov-switching analysis for the G-7 countries show that positive oil price changes, net oil price increases and oil price volatility tend to have a greater impact on output growth. Moreover, their analysis suggests that the role of oil shocks in explaining recessionary episodes have decreased over time. Finally, they conclude that oil shocks tend to be asymmetric.

Hamilton (2008) explores similarities and differences between the run-up of oil prices in 2007–08 and earlier oil price shocks, looking at what caused these price increases and what effects they had on the economy. Whereas previous oil price shocks were primarily caused by physical disruptions of supply, the price run-up of 2007–08 was caused by strong demand confronting stagnating world production. Although the causes were different, the consequences for the economy appear to have been similar to those observed in earlier episodes, with significant effects on consumption spending and purchases of domestic automobiles in particular. Absent those declines, it is unlikely that the period 2007Q4–2008Q3 would have been characterized as one of recession for the United States. This episode should thus be added to the list of U.S. recessions to which oil prices appear to have made a material contribution.

Mehrara and Mohaghegh (2011) study the macroeconomic dynamics in oil exporting countries using Panel VAR approach. On the basis of Impulse Response and Variance Decompositions analysis in a system included economic output, money supply, price index and oil price, we found that: (1) oil shocks are not necessarily inflationary; (2) money is not neutral in these countries; (3) money is the main cause of macroeconomic fluctuations; (4) oil shocks significantly affect economic output and money supply; (5) though oil price is highly driven by its own shocks, domestic shocks, particularly output and money shocks, can sizably affect oil price in the world market.

3.2. Africa

Nchor *et al.* (2016) analysis effect of oil price shocks on the Ghanian economy. This is achieved through the use of Vector Autoregressive (VAR) and Vector Error Correction (VECM) models. The variables considered in the study include: real oil price, real government expenditure, real industry value added, real imports, inflation and the real effective exchange rate. The study points out the asymmetric effects of oil price shocks; for instance, positive as well as negative oil price shocks on the macroeconomic variables used. The empirical findings of this study suggest that both linear and nonlinear oil price shocks have adverse impact on macroeconomic variables in Ghana. Positive oil price shocks are stronger than negative shocks with respect to government expenditure, inflation and the real effective exchange rate. Industry value added and imports have stronger responses to negative oil price shocks.

Sanchez (2011) analyzed the welfare effects of rising oil prices in oil-importing countries using dynamic Computable General Equilibrium (CGE) model on six oil-importing counties (Bangladesh, El Salvador, Kenya, Nicaragua, Tanzania, and Thailand) for the period 1990–2008. He argues that oil price rise has significant adverse impact on GDP with an average annual GDP loss varying from 0.1% for Tanzania to 20% for Kenya.

Akinleye and Ekpo (2013) examine the macroeconomic implications of symmetric and asymmetric oil price and oil revenue shocks in Nigeria, using the vector autoregressive (var) estimation technique. The paper finds that both positive and negative oil price shocks influence real government expenditure only in the long run rather than in the short run, while examining positive and negative shocks to external reserves revealed stronger implications for expenditure in the long run, with positive rather than negative oil price shocks having stronger short and long run effects on real GDP, and therefore triggering inflationary pressure and domestic currency depreciation as importation rises.

Apere and Ijomah (2013) investigates the time-series relationship on the impact of oil price volatility on macroeconomic activity in Nigeria using exponential generalized autoregressive conditional heteroskedasticity (EGARCH), impulse response function and lag-augmented VAR (LA-VAR) models. We found evidence that there is a unidirectional relationship exists between the interest rate, exchange rate and oil prices, with the direction from oil prices to both exchange rate and the interest rate. However, a significant relationship between oil prices and real GDP was not found.

Berument *et al.* (2010) in a study on Middle East and North African countries found the asymmetric effects of world oil price shocks on the GDP of Algeria, Iraq, Jordan, Kuwait, Oman, Qatar, Syria, Tunisia, and UAE to be positive and statistically significant, while positive but insignificant results were reported for Bahrain, Egypt, Lebanon, Morocco and Yemen.

4. Methodologies

To investigate the sources of macroeconomics fluctuations in CEMAC countries, specifically in Cameroon, Chad, Congo Republic, Gabon, and Equatorial Guinea, with Panel VAR model.

Times series Vector Auto-Regression (VAR) models originate in the macro econometrics literature as an alternative to multivariate simultaneous equation models (Sims 1980). In VAR models all variables are treated as endogenous and interdependent, both in a dynamic and in static sense, although in some relevant cases, exogenous variables could be included (Canova and Ciccarelli 2013). Panel VAR have the same structure as VAR models, in the sense that all variable are assumed to be endogenous and interdependent, but cross sectional dimension is added to the representation (Canova and Ciccarelli 2013).

Panel VAR have been used to address a variety of issues of interest to applied macroeconomists and policymakers. Bremond and al. (2014) studies the link between oil and Commodity prices with a panel VAR approach, Mehrara and Mohaghgeh (2011) studies the macroeconomic dynamics in oil exporting countries using Panel VAR approach.

4.1. Data

This paper uses four macroeconomics variables including real GDP growth, real oil prices, consumer index, and Supply money. Annually data from 2000 to 2015 is used for the CEMAC countries. All data gathered from World Development indicator (WDI) database, but oil price of INSEE (Institut National de la Statistique et des Etudes Economiques) in France.

4.2. Model specification

To explore the importance of heterogeneities, dynamics, and simultaneous determination of oil price, real GDP growth, consumer index, Money Supply. We begin with the following baseline panel autoregressive distributed lag (ARDL) specification (Bremond *et al.* 2014):

$$Y_{i,t} = \alpha_i + A(L)Y_{i,t} + \varepsilon_{i,t} \quad (1)$$

where i indicate the country, t runs from 1 to T, Y_{it} is the vector of endogenous variables, $\varepsilon_{i,t}$ is the vector of errors terms, $\alpha_i$is the country-group specific intercept matrix, and $A(L)$ is the matrix polynomial in the lag operator. The estimation is by generalized method of moment (GMM).

$$(Y_{it} = GGDP_{it}, oil\ prince_{it}, INFL_{it}, M2_{it})' \quad (2)$$

where I denote the individual dimension composed by CEMAC country, and t=2000, …2015 the time. $GGDP_{it}$, oil $price_{it}$, $INFL_{it}$, and $M2_{it}$, denote real GDP growth, oil price, consumer index, Money Supply.

5. Results

5.1. Data description, Unit Root, stability tests and cointegration test

5.1.1 Unit Root

Annexes 1 provide the unit root regression results of the variables entered in the model. We find that oil price, gross domestic product growth and money supply are stationary in first difference. Inflation is stationary at level.

5.1.2 Stability test

All the eigen values lie inside the unit circle, Panel Var satisfies stability condition.

Table 2 Stability test

Eigenvalue Real	Imaginary	Modulus
.8506035	**-.2951372**	**.9003512**
.8506035	**.2951372**	**.9003512**
.4237051	**0**	**.4237051**
.0265475	**0**	**.0265475**

5.2. Panel VAR estimation

From our estimations, the GDP growth rate is negatively influenced by the price of oil and positively influenced by the level of inflation and money supply. Inflation is positively influenced by the price of oil and negatively by the money supply.

5.2.1. Impulse response function (IRF) Analysis

It is brought out of the functional analysis of impulse response that, the shocks on the prix of oil, on inflation and on money supply weakly contribute to fluctuations of the GDP growth rate. But, the shocks on the price of oil contributes more than the others. It is the main macroeconomic variable which influences the fluctuations of the GDP growth rate. From observations, the growth rate of NDP reacts to the shock as from the first periods. Later on, it starts stabilizing after the 5th period.

It should also be noticed that, the shock in oil price strongly contributes to the fluctuation of money supply. This is currently observed in the strong reduction of liquidity in the BEAC zone. Inflation is also influenced by the fluctuations of oil price but slightly less than the money supply.

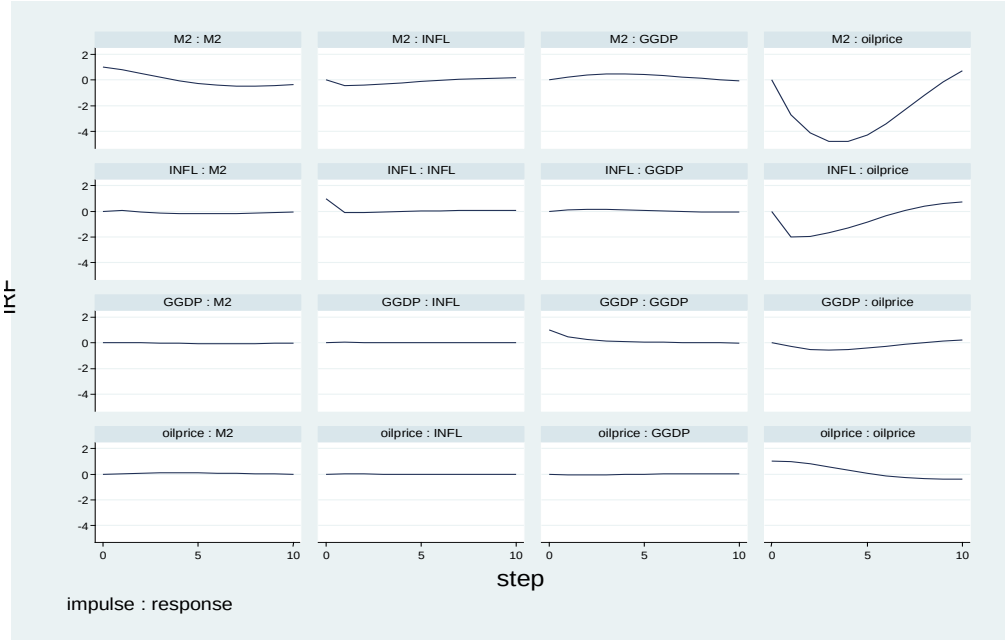


Figure 1. Impulse Response Function (IRF)

5.2.2. Variance Decompositions

The analysis of the Variance Decompositions shows that at the first period, the fluctuation of the GDP growth rate do not depend only on the lag value of this growth rate and of the oil price. The other factors contribute to the fluctuations in growth rate as from the 2th period. For the rest of the periods, the contribution sum of inflation rates and money supply remains inferior to the contribution of oil price.

Conclusion

It comes from our previous analysis that, the CEMAC countries greatly depend on oil rent. In 2015, oil rent represented 4,7% of the GDP of Cameroon, 20% of the GDP of Chad, 40% of the GDP of Congo, 30% of the GDP of Equatorial Guinea and 31,8% of the GDP of Gabon. In a general manner, the oil rent represents 19,6% of the GDP of the CEMAC zone. Functional analysis of impulsional response and of the decomposition of the variance shows that, the shock on oil price, negatively affects the GDP growth rate. And this shock affects even more inflation and money supply. Moreover, the Variance Decompositions shows that, the shock on the oil price contributes more to fluctuations of the Gross Domestic Product than the inflation rate and money supply. In terms of policy recommendations, we then suggest, (i) to put in place a mechanism of sharing risk towards the exogenous shocks within CEMAC, (ii) reducing the dependency on the exportation of raw material, and densifying the intra CEMAC trade, (iii) diversifying the productions and the destinations of exportations, (iv) diversifying the sources of government revenue, (v) reducing heterogeneities in order to render monetary policies more efficient.

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Annexes

Annex 1: Unit root test

Panel unit root test: Summary				
Series: D(GGDP)				
Date: 12/20/16 Time: 15:01				
Sample: 2000 2015				
Exogenous variables: Individualeffects, individuallinear trends				
User-specifiedlags: 1				
Newey-West automatic bandwidth selection and Bartlett kernel				
Balanced observations for each test				
			Cross-	
Method	Statistic	Prob.**	sections	Obs
Null: Unit root (assumes common unit root process)				
Levin, Lin & Chu t*	-4.35244	0.0000	5	65
Breitung t-stat	-6.34317	0.0000	5	60
Null: Unit root (assumes individual unit root process)				
Im, Pesaran and Shin W-stat	-4.76021	0.0000	5	65
ADF - Fisher Chi-square	37.9061	0.0000	5	65
PP - Fisher Chi-square	89.3865	0.0000	5	70
** Probabilities for Fisher tests are computed using an asymptotic Chi				
-square distribution. All other tests assume asymptotic normality.				

Panel unit root test: Summary				
Series: INFL				
Date: 12/20/16 Time: 20:00				
Sample: 2000 2015				
Exogenous variables: Individualeffects				
User-specifiedlags: 1				
Newey-West automatic bandwidth selection and Bartlett kernel				
			Cross-	
Method	Statistic	Prob.**	sections	Obs
Null: Unit root (assumes common unit root process)				
Levin, Lin & Chu t*	-4.12108	0.0000	5	67
Null: Unit root (assumes individual unit root process)				
Im, Pesaran and Shin W-stat	-3.79980	0.0001	5	67
ADF - Fisher Chi-square	32.9844	0.0003	5	67
PP - Fisher Chi-square	55.8292	0.0000	5	72
** Probabilities for Fisher tests are computed using an asymptotic Chi				
-square distribution. All other tests assume asymptotic normality.				

Panel unit root test: Summary				
Series: D(OIL_PRICE)				
Date: 12/20/16 Time: 16:09				
Sample: 2000 2015				
Exogenous variables: None				
User-specifiedlags: 1				
Newey-West automatic bandwidth selection and Bartlett kernel				
Balanced observations for each test				
			Cross-	
Method	Statistic	Prob.**	sections	Obs
Null: Unit root (assumes common unit root process)				
Levin, Lin & Chu t*	-4.44397	0.0000	5	65
Null: Unit root (assumes individual unit root process)				
ADF - Fisher Chi-square	28.5843	0.0015	5	65
PP - Fisher Chi-square	46.5007	0.0000	5	70
** Probabilities for Fisher tests are computed using an asymptotic Chi				
-square distribution. All other tests assume asymptotic normality.				

Panel unit root test: Summary				
Series: D(M2)				
Date: 12/20/16 Time: 16:05				
Sample: 2000 2015				
Exogenous variables: Individualeffects, individuallinear trends				
User-specifiedlags: 1				
Newey-West automatic bandwidth selection and Bartlett kernel				
Balanced observations for each test				
			Cross-	
Method	Statistic	Prob.**	sections	Obs
Null: Unit root (assumes common unit root process)				
Levin, Lin & Chu t*	-6.14226	0.0000	5	65
Breitung t-stat	-1.86444	0.0311	5	60
Null: Unit root (assumes individual unit root process)				
Im, Pesaran and Shin W-stat	-4.54554	0.0000	5	65
ADF - Fisher Chi-square	38.1602	0.0000	5	65
PP - Fisher Chi-square	77.7335	0.0000	5	70
** Probabilities for Fisher tests are computed using an asymptotic Chi				
-square distribution. All other tests assume asymptotic normality.				

Annex 2: Panel VAR-Granger causality Wald test

Equation/Excluded		chi2	Df	Prob
Oilprice	GGDP	2,418	1	0,12
	INFL	8,947	1	0,003
	M2	2,161	1	0,142
	ALL	17	3	0,001
GGDP	oilprice	1,024	1	0,311
	INFL	0,213	1	0,644
	M2	0,356	1	0,551
	ALL	1,352	3	0,717
INFL	oilprice	1,132	1	0,287
	GGDP	0,934	1	0,334
	m2	1,503	1	0,22
	ALL	3,105	3	0,389
M2	oilprice	7,568	1	0,006
	GGDP	0,053	1	0,818
	INFL	1,011	1	0,315
	ALL	8,305	3	0,04

Annex 3: Panel Vector Autoregression (GMM estimation)

	L1.	Coef.	Std. Err.	z	P>z	[95% Conf.	Interval]
oilprice	Oilprice	.991073	.1817967	5,45	0.000	.6347581	1.347388
	GGDP	-.2978365	.191518	-1,56	0.120	-.6732048	.0775318
	INFL	-1.999249	.6683857	-2,56	0.003	-3.30926	-.6892367
	M2	-2.722157	1.851696	-1,47	0.142	-6.351414	.9071008
GGDP	Oilprice	-.050563	.0499574	-1,01	0.311	-.1484777	.0473517
	GGDP	.4635038	.1740421	2,66	0.008	.1223876	.8046201
	INFL	.1162443	.2518354	0,46	0.644	-.377344	.6098326
	M2	.2344472	.3928493	0,6	0.551	-.5355233	1.004418
INFL	Oilprice	.0411436	.0386783	1,06	0.287	-.0346644	.1169517
	GGDP	.038541	.0398854	0,97	0.334	-.039633	.1167149
	INFL	-.1103765	.1762883	-0,63	0.531	-.4558953	.2351422
	M2	-.4259941	.3474316	-1,23	0.220	-1.106947	.2549592
M2	Oilprice	.0430068	.0156337	2,75	0.006	.0123654	.0736483
	GGDP	.0055645	.0242345	0,23	0.818	-.0419343	.0530632
	INFL	.0597116	.0593917	1,01	0.315	-.0566939	.1761172
	M2	.8072593	.1405489	5,74	0.000	.5317885	1.08273

Annex 4: Response variable and forecast horizon

Response variable and forecast horizon					
		impulse variable			
		Oil price	GGDP	INFL	M2
oil price	0	0	0	0	0
	1	1	0	0	0
	2	.8978778	.0053333	.0745763	.0222126
	3	.8346815	.0133794	.1019746	.0499645
	4	.7846813	.0193035	.1162055	.0798097
	5	.7457001	.0226544	.1235611	.1080845
	6	.7190945	.0239403	.1260716	.1308936
	7	.7057981	.0239276	.1252134	.1450609
	8	.7039962	.0234525	.1227213	.1498299
	9	.7089102	.0231763	.1203021	.1476115
	10	.7150543	.0233783	.1190008	.1425666
GGDP	0	0	0	0	0
	1	.013128	.986872	0	0
	2	.0329328	.963387	.002255	.0014251
	3	.05029	.9387494	.0060088	.0049518
	4	.0602804	.9204852	.0094463	.009788
	5	.0637276	.9098835	.0116871	.0147018
	6	.0638682	.9047521	.0127054	.0186744
	7	.0638477	.9020173	.0129293	.0212058
	8	.0654776	.8992737	.0128903	.0223584
	9	.0689943	.8954341	.0129823	.0225893
	10	.073538	.890632	.0133591	.0224709
INFL	0	0	0	0	0
	1	.0007111	.0099993	.9892896	0
	2	.0415465	.0240811	.906401	.0279715
	3	.0478949	.024275	.877414	.0504161
	4	.0470781	.0239207	.8639327	.0650685
	5	.0530775	.0236785	.8513259	.071918
	6	.069074	.0235808	.8344397	.0729055
	7	.0909836	.0237664	.8141103	.0711397
	8	.1124566	.0242415	.7939152	.0693868
	9	.1288868	.0248659	.7771369	.0691103
	10	.1387492	.0254625	.7653168	.0704714
M2	0	0	0	0	0
	1	.0119416	.0785761	.0037555	.9057269
	2	.0975508	.0614546	.021074	.8199206
	3	.3235916	.0499264	.015244	.611238
	4	.5124594	.0441101	.0273839	.4160466
	5	.6129952	.0428104	.0467568	.2974375
	6	.6516562	.0437013	.0646899	.2399526

7	.6570356	.045243	.0785435	.2191778
8	.6470363	.0466494	.0880173	.2182971
9	.6324363	.0475542	.0934933	.226516
10	.6199761	.0478455	.0956858	.2364927

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THE ECONOMIC BUBBLE AND ITS MEASUREMENT

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Abstract:

In mainstream economics, the sight is restricted to forms of financial bubbles. In Concordian economics, rather than the behavior of the financial markets. instead, a bubble is defined as a separation of monetary values from values of real wealth. Hence, the concern is with the behavior of the entire economic system. Once defined, Concordian economics allows us to measure the bubble. To obtain this result, Concordian economics overcomes one of the major hurdles in economics, that is the measurement of real wealth as an entity separate and distinct from monetary wealth.

Keywords: financial bubbles; economic bubble; Concordian economics

JEL Classification: A10; B40; B59; C18; C60; D20; D30; D84; E01; E19; G01; K40

1. Introduction

The extant economics literature talks of many bubbles, such as the Dutch tulip bubble of 1637, the housing market bubble, the stock market bubble (various years). Common sense suggests to classify these as partial or "commodities" bubbles. As previously seen, in Concordian economics one is concerned with the economic bubble, namely the general bubble that permeates the economic system as a whole.

2. The Economic Bubble and Its Measurement

In Concordian economics, the economic bubble is defined as a separation of monetary wealth from real wealth.

In the following paragraphs, we shall attempt to show that, within Concordian economics, the bubble can be measured both geometrically and mathematically, and measures can be taken for the abatement of its negative effects.

2.1. Concordian Economics and the Economic Process

While modern economics studies a great variety of economic theories, Concordian economics confines itself to the study of the economic process (Gorga 2002 and 2009). The fundamental conception of the economic process is contained in the following figure, which reproduces the three essential component elements of the economic process:

1. The Production of Real Wealth (RW);
2. The Distribution of Ownership (DO) rights over monetary and real wealth; and
3. The Consumption - or expenditure - of Monetary Wealth (MW) to purchase real wealth.

Even in the purchase and sale of a car we have these three elements: the car, the money, and the deed of ownership. Indeed, these three elements exist in the purchase of a chocolate bar as well: Without the proof of purchase, or the sales slip, one risks arrest in transporting the chocolate bar out of the store.

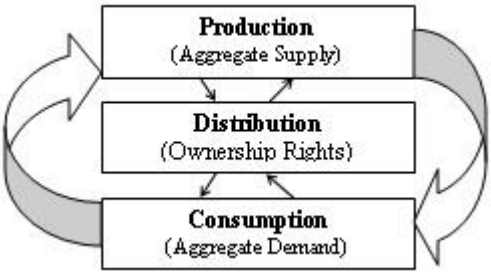

Figure 1. The economic process

This figure reads as follows. As soon as an item of real wealth held by a producer is exchanged for monetary wealth held by a consumer, one cycle of the economic process is completed. For the exchange to occur, in a civilized society both transactors have to be the legal owners of the wealth they exchange. There is much that can be said on this figure. We are using it here to obtain the measurement of the economic bubble.

2.2 The Geometric Measurement of Economic Bubbles

The geometric representation of the bubble is offered in the following figure, which is derived from Figure 1 in this fashion: All three rectangles representing respectively Production, Distribution, and Consumption of wealth are reduced to three segments of a line. When we project these elements over time, we obtain an idealized figure such as follows:

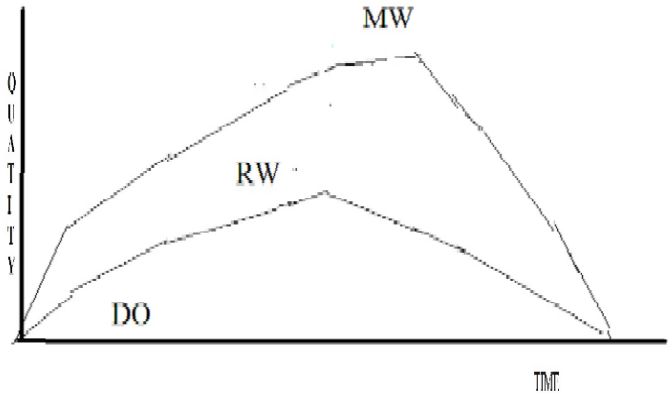

Figure 2. Idealized trajectory of values over time

MW stands for monetary wealth. As we know, over time the value of monetary wealth tends to grow faster than the value of real wealth. Various factors contribute to this result, chief among them today is the greater impact of "financialization". It is in this disjunction that we find the definition of a bubble. A bubble is the separation of monetary values from real economic values.

RW stands for real wealth. As we know, the value of real wealth tends to grow over time. But then, at times it does decrease. When its value decreases, production tends to decrease. As already pointed 4 out,

history tells us that the decrease of real wealth can stop at any time, but there is no assurance as to when or where it will stop. With real wealth gone, some civilizations have even disappeared from the face of the earth.

DO stands for distribution of ownership rights over monetary wealth as well as real wealth. As a first approximation, we assume that notwithstanding significant changes this line remains constant over time. Hence it is depicted as a flat line. Development of this issue will eventually give us information as to who owns what, who even owns which part of the bubble. The line is held constant because at this stage of the discussion we are not concerned with the dynamics of the creation of the bubble, but with the measurement of the size of the bubble - at any moment in its development.

The economic bubble consists of the area between RW and MW. The formulas for the measurement of this area is given in any standard physics textbook. For general reference, see Wikipedia.

2.3 The Mathematical Measurement of the Economic Bubble

In economics, until recently, one ran into a huge snag if one wanted to measure the area between RW and MW of Figure (see Gorga 2012). The reason is that in economics, with everything being measured in "money," up until recently there was no way of distinguishing the numeric values that the three lines represent. Yeoman efforts were carried out by Keynes in his attempt to design "labour units" for the measurement of real wealth. To no avail. Successive attempts have been pursued under the hope to use objective units such as British Thermal Units (BTUs). But none of these attempts have proved to be workable; thus, measurement of the bubble has remained confined to some specialized financial markets. Rather recently, as previously reported, a breakthrough occurred - and has been finalized only in the last few days. As soon I announced to my wife, Joan, that I had solved the problem of measuring wealth by distinguishing values of the real economy (Production), from values of the legal economy (Distribution), and values of the monetary economy (Consumption) by using different colours, she said: *"But you can call them p-values, d-values, and c-values*." I must acknowledge: "Brilliant."

The third stage is this. Up until recently, working under a common misconception I misled himself and other people because conventional economic thinking led me to believe that the problem of the numeraire - yet to be found - is restricted to the measurement of real wealth. Thus p-values were expected to be applicable only to real wealth. Not so. Recently, I have discovered that the problem needs to be transformed from a specific to general task.

In Concordian economics the numeraire remains the same in every field, it is always the currency of the country of observation. All we need to keep distinct is the object that we want to measure. In Concordian economics, we clearly separate real wealth from monetary wealth and we measure real wealth in p-values, while we measure monetary wealth in c-values. The two modules are brought together again by the insertion between them of the value of ownership rights over real and monetary wealth, which are measured in d-values.

In Concordian economics, p-values are different from c-values, and different from d-values, not because they are measured by a different yardstick, but because they themselves are entities different from each other. The yardstick of measurement remains the same, the local currency however denominated. Thus, for the US economy, we obtain this description:

- p-values are dollar values that record purchase and sale of real wealth, such as tables and chairs and services (p-dollar values);
- d-values are dollar values that record the value of distribution of ownership of real and monetary wealth (d-dollar values);
- c-values are dollar values that record purchase and sale of monetary wealth, such as currency and stocks and bonds (c-dollar values).

These numbers represent common economic values - with some major differences from standard economic theory. One difference is that, in Concordian economics, "real" values relate to the value of real wealth, not financial values minus inflation.

The other major difference is that the real economy, the monetary economy, and the legal economy are constantly kept separate from each other. In mainstream economics for the measurement of car production, for instance, one adds all the values in any specific currency of the production of cars in a given unit of time, and then one analyses the behavior of these numbers over time.

However, as soon as one gives values of the production of cars in dollars, for instance, the result is a dollar value. Where is the car? Cars are no longer to be seen. The real economy disappears in the midst of the monetary valuation of real transactions.

In Concordian economics, instead, p-values are identified and remain forever distinguished from d-values and c-values. To repeat, p-values measure real wealth; d-values measure the value of ownership rights over real and monetary wealth; c-values measure the value of monetary instruments used to purchase real wealth.

Also, in Concordian economics, "money" is identified as the sum of all financial instruments that are used to purchase real wealth. Specifically, contrary to all extant standard definitions, money is assumed to be the sum of the value of coins, currency, mortgages, stocks, bonds, derivatives, and all conceivable financial instruments. Stocks and bonds, in particular are used to purchase entire factories, and should they not be counted as money? To my knowledge, only Murray Rothbard (2010, 259-265) includes stocks and bonds in the definition of money.

2.4. How Are these Values Determined?

These are not arbitrary values determined by a coterie of friends or market conspirators. They are values determined by the market as a whole, and denominated in the local currency of each country of the world. There are, of course, enormous controversies surrounding any issue of valuation in economics. Eventually, the framework of analysis of Concordian economics will resolve many of these controversies, because we are going to have a continuing triple check on our deliberations. We will see bubbles develop; we will become more careful with our money and other people's lives.

2.5. Better than Gold

As soon as Michael David Rubin (2016) read the previous paper on the definition of the bubble, he urged me to advocate for the adoption of a gold standard. The purpose of the recommendation, of course, is to develop a monetary system that is stable. Arguably, the Gold Standard as it worked mainly during the 19th Century was better than the Fiat Money system that rules today. Since even the best designed Gold Standard has the essential drawback of being tied to a scarce commodity that is not flexible enough to accommodate the needs of the market, and since the oscillations in the price of gold can be rather drastic at times, I am utterly confident that p-values are better than gold.

As conceived long ago, see Gorga and Kurland (1987, 83-86), p-values are measurements suited to the creation of a Golden Standard. This is the creation and measurement of a monetary system that reflects and fosters the creation of real wealth. Under a Golden Standard, monetary resources are always 6 sufficient to satisfy the needs of the market, and are flexible enough to accommodate the needs of volatile human beings, but will automatically curb the urges of market speculators and manipulators. Today, market speculators and manipulators can even be excused because no one knows the extent of the damage they cause; they are covered by a veil of ignorance. With the assistance of p-, d-, and c-values, they will see, as the Russians say, that "Greed ruins the greedy". As currently proposed on the pages of Mother Pelican and Econintersect, the Golden Standard will eventually be implemented through three recommended changes in the rules of the Federal Reserve System and a petition currently circulating on the Internet as well as another petition advocating the Jubilee Solution or a systematic reduction of debt to avoid unruly crashes in financial markets.

Conclusions

It is not the valuation of a car, new or used, that is difficult to determine nor is the aggregation of these values a problem. (Speaking more comprehensively, in Concordian economics one uses the same models in microeconomics as in macroeconomics.) It is the value of financial instruments that is so very difficult to determine. One of the major functions of Concordian economics will be to resolve these issues of valuation of c-values in the future. It is p-values that ought to indicate c-values - not the other way around. In other words, while still subject to the vagaries of human expectations, p-values can be objectively verified - as well as c-values, as well as the separation of monetary values from real values. The bubbles will become manifest and measurable on an ongoing basis. Measured bubbles are a warning to the greedy.

Needless to say, the quantification project cannot be carried out by any single person or even single institution. Most economists have to agree to the scientific necessity of collecting data in accordance with the new categories of thought outlined by Concordian economics; most governments of the world will have to implement this quantification project.

The sooner, of course, the better.

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TRENDS IN THE DEVELOPMENT OF INTERNATIONAL TRADE IN 2011-2016, AND FORECASTS FOR 2017

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Abstract: *This paper analyzes the developments in world trade in goods and services in 2015 - 2016, as well as the prospects for 2017 - 2020. It highlights both the main determinants and salient features of merchandise trade in 2015, and its dynamics in terms of volume and value. It also sheds light on the geographical composition of merchandise trade and the major traders worldwide. Similarly, the paper outlines the main trends shaping trade in services in 2015. According to the findings of the paper, 2017 marked the sixth consecutive year in which trade growth has been disappointing, with trade expanding less than 3%, and exceeding only marginally global GDP growth.*

All forecasts by international economic organizations in early 2016, leading to the conclusion that the expansion of international trade from 2016 to 2017 will grow moderately outrunning dynamic global GDP, still below the annual average rate recorded in the pre-crisis period.

Keywords: global trade, merchandise trade, trade in services, World Trade Organization (WTO), forecasts

JEL Classification: F10, F13, F17, O19

1. Trends in the evolution of international trade in 2011 - 2016

In the presentation that follows, based on data provided by: European Commission, WTO, OECD, IMF, WTO, World Bank, National Institute of forecasts or statistics, etc. and we have made a brief analysis of trends in trade international 2011-2016 and forecasts for 2017-2020 (2016 is considered the year of forecast).

Growth rate in international trade since 2012 and has slowed considerably compared to previous pre-financial crisis global economic registering Annual evolution that was not expected, led to adjustments systematic both estimates and forecasts elaborate economic organizations level international.

According to preliminary estimates of the WTO, published in April 2016, international trade grew in 2015 at a rate of only 2.8% in terms of volume and dynamics was less 3.3% in April forecasted in 2015, which recorded the fourth consecutive year as the growth of international trade has failed to advance significantly than global GDP growth unlike the years before the crisis, when the ratio was 2 1. Changing the ratio of 2: 1 was assigned to a complex of factors is not exclusive and are not exhaustive meeting: macroeconomic conditions unfavorable economic growth sluggish in advanced economies, changing competition worldwide demand and a faint investment, changes in production structure, maturation global value chains, weak trade finance, the slow process of liberalization of international trade and accumulation of protectionist measures in the post-crisis. The

calculations made by the WTO, was concluded that the average of the years 2012-2015, international trade grew only 2.6%, with GDP growth comparative to International 2.3% throughout the same period (WTO 2016).

Table 1. Evolution of world trade in 2011-2015 and perspectives for 2016/2017 (annual change in %).

Worldwide / Regions	2011	2012	2013	2014	2015	2016	2017-1
Worldwide trade of goods-2							
Exports	5,4	2,2	2,4	2,8	2,8	2.8	3.6
Developed economies	-						
Economies in development/ emerging economies	5.2	1.1	1,7	2,4	2,6	2,9	3,8
North America	5,8	3,8	3,8	3,1	3,3	2,8	3,3
South America / Central America	6,6	4,5	2,8	4,1	0,8	3,1	4,0
Europe	6,6	0,9	1,2	1,8	1,3	1,9	1,9
Asia	5,6	0,8	1,7	2,0	3,7	3,1	4,1
Another regions-3	6,4	2,7	5,0	4,8	3,1	3,4	4,0
Imports	2,0	3,9	0,7	0,0	3,8	0,4	0,4
Developed economies	-						
Economies in development/ emerging economies	3,4	-0,1	-0,2	3,5	4,5	3,3	4,1
North America	7,8	4,9	5,0	2,1	0,2	1,8	3,1
South America / Central America	4,4	3,2	1,2	4,7	6,5	4,1	5,3
Europe	12,6	0,7	3,6	-2,2	-5,8	-4,5	5,1
Asia	3,2	-1,8	-0,3	3,2	4,3	3,2	3,7
Another regions	6.5	3,7	4,8	3,3	1,8	3,2	3,3
	7,9	9,9	3,7	-0,5	-3,7	-1,0	1,0

Note. 1- forecast for the 2016/2017; 2 Average global exports and imports in volume terms; 3 - Other regions include: Africa, Comunitatzea Independent States (CIS) and Middle East

Source: Table prepared based on data WTO Secretariat (WTO, 2015 to 2016)

As a result, after many years after the global economic and financial crisis, international trade assessed and evaluated in um slow, far enough from decades earlier 1990-2008 global financial crisis when their growth amounted to 6% , annual average. In the coming years the global financial crisis, there has been a rather slow recovery of developed economies and moderate developing countries / emerging. Thus, in 2010 increased by 13.9% in volume terms compared with the decline of 12.5% in 2009, 5.4% ritmulu growth of international trade in 2011, with a reduction of 2.6% the average of the years 2012-2015 (Table T1).

By 2013, a reduction but still sluggish economy UE had influence on the expansion of world trade, with a significant share of EU imports 33% worldwide, including intra-EU world total and 15 % for exclusion thereof, as shown in Dare WTO (WTO) import growth in developing countries / emerging prompted urge expansion of trade in the post-crisis, a substantial slowdown after 2013, reaching in 2015 the lowest 0.2% of the global crisis 2008/2009 (table T1).

The adverse effects of the global financial crisis and the eurozone crisis that arose subsequently continued to affect the global economic recovery and obviously the trade international recovery. Adjustments cumbersome post-crisis and the prolonged economic recovery faced with new challenges: deepening conflicts geo political in various parts of the world (potential threats of economic nature, the prospect of economic growth, geopolitical uncertainties, 74% globally and 83% Romania), uncertainties macro economic, low prices for commodities and economic flows declining, emphasizing volatility exchange rate and capital flows, (threat: 73% globally and 68% in Romania), stagnation of investments and decrease productivitîţii growth and finally permanent decoupling of financial activities of the real sectors of the economy (UN / DESA 2016).

During the years of the global financial crisis, policy makers from around the world is facing great challenges intention to stimulate investment on the one hand and renewed growth and global trade, on the other hand.

Slowdown in world trade is a mere reflection of the current state of the world economy or other forces around? The investigations were concluded that the game is a combination of factors, both cyclical short-term and structural in nature, with long-term impact. There was fear generated by slow growth of world trade, the relationship between trade and global GDP was an upper limit. The statistical data shows that international trade has increased 27 times between 1950 and 2008, which would return to growth three times faster than global GDP.

Therefore, based on World Bank data, the relationship between trade and overall world GDP rose from 25% in 1960 to 60% in 2015/2016. On the other hand, data from the WTO confirms that the reduction in the elasticity of trade to global income did not involve a lower ratio between trade and world GDP, keeping at a record low in appropriate their (WTO 2016).

A sharp deceleration in world trade was noted in the second quarter of 2015 that was reversed by the end of the developed countries and the developing / emerging. This rebound in the first half of 2015, according to quarterly data provided by the WTO, has attenuated on exports that imports of goods in volume terms also geographical regions in a lesser or greater, although the impact was felt the strongest in the second quarter.

WTO experts failed to predict many of developments at the regional level. North American export growth, projected to be 4.4%, was well below expectations, 0.8% in 2015 while exports of oil-exporting regions (Africa, Middle East and CIS) were more robust, increasing by 3.9% to 0.5% of the projected figure (Table 2.1). Then the dynamics of the region Europe exports and imports of 3.5% and of 4.3% in 2015 from 2.8% projected upper level and 3.2%, respectively. The decline in imports was oil-producing regions of -3.7% in 2015 which exceeded the forecast of -1.5%. Recovery of intra-EU trade contributes to a better situation in the area of trade in Europe. Severe recession in Brazil and other countries in the region in difficulty decreased the Central and South American imports in 2015, -5.8%.

Although developments in the global volume of trade in goods has grown at a steady pace and quite slow in previous years, this development conceals significant changes in terms of major contributions to this growth geographies. For example, Europe contributed to the 1.3% which brought to 44% of that growth. Declining energy prices in 2015, while oil supply that surpass demand was generated by the combination, on the one hand, the increase in exports generated by oil-producing countries and on the other hand, imports decreased in Asia (WTO, 2016).

From the data in Table 1 we see that:

- if that imports exports of developed countries increased in 2015 compared to 2014, export growth economies of developing / emerging was insignificant with the growth rate of imports fell further.
- increasing developed country imports 4.5% in 2015, it is for the second year higher in developing countries imports / emerging at a rate of 0.2%.

In summary, the developed economies for the second consecutive year, they played base Power TCP in boosting global trade and economies of developed countries in Europe helping to increase global imports in 2015, accounting for 70.3% of this increase.

Anemia increasing volumes of trade in goods in 2015 contrasts with the development value of these exchanges in dollars, which compared to the previous year sank by 13.5% (from 19,000 billion to 16,000 billion), is explained on the one hand, the further collapse of commodity and exchange rate fluctuations due to the economic downturn in China, on the other hand (WTO 2016).

Hence, a feature of the evolution of trade in goods in nominal terms in 2015 is deteriorating commodity prices and the appreciation of the US dollar with strong role in the sharp decline in the value of international trade in 2015.

Another feature of the evolution of trade in goods in nominal terms in 2015 generalized decline of international trade flows, covering all geographical regions in terms of exports and imports decreased. For example, -34.7% Middle East, CIS -32% -29.6% Africa, Central and South America -21.2%.

Regarding international trade in services, the value of world trade in services in 2015, it has declined much less than that of trade in goods, 13.5%. An increase of 6% in the years 2013 to 2014, exports of services decreased in 2015 by 6.4% compared to 2014, totaling USD 4.675 billion dollars (estimates WTO 2016).

Excluding trade in services conducted between Member States, as the EU as a single entity, the EU-28 remains the largest exporter of services in the world. EU exports of services to the world in 2015 amounted to 890 billion obtaining a 24.7% share in total world exports, 690 billion US dollars with a share of 19.1%, China 229 billion dollars and 6.4%, Japan with 158 billion dollars and 4.4%, then the fifth India on par with Japan.

If we exclude imports and intra-EU trade in global services trade, the EU is the biggest importer of services from around the world. In 2015, EU imports figure was raised to 728 billion figure which represents 20.3% of total imports global services. Next place rests with 469 billion US dollars and 13.1%, followed by China with 437 billion of dollars and 12.2%, Japan with 174 billion of dollars and 4.8%, Singapore 144 billion dollars and 4%.

With 8.6% in 2015, decreased non-EU services exports and imports by 6.3% over 2014, totaling 890 billion dollars respectively 728 billion dollars. In relations with the wider world, the EU services balance surpluses in 2015 with a balance of 162 respectively (213 billion dollars in 2013 and 255 respectively 2014).

Amplification final consumption contribution to economic growth leads to a closing of the output gap (output gap) in 2017, even if the GDP growth potential reach to exceed 3% in the medium term. Improving particular potential growth during 2015- 2018 is due to a conscious contribution in total factor productivity growth, showing character and reflect the economic intensive growth of structural changes that have occurred in previous years in the Romanian economy. After subtracting 2013-2014, fixed capital formation, it gradually increases the contribution which results in an improvement in the potential increase of capital stock. Decrease in net investments in machinery and transport equipment was improved from 2014 (-6.4% -13.1% facades), signaling a trend reversal of the establishment of production facilities.

The contribution of labor (expressed only hours worked in the economy) will return to positive values within the expected range. A natural rate of unemployment decreased slightly positive contribution of hours worked is the result of favorable trends in employment and hours worked in the economy (Annex 2).

If the latest forecasts by the EC and the IMF reveal an acceleration of economic activity in the EU and domestic economic environment will improve for reduction of VAT, it is estimated that Romania will record in the next period until 2018 a gradual improvement Romania's economic performance so that they can recover the decline of 2009-2010.

GDP growth of 3.2% in 2015 due to the positive contribution of domestic demand for the coming years is expected to gradually accelerate the pace of GDP growth to about 4.0% in 2018. Private consumption will record annual growth rates give 3.5% in 2015 to 4.1% in 2018. Considering the evolution of gross fixed capital formation in 2014, it is estimated by 2018 to register growth that can be supported by more substantial inflows of funds Europe. Thus, annual increases will accelerate from 4.5% in 2015 to 6.8% in 2018.

Net exports will reduce its contribution to real GDP growth in 2014 compared with a negative contribution until 2018 or from 0.2% in 2015 to 0.4% in 2018 due to increases in imports of goods and the superior service of exports, rising imports generated investment (Annex 3).

In 2014, Romanian exports growth compared to 2013 was 5.8% while imports increased by 5.9%. In early 2015 Romania's exports increased by 3.6%, while imports increased by 3.3%.

Intra-EU exports recorded a slowdown in growth from 19.1% in 2011 to 8.1% in 2014. Imports intra had a growth rate of 6.8% in 2015 compared to 2014, and the extra recorded in the same period, an increase of 9.9%, which resulted in the intra trade deficit of 6.7 billion and in the extra-communitarian zone a non-surplus. In this situation, the global economy in the period 2017-2018, annual average growth is expected exports goods 7.3% and imports by 7.5%. After 2014, the deficit FOB-CIF trade in GDP was 4.0% in 2018 is expected to reach 5.1%. For exports to the EU, it is estimated an average annual rate of increase of 8.4% and 7.4% for intra-EU imports. For extra imports and exports, it is estimated an average increase of 4.5% and 9.5%. In the medium term, the current account deficit will remain at a value between 2.3-2.8 billion. Euro with a share of 1.5% of GDP in 2018 (Annex 2).

Europe is considered the world's largest exporter of processed goods and services representing in turn, the largest export market for 80 countries. In 2015, EU trade in goods and rest of the world reached 3517 billion.

Table 2. EU-trade commodity trading power in 2015 (million)

Country	Exports	Imports	Total	Trade balance
EU	1.791534	1.727125	3.518659	64.410
China	170484	350424	520909	-179940
USA	371223	248437	619660	122786
Japan	56550	507568	116318	-3217

Source: European Commission, 2016

In terms of real convergence, measured by differences from the EU average GDP per capita expressed in purchasing power standard (PPS), Romania has made significant advances in previous years, hovering in the years 2015/2016 to about 55 % of the average EU-28 compared to 54.5% in 2013 to about 2% more than in 2012. Moreover, real convergence of Romanian economy continued in the years of crisis, the pace of GDP per capita throughout the period 2009-2013 was 2% above the EU.

Experience the new Member States which joined the euro in this program (convergence program of Romania to adopt the euro from data 2019, 1 January) real convergence of Romania will reach 2018 to about

65%, with the prospect of 2020 when GDP per capita in PPS in Romania to represent 71% of the European average.

Table 3. Prospects for GDP per inhabitant in PPS

	2013	2018	2020
GDP per capita in PPS.	14.500	18.000	21.100
% of the EU-28	54, 5	65,5	71,0

Source: Eurostat and the National Commission for Prognosis

2 Trends in the evolution of the international trade structure

From forecasts of international economic organizations conclude that the expansion of world trade in 2017 will accelerate moderately, slightly outpacing world GDP growth being below the annual average rate recorded pre-crisis growth. Studying the forecasts issued by the WTO and OECD find to be retained on the future development of international trade. In 2017, the growth of international trade would be accelerating slightly, supported by stimulating investment in China, the gradual stabilization of demand countries producing commodities, which could add an improvement in the investment cycle in advanced economies (OECD 2016).

Table 4. Forecast of international trade growth in volume terms, 2017 (in%)

Source/Date	World economy / savings groups -1	Annual adjustments			
		2014	2015-2	2016	2017-3
UNO January 2016	World trade in goods and services -4	3,3	2,7	4,0	4,7
	World exports	3,6	2,7	4,0	4,5
	World imports	3,9	2,6	4,0	4,9
World Bank	World trade in goods and services	3,6	3,6	3,8	4,3
WTO April 2016	International trade in goods and services -5	2,8	2,8	2,8	3,6
	Exports	-	-	-	-
	Developed economies	2,4	2,6	2,4	3,8
	Emerging economies	3,1	3,3	2,8	3,7
	Imports	-	-	-	-
	Developed economies	3,5	4,5	3,3	4,1
	Emerging economies	2,1	0,2	1,5	3,1
IMF April 2016	World trade in goods and services	3,5	2,8	3,1	3,6
	Exports	-	-	-	-
	Advanced economies	3,5	3,4	2,5	3,8
	Emerging economies	3,1	1,2	3,8	3,3
	Imports	-	-	-	-
	Developed economies	3,5	4,3	3,4	4,1
	Emerging economies	3.7	0,5	3,0	3,1
European Commission May 2016	World trade in goods and services-6	3,2	2,5	2,5	3,6
	Exports EU-6	4,0	5,3	3,5	5,4
	Exports in Euro Zone	4,1	5,2	3,5	4,7
	World imports of goods and services-6	3,4	2,3	2,0	4,0
	Imports EU-6	4,7	5,9	4,7	5,1

	Imports in Euro Zone	4,5	6,0	4,6	5,3
OCD June 2016	World trade -7	3,1	2,6	2,1	3,7
	OECD Exports	3,9	3,9	2,6	4,0
	OCDE Imports	3,7	4,5	3,1	4,2
	Non-OECD Exports	3,3	0,7	0,9	1,7
	Non-OECD Imports	3,5	0,7	0,9	1,8

Note: 1. Each institution uses its own classification and methodology 2. Estimates. 3. Prognosis. 4. Average of exports and imports of services. 5. Average of exports and imports of goods. 6. Including intra-EU and extra-UE. 7. Arithmetic rhythm average growth of exports and imports in terms of volume.
Sources: based on reports UN / DESA 2016 World Bank 2016 2016 WTO, IMF 2016, the European commission OECD 2016 and 2016.

Since GDP (WTO predicts an increase of 2.7% in 2017, WTO, 2016) in developed economies is expected to revive slightly, import growth in developed countries is expected to decline and demand for goods imported economies in D/ emergencies increase due mainly to revive Asian imports.

Although for 2017, WTO economists forecast a 3.6% increase in the dynamics of global trade, it will remain below the average further 5.5% in 1990-2015 and obviously will be below the average annual 6% for the period post-global financial crisis.

Conclusion

WTO forecast risks, just like those of other organizations involved in international economic forecasts, tilts the balance towards a downward revision in the forecast figures. In their view, these risks include: slowing further growth in the economies of developing / emerging weakening growth momentum of the Chinese economy, increased financial market volatility and exposure of countries with high external debt to strong fluctuations in exchange rates.

World Bank estimates show that 1% decrease in the growth rate of the BRICS (Brazil, Russian Federation, India, China and South Africa) would have the consequences a reduction of 0.8% growth in other economic development / emerging and 0.4% growth globally (World Bank 2016). The fact that China imports 10% of world imports of goods made us to assert that there is a considerable risk of contamination by other emerging markets with potential negative effects including some advanced economies.

Moreover, in assessing forecasts growth of international trade must be considered and conclusions from the literature, which reveal that, while cyclical factors, short-term (including weak demand) prevailed during the global financial crisis in the first-year economic recovery, their contribution would be gradually reduced in recent years, which means that after the global financial crisis, the growth of trade is slower not only because the dynamics lowest of global GDP, but that commercial change itself became less responsive to GDP and in part structural phenomenon will persist in the future.

In summary, all forecasts by international economic organizations in early 2016, leading to the conclusion that the expansion of international trade from 2016 to 2017 will grow moderately outrunning dynamic global GDP, still below the annual average rate recorded in the pre-crisis period (Annexes 1-3).

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Annex 1

The share of the world economy the main countries / groups of countries in % of world GDP

Type of country/ years	1990	2000	2011	2017
Developed countries	69,2	62,8	51,1	45,7
USA	24,7	23,5	19,1	17,7
EU	28,5	24,9	20,1	17,2
Japan	10,1	7,7	5,6	4,8
Emerging countries	30,8	37,2	49,9	54,3
Brazil	3,3	2,9	2,9	2,9
China	3,9	7,1	14,3	18,3
India	3,2	3,7	5,7	6,8
Russia	---	2,7	3,0	3,0

Source: Based on data provided by the IMF (IMF 2016)

Annex 2

Contribution of potential GDP growth (2014-2018) (Contribution%)

Years	Potetial GDP	Capital	Work	IFP	Output Gap
2014	2,0	0,6	-0,1	1,5	-1,7
2015	2,5	0,7	0,2	1,5	-1,0
2016	2,9	0,9	0,4	1,6	-0,6
2017	3,1	1,0	0,5	1,6	0,0
2018	3,2	1,2	0,4	1,7	0,7

Source: National Commission for Prognosis

Annex 3

Components of GDP, 2015-2018

-	Annual percentage changes			
Years	2015	2016	2017	2018
Real GDP	3,2	3,4	3,7	4,0
Private consumption expenditure	3,5	3,6	3,9	4,1
Public consumption expenditure	1,0	2,0	1,6	1,5
Gross fixed capital formation	4,5	5,5	6,3	6,8
Exports of goods and services	6,1	5,8	6,2	6,3
Imports of goods and services	6,5	6,8	7,2	7,2

Source: National Commission for Prognosis

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FUZZINESS AND STATISTICS – MATHEMATICAL MODELS FOR UNCERTAINTY

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Abstract:

Real data from continuous quantities, considered under different models in economic theory, cannot be measured precisely. As a result, measurement results cannot be accurately represented by real numbers, as they contain different kinds of uncertainty. Beside errors and variability, individual measurement results are more or less fuzzy as well. Therefore, real data have to be described mathematically in an adequate way. The best up-to-date models for this are so-called fuzzy numbers, which are special fuzzy subsets of the set of real numbers. Based on this description, statistical analysis methods must be generalized to the situation of fuzzy data. This is possible and will be explained here for descriptive statistics, inferential statistics, objective statistics, and Bayesian inference.

Keywords: Bayesian analysis; descriptive statistics; fuzzy information; fuzzy numbers; statistical inference

JEL Classification: C11; C13; C15

1. Introduction

Measurement results of continuous variables are often clouded with uncertainty. In addition, many data are not exact numbers but more or less fuzzy. This type of uncertainty differs from measurement errors. In fact, the obtained data are generally associated with various types of uncertainty. There are multiple components to uncertainty in economic analyses, and it is a challenge for researchers to characterize the full nature and magnitude of these components (Hansen 2017). This challenge should be approached with caution for an accurate data analysis. Data with uncertainty (fuzzy data) are common in economic analysis, *i.e.* economic indicators such as measures of trade, labor force, and stock market. In these cases, the data to be reviewed and/or further analyzed are often presented with considerable uncertainties. Nevertheless, such data, despite their uncertainties, are essential for decisions and often critical. Different methods have been developed for analyzing or correcting results from the use of incomplete data (Krasker 1983). The description of fuzzy data and their statistical analysis also form an active field of research. The most up-to-date mathematical model to describe the fuzziness is fuzzy numbers and their characterizing functions (Viertl 2015).

In this contribution, the generalized statistical methods to handle fuzzy data, common in economic analysis, are described. In section 2, definition of fuzzy data, fuzzy probability densities, and fuzzy-valued functions are explained. Definition of fuzzy sample and some of generalized methods for measures of location

(central tendency) as well as dispersion are described in section 3. Some additional useful descriptive statistics for fuzzy data are explained in section 4. In section 5, inferential statistics based on fuzzy information are described. In section 6, the generalized method for Bayesian Inference with fuzzy data is presented. An open-end and critical research area, *i.e.* fuzzy stochastic processes, is suggested in section 7. The contribution is concluded with final remarks in section 8.

2. Fuzziness

Many important economic information is not obtained as precise data, but rather imprecise, e.g. high income, low interest rate, good quality, and many more. A modern quantitative description of such linguistic variables is fuzzy sets. Hence, the occurring uncertainty can be modelled using the theory of fuzzy random functions (Möller 2009). A fuzzy subset of a given set *M* is a generalization of the *indicator function* $I_A(\cdot)$ of a classical subset $A \subseteq M$. These generalized functions are called membership functions ξ(·), which are functions ξ: $M \rightarrow$ [0,1]. The value ξ(x) for $x \in M$ is the degree to which x belongs to the fuzzy set defined by ξ(·).

Remark 2.1 Indicator functions are special forms of membership functions obeying $I_A: M \rightarrow \{0,1\}$, *i.e.* they allow only two possible values of 0 and 1.

General membership functions can assume all values from the closed unit interval [0,1], however. Fuzzy subsets of a universal set M are determined by a family of classical subsets of M, *i.e.* the so-called δ-cuts.

Definition 2.1 Let ξ(·) be the membership function of a fuzzy subset of the universal set M. For all δ∈(0,1], the δ-cut $C_\delta[\xi(\cdot)]$ is defined by $C_\delta[\xi(\cdot)] := \{x \in M: \xi(x) \geq \delta\}$.

Now the following representation lemma is valid (Viertl 2011).

Lemma 2.1 For every membership function ξ(·), the following holds true:

$\xi(x) = max\{\delta \cdot I_{C_\delta[\xi(\cdot)]}(x_0): \delta \in [0,1]\} \quad \forall x \in M$

Proof: For fixed $x_0 \in M$ and δ∈[0,1], we have

$$\delta \cdot I_{C_\delta[\xi(\cdot)]}(x_0) = \delta \cdot I_{\{x;\xi(x) \geq \delta\}}(x_0) = \begin{cases} \delta \text{ for } \xi(x_0) \geq \delta \\ 0 \text{ for } \xi(x_0) < \delta \end{cases}.$$

Therefore, we obtain for every δ∈[0,1], $\delta \cdot I_{C_\delta[\xi(\cdot)]}(x_0) \leq \xi(x_0)$, and further,

$sup\{\delta \cdot I_{C_\delta[\xi(\cdot)]}(x_0): \delta \in [0,1]\} \leq \xi(x_0)$. On the other hand, we have for $\delta_0 = \xi(x_0)$:

$\delta_0 \cdot I_{C_{\delta_0}[\xi(\cdot)]}(x_0) = \delta_0$ and, therefore, $sup\{\delta \cdot I_{C_\delta[\xi(\cdot)]}(x): \delta \in [0,1]\} \geq \delta_0$,

which implies $sup\{\delta \cdot I_{C_\delta[\xi(\cdot)]}(x_0): \delta \in [0,1]\} \leq \xi(x_0) = max\{\delta \cdot I_{C_\delta[\xi(\cdot)]}(x_0): \delta \in [0,1]\} = \delta_0$.

Remark 2.2: Fuzzy sets are determined by the family of their δ-cuts $C_\delta[\xi(\cdot)]; \delta \in (0,1]$. The family of δ-cuts is a nested family of subsets of M, *i.e.* $\delta_1 < \delta_2 \Rightarrow C_{\delta_1} \supseteq C_{\delta_2}$. The question is whether every nested family of subsets of M are the δ-cuts of a fuzzy set in M. There are counterexamples, but the following construction lemma holds:

Lemma 2.2 Let $(A_\delta; \delta \in (0,1])$ be a nested family of subsets of a given set M. Then, a fuzzy subset of M is generated, whose membership function is defined by

$\xi(x) = sup\{\delta \cdot I_{A_\delta}(x_0): \delta \in [0,1]\} \qquad \forall x \in M,$

for which the following holds true:

$$C_\delta[\xi(\cdot)] = A_\delta \text{ IFF } \bigcap_{\beta < \delta} A_\beta = A_\delta$$

Proof: First we extend the family $(A_\delta; \delta \in (0,1])$ by the element $A_0 = M$. Then, the proof is in the following three steps:

(1) $A_\delta \subseteq C_\delta[\xi(\cdot)] \quad \forall \delta \in (0,1]$:

For $x \in A_\delta$, we have $\delta \cdot I_{A_\delta}(x) = \delta$ and, thus, $sup\{\beta \cdot I_{A_\beta}(x): \beta \in (0,1]\} \geq \delta$. By definition of $\xi(\cdot)$, we have $\xi(x) \geq \delta$ and, therefore, $x \in C_\delta[\xi(\cdot)]$.

(2) $A_\delta = \bigcap_{\beta<\delta} A_\beta \Rightarrow C_\delta[\xi(\cdot)] = A_\delta$:

For $x \notin A_\delta$, from $A_\delta = \bigcap_{\beta<\delta} A_\beta$, we know there exists $\alpha < \beta$ with $x \notin A_\alpha$, and by the nested structure of the generating family $(A_\delta; \delta \in (0,1])$, we know that $x \notin A_\beta \; \forall \beta \in (\alpha, 1]$. Therefore,

$\xi(x) = sup\{\beta \cdot I_{A_\beta}(x): \beta \in (0,1]\} \leq \alpha < \delta$, and therefore $x \notin C_\delta[\xi(\cdot)]$.

(3) $C_\delta[\xi(\cdot)] = A_\delta \Rightarrow A_\delta = \bigcap_{\beta<\delta} A_\beta$:

$A_\delta \subseteq \bigcap_{\beta<\delta} A_\beta$ holds by the nested structure of $(A_\delta; \delta \in (0,1])$. For $x \notin C_\delta[\xi(\cdot)]$, assuming $A_\delta = C_\delta[\xi(\cdot)]$, we obtain $\xi(x) = sup\{\beta \cdot I_{A_\beta}(x): \beta \in (0,1]\} < \delta$. Choosing $\alpha \in (\xi(x), \delta)$, we have $x \in A_\delta$ and, therefore $x \notin \bigcap_{\beta<\delta} A_\beta$.

Remark 2.3 For a given nested family of subsets of M, the generated fuzzy subset contains δ-cuts which are equal to the generating sets as far as possible. See also (Zadeh 1965 and Dubois 1987) for related mathematical definitions.

2.1. Fuzzy Data

Measurement data from continuous quantities are always more or less imprecise, *i.e.* they cannot be represented by precise numbers. Therefore, a more general concept than real numbers is necessary. The best up-to-date such models are so-called fuzzy numbers.

Definition 2.2 A fuzzy number x^* is a fuzzy subset of the set of real numbers $\mathbb{R}$, whose membership function $\xi(\cdot)$ obeys the following:

(1) supp[$\xi(\cdot)$] is a bounded set, i.e. supp[$\xi(\cdot)$] $\subseteq$ [a,b] with $-\infty < a < b < \infty$.

(2) $C_\delta[\xi(\cdot)]$ is a finite union of compact intervals, i.e.

$C_\delta[\xi(\cdot)] = \bigcup_{j=1}^{k_\delta} [a_{\delta,j}, b_{\delta,j}] \neq \emptyset$ for all $\delta \in (0,1]$.

Membership functions obeying the conditions (1) and (2) are called *characterizing functions*. If all δ-cuts of a fuzzy number are compact intervals, the corresponding fuzzy number is called *fuzzy interval*.

Remark 2.4 Methods for obtaining the characterizing function of fuzzy measurement data can be found in (Klir 1995, Viertl 2011 and Kovarova 2015).

For multivariate data and their statistical inference, the following concept of fuzzy vectors is necessary:

Definition 2.3 A fuzzy subset $\mathbf{x}^*$ of the Euclidean space $\mathbb{R}^n$ is called *n*-dimensional *fuzzy vector* if the membership function $\zeta(\cdot)$ of $\mathbf{x}^*$ fulfils the following:

(1) supp[$\zeta(\cdot)$] is a bounded set, *i.e.* it is contained in an *n*-dimensional interval $\bigtimes_{i=1}^n [a_i, b_i]$ of finite volume.

(2) $C_\delta[\zeta(\cdot)]$ is non-empty for all $\delta \in (0,1]$, and it is a finite union of simply connected and closed subsets of $\mathbb{R}^n$.

Remark 2.5 A vector $(x_1^*, \ldots, x_n^*)$ of fuzzy numbers is not a fuzzy vector. In this case, it is necessary to combine the fuzzy numbers $x_1^*, \ldots, x_n^*$ to obtain a fuzzy element $(x_1, \ldots, x_n)^*$ of the sample space $M_x^n \in \mathbb{R}^n$ and, then, the following holds:

Lemma 2.3 Let $x_1^*, \ldots, x_n^*$ be fuzzy numbers with corresponding characterizing functions $\xi_1(\cdot), \ldots, \xi_n(\cdot)$. Then the function $\zeta(\cdot, \ldots, \cdot)$, defined by $\zeta(x_1, \ldots, x_n) = min\{\xi_1(x_1), \ldots, \xi_n(x_n)\}$ $\forall (x_1, \ldots, x_n) \in \mathbb{R}^n$, is the vector-characterizing function of an *n*-dimensional fuzzy vector $(x_1, \ldots, x_n)^*$.

Proof: By the validity of $C_\delta[\zeta(\cdot, \ldots, \cdot)] = \bigtimes_{i=1}^n C_\delta[\xi_i(\cdot)]$, the δ-cut $C_\delta[\zeta(\cdot, \ldots, \cdot)] \neq \emptyset \;\; \forall \delta \in (0,1]$. Moreover, this δ-cut is a finite union of Cartesian products of compact intervals and the supp[$\zeta(\cdot, \ldots, \cdot)$] is contained in $\bigtimes_{i=1}^n \text{supp}[\xi_i(\cdot)]$, as a result, the proof is concluded. Finally, if all δ-cuts of an *n*-dimensional fuzzy vector are simply connected, then the fuzzy vector is called *n*-dimensional fuzzy interval.

2.2. Fuzzy Probability Densities

Prior information in Bayesian inference is usually assumed in form of probability distributions on the parameter space $\ominus$ of stochastic models $X \sim f(\cdot|\theta)$; $\theta \in \ominus$. In case of fuzzy data, it turns out that a more general concept of probability is necessary. This leads to so-called fuzzy probability densities.

Definition 2.4 Let $(M, \mathcal{B}, \mu)$ be a measure space and $f^*: M \to \mathcal{F}_I(\mathbb{R}_+)$ be a function which assigns to $x \in M$ a fuzzy interval $f^*(x)$ for all $x \in M$, such that the so-called δ-level functions $\underline{f}_\delta(\cdot)$ and $\bar{f}_\delta(\cdot)$, defined by $C_\delta[f^*(x)] = [\underline{f}_\delta(x), \bar{f}_\delta(x)]$ $\forall x \in M$, $\forall \delta \in (0,1]$, are integrable functions, and there exists some classical probability density $g: M \to \mathbb{R}_+$ obeying $\underline{f}_1(x) \le g(x) \le \bar{f}_1(x)$ $\forall x \in M$, then f^* is called *fuzzy probability density*.

Based on fuzzy probability densities, a generalized concept of probability for event B in $\mathcal{B}$ can be defined, *i.e.* these generalized probabilities are fuzzy intervals $P^*(B)$, whose generating families are defined in the following way:

Definition 2.5 Based on a fuzzy probability density f^*, for $\delta \in (0,1]$, the system of classical probability densities g obeying $\underline{f}_\delta(x) \le g(x) \le \bar{f}_\delta(x)$ $\forall x \in M$ is denoted by $\mathcal{D}_\delta$. Then for an event B, the fuzzy probability $P^*(B)$ has generating family of intervals, $([a_\delta, b_\delta]; \delta \in (0,1])$, given by

$$a_\delta := inf\{\int_B g \, d\mu : g \in \mathcal{D}_\delta\} \text{ and } b_\delta := sup\{\int_B g \, d\mu : g \in \mathcal{D}_\delta\}.$$

The characterizing function of the fuzzy interval $P^*(B)$ is given by the construction Lemma 2.2.

Remark 2.6 Fuzzy probability densities are basic for Bayesian inference with fuzzy data (Viertl and Sunanta 2013).

2.3. Integration of Fuzzy-valued Functions

Let $(M, \mathcal{B}, \mu)$ be a measure space and f^* a fuzzy valued function, where all values $f^*(x)$ are fuzzy intervals, and all δ-level functions be μ-integrable with finite integral. Then, it is possible to integrate f^* and this integral is a fuzzy interval, defined in the following way:

Definition 2.6 The fuzzy valued integral $J^* = \int_M f^* d\mu$ is the fuzzy interval, which has generating family $(A_\delta; \delta \in (0,1])$ where $A_\delta = [\underline{J}_\delta, \bar{J}_\delta]$, given by $\underline{J}_\delta := \int_M \underline{f}_\delta d\mu$ and $\bar{J}_\delta := \int_M \bar{f}_\delta d\mu$. The characterizing function $\eta(\cdot)$ of J^* is given by $\eta(x) = sup\{\delta . \mathbb{1}_{[\underline{J}_\delta, \bar{J}_\delta]}(x): \delta \in [0,1]\}$ $\forall x \in \mathbb{R}$.

Remark 2.7 The integration of fuzzy valued functions is basic for the generalization of predictive distributions in Bayesian inference.

3. Fuzzy Samples and Foundations of Statistical Inference

Samples of continuous stochastic quantities X consist of a finite sequence of fuzzy numbers. In order to generalize statistical methods for fuzzy data, functions of data are important. Let M_X be the observation space of X, i.e. the set of possible values for X. Then the *sample space* for standard samples is the Cartesian product of n copies of the observation space M_X, *i.e.* the sample space is M_X^n. Classical statistical functions are mappings from the sample space to some measurable space $(N, \mathcal{A})$. For a classical statistical function $f: M \to N$, in case of fuzzy sample $x_1^*, \dots, x_n^*$, the generalized value $f(x_1^*, \dots, x_n^*)$ has to be defined in a reasonable way. This is possible by application of the so-called *extension principle* (see also Zadeh 1975) from the theory of fuzzy sets.

Definition 3.1 (*Extension principle*) Let $f: M \to N$ be any function, and x^* a fuzzy subset of M with membership function $\xi(\cdot)$. Then, the generalized value $f(x^*)$ is the fuzzy subset of N whose membership function $\eta(\cdot)$ is defined in the following way:

$$\eta(y) = \begin{cases} sup\{f(x): x \in M, f(x) = y\} & \text{if } \exists x \in M: f(x) = y \\ 0 & \text{if } \nexists x \in M: f(x) = y \end{cases} \quad \forall y \in N$$

Remark 3.1 For $N = \mathbb{R}$, $\eta(\cdot)$ need not be a characterizing function. However, for continuous functions f, the following theorem holds:

Theorem 3.1 Let $f: \mathbb{R}^n \to \mathbb{R}$ be a continuous function and **x*** be an *n*-dimensional fuzzy vector. Then, the generalized value $f(x^*)$, obtained by application of the extension principle, is a fuzzy number. A detailed proof can be found in (Viertl 2011).

Remark 3.2 In order to apply the extension principle to functions of fuzzy samples, first, the vector of fuzzy data has to be combined into a fuzzy vector. This is done by the application of Lemma 2.3, *i.e.* the fuzzy sample with corresponding characterizing function $\xi_1(\cdot), \dots, \xi_n(\cdot)$ is combined into a fuzzy vector **x*** whose vector-characterizing function $\zeta(\cdot,\dots,\cdot)$ has values

$$\zeta(x_1, \dots, x_n) = min\{\xi_1(x_1), \dots, \xi_n(x_n)\} \quad \forall (x_1, \dots, x_n) \in \mathbb{R}^n.$$

Based on this combined fuzzy sample **x***, statistical functions $\mathcal{S}: M_X^n \to N$ can be generalized by application of the extension principle. The fuzzy value $\mathcal{S}(x_1^*, \dots, x_n^*)$ is defined by $\mathcal{S}(\mathbf{x}^*)$, *i.e.* the membership function $\eta(\cdot)$ of $\mathcal{S}(x_1^*, \dots, x_n^*)$ is given by its values

$$\eta(y) = \left\{ \begin{array}{ll} sup\{\zeta(\mathbf{x}): \mathbf{x} \in M_X^n, \mathcal{S}(\mathbf{x}) = y\} & \text{if } \exists \mathbf{x} \in M: \mathcal{S}(\mathbf{x}) = y \\ 0 & \text{if } \nexists \mathbf{x} \in M: \mathcal{S}(\mathbf{x}) = y \end{array} \right\} \quad \forall y \in N.$$

Example 3.1 For the sample mean $\overline{x} = \mathcal{S}(x_1, \dots, x_n) = \frac{1}{n}\sum_{i=1}^n x_i$ in case of *n* fuzzy numbers $x_1^*, \dots, x_n^*$, the fuzzy value $\overline{x}^* = \mathcal{S}(x_1^*, \dots, x_n^*)$ is also a fuzzy number by Theorem 3.1. Moreover, the generalized sample dispersion s^* is a fuzzy number. In figure 3.1, the characterizing functions of a fuzzy sample are depicted. In figure 3.2, the characterizing function of the corresponding fuzzy sample mean is given, along with the characterizing function of the fuzzy sample dispersion in figure 3.3 respectively.

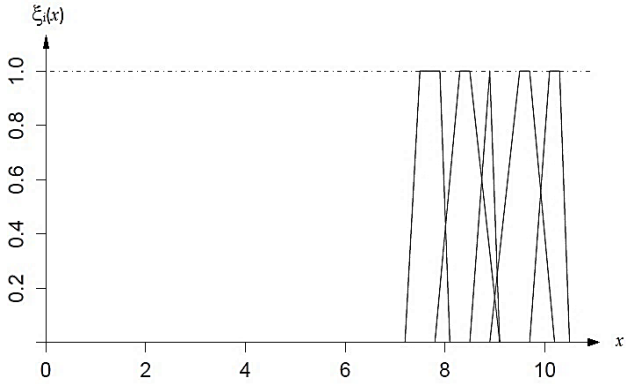

Figure 3.1. Fuzzy sample

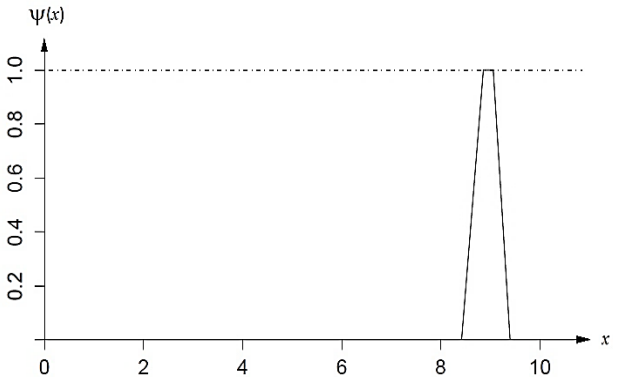

Figure 3.2. Fuzzy sample mean

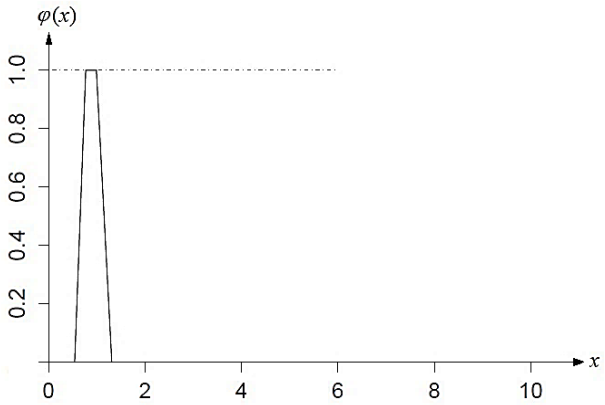

Figure 3.3. Fuzzy sample dispersion

4. Descriptive Statistics for Fuzzy Data

For fuzzy data, descriptive statistics has to be generalized. Mean values and empirical variances of fuzzy data are already introduced in Example 3.1.

4.1 Maxima and minima of fuzzy data

For fuzzy data $x_1^*, \dots, x_n^*$ with characterizing functions $\xi_1(\cdot), \dots, \xi_n(\cdot)$ and δ-cuts $C_\delta[\xi(\cdot)] = \bigcup_{j=1}^{k_{i,\delta}} [a_{i,\delta,j}, b_{i,\delta,j}]$, the fuzzy numbers x_{max}^* and x_{min}^* are defined by application of the extension principle for the functions $max\{x_1, \dots, x_n\}$ and $min\{x_1, \dots, x_n\}$ to the fuzzy combined sample $\mathbf{x}^*$ (see also remark 3.2).

Definition 4.1 Let $x_1^*, \dots, x_n^*$ be *n* fuzzy numbers of the observation space $M_x \subseteq \mathbb{R}$ with corresponding characterizing functions $\xi_1(\cdot),\dots,\xi_n(\cdot)$. To obtain a fuzzy vector $\mathbf{x}^*$, the fuzzy numbers $x_1^*, \dots, x_n^*$ have to be combined. Through construction of an *n*-dimensional vector-characterizing function $\zeta(\cdot,\dots,\cdot)$ via a triangular norm (t-norm *T*), the combined fuzzy sample $\mathbf{x}^*$ forms a fuzzy element $(x_1, \dots, x_n)^*$ of the sample space M_x^n, *i.e.* $x_i^* \triangleq \xi_i(\cdot), i = 1(1)n \xrightarrow{t-norm\ T}$ combined fuzzy sample $\mathbf{x}^*$ and vector-characterizing function $\zeta(\cdot,\dots,\cdot)$ where $\zeta(x_1, \dots, x_n) := T_n[\xi_1(x_1),\dots, \xi_n(x_n)] \quad \forall (x_1, \dots, x_n) \in \mathbb{R}^n$ and the combination T_n, which is the *n*-dimensional extension of the t-norm *T* by its associativity, *i.e.* $T_n(y_1, \dots, y_n) = T\big(y_1, T(\dots, T(y_{n-1}, y_n) \dots)\big) \qquad \forall (y_1, \dots, y_n) \in [0,1]^n$.

For statistical and algebraic calculations with fuzzy data, the minimum t-norm *T* is optimal (Viertl 2011), *i.e.* $\zeta(x_1, \dots, x_n) = T_n\big(\xi_1(x_1), \dots, \xi_n(x_n)\big) = min\{\xi_1(x_1), \dots, \xi_n(x_n)\} \qquad \forall (x_1, \dots, x_n) \in \mathbb{R}^n$.

Lemma 4.1 A fuzzy vector $\mathbf{x}^*$ is obtained via minimum t-norm when the individual values of the variables x_i are fuzzy numbers x_i^*. Through the minimum-t-norm, the combination of *n* fuzzy numbers with characterizing functions $\xi_i(\cdot)$, $i = 1(1)n$, a fuzzy vector $\mathbf{x}^* = (x_1, \dots, x_n)^*$ is obtained. In this case, the following holds:

$$C_\delta[\zeta(\cdot,\dots,\cdot)] = X_{i=1}^n C_\delta[\xi_i(\cdot)] \qquad \forall\ \delta \in (0,1]$$

In words, the δ-cuts of the fuzzy vector $\mathbf{x}^* = (x_1, \dots, x_n)^*$ are the Cartesian products of the δ-cuts of the fuzzy numbers x_i^*, *i* =1(1) *n*.

Proof:

$$\begin{aligned} C_\delta[\zeta(\cdot,\dots,\cdot)] &= \{\mathbf{x} \in \mathbb{R}^n : \zeta(\mathbf{x}) \geq \delta\} \\ &= \{\mathbf{x} : min\{\xi_1(x_1),\dots, \xi_n(x_n)\} \geq \delta\} \\ &= \{\mathbf{x} = (x_1,\dots,x_n) : \xi_i(x_i) \geq \delta \quad \forall i = 1(1)n\} \\ &= X_{i=1}^n C_\delta[\xi_i(\cdot)] \end{aligned}$$

The concepts of combined fuzzy samples and triangular norms are useful for succinct multivariate statistical analysis of fuzzy data.

4.2 Histograms for fuzzy data

For a given partition of the observation space and fuzzy data in some cases, it is not possible to decide to which class a fuzzy observation belongs. This makes it necessary to generalize the concept of histograms. A first step is to construct lower and upper values of the frequencies $h_n(K_j)$ of class K_j, j = 1(1)k.

Let $x_1^*, \dots, x_n^*$ be fuzzy data with corresponding characterizing functions $\xi_1(\cdot),\dots,\xi_n(\cdot)$. Then, based on supp[$\xi_i(\cdot)$], the lower value $\underline{h}_n(K_j)$ is determined by

$$\underline{h}_n(K_j) := \frac{\{\#x_i^* : \text{supp}[\xi_i(\cdot)] \subseteq K_j\}}{n} \quad \forall j = 1(1)k,$$

And the upper value $\overline{h}_n(K_j)$ is defined by

$$\overline{h}_n(K_j) := \frac{\{\#x_i^* : \text{supp}[\xi_i(\cdot)] \cap K_j \neq \emptyset\}}{n},$$

where # indicates cardinality. Consequently, an interval valued histogram is obtained, whose frequencies are the intervals $h_n(K_j) = [\underline{h}_n(K_j), \overline{h}_n(K_j)]$ for j = 1(1)k.

Remark 4.1 For two disjoint classes K_j and K_l, the lower values of the frequencies are super-additive, *i.e.* $\underline{h}_n(K_j \cup K_l) \geq \underline{h}_n(K_j) + \underline{h}_n(K_l)$, while the upper values are sub-additive, *i.e.* $\overline{h}_n(K_j \cup K_l) \leq \overline{h}_n(K_j) + \overline{h}_n(K_l)$.

This is, in fact, easily seen for $\underline{h}_n(\cdot)$ by

$$\#x_i^* : \text{supp}[\xi_i(\cdot)] \subseteq [K_j \cup K_l] \geq \#x_i^* : \text{supp}[\xi_i(\cdot)] \subseteq K_j + \#x_i^* : \text{supp}[\xi_i(\cdot)] \subseteq K_l,$$

and for $\overline{h}_n(\cdot)$ by

$$\#x_i^* : \text{supp}[\xi_i(\cdot)] \cap [K_j \cup K_l] \neq \emptyset \leq \#x_i^* : \text{supp}[\xi_i(\cdot)] \cap K_j \neq \emptyset + \#x_i^* : \text{supp}[\xi_i(\cdot)] \cap K_l \neq \emptyset.$$

In Figure 4.1, an example of an interval-valued histogram is depicted.

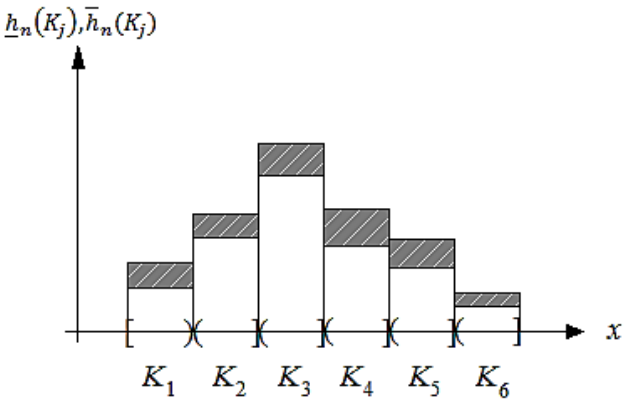

Figure 4.1. Interval-valued Histogram

Remark 4.2 Similar to the conditions for classical relative frequencies, whose abstraction leads to the axioms of probability distributions, the conditions of interval-valued frequencies are abstracted as axioms for so-called *interval probabilities*.

A more informative generalization of histograms is obtained when the considerations above are made for each δ-level. Then the results are fuzzy numbers $h_n^*(K_j)$ as fuzzy frequencies. The construction is the following:

For each $\delta \in (0,1]$, the generating family of intervals for the characterizing function of the fuzzy frequency $h_n^*(K_j)$ is $A_{j,\delta} = [\underline{h}_{n,\delta}(K_j), \overline{h}_{n,\delta}(K_j)]$, where $\underline{h}_{n,\delta}(K_j) := \frac{\{\#x_i^* : C_\delta[\xi_i(\cdot)] \subseteq K_j\}}{n}$ and $\overline{h}_{n,\delta}(K_j) := \frac{\{\#x_i^* : C_\delta[\xi_i(\cdot)] \cap K_j \neq \emptyset\}}{n}$.

By application of the construction lemma 2.2, the characterizing function of $h_n^*(K_j)$ is obtained.

Remark 4.3 The characterizing functions of fuzzy relative frequencies are step functions. Moreover, fuzzy histograms are examples of fuzzy valued functions:

$$f^*(x) := h_n^*(K_j) \quad \forall x \in K_j$$

Similar to the interval-valued relative frequencies, i.e. for the generating values $\underline{h}_{n,\delta}(K_j)$ and $\overline{h}_{n,\delta}(K_j)$, the following holds: For $K_j \cap K_l = \emptyset$,

$$\underline{h}_{n,\delta}(K_j \cup K_l) \geq \underline{h}_{n,\delta}(K_j) + \underline{h}_{n,\delta}(K_l)$$
$$\overline{h}_{n,\delta}(K_j \cup K_l) \leq \overline{h}_{n,\delta}(K_j) + \overline{h}_{n,\delta}(K_l).$$

Figure 4.2 displayed the axonometric picture of a fuzzy histogram (Viertl 2011).

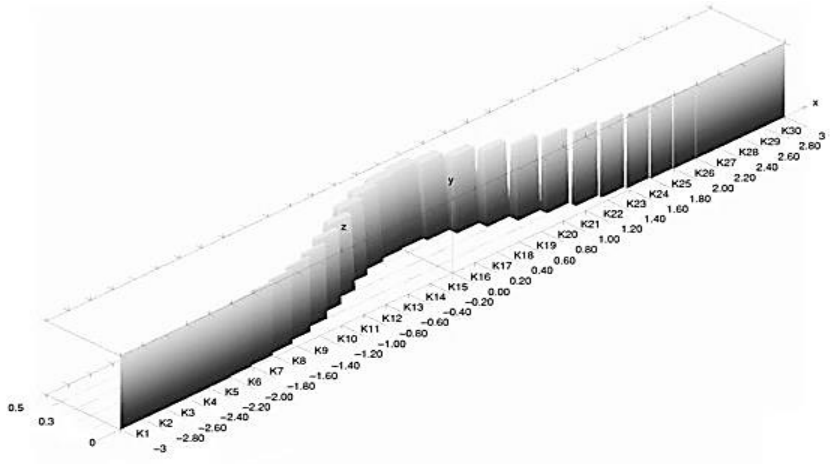

Figure 4.2. Fuzzy Histogram

4.3 Empirical distribution functions for fuzzy data

In order to generalize the empirical distribution function $\hat{F}_n(\cdot)$ to the situation of fuzzy data $x_1^*, \ldots, x_n^*$ with characterizing functions $\xi_1(\cdot), \ldots, \xi_n(\cdot)$, the following construction, using δ-cuts is useful which yields a fuzzy valued function $\hat{F}_n^*(\cdot)$: For fixed $x \in \mathbb{R}$ and $\delta \in (0,1]$, we define

$$\hat{F}_{\delta,U}(x) := \frac{\#x_i^*: C_\delta[\xi_i(\cdot)] \cap (-\infty, x] \neq \emptyset}{n}$$

$$\hat{F}_{\delta,L}(x) := \frac{\#x_i^*: C_\delta[\xi_i(\cdot)] \subseteq (-\infty, x]}{n}.$$

The generating system of intervals for the fuzzy value $\hat{F}_n^*(x)$ is $A_\delta(x) = [\hat{F}_{\delta,L}(x), \hat{F}_{\delta,U}(x)]$, $\delta \in (0,1]$. The characterizing function of $\hat{F}_n^*(x)$ is obtained through lemma 2.2.

For variable $x \in \mathbb{R}$, two functions $\hat{F}_{\delta,L}(\cdot)$ and $\hat{F}_{\delta,U}(\cdot)$ are obtained, which in turn determine the $\hat{F}_n^*(\cdot)$ for variable δ. Based on $\hat{F}_n^*(\cdot)$, empirical fractiles for the fuzzy empirical distribution can be defined.

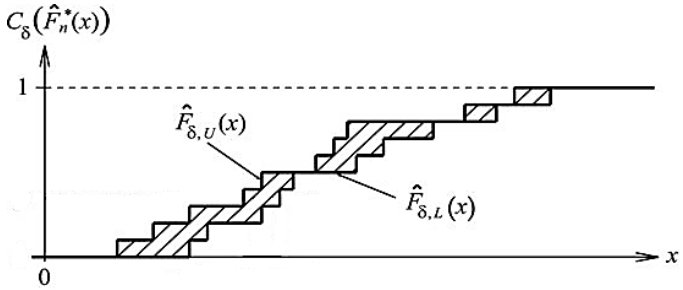

Figure 4.3. δ-Cut of $\hat{F}_n^*(\cdot)$

For $p \in (0,1)$, the lower and upper δ-level curves $\hat{F}_{\delta,L}(\cdot)$ and $\hat{F}_{\delta,U}(\cdot)$ are used to define the generating family of intervals $(A_\delta; \delta \in (0,1))$ for the fuzzy interval q_p^*, which is the empirical fractile of $\hat{F}_n^*(\cdot)$:

$$A_\delta := \left[\hat{F}_{\delta,U}^{-1}(p), \hat{F}_{\delta,L}^{-1}(p)\right] \quad \forall \delta \in (0,1)$$

where $\hat{F}_{\delta,U}^{-1}(p)$ and $\hat{F}_{\delta,L}^{-1}(p)$ are defined by

$\hat{F}_{\delta,U}^{-1}(p) := min\{x \in \mathbb{R}: \hat{F}_{\delta,U}(x) = p\}$ and $\hat{F}_{\delta,L}^{-1}(p) := max\{x \in \mathbb{R}: \hat{F}_{\delta,L}(x) = p\}$.

The characterizing function of the fuzzy fractile q_p^* is obtained from the construction lemma 2.2. An example for the definition of A_δ is given in Figure 3.4.

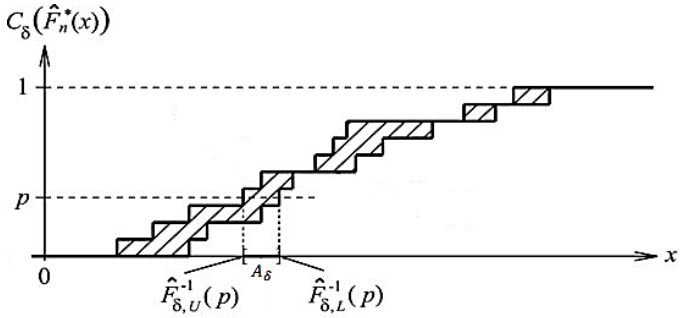

Figure 4.4. The definition of A_δ for q_p^*

4.4 Fuzzy empirical correlation

In case of fuzzy 2-dimensional data given as 2-dimensional fuzzy vector $(x_i, y_i)^*$, *i* = 1(1)*n* with vector-characterizing functions $\zeta_i(\cdot,\cdot)$, the classical correlation coefficient can be generalized by application of the extension principle (Definition 3.1). First, the fuzzy vectors have to be combined into a fuzzy vector in the sample space $\mathbb{R}^{2n}$ by application of the minimum-t-norm, *i.e.* the vector-characterizing function $\zeta(\cdot,\ldots,\cdot)$ of the fuzzy combined sample is given by its values

$$\zeta(x_1, y_1, \ldots, x_n, y_n) := min\{\zeta_i(x_i, y_i): i = 1(1)n\} \quad \forall (x_1, y_1, \ldots, x_n, y_n) \in \mathbb{R}^{2\cdot n}.$$

Now the extension principle is applied to the function

$$g(x_1, y_1, \ldots, x_n, y_n) = \frac{\sum_{i=1}^n (x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^n (x_i - \overline{x})^2}\sqrt{\sum_{i=1}^n (y_i - \overline{y})^2}}$$

and the characterizing function $\psi_{r^*}(\cdot)$ of the generalized (fuzzy) empirical correlation coefficient r^* is given by its values

$$\psi_{r^*}(r) := \begin{Bmatrix} sup\{\zeta(x_1, y_1, \ldots, x_n, y_n): \text{for } g(x_1, y_1, \ldots, x_n, y_n) = r\} \\ 0 \text{ for } \nexists\ (x_1, y_1, \ldots, x_n, y_n): g(x_1, y_1, \ldots, x_n, y_n) = r \end{Bmatrix} \quad \forall r \in \mathbb{R}.$$

Remark 4.4 The support of $\psi_{r^*}(\cdot)$ is a subset of the interval [-1,1].

In figure 4.5, a fuzzy sample and the corresponding generalized empirical correlation coefficient r^* is given.

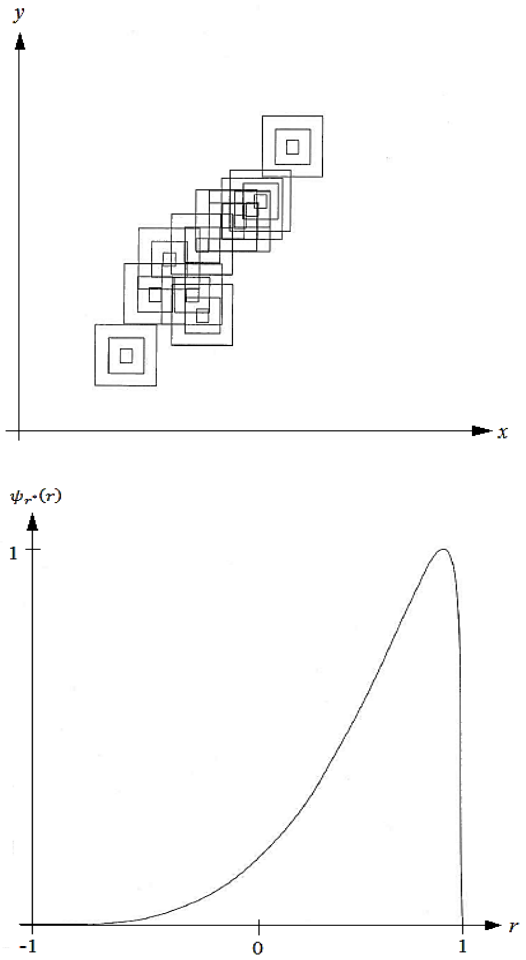

Figure 4.5. Fuzzy 2-dimensional data and its empirical correlation coefficient r^*

Remark 4.5 The definition of r^* allows different types of data. These data can be in form of fuzzy components x_i^* and y_i^*.

5. Objectivist Statistical Inference for Fuzzy Data

In objectivist statistics, it is assumed that a true underlying distribution exists which has to be estimated or tested. For parametric stochastic model $X \sim f(\cdot\,|\theta)$, $\theta \in \ominus$, the existence of a true value θ_0 of the parameter is assured, as a result, this θ_0 has to be estimated as good as possible.

5.1 Generalized Point Estimators

This is the generalization of so-called point estimators from standard statistics. For a stochastic quantity *X* with observation space M_X a certain characteristic value should be an element of N, where (N, ϱ) is a measurable space. Let $\tau: M_X^n \to N$ be a standard point estimator, which is a measurable function from the sample space M_X^n to the space of possible values of the characteristic value. This estimator is generalized by application of the extension principle in the following way:

Let $x_1^*, \ldots, x_n^*$ with $x_i^* \mathrel{\widehat{=}} \xi_i(\cdot)$ be a fuzzy sample; first the vector-characterizing function $\zeta(\cdot,\ldots,\cdot)$ of the combined fuzzy sample $\mathbf{x}^*$ is determined by its values

$$\zeta(x_1, \ldots, x_n) := min\{\xi_1(x_1), \ldots, \xi_n(x_n)\} \quad \forall (x_1, \ldots, x_n) \in \mathbb{R}^n.$$

Then, the membership function of the fuzzy estimate $\tau(x_1^*, \ldots, x_n^*)$ is given by

$$\eta(s) = \begin{cases} sup\left\{min\{\xi_1(x_1), \ldots, \xi_n(x_n)\} : \tau(x_1, \ldots, x_n) = s\right\} \\ 0 \quad \text{if } \nexists (x_1, \ldots, x_n) \in \mathbb{R}^n : \tau(x_1, \ldots, x_n) = s \end{cases} \quad \forall s \in N.$$

Remark 5.1 If the parameter θ_0 of a parametric stochastic model $X \sim f(\cdot\,|\theta)$ has to be estimated, a generalized estimator $\vartheta(x_1^*, \ldots, x_n^*)$ is a fuzzy element of the parameter space $\ominus$.

Example 5.1 Let X be a stochastic quantity whose expectation $\mathbb{E}(X)$ exits. Then, for the fuzzy sample given in Figure 5.1, the fuzzy estimate for this sample is the generalization of the standard estimator $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. The characterizing function of the fuzzy estimate $\overline{x}^* = \frac{1}{n}\sum_{i=1}^{n} x_i^*$ is depicted in Figure 5.2.

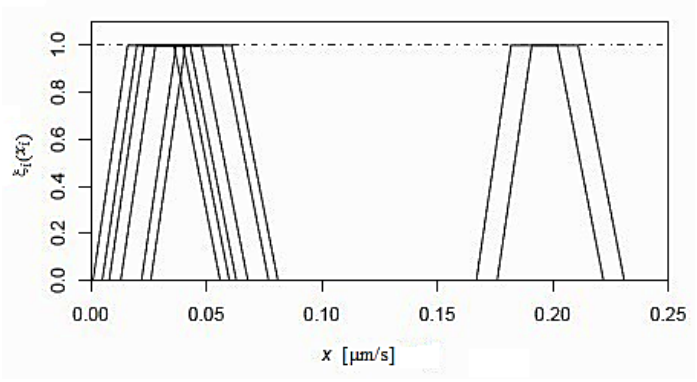

Figure 5.1. Characterizing functions of a fuzzy sample

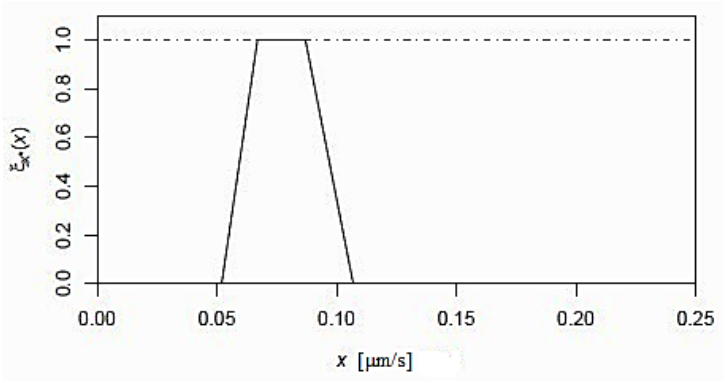

Figure 5.2. Characterizing function of the fuzzy sample mean

5.2 Generalized Confidence Sets

For a given parametric stochastic model $X \sim f_\theta$, $\theta \in \ominus$ and a standard confidence function $\boldsymbol{\kappa}: M_X^n \to \mathcal{P}(\ominus)$, where $\mathcal{P}(\ominus)$ denotes the power set of the parameter space $\ominus$, in case of fuzzy sample $x_1^*, \ldots, x_n^*$, the concept of confidence set has to be generalized. This is possible in the following way:

Based on the combined fuzzy sample $\mathbf{x}^*$, with vector-characterizing function $\zeta(\cdot,\ldots,\cdot)$, the membership function $\varphi(\cdot)$ of the generalized (fuzzy) confidence set $\ominus_{1-\alpha}^*$ is defined by

$$(\theta) = \begin{cases} sup\{\zeta(\mathbf{x}): \theta \in \boldsymbol{\kappa}(\mathbf{x})\} & \text{if } \exists \mathbf{x} \in M_X^n: \theta \in \boldsymbol{\kappa}(\mathbf{x}) \\ 0 & \text{if } \nexists \mathbf{x} \in M_X^n: \theta \in \boldsymbol{\kappa}(\mathbf{x}) \end{cases} \quad \forall \theta \in \ominus, \text{ where } \mathbf{x} = (x_1, \ldots, x_n) \in M_X^n.$$

For the membership function (·), the following holds true:

$$I_{\bigcup_{\mathbf{x}:\zeta(\mathbf{x})=1} \boldsymbol{\kappa}(\mathbf{x})}(\theta) \leq \varphi(\theta) \qquad \forall \theta \in \ominus,$$

which can be proven by observing $sup\{\zeta(\mathbf{x}): \theta \in \boldsymbol{\kappa}(\mathbf{x})\}$. Assuming $\theta \in \bigcup_{\mathbf{x}:\zeta(\mathbf{x})=1} \boldsymbol{\kappa}(\mathbf{x})$, then

$$\exists \mathbf{x} \in M_x^n: \theta \in \boldsymbol{\kappa}(\mathbf{x}) \Rightarrow \zeta(\mathbf{x}) = 1 \Rightarrow \varphi(\theta) = sup\{\zeta(\mathbf{x}): \theta \in \boldsymbol{\kappa}(\mathbf{x})\} = 1$$

$$\Rightarrow \varphi(\theta) \geq 1 \Rightarrow \varphi(\theta) \geq I_{\bigcup_{\mathbf{x}:\zeta(\mathbf{x})=1} \boldsymbol{\kappa}(\mathbf{x})}(\theta)$$

5.3 Statistical Tests based on Fuzzy Data

Classical test statistics $T = t(x_1, \dots, x_n)$ based on standard samples $x_1, \dots, x_n$ are measurable functions from the sample space M_X^n to a suitable measurable space (N, ϱ) which is partitioned into an *acceptance region A* and its complement *N**A*, i.e. the *rejection region*.

For fuzzy samples $x_1^*, \dots, x_n^*$, the value $t^* = t(x_1^*, \dots, x_n^*)$ becomes fuzzy, and therefore it can be ambiguous to which region the value t^* belongs. Therefore, statistical tests have to be adapted accordingly.

A first solution would be to take observations until the support of t^* is a subset of *A* or A^c. Another method is to determine a *p*-value based on the fuzzy value t^*, where this *p*-value is defined as the probability of an error of the first type (rejecting a true hypothesis) for which the support of t^* is only contained in the rejection region A^c of the corresponding classical standard test.

Remark 5.2 For fuzzy samples, a more natural approach is the generalization of *p*-values in form of fuzzy numbers, as explained here:

Let $\eta(\cdot)$ be the characterizing function of the fuzzy value t^* of the generalized test statistic $t(x_1^*, \dots, x_n^*)$. Considering the δ-cuts of t^*, $C_\delta[t^*] = [t_1(\delta), t_2(\delta)] \ \forall \delta \in (0,1]$, the fuzzy *p*-value p^* for a given standard test is defined in the following way:

For one-sided tests with test statistic T and decision rule "rejection for $T \le t_{critical}$", the generating family of intervals for the fuzzy *p*-value p^* is defined by

$$A_\delta := [Pr\{T \le t_1(\delta)\}, Pr\{T \le t_2(\delta)\}] \qquad \forall \delta \in (0,1].$$

For one-sided tests with decision rule "rejection for $T \ge t_{critical}$", the generating family $(A_\delta; \delta \in (0,1])$ for p^* is defined by

$$A_\delta := [Pr\{T \ge t_2(\delta)\}, Pr\{T \ge t_1(\delta)\}] \qquad \forall \delta \in (0,1].$$

In case of two-sided tests, *i.e.* acceptance for $t_l \le T \le t_u$, and fuzzy value t^* of the test statistic T, first, it has to be decided on which side, of the median *m* of the distribution of T, the main part of fuzziness of t^* is located. Therefore, the areas under the characterizing function $\eta(\cdot)$ of t^* have to be computed, which are on both sides of the median *m*. Denoting these areas by F_1 and F_2 respectively, the generating family $(A_\delta; \delta \in (0,1])$ for the fuzzy *p*-value p^* are defined by

$$A_\delta := \begin{cases} [2Pr\{T \le t_1(\delta)\}, min\{1, Pr\{T \le t_2(\delta)\}\}] \text{ if } F_1 > F_2 \\ [2Pr\{T \ge t_2(\delta)\}, min\{1, Pr\{T \ge t_1(\delta)\}\}] \text{ if } F_1 \le F_2 \end{cases} \forall \delta \in (0,1].$$

The δ-cuts of p^* are denoted by $[p_1(\delta), p_2(\delta)]$, and can be interpreted in terms of generalized probabilities, and can be compared with the significance level α of the test. The decision is made according to a three-decision testing problem:

If, for all $\delta \in (0,1]$ and $p_1(\delta) \le p_2(\delta)$,

$p_2(\delta) < \alpha$: reject $\mathcal{H}_0$ and accept $\mathcal{H}_1$

$p_1(\delta) > \alpha$: accept $\mathcal{H}_0$ and reject $\mathcal{H}_1$

$\alpha \in [p_1(\delta), p_2(\delta)]$: both $\mathcal{H}_0$ and $\mathcal{H}_1$ are neither accepted nor rejected.

In the third case, the uncertainty of making a decision is expressed by the characterizing function $\xi(\cdot)$ of p^*.

The case that $t_1(\delta) = t_2(\delta)$ for all $\delta \in (0,1]$ implies $p_1(\delta) = p_2(\delta)$, *i.e.* we have a two-decision problem similar to tests based on precise data (Filzmoser 2004).

Example 5.2 Let the test statistic T have a standard normal distribution with density $f(\cdot)$ and the characterizing function $\eta(\cdot)$ of t^*(0.2,0.7,1.2) is of symmetric triangular shape as shown in Figure 4.3a. The following one-sided statistical hypotheses are to be tested at the significance level α = 0.05,

$\mathcal{H}_0$: $\theta \le \theta_0$ vs. $\mathcal{H}_1$: $\theta > \theta_0$; θ is an unknown parameter and θ_0 = 0 in this example, *i.e.* $\mathcal{H}_0$: $\theta \le 0$ vs. $\mathcal{H}_1$: $\theta > 0$. The δ-cuts of the fuzzy *p*-value p^* as defined above are to be determined for decision.

In Figure 4.3a, the δ-cut of t^*, $C_\delta[t^*] = [t_1(\delta), t_2(\delta)]$, for δ = 0.5 is derived as $[t_1(0.5), t_2(0.5)] = [0.45, 0.95]$ accordingly. The *p*-values corresponding to $t_1(0.5)$ and $t_2(0.5)$ are

0.33 and 0.17 respectively as shown in shaded areas in Figure 4.3a and the construction of their characterizing function in Figure 4.3b. Finally, the resulting fuzzy *p*-value is compared to the significance level α = 0.05 and $\mathcal{H}_0$ conclude that is not rejected.

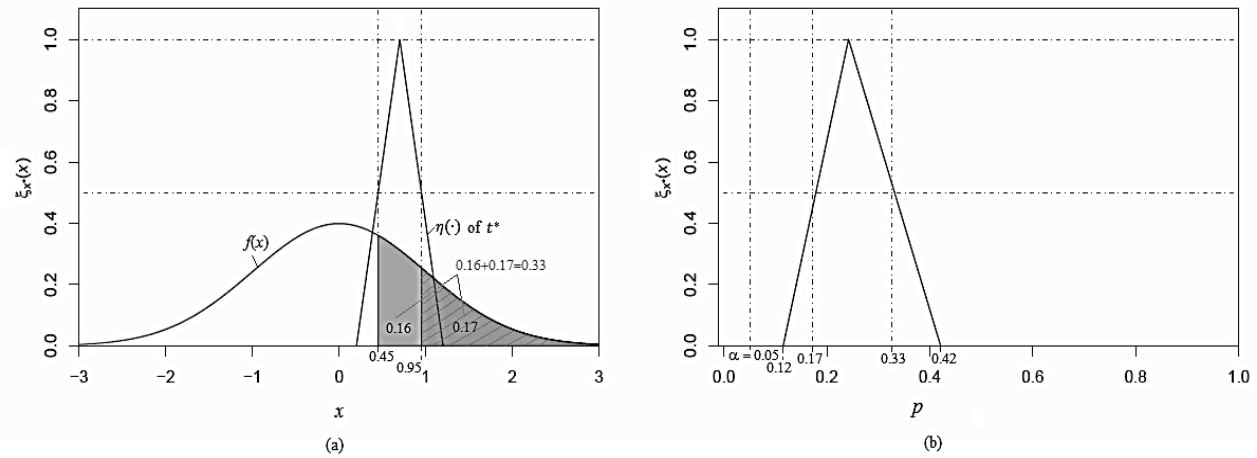

Figure 4.3: Construction of the Characterizing Function of the Fuzzy *p*-value

6. Bayesian Inference and Fuzzy Information

Bayesian inference uses a-priori information of specific parameters in stochastic models. In other words, Bayes' theorem formulates the transition from the a-priori distribution $\pi(\theta)$ of the stochastic quantity, describing parameters of interest, to the so-called a-posteriori distribution $\pi(\theta|D)$ based on data *D*. In case of continuous stochastic models *X* ~ $f(\cdot|\theta)$; $\theta \in \ominus$, based on observations $x_1, \dots, x_n$ of *X*, the transition from an a-priori density to an updated information with the distribution of the stochastic quantity describing the parameter θ is given by the conditional density $\pi(\cdot \mid x_1, \dots, x_n)$ of $\tilde{\theta}$, *i.e.* Bayes' theorem

$$\pi(\theta|x_1,\dots,x_n) = \frac{\pi(\theta)\cdot l(\theta; x_1,\dots,x_n)}{\int_{\ominus} \pi(\theta)\cdot l(\theta; x_1,\dots,x_n)d\theta} \text{ or } \pi(\theta|x_1,\dots,x_n) \propto \pi(\theta)\cdot l(\theta; x_1,\dots,x_n) \quad \forall\theta \in \ominus,$$

where $l(\theta; x_1, \dots, x_n)$ is the likelihood function defined on the parameter space $\ominus$.

In standard Bayesian inference, it is assumed here that $\pi(\theta)$ is a standard (classical) probability density, and the data are given as numbers or vectors.

For a more realistic Bayesian statistical inference, the inevitable uncertainties (including fuzziness) have to be addressed, *i.e.* the fuzziness of continuous quantities and of the a-priori knowledge, however. As explained in Section 2, this fuzziness can be defined in form of fuzzy numbers or vectors and modelled by the so-called fuzzy probability densities, respectively. Finally, Bayes' theorem can be generalized to handle the situation of fuzzy a-priori density and fuzzy data.

6.1 Fuzzy a-priori densities

Based on fuzzy probability densities (Section 2.2), generalized fuzzy probabilities of events $A \in \mathcal{A}$, $P^*(A)$ are fuzzy intervals. The characterizing function $\xi(\cdot)$ of $P^*(A)$ is given by the generation lemma (Viertl 2011), *i.e.* $\xi(y) = sup\{\delta.I_{[a_\delta, b_\delta]}(x)\colon \delta \in (0,1]\}$ for all $y \in \mathbb{R}$, where $I_B(\cdot)$ denotes the indicator function of the set *B*, and $[a_0, b_0] := \mathbb{R}$.

Definition 6.1: A fuzzy a-priori density on the parameter space $\ominus$ is a fuzzy density defined on $\ominus$.

6.2 Generalized likelihood function

The likelihood function $l(\theta; x_1, \dots, x_n)$ in Bayes' theorem can be generalized for fuzzy data $x_1^*, \dots, x_n^*$ through the extension principle and based on the combined fuzzy sample $\mathbf{x}^*$ and its vector-characterizing function $\zeta(\cdot,\dots,\cdot)$. The characterizing function $\psi(\cdot)$ of the fuzzy value $l(\theta; \mathbf{x}^*)$ is, then, given by its values as

$$\psi_{l(\theta;\mathbf{x}^*)}(y) = \begin{cases} sup\{\zeta(\mathbf{x})\colon \mathbf{x} \in M_X^n \wedge l(\theta;\mathbf{x}) = y\} & \text{if } \exists \mathbf{x} : l(\theta;\mathbf{x}) = y \\ 0 & \text{if } \nexists \mathbf{x} \in M_X^n \colon l(\theta;\mathbf{x}) = y \end{cases} \quad \forall y \in \mathbb{R}.$$

Based on the generalized likelihood function, Bayes' theorem is generalized for fuzzy a-priori densities $\pi^*(\cdot)$ on the parameter space $\ominus$ and fuzzy data.

6.3 Generalized Bayes' theorem for fuzzy data

The following description of Bayes' theorem is generalized for the continuous case. However, for discrete variables, Bayes' formula can also be generalized similarly (Sunanta 2016). Let $X \sim f(\cdot|\theta)$; $\theta \in \ominus$ be a continuous stochastic model with continuous parameter space $\ominus \subseteq \mathbb{R}^k$, and $\pi^*(\cdot)$ a fuzzy a-priori density on the parameter space $\ominus$. The sequential updating from standard Bayes' theorem is generalized as following:

Given a fuzzy a-priori density $\pi^*(\cdot)$ and fuzzy sample $x_1^*, \dots, x_n^*$, the generalized a-posteriori density $\pi^*(\cdot \mid x_1^*, \dots, x_n^*)$ is then calculated, from which the result is the same as if the sample were separated in two parts $x_1^*, \dots, x_m^*$ and $x_{m+1}^*, \dots, x_n^*$, *i.e.* the a-posteriori density of the first partition $\pi^*(\cdot \mid x_1^*, \dots, x_m^*)$ is obtained. Then, this a-posteriori density is taken as new a-priori density, based on which the a-posteriori density $\pi^*(\cdot \mid x_{m+1}^*, \dots, x_n^*)$ is finally calculated.

The generalized fuzzy Bayes' theorem is based on δ-level functions of the fuzzy a-priori density and of the generalized likelihood function, as well as the vector-characterizing function of the combined fuzzy sample $\mathbf{x}^*$.

Using the above notation, the δ-level functions of the fuzzy a-posteriori density $\pi^*(\cdot \mid x_1^*, \dots, x_n^*)$, with combined fuzzy sample $\mathbf{x}^*$, obtained by minimum-combination rule, are defined in the following way:

$$\overline{\pi}_\delta(\theta|\mathbf{x}^*) = \frac{\overline{\pi}_0(\theta)\cdot \overline{l}_\delta(\theta;\mathbf{x}^*)}{\int_\ominus \frac{1}{2}\left[\underline{\pi}_0(\theta)\cdot \underline{l}_\delta(\theta;\mathbf{x}^*) + \overline{\pi}_0(\theta)\cdot \overline{l}_\delta(\theta;\mathbf{x}^*)\right] d\theta}$$

and $\forall \theta \in \ominus$

$$\underline{\pi}_\delta(\theta|\mathbf{x}^*) = \frac{\underline{\pi}_0(\theta)\cdot \underline{l}_\delta(\theta;\mathbf{x}^*)}{\int_\ominus \frac{1}{2}\left[\underline{\pi}_0(\theta)\cdot \underline{l}_\delta(\theta;\mathbf{x}^*) + \overline{\pi}_0(\theta)\cdot \overline{l}_\delta(\theta;\mathbf{x}^*)\right] d\theta}$$

The averaging in the integral is necessary to keep the sequential updating condition (Viertl 2011).

6.4 Fuzzy predictive distribution

In Bayesian inference, the predictive density based on the a-posteriori density $\pi(\cdot|D)$ is defined to be the marginal density of X from $(X,\tilde{\theta})$ denoted by $p(\cdot|D)$, whose values are given by

$$p(x|D) := \int_\ominus f(x|\theta)\pi(\theta|D)\, d\theta \quad \forall x \in M_X.$$

The generalization for fuzzy a-posteriori densities $\pi^*(\cdot|\,\mathbf{x}^*)$ is obtained through a construction similar to the calculation of probabilities based on fuzzy densities (section 2.2).

Defining a continuous stochastic model $X \sim f(\cdot|\theta)$; $\theta \in \ominus$ and the fuzzy a-posteriori density $\pi^*(\cdot|\,\mathbf{x}^*)$, the generalized integral $\int_\ominus f(x|\theta)\pi^*(\theta|\,\mathbf{x}^*)\, d\theta \quad \forall x \in M_X$ is a fuzzy interval $p^*(x|\mathbf{x}^*)$ whose characterizing function is generated by the following family $(A_\delta; \delta \in (0,1])$ of intervals A_δ:

For all $\delta \in (0,1]$, let $\underline{\pi}_\delta(\cdot|\mathbf{x}^*)$ and $\overline{\pi}_\delta(\cdot|\mathbf{x}^*)$ be the δ-level functions of the fuzzy a-posteriori density $\pi^*(\cdot|\mathbf{x}^*)$, and D_δ the set of all classical probability densities $g(\cdot)$ on $\ominus$, where $\underline{\pi}_\delta(\theta|\mathbf{x}^*) \le g(\theta) \le \overline{\pi}_\delta(\theta|\mathbf{x}^*)$ $\forall \theta \in \ominus$.

The interval $A_\delta = [c_\delta, d_\delta]$, $\delta \in (0,1]$ is defined by

$c_\delta := inf\{\int_\ominus g(\theta)d\theta : g \in D_\delta\}$ and $d_\delta := sup\{\int_\ominus g(\theta)d\theta : g \in D_\delta\}$. The system of intervals $[c_\delta, d_\delta]$, $\delta \in (0,1]$, is nested by the validity of $\delta_1 < \delta_2 \Rightarrow D_{\delta_1} \supseteq D_{\delta_2}$.

The characterizing function of $p^*(x|\mathbf{x}^*)$ is given by the generation lemma from Section 6.1, *i.e.*

$\psi_x(y) = sup\{\delta . I_{[c_\delta, d_\delta]}(y) : \delta \in [0,1]\}$ $\forall y \in \mathbb{R}$, where $[c_0, d_0] := \mathbb{R}$.

The fuzzy predictive density is, then, the family of $p^*(x|\mathbf{x}^*)$, $x \in M_X$.

7 Fuzzy Stochastic Processes

Stochastic processes are families of stochastic quantities $X_t, t \in T$ where T is an appropriate index set, frequently T is a time interval, *i.e.* $T \subseteq \mathbb{R}$. The occurring of uncertainty can be modelled using the theory of *fuzzy random processes*, which is derived from the theory of fuzzy random functions (Möller 2009). By the imprecision of data from continuous quantities, the observations (trajectories) of the process are fuzzy. Therefore, the sample paths are fuzzy valued functions. For the so-called *fuzzy stochastic process* $X_t^*, t \in T$, different mathematical models exist, i.e. X_t^* can be modelled as *fuzzy random variables*.

A special case would be time series, in which case the uncertainty of the individual observed value is modeled as a fuzzy variable $X_n^*, n = 1(1)N$. A time series of fuzzy data may be viewed as a random realization of a *fuzzy stochastic process*, of which the generalized model is necessary for further forecasting (Möller and Reuter 2007). However, for fuzzy stochastic processes with continuous time, many challenging research topics are still open, *e.g.* the first hitting times, predictions, an analogue of convergence theorems.

Conclusions

For observations and measurements of continuous quantities, fuzziness is unavoidable. Therefore, suitable mathematical models are necessary to describe real data. This is possible and many research topics related to this are still waiting to be solved. The fuzziness of individual measurement results can be described by so-called fuzzy numbers, whereas the variability and errors are described by stochastic models.

In this contribution, some generalized statistical methods for fuzzy data, *i.e.* descriptive statistics, statistical inference, and Bayesian inference are described. Descriptive statistics provide simple summaries of the collected samples and measures (data). They form the basis of virtually every quantitative analysis of the data. Through concepts of fuzzy numbers and characterising functions, fuzzy data are summarised and represented in form of fuzzy histograms. Some other statistics, such as fuzzy empirical distribution functions and correlation coefficients, are also useful for preliminary data analysis. For realistic projection of the behaviours of the variables under analysis, models for prediction based on fuzzy information, through Bayesian inference and fuzzy predictive density, are introduced.

Fuzziness is everywhere in the physical world, including in economics arena. In order to describe different facets of reality, the analysis methods have to capture this type of uncertainty. The related methods are available through mathematical models for fuzzy data. Accordingly, application of such methods results in more realistic models for data analysis and, subsequently, better understanding of the collected data for further use of such information.

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POLICY ON BALANCED REGIONAL DEVELOPMENT IN MACEDONIA - GOALS, CHALLENGES, TRENDS

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Abstract

In this analysis based on available statistics per different regions in Macedonia, we make attempt first to establish the actual situation in terms of distribution of regional development and identify tendencies in individual regions, which will later serve as solid baseline for establishing possible positive movements in individual regions and for identifying key challenges in the policy on balanced regional development, as well as for testing the policy's effectiveness in the course of years. Moreover, our focus will be on legislative establishment of the policy on balanced regional development in the past decade, assessment of what has been planned and realized, identification of weaknesses and shortfalls in policy implementation, and provision of recommendations aimed at better implementation of these policies in the future.

Key words: balanced regional development; state and local government; intergovernmental relations; public economics; Macedonia

JEL Classification: H11; H72; H73; D78; H77

Introduction

As a concept, regional development implies continuous financial support from the state and high degree of coordination between the ministries, donors and stakeholders at regional and at local level. Regional policy is one of most prominent features in functioning of the European Union, where it is called the Cohesion Policy[1] and whose goal is to improve welfare of the regions across Europe and reduce regional imbalances.

Adoption of the regional development policy in Republic of Macedonia implied an attempt for alignment of national priorities with those defined under EU's policy on economic and social cohesion and those identified in the Lisbon Strategy.[2] This alignment was made for the purpose of stimulating development of planning regions in line with the EU guidelines, and for capacity building of planning regions and local self-government units for utilization of relevant components under IPA available to Macedonia.

In Republic of Macedonia, the Law on Balanced Regional Development was adopted in 2007 and implied one of the most important steps towards introduction of new approach to address the problem that, for decades

[1] More than one third of the EU Budget is allocated in support of this policy.

[2] More information on the Lisbon Strategy are available at: http://www.consilium.europa.eu/en/uedocs/cms_data/docs/pressdata/en/ec/00100-r1.en0.htm

had been treated as underdevelopment of certain specific areas, underdeveloped regions, etc.[3] This piece of legislation explicitly defined the policy on stimulating balanced regional development as system of goals, instruments and measures aimed at addressing regional disparities and attaining sustainable development in Republic of Macedonia.

The need for adoption of this law and policy was justified with long-standing absence of regional development policy and dominant concentration of most economic activities in Skopje region and partial development of greater urban centres in other planning regions. High concentration of population and economic activities in urban centres has negative effects on social and spatial aspects of development, ultimately resulting in extinction of large portion of rural settlements, while creating problems in operation of urban centres due to the lack of relevant technical and social infrastructure that would sustain higher population density. Therefore, disparities emerged in economic, social and other aspects of development between and within planning regions and provided the starting point for regional development planning.

All strategic, programme and planning documents that followed after the law's adoption, as well as projects for implementing the policy on balanced regional development (hereinafter: ERD) in the last 10 years have brought to the surface *numerous shortfalls and problems* in policy performance, marked by many delays and breaches of law-stipulated deadlines.

As part of this analysis and based on available statistics per region, we will attempt first to establish the actual situation in terms of distribution of regional development and identify tendencies in individual regions, which will later serve as solid baseline for establishing possible positive movements in individual regions and for identifying key challenges in the policy on balanced regional development, as well as for testing the policy's effectiveness during years. Moreover, our focus will be on legislative establishment of the policy on balanced regional development in the past decade, assessment of what has been planned and realized, identification of weaknesses and shortfalls in policy implementation, and provision of recommendations aimed at better implementation of these policies in the future.

1. Why we need policy on balanced regional development – statistical indicators per region

In its recent editions of the publication called "Macedonia in Figures", the State Statistical Office publishes data on regional dispersion of GDP. Value of this indicator is zero when GDP per capita in all regions across the country is identical, and increases in proportion with growing differences between regional GDP per capita and average GDP per capita at national level. 2014 edition of this publication presented data that regional dispersion of GDP was 31.0% in 2010, 29.3% in 2011 and 31.5% in 2012.[4] Next year's edition of "Macedonia in Figures 2015" corrected these figures on the basis of the new methodology, whereby regional dispersion of GDP accounted for 28.9% in 2010 (correction by 2.1%), 26.5% in 2011 (correction by 2.8%), and 29.1% in 2012 (correction by 2.4%).[5] The most recent edition published in 2016 shows that, in 2013, regional dispersion of GDP was 29.0%.[6]

Although action plans and other government documents emphasize that measure and activities aimed at stimulating balanced regional development were implemented in the period after the regional development policy was adopted, and resulted in reduced development disparities between the City of Skopje and other planning regions,[7] many indicators confirm that these differences are not reduced, and in some regions they continue to increase compared to Skopje region.

In continuation of this analysis, we present and elaborate several tables with data that provide general conclusions in terms of regional development, as well as specific conclusions on the needs, challenges and effects of the policy on balanced regional development. Without any intention to relativize the classification of planning regions according to their development, designed in 2008 and 2012 by experts with exceptional knowledge in economic, demographic and developmental sciences, we enlist sets of statistical data showing that certain important segments of regional development are deviating, sometimes by large margins, from dominant tendencies that have been taken as decisive for establishing development in individual regions.

[3] More information for the policy on supporting underdeveloped regions is presented in Frame 1: Previous Policies on Regional Development.

[4] Macedonia in Figures 2014, State Statistical Office, June 2014

[5] Macedonia in Figures 2015, State Statistical Office, June 2015

[6] Macedonia in Figures 2016, State Statistical Office, June 2016, available at: http://www.stat.gov.mk/Publikacii/MakedonijaVoBrojki2016_mk.pdf

[7] Action Plan for Implementing the Strategy on Regional Development 2016-2018, "Official Gazette of the Republic of Macedonia" no. 123/2016, p. 3

Table 1. Basic characteristics of planning regions

Region	Area (in km²)	Number of municipalities	Population (2006)	Population (2015)
Macedonia	25,713	80	2,040,228	2,071,278
Vardar	4,042	9	154,230	152,917
East	3,537	11	180,938	176,877
Southwest	3,340	9	222,385	219,718
Southeast	2,739	10	171,972	173,552
Pelagonija	4,717	9	236,088	230,771
Polog	2,416	9	310,178	320,299
Northeast	2,310	6	173,982	176,231
Skopje	1,812	17	590,455	620,913

Source: State Statistical Office

If population growth is used as parameter to assess progress in regional development, only half of regions could be considered as relatively progressed in the last 10 years: Skopje, with more than 30,000 new inhabitants; Polog, with more than 10,000 new inhabitants; Northeast and Southeast region, with minimum increase of 1,500 to 2,500 inhabitants. Of course, these data are based on population growth projections, as the population census has not been organized for 14 years, and additional limiting factor is lack of comprehensive and accurate records on people that have left the state. Nevertheless, based on data available, even under such modest population growth per region (in some regions we observe mild population decrease), there is strong and evident difference among regions and enhanced dominant position of Skopje region, whose population number is three times higher compared to other regions.

Table 2. Population's ageing, per region

	2005			2015		
	Population above 65 years (%)	Population below 15 years (%)	Population growth	Population above 65 years (%)	Population below 15 years (%)	Population growth
Macedonia	11.0	19.2	4,076	13	16.7	2,614
Pelagonija	15.2	16.6	-557	15.7	15.4	-720
Vardar	11.9	17.5	-12	14.1	15.8	-173
Southeast	10.5	21.1	410	13.2	16.4	50
Southwest	10.3	20.8	309	11.3	15.4	184
Skopje	10.6	18.5	2,576	13.7	18.3	2,214
Northeast	11.3	18.3	221	11.9	17.5	137
Polog	8.2	23.3	1,366	9.0	17.0	1,197
East	11.9	16.7	-237	14.5	14.3	-275

Source: National Development Plan 2007-2009 and State Statistical Office

Analysis of the population's ageing per region shows that population is ageing in all regions, i.e. the share of young population is decreasing, with the most dramatic examples observed in Polog, Southeast and Southwest region, where difference in shares of young population in total population has changed by around 5% to 6%, while the biggest growth of elderly population is noted in Skopje, Southeast, East and Vardar region, by 2% to 3%. These worrying trends are confirmed by data on population growth, *i.e.* population growth is decreased in all regions compared to the levels recorded 10 years ago, and more than 50% of national population growth comes from Skopje, with Skopje and Polog region accounting for around 90% of national population growth.

Data presented in Table 3 show major changes in terms of the socially most endangered population, whereby biggest decrease is noted in Pelagonija, where the number of social allowance beneficiaries has been reduced almost threefold in the period 2006-2014. Over the period of 8 years, Northeast, Vardar and Southeast region have reduced their numbers of social allowance beneficiaries by more than two times, while Southwest and East region have almost two times less social allowance beneficiaries compared to their relevant 2006 figures. At the same time, significant decrease of socially endangered population is noted in Polog (-43%) and Skopje (-30.1%) region.

Table 3. Social allowance beneficiaries

	2006	2010	2014	Difference 2006-2014 (%)
Macedonia	249,619	176,431	128,679	- 48.4%
Vardar	14,936	9,183	5,977	- 60%
East	19,228	11,322	9,694	- 49.6%
Southwest	22,989	16,233	11,615	- 49.5%
Southeast	14,029	8,058	6,331	- 54.9%
Pelagonija	31,531	14,649	10,811	-65.7%
Polog	44,694	41,391	25,492	-43%
Northeast	41,685	20,053	16,440	-60.6%
Skopje	60,527	55,542	42,319	-30.1%

Source: State Statistical Office

Table 4. Activity rate of the population aged above 15 years, per region and per year

	2009			2012			2015		
	Activity	**Employment**	**Unemployment**	**Activity**	**Employment**	**Unemployment**	**Activity**	**Employment**	**Unemployment**
Macedonia	56.7	38.4	32.2	56.5	39	31	57	42.1	26.1
Vardar	58.3	35.2	39.7	59.1	37.9	35.9	60.7	45.8	24.5
East	59.5	49.4	17	61.5	50.1	18.5	62.5	51.6	17.5
Southwest	55.8	37.5	32.7	56.2	32.4	42.3	54.9	36.2	33.9
Southeast	69.6	59.6	14.4	70.7	60.9	13.8	68.4	56.9	16.7
Pelagonija	63.8	42.6	33.2	62.8	46.9	25.3	66.3	52.2	21.1
Polog	46.1	33.5	27.3	44.5	29.3	34.2	47.1	33.2	29.6
Northeast	56.9	20	64.8	52.1	24.6	52.8	54	30.6	43.2
Skopje	54.4	36.2	33.5	55.3	38	31.3	54.4	40.4	25.7

Source: State Statistical Office

Population activity, employment and unemployment rates (see Table 4) show certain positive trends in terms of employment of working age population, especially in Northeast and Vardar region, where employment has increased by more than 10 pp in the period of 6 years, while unemployment has decreased by more than 20 pp and 15 pp, respectively. Pelagonija is marked by positive indicators in this period (almost 10% more employed and around 12% less unemployed people). Unlike them, Southwest, Southeast and Polog region are marked by insignificant decrease in employment and insignificant increase in unemployment in the last 6 years, while East region is marked by increase in both, employment and unemployment. Regional dispersion of unemployment provides the conclusion that it continues to be the major challenge for the policy on balanced regional development, having in mind that unemployment has been modestly decreased in most regions, but great differences remain and should provide the basis for various measures and activities in different regions. In 2015, the unemployment rate in some regions, such as in the Northeast, is higher by almost 16%, and in Southwest is higher by 7% than the national average.

GDP per region is one of the most relevant indicators on overall regional development. Data for this category show that compared to 2006 figures the share of Skopje region in total national GDP has been decreased, and that share in GDP of the least developed region (Northeast) is marked by minimal, but continuous increase, while GDP per capita in Southeast is characterized by steady increase in the last consecutive years above the national average of GDP per capita and its share in total national GDP has increased from 7.6% in 2006 to 10% in 2014. Vardar, Southwest, Pelagonija and Polog region are marked by oscillations under this parameter, as shown with comparison of datasets for the years 2006, 2010 and 2014. Comparison of relevant figures for the years 2006 and 2014 provides the conclusion that the share in national GDP of Skopje region has decreased by 4.6 pp and reduction of relevant shares of Pelagonija, Polog and Vardar region accounting for 0.2 pp has been "compensated" with growth in Southeast (by 2.4 pp), East (by 1.8 pp), Northeast (by 0.6 pp) and Southwest (by 0.5 pp) region.

Table 5. GDP per region

2006	GDP (in million MKD)	Structure of GDP MK = 100%	GDP per capita (in MKD)	GDP per capita MK =100
Macedonia	310,915	100	152,392	100
Vardar	25,498	8.2	165,327	108.5
East	19,913	6.4	110,055	72.2
Southwest	22,855	7.4	102,774	67.4
Southeast	23,670	7.6	137,640	90.3
Pelagonija	35,238	11.3	149,258	97.9
Polog	22,658	7.3	73,047	47.9
Northeast	13,612	4.4	78,240	51.3
Skopje	147,470	47.4	249,756	163.9
2010	**GDP (in million MKD)**	**Structure of GDP MK = 100%**	**GDP per capita (in MKD)**	**GDP per capita MK = 100**
Macedonia	437,296	100	212,795	100
Vardar	31,249	7.1	203,102	95.4
East	37,850	8.7	210,546	98.9
Southwest	35,828	8.2	161,492	75.9
Southeast	39,161	9	226,550	106.5
Pelagonija	52,923	12.1	226,036	106.2
Polog	33,707	7.7	107,074	50.3
Northeast	20,671	4.7	118,092	55.5
Skopje	185,906	42.5	308,467	145
2014	**GDP (in million MKD)**	**Structure of GDP MK = 100%**	**GDP per capita (in MKD)**	**GDP per capita MK =100**
Macedonia	**527,632**	**100.0**	**255,206**	**100.0**
Vardar	42,079	8.0	274,404	107.5
East	43,407	8.2	244,272	95.7
Southwest	41,629	7.9	189,109	74.1
Southeast	52,775	10.0	304,140	119.2
Pelagonija	58,412	11.1	251,988	98.7
Polog	37,413	7.1	117,284	46.0
Northeast	26,182	5.0	148,745	58.3
Skopje	225,734	42.8	366,482	143.6

Source: State Statistical Office

Structure of GDP according to regions provides the best image of imbalanced development among regions in Macedonia – Skopje region (which is the residence of 1/3 of total population) creates as much as 42.8% of GDP and has higher GDP per capita compared to the national average, followed by Southeast and Vardar region whose GDP per capita are also higher than the national average. GDP per capita in Polog is below half (46%) of the national average, whereas GDP per capita in Northeast region is around 58% of the national average, while this parameter in Southwest region is 74%. This should be straightforward signal for policy-makers and implementers of balanced regional development that these three regions need greater attention in the short term, having in mind that they are also affected by the highest unemployment rates (Table 4) and unless they catch up the pace with the more developed regions, these regions could remain black spots on the map of the least developed regions in Europe.

Table 6. Equity investments (in million MKD)

	2010	2011	2012	2013	2014	Total	%
Macedonia	100,851	109,219	109,071	119,003	123,549	561,693	100
Vardar	6,621	5,642	5,016	8,179	7,350	32,808	5.8
East	9,078	5,070	5,964	10,462	10,175	40,749	7.3
Southwest	5,095	4,241	5,352	5,597	9,473	29,758	5.3
Southeast	7,148	5,355	4,728	6,390	6,940	30,560	5.4
Pelagonija	7,690	5,765	10,236	10,745	8,329	42,765	7.6
Polog	7,105	7,708	9,059	7,558	7,058	38,487	6.9
Northeast	2,905	1,353	2,645	2,482	2,543	11,927	2.1
Skopje	55,210	74,086	66,072	67,589	71,681	334,638	59.6

Source: State Statistical Office

Data on equity investments (see Table 6) show the most defeating results in terms of the policy on balanced regional development. The share of individual regions in total investments shows that more than half (sometimes around 70%) of investments are made in the most developed region and that almost no investments are made in the least developed region (Northeast), whose share in total equity investments is constant and stands below 3%.

Table 7. Active business entities

	2008	2012	2015
Macedonia	63,193	74,424	70,139
Vardar	4,828	5,975	5,470
East	5,299	5,913	5,692
Southwest	6,484	7,564	7,127
Southeast	5,503	6,373	5,889
Pelagonija	7,523	8,468	8,071
Polog	6,050	7,285	7,554
Northeast	3,691	4,283	4,139
Skopje	23,815	28,563	26,197

Source: State Statistical Office

Data on active business entities (as presented in Table 7) show positive tendencies in all regions, especially in terms of the total number of registered entities. As regards the share of individual regions in total number of active entities, the situation is rather stale throughout the years: Skopje region maintains its share of around 37-38%, Pelagonija – around 11-12% and Southwest – around 10%. Slightly more noticeable progress is observed in Polog, whose share increased from 9.6% in 2008 to 10.8% in 2015, climbing before Southwest region on the third place under this indicator for regional development. However, comparison of regions in terms of the number of employees shows that, in 2015, Southeast, Pelagonija and Polog region had fewer companies employing more than 250 employees, while Southeast and Pelagonija regions had fewer companies employing 50 to 249 people.

Table 8. Construction works performed (in thousand MKD)

	2007	%	2010	2013	2015	%
Macedonia	22,258,526	100	25,025,816	37,750,104	41,209,648	100
Vardar	942,246	4.2	1,184,760	4,193,177	4,158,385	10.1
East	811,578	3.6	1,839,722	1,614,495	4,176,346	10.1
Southwest	3,140,527	14.1	2,378,635	2,754,445	5,386,551	13.1
Southeast	1,756,640	7.9	2,195,692	2,127,929	1,421,037	3.4
Pelagonija	1,635,286	7.3	2,475,012	5,107,026	4,788,052	11.6
Polog	7,470,033	33.6	5,068,724	4,352,401	2,555,666	6.2
Northeast	288,531	1.3	633,970	1,073,275	4,549,361	11
Skopje	6,213,685	27.9	9,249,301	16,527,356	14,174,250	34.4

Source: State Statistical Office

Value of construction works performed is another important parameter used to compare development in regions, although this component is also marked by certain major oscillations. If the main "construction site" in 2007 was Polog region, with almost one third of total value of construction works performed (and was ranked second in the next five years, immediately after Skopje), in 2015 this region is second to last under this parameter, demonstrating slightly better performance than Southeast, which has been continuously ranked low on this list. In this period of 9 years, Pelagonija and Southwest are ranked among regions marked by higher value of construction works performed, while Vardar and East region continuously demonstrate lower value of construction works performed. The lowest value of construction works performed in the last 9 years is recorded in Northeast region which, except in the year 2015, is ranked last under this parameter.

Table 9. Total municipal revenue and expenditure (in thousand MKD)

	2013		2014		2015	
	revenue	expenditure	revenue	expenditure	revenue	expenditure
Macedonia	28,136,999	27,892,548	28,253,055	27,732,155	30,132,499	29,251,474
Vardar	1,899,154	1,890,153	2,042,024	2,005,454	2,156,996	2,096,797
East	2,352,714	2,328,043	2,532,933	2,452,526	2,536,120	2,509,826
Southwest	2,648,840	2,603,786	2,624,033	2,594,441	2,793,562	2,759,446
Southeast	2,242,866	2,232,939	2,408,557	2,337,640	2,543,821	2,483,198
Pelagonija	3,024,743	3,007,182	3,006,346	2,958,032	3,111,393	3,058,696
Polog	3,108,638	3,072,581	3,270,648	3,207,566	3,353,193	3,326,890
Northeast	2,031,806	2,023,025	2,113,130	2,084,340	2,028,421	2,009,295
Skopje	10,816,665	10,734,839	10,255,383	10,092,156	11,608,994	11,007,326

Source: sobranie.mk

As regards total municipal revenue, local self-government units in Skopje region, including the City of Skopje, have generated 37.8% of total revenue of all LSGUs in Macedonia in the last three years. Lowest revenue was generated by municipalities in Vardar (7%) and Northeast (7.1%), followed by municipalities in Southeast (8.3%), East (8.6%), Southwest (9.3%), Pelagonija (10.6%) and Polog (11.2%) region.

Previous data were presented and elaborated for the purpose of comparing development in the eight planning regions according to some of the more important development parameters. A more detailed analysis of components under these data and comparison of datasets for longer period of time provide a more accurate image about the strengths and weaknesses of individual regions, especially in relation to their growth potential. Some parameters used to assess development show negative tendencies in respect to the policy on balanced regional development (in particular, data on population's ageing and total equity investments), while other data provide small hope for possible mitigation of consequences from long-standing dominant investments in development of the capital (data on active business entities and construction works performed).

2. Balanced regional development – legislative framework

Parliament of the Republic of Macedonia adopted the Law on Balanced Regional Development on its session held on 15th May 2007. This piece of legislation stipulates goals, principles and policy holders in balanced regional development, planning of regional development, financing and allocation of funds to stimulate balanced regional development, monitoring and assessment for implementation of planning documents and projects, and other issues pertaining to regional development.[8]

By means of this law, regional development was defined as process on identification, stimulation, management and utilization of potentials of planning regions and areas with specific developmental needs, and established the policy on balanced regional development as system of goals, instruments and measures aimed at reducing regional disparities and attainment of balanced and sustainable development in Republic of Macedonia. Planning regions are defined as functional territorial units established for the purpose of development planning and policy implementation for stimulating balanced regional development.

Key goals of the policy on stimulating balanced regional development, as defined in this law, include:

- balanced and sustainable development on the entire territory of Republic of Macedonia, based on the model of polycentric development;
- reduced disparities between and within planning regions, and improved quality of life for all citizens;

[8] Law on Balanced Regional Development, consolidated text, "Official Gazette of the Republic of Macedonia" no. 63/2007, 187/2013, 43/2014 and 215/2015

- increased competitiveness of planning regions, by strengthening their innovative capacity, optimal use and valuation of natural wealth, human capital and economic specificities of different regions;
- maintained and developed unique identity of planning regions, as well their affirmation and development;
- revival of villages and development of areas with specific developmental needs; and
- support for inter-municipal and cross-border cooperation of local self-government units, for the purpose of stimulating balanced regional development.

Frame 1: previous policies on regional development

According to some documents, the policy on balanced regional development is a result of "non-existing regional development policy in the period until 2007", although the state had been implementing the policy on "faster development of economically underdeveloped areas", and had established criteria on underdevelopment, the Fund for Crediting Faster Development of Economically Underdeveloped Areas, and the Parliament adopted the Law on Stimulating Development in Economically Underdeveloped Areas in 1994,[9] *which anticipated transfer of funds in the amount of 1% of GDP to economically underdeveloped regions (around 6 million USD were planned for this purpose in 1994*[10]*).*

In the first years after Republic of Macedonia declared its independence, the Ministry of Development was competent to administer support for underdeveloped areas and then current Minister of Development, Sofija Todorova, at the parliament session organized for adoption of this law, claimed that legal provisions anticipate state interventionism for the purpose of stimulating development in underdeveloped areas.[11]

Hence, according to data from the Bureau for Economically Underdeveloped Areas, funds transferred for this purpose in 2000 accounted for 0.30% of GDP, in 2001 - 0.2%, in 2002 - 0.4%, in 2003 - 0.1%, in 2004 – 0.08%, and in 2005 they accounted for 0.06%.[12]

Sources of funds for regional development included the Budget of Republic of Macedonia, budgets of local self-government units and funds of the European Union, other international sources, donations and sponsorships from natural and legal entities, and other funds, as stipulated by law. For the purpose of stimulating balanced regional development, annual funds allocated from the Budget of Republic of Macedonia should amount to at least 1% of the Gross Domestic Product.

Funds intended for balanced regional development are allocated by the Government of Republic of Macedonia, as follows:

- 70% to finance development projects of planning regions;
- 20% to finance development projects of areas with specific developmental needs; and
- 10% to finance development projects of villages.

Funds intended to finance development projects of planning regions are allocated according to the classification of development in planning regions. Establishment of development level in planning regions is pursued on the basis of economic development index and the demographic index. Criteria and indicators on development of planning regions are established by means of an act adopted by the Government of Republic of Macedonia. Classification of planning regions according to their development is determined by means of an act adopted by the Government of Republic of Macedonia and is valid for a period of five years.[13] Funds are

[9] Economically underdeveloped areas covered 64% of the entire territory of the Republic of Macedonia and accounted for 22% of total population when this law was adopted in 1994.

[10] Shorthand notes from the parliament discussion, available at: http://www.sobranie.mk/WBStorage/Files/71sednica7prod12jan94god.pdf

[11] Shorthand notes from the parliament discussion, available at http://www.sobranie.mk/WBStorage/Files/71sednica7prod12jan94god.pdf, p. 51

[12] National Development Plan 2007-2009, Government of the Republic of Macedonia, Skopje, February 2007, p. 57

[13] First classification on development of planning regions was made on the basis of the Decision on detailed criteria and indicators on development of planning regions ("Official Gazette of the Republic of Macedonia" no. 162/08). According to this decision, the development index was established as weighted average of the economic-social and demographic indices, whereby these two indices have equal weighted shares in creation of the development index. Establishment of economic-social indicators relies on parameters such as: GDP per capita, budget revenue per capita, growth of added value in non-financial sector and unemployment rate, while the demographic index is established on the basis of population growth, population's ageing coefficient, migration rate per 1000 inhabitants and graduated students per 1000 inhabitants.

transferred to the account of the Bureau of Regional Development (hereinafter: the Bureau) which only implements the decision on their distribution.[14]

At the time when the Law on Balanced Regional Development was adopted, the average GDP per capita in Republic of Macedonia accounted for 30% of the EU-27 average in 2007, and the country was categorized in the group of country with low economic development. GDP in the most developed planning region (Skopje) accounted for 44.5% of the EU-27 average and GDP of the least developed region (Northeast) was only 13.1%. In that period, Skopje region had 3.4 times higher GDP per capita compared to Northeast, representing a significant disparity and challenge for efficiency of measures, goals and activities on balanced regional development that provided the framework for adoption of this law. Major disparity in development of planning regions across Macedonia is seen also from the comparison of developmental, economic-social and demographic indices (see Chart 1 and Table 10 below), with prominent difference between Skopje and other regions, especially in terms of economic-social development.

Chart 1. Comparison of development per region for the period 2008-2012

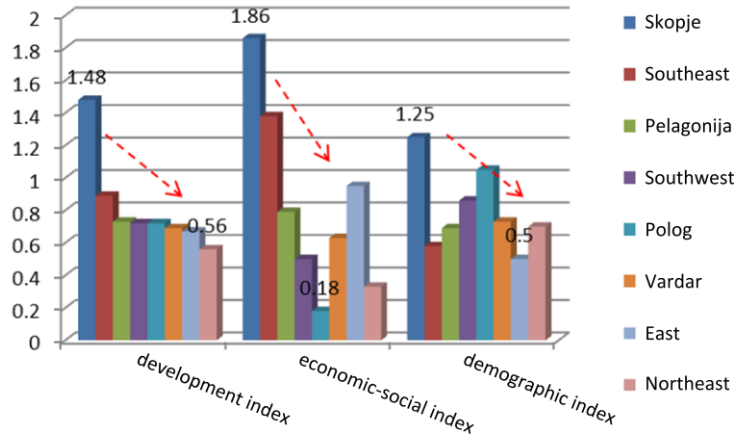


Table 10. Classification of planning regions according to their development for the period 2008-2012

Planning region	According to development index	According to economic-social index	According to demographic index
Skopje	1.48	1.86	1.25
Southeast	0.89	1.38	0.58
Pelagonija	0.73	0.79	0.69
Southwest	0.72	0.50	0.86
Polog	0.72	0.18	1.05
Vardar	0.69	0.63	0.73
East	0.67	0.95	0.50
Northeast	0.56	0.33	0.70

Source: Decision on classification of planning regions according their development for the period 2008-2012, "Official Gazette of the Republic of Macedonia" no. 162/2008

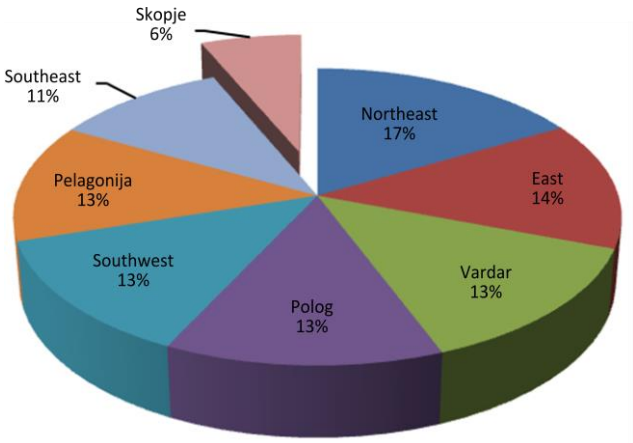
These indices (developmental, economic-social and demographic) how much planning regions differ in terms of development compared to the national average, whereby:

- **index value of 1** means that planning region's development is equal to the average development at the level of Republic of Macedonia;
- **index value higher than 1** means that planning region's development is higher than the average development at the level of Republic of Macedonia; and
- **index value lower than 1** means that planning region's development is lower than the average development at the level of Republic of Macedonia.

[14] Law on Balanced Regional Development, consolidated text ("Official Gazette of the Republic of Macedonia" no. 63/2007, 187/2013, 43/2014 and 215/2015), Article 33

Classification of planning regions' development served as basis for calculation of their shares in distribution of funds intended to finance development projects of planning regions in the period 2008-2012. According to this calculation, almost all regions are anticipated to receive at least twice as many funds than Skopje region that will benefit from 6.4% of total funds allocated on annual basis from the Budget of Republic of Macedonia.

Share of planning regions in distribution of funds for regional development, for the period 2008-2012 (%)



Source: Decision on classification of planning regions according to their development for the period 2008-2012, "Official Gazette of the Republic of Macedonia" no. 162/2008

Funds intended for stimulating regional development should reduce disparities among and within regions. According to primary and secondary legislation, utilization of funds is directly conditioned with submission and implementation of quality projects for stimulating regional development which, on the other hand, depends on development project planning and implementation capacity of individual regions. Therefore, establishment of the system on regional development must pay special attention to the component on developing capacities of relevant institutions, which is an important determinant for planning regions to obtain/utilize funds.[15]

In 2013, the Government of RM adopted new classification of planning regions according to their development for the period 2013-2017.[16]

Table 11. Classification of planning regions according to their development, for the period 2013-2017

Planning region	According to development index	According to economic-social index	According to demographic index
Skopje	1.51	1.48	1.53
Southeast	0.97	1.29	0.72
East	0.96	1.36	0.65
Pelagonija	0.91	1.09	0.80
Polog	0.82	0.50	1.07
Southwest	0.81	0.98	0.69
Vardar	0.73	0.70	0.76
Northeast	0.63	0.27	0.90

According to this classification, the share of planning regions in distribution of funds intended for regional development for these four years has been moderately changed and should follow the ratio presented on the chart.

[15] Strategy on Regional Development 2009-2019, "Official Gazette of the Republic of Macedonia" no. 119 from 30.9.2009, pp. 13-14

[16] Decision on classification of planning regions according to their development, for the period 2013-2017, "Official Gazette of the Republic of Macedonia" no. 88/2013

Chart 2. Share of planning regions in distribution of funds intended for regional development (%), for the period 2013-2017

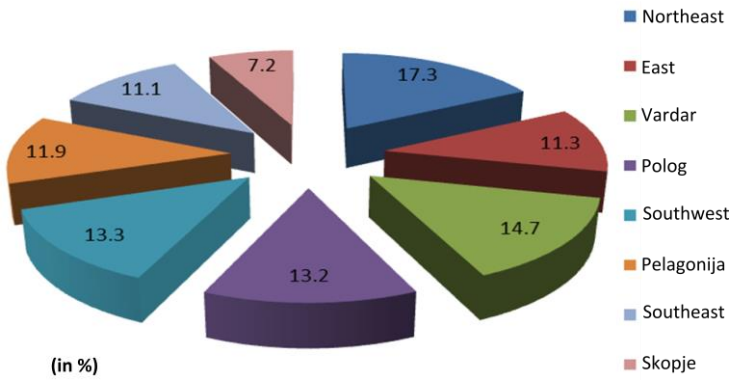


Table 12. Share of planning regions in distribution of funds for regional development

Region (share in distribution of funds)	2008-2012 (%)	2013-2017 (%)	Difference
Northeast	16.7	17.3	+ 0.6
East	14	11.3	-2.7
Vardar	13.5	14.7	+1.2
Polog	13	13.2	+0.2
Southwest	13	13.3	+0.3
Pelagonija	12.9	11.9	- 1
Southeast	10.6	11.1	+0.5
Skopje	6.4	7.2	+0.8

Development indices of planning regions in the Republic of Macedonia show that only Skopje region is characterized by above average development, while development in all other regions is below the national average. In that, difference between Skopje and the second most developed region (Southeast) is significant (index value of 1.48 and index value of 0.89, respectively), but difference between Skopje and the least developed region (Northeast) is exceptionally great (index value of 1.48 and index value of 0.56, respectively).

Frame 2: Important dates for the policy on balanced regional development

✓ ***1994*** *–Adoption of the Law on Stimulating Development of Economically Underdeveloped Areas, which anticipated generation of funds for development of economically underdeveloped areas from the central budget, in the amount of 1% of GDP on annual level;*

✓ ***2001*** *–Adoption of the nomenclature of units for territorial statistics (NUTS) for Republic of Macedonia, according to which the entire territory of Macedonia is NUTS level 1 and 2, while NUTS level of 3 is assigned to eight statistical regions.*

✓ ***15 May 2007*** *– Parliament of the Republic of Macedonia adopted the Law on Balanced Regional Development.*

✓ ***2007-2009*** *– Establishment of eight Centres on Development of Planning Region. First centre was established in Polog region within the law-stipulated deadline of 9 months from entry in effect of the Law on Balanced Regional Development, and the last centre was established in Skopje region, in May 2009.*

✓ ***August 2008*** *– Council on Balanced Regional Development adopted the decision on classification of planning regions according to their development for the period 2008 – 2012 and the decision on detailed criteria and indicators on establishing development of planning regions.*

✓ ***December 2008*** *– Planning regions were classified according to their development.*

✓ ***January 2009*** *– The Government adopted 2009 Programmes on Financial Support for Regional Development in 2009 ("Attractive Planning Regions for 2009", implemented by the Ministry of Local Self-*

Government with total budget of 150,000,000 MKD and Programme on Balanced and Sustainable Regional Development implemented by the Bureau for Regional Development in the amount of 166,500,000 MKD).

✓ ***May 2009*** *– Council on Balanced Regional Development adopted the decision on detailed criteria and indicators for establishment of areas with specific developmental needs and the decision on establishing areas with specific developmental needs in the Republic of Macedonia, for the period 2009 - 2013.*

✓ ***August 2009*** *– Bureau for Regional Development announced the first open call for proposals on development of areas with specific development needs and development of villages that will be financed by MLSG programmes, and issued circulatory letter to presidents of planning regions about their preparedness to receive project proposals on development of planning regions. Although the Government adopted decisions on project financing, sufficient funds were not paid from the budget to the centres on development of planning regions, so funds anticipated for the year 2010 were redirected to finance projects approved in 2009. Projects submitted for the year 2010 were transferred for implementation in 2011 and the Bureau did not announce the 2011 open call for proposals.*

✓ ***29 September 2009*** *– Parliament of the Republic of Macedonia adopted the Strategy on Regional Development.*

✓ ***November-December 2009*** *– Centres on Development of Planning Regions adopted the programmes on development of planning regions. Programmes adopted by Skopje and Northeast region covered the period 2009-2014, those adopted by East and Southeast region covered the period 2009-2013, programme for Vardar region covered the period 2008-2013, and those adopted by Pelagonija and Southwest region concerned the period 2010-2015.*

✓ ***December 2009*** *– Government of the Republic of Macedonia adopted the decision on financing development projects of planning regions and allocated 131,810,000 MKD from the Budget of RM.*

✓ ***March 2010*** *– Government of the Republic of Macedonia adopted the Action Plan for Implementing the Strategy on Regional Development for the period 2010–2012.*

✓ ***2012*** *– Government of the Republic of Macedonia integrated balanced regional development in its priorities.*

✓ ***April 2013*** *– Government of the Republic of Macedonia adopted the decision on classification of planning regions according to their development for the period 2013-2017, according to which Skopje region has development index value of 1.51 in regard to the national average, Southeast – 0.97, East – 0.96, Pelagonija – 0.91, Polog – 0.82, Southwest – 0.81, Vardar – 0.74 and Northeast – 0.63. Based on this classification, calculation was made of relevant shares of planning regions in distribution of funds intended to finance development projects of planning regions for the period 2013-2017.*

✓ ***June 2013*** *– Government of the Republic of Macedonia adopted the Action Plan on Implementing the Strategy on Regional Development for the period 2013-2015.*

✓ ***February 2014*** *– Government of the Republic of Macedonia adopted the decision on establishing areas with specific developmental needs for the period 2014-2018*

✓ ***2014*** *– Adoption of the Strategy on Amending the Strategy on Regional Development by means of which parameters on targets for the year 2019 were reduced (initially planned average GDP per capita of 50% from the EU average was reduced to 42%; planned development of the least developed region expressed as GDP per capita to be at least 35% of the EU average was reduced to 26%; planned difference in GDP per capita between the most and the least developed region not to exceed 2.5 times was reduced to 2.2 times).*

✓ ***November-December 2014*** *– Centres on Development of Planning Regions adopted the programmes on development of planning regions for the period 2014 - 2019.*

✓ ***June 2016*** *– Government of the Republic of Macedonia adopted the Action Plan on implementing the Strategy on Regional Development for the period 2016-2018.*

3. Strategy and action plans on balanced regional development

Strategy on Balanced Regional Development, which in many aspects was based on the National Development Plan, was planned according to macroeconomic policies in the state geared towards ensuring annual growth rate of around 6.5% in the indicated period, and should growth continued under the same dynamics, following outcomes would be attained:

- average GDP per capita (according to purchase power parity - PPP) in Republic of Macedonia should reach 50% of the EU average in 2019;

- the least developed planning region in Republic of Macedonia should reach GDP per capita (according to PPP) of at least 35% of the EU average in 2019; and
- difference between GDP per capita of the most and the least developed region should not exceed 2.5 times in 2019.

Unfortunately, these overly-ambitious projections were not attained, although majority of them were developed at the time when the world was affected by the major financial crisis that had inevitable effects on the Macedonian economy and therefore they should have been more realistic. In the next years, GDP growth in the state never even approach the level projected at 6.5% (according to data from the Ministry of Finance, the highest growth rate was attained in 2008 with GDP growth of 5.5% and in 2015 with GDP growth of 3.8%),[17] while according to the EC's last progress report for Macedonia GDP per capita in 2014 reached 37% of the EU-28 average.[18]

In 2014, the Government, *i.e.* the Ministry of Local Self-Government revised the Strategy on Regional Development and the Parliament adopted the Strategy on Amending the Strategy on Regional Development. Some of more important changes were made in regard to strategic goals, whereby instead of average GDP per capita of 50% of the EU average, as planned in 2009, the new document revised this projection to 42% of the EU average. Instead of the least developed planning region to reach GDP per capita of at least 35% of the EU average, in the new document this parameter was reduced to 26%. Moreover, instead of difference in GDP per capita between the most and the least developed region not to exceed 2.5 times (as anticipated under 2009 Strategy), in 2014 this difference was corrected downward to 2.2 times.[19]

In addition to failed projections, this policy was also faced with many delays in implementation of its main components. One of the most important segments in implementation of this policy are the Centres on Development of Planning Regions, which should have been established within 9 months from the law's adoption, but with the exception of Polog, all other regions have breached this deadline. The last centre was established in Skopje region, in May 2009, *i.e.* 14 months beyond the law-stipulated deadline. Their function, inter alia, is to coordinate assistance from the Government and donors intended for regional development, as well as adoption of five-year Programmes on Development of Planning Regions. Deadlines stipulated for adoption of these programmes were also breached (they should have been completed within 9 months from the law's entry in effect, *i.e.* in March 2008, but all programmes were adopted in late 2009).

From their start of operation, the Centres on Development of Planning Regions faced problems in terms of their financing. According to the law, 50% of their revenue should come from the Budget of RM and remaining 50% should be transferred from the budgets of local self-government units (hereinafter: LSGUs) covered within the region for which each of these centres were established. Significant portion on LSGUs did not comply with this obligation in timely manner (and some of them have not paid these funds for years back), thus hindering the centres' operation. In the case of several centres, this problem is still slowing down their operation and results in lower efficiency in terms of fundraising from foreign donors intended for development of particular regions.

On the other hand, despite the fact that in August 2009 the Bureau announced an open call for proposals related to development of areas with specific developmental needs and development of villages that should be financed by the Budget of RM (and at the same time issued circulation letter to presidents of planning regions about their preparedness to receive project-proposals for development of planning regions) and despite the fact that the Government adopted decision on project financing, sufficient budget funds were not disbursed to the Centres on Development of Planning Regions, as a result of which funds anticipated for 2010 were reassigned to finance projects approved in 2009. Project applications for 2010 were transferred for financing in 2011, on the account of which open call for proposals was not announced in 2011.[20] Afterwards, open calls for proposals were announced under relatively regular dynamics.

Official website of the Bureau for Regional Development hosts data about the amount of funds awarded by the Government through the Bureau in the last years (see Table below).[21]

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[17] Data are taken from the website of the Ministry of Finance, Category: Macroeconomy, Subcategory: Indicators and Projections, available at: http://finance.gov.mk/mk/node/401, last accessed on 5th November 2016

[18] The former Yugoslav Republic of Macedonia 2016 Report, Brussels, 9.11.2016, available at: http://ec.europa.eu/enlargement/pdf/key_documents/2016/20161109_report_the_former_yugoslav_republic_of_macedonia.pdf

[19] Strategy on Amending the Strategy on Regional Development 2009-2019, Ministry of Local Self-Government, September 2014

[20] Development and classification of planning regions (expert study) Vlabor, Skopje, May 2013, p. 17

[21] www.brr.gov.mk (last accessed on 24.10.16)

Table 13. Funds from the Budget of RM intended for balanced regional development through the Bureau for Regional Development

Year	For development projects of planning regions	For development of areas with specific developmental needs	For projects on development of villages
2016	179,315,540	51,233,010	25,118,635
2015	91,276,253	26,385,800	13,192,900
2014	42,095,533	12,027,295	6,013,648
2013**		14,845,732	7,422,866
2012	13,322,925	3,806,550	1,903,000
2011	92,065,766	26,304,505	13,152,252
2010*			
2009	131,810,000	37,660,000	18,481,250

Source: Bureau for Regional Development
*In 2010 no funds were allocated for development projects of planning regions
**Official website of the Bureau for Regional Development does not host data concerning funds allocated for development projects of planning regions in 2013.

One of the most important components in the legislative framework for financing this policy is the transfer of funds in the amount of 1% of GDP to the Ministry of Local Self-Government and the Bureau which are later, on the basis of development classification of the regions, transferred in relevant percentages to individual planning regions (highest share of funds is allocated for the least developed and lowest share of funds to the most developed region). With every new budget, budget adjustment and final balance sheet for the Budget of RM, as well as new Action Plans on Implementing the Strategy on Regional Development we were able to learn that, contrary to the legislative framework in place, the Government is applying different methodology to calculate shares of GDP that will be invested in balanced regional development.

3.1. Action Plans on Implementing the Strategy on Regional Development

To date, a total of three Action Plans on Implementing the Strategy on Regional Development were adopted, all with delays in terms of law-stipulated deadline defined as at least six months prior to expiration of the current planning period.[22]

First Action Plan on Implementing the Strategy on Regional Development 2010–2012 was adopted by the Government of Republic of Macedonia on its session held on 20th April 2010.

This document confirmed what could have been only sensed in the previous two years with postponement of procedures on implementing law-stipulated obligations on balanced regional development and allocation of funds for this purpose from state budgets and budget adjustments in 2008 and 2009. Instead of complying with the law obligation on allocating 1% of GDP for regional development through MLSG (or the Bureau for Regional Development), this document endorsed the position of the Government that these funds should be disbursed to the budgets of several ministries and agencies whose competences include regional components, without any obligation on disbursing said funds according to the methodology on shares of planning regions in distribution of funds intended to finance development projects of planning regions. With this approach, the policy maker has actually reinstituted one segment of the process on development of underdeveloped regions to the level that existed prior to adoption of the law and strategy on balanced regional development. Exactly the methodology on stimulating development of the regions in proportion to their level of (under)development was the new specific difference compared to the previous policy, for which holders of the new policy claimed it had been "monocentric". On that account, most independent reports on the policy on balanced regional development present data according to which financial funds from the Government intended to support this policy have not reached the famous 1% of GDP in any of the years, while government reports operated with data that this share ranges from around 1% to 3.25% of GDP, although it is interesting to note that these shares concern only funds planned, but not funds realized, as confirmed with specific data from the budget's final balance sheets.

These data allow the conclusion that not a single action plan presents data on share of funds awarded according to the methodology stipulated by the law and the strategy on balanced regional development which, in the period 2008 – 2015, ranged from 0.002 to 0.3% of GDP.

[22] Law on Balanced Regional Development, consolidated text ("Official Gazette of the Republic of Macedonia" no. 63/2007, 187/2013, 43/2014 and 215/2015), Article 10

Here, it should be noted that the Action Plan 2010-2012 featured a paragraph that reads: "2009 Budget of the Republic of Macedonia planned funds in the amount of 4,568 million MKD intended to support regional development, which accounted for 1.15% of GDP. Although these funds were allocated (labelled as "regional") and were not managed by the Ministry of Local Self-Government, they still represent certain improvement, i.e. different practice and serious effort of the country to support regional development. It will take time for mechanisms to be developed and regional policy to strengthen in order to enable distribution of these funds directly for support of balanced regional development".[23]

Below are several quotations from this document that confirm the government is applying different methodology for calculation of investments in balanced regional development from the methodology stipulated by the law.

"2010 Budget of the Republic of Macedonia secured funds in total amount of 6,307.85 MKD, representing 1.53% of estimated GDP for the year 2010 in the amount of 413,066 million MKD. These are total funds planned for support of projects with regional developmental component and distributed under individual programmes and sub-programmes of different ministries, which means that they will be implemented according to their policies on development support."[24]

Table 14. Funds from the Budget of RM planned for regional development, for the period 2009-2012 (in million MKD)

Source of funding	2009	2009 (budget adjustment)	2010	2011	2012	Total 2010-2012
IPA*						
Ministry of Local Self-Government	425.23	372.13	333.52	442.24	464.35	1240.11
Other line ministries	4,143.34	3,397.51	5,974.33	6,213.30	6,523.97	18,711.60
Total	4,568.57	3,769.64	6,307.85	6,655.54	6,988.32	19,951.71

Source: Action Plan on Implementing the Strategy on Regional Development 2010-2012
*Action Plan does not include data on funding awarded under IPA fund.

Table 15. Funds planned under budgets of line ministries including regional development component and intended for support of regional development (in million MKD)

Institution	2009	2009 (budget adjustment)	2010	2011	2012	Total 2010 - 2012
Ministry of Finance	26.74	19.99	442.29	459.98	482.98	1,385.25
Ministry of Economy	233.13	199.83	590.00	613.60	644.28	1,847.88
Ministry of Environment and Spatial Planning	248.40	243.35	215.50	224.12	235.33	674.95
Ministry of Transport and Communications	823.78	607.70	1,616.60	1,681.26	1,765.33	5,063.19
Ministry of Agriculture, Forestry and Water Economy	654.47	276.43	770.62	801.44	841.52	2,413.58
Ministry of Agriculture, Forestry and Water Economy	500.00	500.00	632.00	657.28	690.14	1,979.42
Agency for	74.00	66.29	75.48	78.50	82.42	236.40

[23] Action Plan on Implementing the Strategy on Regional Development 2010– 2012, Ministry of Local Self-Government, March 2010, pp. 1-2

[24] Action Plan on Implementing the Strategy on Regional Development 2010– 2012, Ministry of Local Self-Government, March 2010, p. 42

Institution	2009	2009 (budget adjustment)	2010	2011	2012	Total 2010 - 2012
Agriculture Development						
Ministry of Education and Science	642.90	804.00	628.89	654.05	686.75	1,969.68
Agency for Youth and Sports	390.00	300.00	471.00	489.84	514.33	1,475.17
Ministry of Health	49.92	49.92	201.95	210.03	220.53	632.51
Agency for State Roads	500.00	330.00	330.00	343.20	360.36	1,033.56
Total:	4,143.34	3,397.51	5,974.33	6,213.30	6,523.97	18,711.60

Source: Action Plan on Implementing the Strategy on Regional Development 2010-2012

Explanation provided in this action plan why these funds are shown in this table implied the fact that they will be used to finance projects focused on regional development, *i.e.* contributing to stimulation of balanced regional development on the entire territory of Republic of Macedonia, which means that these projects have developmental character and could contribute to increased GDP and employment in individual regions.[25]

In August 2013, the Government of RM adopted the second Action Plan on Implementing the Strategy on Regional Development 2013-2015. This document continued the same methodology on calculation of law-stipulated 1% of GDP for balanced regional development. "2012 Budget (after the budget adjustment) secured 4,259.98 million MKD, representing 0.88% of estimated GDP in 2012 (481,808 million MKD). Although these funds are allocated to the budgets of several ministries and agencies (labelled as "regional") and are not managed by the Ministry of Local Self-Government, they still represent serious effort of the country to support regional development."[26]

Table 16. Funds intended for support of regional development 2012-2015 (in thousand MKD)

Source of funding	2012	2012 (budget adjustment)	2013	2014	2015	Total (2013-2015)
MLSG and BRD	175,600	77,360	99,035	103,780	118,880	321,695
Other line ministries	4,774,500	4,181,520	4,492,836	4,331,619	4,835,600	13,661,055
Total	4,950,100	4,258,880	4,591,871	4,430,654	4,954,480	13,982,750

Source: Action Plan on Implementing the Strategy on Regional Development 2013-2015

Table 17. Funds for support to regional development through MLSG and the Bureau, for the period 2012-2015 (in thousand MKD)

Account	Institution	Purpose	2012	2012 (budget adjustment)	2013	2014	2015	Total
19101	MLSG	balanced regional development	109,000	27,860	49,992	53,780	63,880	167,652
28001	BRD	sustainable balanced regional development	65,500	49,500	49,043	50,000	55,000	154,043
	Total		174,500	77,360	99,035	103,780	118,880	321,695

[25] It should be noted that this document did not include explanation whether funds from budgets of "other line ministries" are distributed according to the decision on classification of planning regions according to their development. If that was the case, one could accept arguments offered by authors of the Action Plan that these funds contribute to stimulating balanced regional development on the entire territory of Republic of Macedonia.

[26] Action Plan on Implementing the Strategy on Regional Development 2013-2015, "Official Gazette of the Republic of Macedonia" no. 122/2013, p. 2

In addition to above-enlisted funds for regional development, 2013 Budget of RM, under items related to budgets of other line ministries, indicated funds that include regional development component and are intended to support regional development.[27]

The last action plan published in June 2016 assessed that 2016 Budget of RM anticipates funds in the amount of 19,000,512,000 MKD (or around 3.25% of estimated GDP in 2016).[28] It should be noted that authors of that strategic document integrated in these figures funds from projects of ministries that are not members of the Council on Balanced Regional Development of Republic of Macedonia (such as the Ministry of Education and Science and the Ministry of Health), so it should not come as surprise that in its next document the Government claims that larger share of the Budget of RM is in function of implementing the policy on balanced regional development.

This action plan stressed that, in the period after adoption of the regional development policy, measures and activities were taken to stimulate balanced regional development, on the basis of which disparity in development of the City of Skopje and other planning regions was reduced. It also presents data that 230 projects are approved for the purpose of stimulating balanced regional development, in total value of 264,820,014 MKD, and that 390 developmental projects were financed, in total value of 602 million MKD (almost 10 million EUR), by MLSG on the basis of identified development priorities of planning regions and under development strategies.[29]

These data provide the conclusion that none of the actions plans for implementation of the Strategy on Regional Development do not present shares according to the methodology anticipated in the Law and Strategy on Balanced Regional Development, while funds indicated for the period 2008-2015 range from 0.002% to 0.3% of GDP.

4. Efficiency of the policy on balanced regional development

Having in mind that data on efficiency of the policy on balanced regional development are relatively limited, in continuation we make an attempt to present and analyse available data related to impact indicators from implementation of the Strategy on Balanced Regional Development 2009–2019 presented in three action plans.

The first action plan included the following table:[30]

[27] Action Plan on Implementing the Strategy on Regional Development 2013-2015, "Official Gazette of the Republic of Macedonia" no. 122/2013, pp. 45-46. This document enlisted projects planned to be financed with funds from the Budget of RM and are focused on regional development, i.e. contribute to stimulation of balanced and sustainable regional development on the entire territory of Macedonia. Explanation for their inclusion implies the fact these are projects with developmental character and could contribute to increased GDP and employment in certain regions. They also include budget items which, according to account, programme, sub-programme, category and item, have regional component in their description, but are not accompanied with clear justification in terms of their regional effect and impact. For example, they include funds planned for Technology and Industrial Development Zone falling within competences of the Ministry of Economy, funds for environmental investments falling within competences of the Ministry of Environment and Spatial Planning, investments in railway infrastructure falling within competences of the Ministry of Transport and Communications, funds intended to stimulate employment falling within competences of the Ministry of Labour and Social Policy, funds for human resource development under budget programme MG falling within competences of the Ministry of Finance, etc.

[28] This budget increase was noted in EC's last progress report for Macedonia published in 2016, where it is said that balanced regional development is an integral part of the Ohrid Framework Agreement, that Action Plan 2016-2019 anticipated more transparent and balanced distribution of state funds, that budget on regional development is doubled in 2016 and that 2017 budget anticipates another increase, but that would again be insufficient.

[29] Action plan for Implementing the Strategy on Regional Development 2016-2018, "Official Gazette of the Republic of Macedonia" no. 123/2016, p. 2

[30] Action Plan for Implementing the Strategy on Regional Development 2010– 2012, Ministry of Local Self-Government, March 2010 (p. 20)

Table 18. Impact indicators from implementation of the Strategy 2009 – 2019

	Indicator	2009 value	2019 value
1	Average regional GDP per capita in Republic of Macedonia (according to PPP)		at least 50% of the EU average
2	GDP per capita (according to PPP) in the least developed planning region in Republic of Macedonia		at least 35% of the EU average
3	Difference in GDP per capita between the most and the least developed planning region	3.6 times *	maximum 3 times
4	Life expectancy at birth	73.4 years*	75 years
5	Regional share of population growth total population growth in Republic of Macedonia		
	Region with biggest growth in total growth in Republic of Macedonia	3.96%***	2%****
	Region with lowest growth in total growth in Republic of Macedonia	MK – 5.68%***	-2%****
6	Education level of the population		
		primary 53.1%	35% (reduction by 40% at regional level)
		secondary 36.9%	45% (increase by 20% at regional level)
		higher 10%	20% (increase by 100% at regional level)
7	Number of unemployed	341,893**	250,000 (reduction by 30% at regional level)

* 2007; **September 2009 ; *** 1994 – 2002 ; ****2002 – 2012
Source: Action Plan for Implementing the Strategy on Regional Development 2010–2012

The second action plan featured similar table with same title, but changed structure.[31] Data in the second table are updated and compared to those presented in the first table and demonstrate more specific effects in terms of projected values for 2019, especially in regard to two from seven targets: difference in GDP per capita between the most and the least developed planning region and number of unemployed. Data presented in actions plans provide the conclusion on modest progress in attainment of defined targets, but there is significant space for improvement in the field of reducing unemployment, difference in GDP per capita compared to the EU and among regions, population growth in regions and life expectancy.

Table 19. Impact indicators from implementation of the Strategy 2009 – 2019

	Indicator	2009	2010	2011	2019 value
1	Average GDP per capita in Republic of Macedonia	3,269 EUR	3,434 EUR	3,630 EUR	at least 50% of the EU average
2	GDP per capita in the least developed planning region in Republic of Macedonia	1,527 EUR	1,606 EUR	1,718 EUR	at least 35% of the EU average
3	Difference in GDP per capita between the most and the least developed planning region	3.32 times	3.18 times	3.04 times	maximum 3 times
7	Number of unemployed	341,295***	321,341****	281,341*****	250,000 (reduction by 30% at regional level)

December 2009; * December 2010; ***** December 2011
Source: Action Plan for Implementing the Strategy on Regional Development 2012– 2015

[31] Action Plan for Implementing the Strategy on Regional Development, "Official Gazette of the Republic of Macedonia" no. 122/2013, p. 23

Data presented in the next table provide the impression that funds anticipated for implementation of the policy on balanced regional development are gradually disappearing in the public finance management process, under "allocation", "approval" and "disbursement" from the Budget of RM to their realization through the Ministry of Local Self-Government. Hence, for example, the share of funds realized by MLSG in those planned and allocated under the Budget of RM in the period 2009-2012 accounted for only 45%, while the share of funds realized from those planned under the Action Plan 2009-2012 is even lower and accounted for less than 1/3 (29%). In this context, there is another important information missing in relation to the share of these funds in total GDP at national level for the period 2009–2012 and the possible deviation of this share against the obligation stipulated under Article 27, paragraph 2 of the Law on Balanced Regional Development that reads: "In order to stimulate balanced regional development, funds in the amount of at least 1% of GDP shall be allocated from the Budget of Republic of Macedonia".

Table 20. Impact indicators and outcomes from implementation of measures under the Action Plan 2010-2012[32]

Indicator	2010	2011	2012
	in million MKD		
Funds allocated from the Budget of RM and planned for realization through the Ministry of Local Self-Government	333.52	451.08	473.64
Approved (planned for 2012) funds from the Budget of RM and planned for realization through the Ministry of Local Self-Government	246	202.29	77.4
Ratio of planned and allocated funds from the Budget of RM and intended for realization through MLSG	73.8%	44.8%	16.3%
Disbursed funds from the Budget of RM and intended for realization through MLSG	138.8	140.6	67.3
Utilization of funds allocated from the Budget of RM and intended for realization through MLSG	56.4%	69.4%	87%
Ratio of utilized and planned funds under the Action Plan 2009-2012	41.6%	31.1%	14.2%

Source: Action Plan for Implementing the Strategy on Regional Development 2012– 2015

As regards attainment of targets defined under the development index used to establish development of planning regions, the second action plan presented data indicating that planned targets for development of regions in 2008 were approximately attained by majority of regions in 2012.[33]

Table 21. Planned and attained targets under the development index of regions in 2012

Planning region	2008	2012 (planned)	2012 (attained)
Skopje	1.48	1.62	1.51
Southeast	0.89	1.03	0.97
Pelagonija	0.73	0.80	0.91
Southwest	0.72	0.74	0.81
Polog	0.72	0.75	0.82
Vardar	0.69	0.75	0.73
East	0.67	0.72	0.96
Northeast	0.56	0.60	0.63

Source: Action Plan for Implementing the Strategy on Regional Development 2012-2015

As regards the last action plan, we underline several datasets related to impact indicators and their attainment.

Table 22 includes much more detailed information about attainment of seven main indicators from implementation of the Strategy on Balanced Regional Development by 2019 compared to previous tables with same indicators presented in two action plans.

[32] Action Plan for Implementing the Strategy on Regional Development 2013-2015, "Official Gazette of the Republic of Macedonia" no. 122/2013, p. 25

[33] Action Plan for Implementing the Strategy on Regional Development 2013-2015, "Official Gazette of the Republic of Macedonia" no. 122/2013, p. 26

According to the first indicator defined as average GDP per capita in Republic of Macedonia (according to PPP), there is evident progress, but most probably it is insufficient for attainment of the target defined as at least 50% of the EU average under the 2009 Strategy. Considering that this strategy was revised in 2014 and the target was corrected downward to 42% of the EU average, the revised target could be attained is the state attains solid GDP growth in following years.

The second target, GDP per capita (according to PPP) of the least developed planning region in Macedonia to be at least 35% of the EU average, seems impossible because the two least developed regions (Polog – according to lowest GDP per capita, and Northeast – according to the development index) have GDP per capita that is 17.7%, *i.e.* 22.4% of the EU average. However, given that the revised strategy anticipates the least developed planning region to have GDP per capita that is 26% of the EU average in 2019, there is a possibility for this revised target to be attained.

Third target concerning difference in GDP per capita between the most and the lest developed region to be maximum three times is already attained in 2013, although the strategy revised in 2014 reduced this difference to 2.2 times, which seems overly-ambitious, having in mind data presented in tables above, according to which the least developed region (Northeast) continues to lag behind in comparison to majority of regions in terms of almost all parameters.

As regards the fourth target, data are not segregated by region, i.e. life expectancy is presented only in terms of the national average.

In terms of the fifth target (ratio of regional population growth and national population growth), data presented concern years 2009 and 2013 and provide the impression that demographic situation is starting to stabilize, without major oscillations in population growth in individual regions as was the case in 2009. In 2013, biggest population growth against the national average was noted in Skopje region, with population growth higher by 1.63% compared to the national rate. Biggest reduction in population in the same year was noted in East region, marked by population growth lower by 1.85 % compared to the national rate. If this growing trend in regions continues, it is possible for the desired situation of maximum 2% difference in population growth among regions against the national rate to be attained in 2019.

Data related to the sixth and seventh target (education level of work-able population and number of unemployed) and presented in the most recent Action Plan for Implementing the Strategy on Regional Development 2009-2019 show positive developments, although similarly to the situation under the fourth target, data presented concern only the national average.

Table 22. Impact indicators from implementation of the Strategy 2009-2019[34]

	Indicator	2007	2009	2013	2019 value
1	**Average GDP per capita in Republic of Macedonia (according to PPP)**	7,512 EUR (30% of the EU-27 average	8,424 EUR (35% of the EU-27 average)	9,500 EUR (36% of the EU-28 average)	at least 50% of the EU average
2	**GDP per capita (according to PPP) of the least developed region in Republic of Macedonia**	3,571 EUR	3,952 EUR	4,636 EU (Polog has the lowest GDP per capita that is 17.7% of the EU-28 average) 5,918 EUR (Northeast as the least developed region overall has GDP per capita that is 22.4% of the EU-28 average)	at least 35% of the EU average
3	**Difference in GDP per capita of the most and the least developed region**	3.47 times	3.32 times	2.94 times	maximum 3 times
4	**Life expectancy at birth**	73.4 years	74.7 years	74.98 total 77.05 for women* 72.97 for men	80 years for women 75 years for men

[34] Action Plan for Implementing the Strategy on Regional Development 2016-2018, "Official Gazette of the Republic of Macedonia" no. 123/2016, p. 21

5	**Ratio of regional and national population growth**				
	Region with highest population growth (difference against the national population growth)	/	3.96%	1.63%** (Skopje planning region)	2%
	Region with lowest population growth (difference against the national population growth)	/	-5.68%	-1.85%** (East planning region)	-2%
6	**Education level of work-able population**				
	Primary	/	53.1%	38.92%***	35% (decrease by 40%)
	Secondary	/	36.9%	46.11%***	45% (increase by 20%)
	Higher	/	10%	14.97%***	20% (increase by 100%)
7	**Number of unemployed (and other people seeking job)**	341,893****	272,392*****	223,808******	250,000 (decrease by 30%)

Source: State Statistical Office:
*2010 **2010 in relation to 2014 ***2014
Source: Employment Agency of the Republic of Macedonia
****September 2009 ***** April 2012 ******December 2014

Tables 23 and 24 present data on funds anticipated for implementation of the policy on balanced regional development from the moment they were planned, by means of three-year action plans or programmes on balanced regional development, until the moment they are realized from the Budget of RM through the Ministry of Local Self-Government and the Bureau for Regional Development. They confirm the thesis on inefficient financial management and "loss/reduction" of funds in the process of planning and realization. However, analysis of the indicator on funds disbursed from the Budget of RM and realized by MLSG against funds planned under action plans provides the conclusion that their share in the period 2013-2015 has increased and reached 53% compared to the period 2009-2012 when this share amounted to only 29%. Solid impression is made by major increase of funds planned for this purpose in 2016, 2017 and 2018, which is welcomed in the EC's most recent progress report for Macedonia, but having in mind previous trends there is a possibility for these funds to also "disappear" under budget adjustments in "allocation", "approval" or "disbursement".

Table 23. Impact indicators from implementation of measures under the Action Plan 2013-2015

Realized funds (in million MKD)

Indicator	2013	2014	2015
Funds planned by MLSG and BRD under Action Plan 2013-2015	99.4	206.5	227.2
Funds planned by MLSG and BRD under Annual Programmes on Balanced Regional Development	99.41	117.11	147.71
Funds disbursed from the Budget of RM and realized through MLSG and the Bureau	50.91	112.33	123.56

Table 24. Impact indicators and outcomes from implementation of the Action Plan 2016-2018

Planned funds

	Indicator[35]	2016	2017	2018
		in million MKD		
1	Funds allocated from the Budget of RM and intended for realization through MLSG and the Bureau for Regional Development	570.17	870.17	980.98
	Utilization rate of funds allocated from the Budget of RM and intended for realization through MLSG	90%	90%	95%
2	Funds allocated from the Budget of RM and intended for realization through other line ministries **	17,157.88	19,509.71**	19,962.42***
	Utilization rate of funds allocated from the Budget of RM and intended for realization through other line ministries	70%	75%	80%

***Funds presented in the table are from the following budget accounts: OA- balanced regional development, MB - cross-border cooperation, except for item 420 on goods and services from the general budget, and 4A - square.

**Funds allocated under IPA and IPA2 and related to regional development are adequately presented in the budget of institutions responsible to manage these funds, and are allocated from 2016 Budget of RM.

***Sum of funds allocated from the Budget of RM for implementation of developmental activities relevant for balanced regional development and intended for realization by line ministries are not included in funds planned under programmes with development component for the year 2017 and 2018, because such data do not exist.

The last action plan presents new targets for developmental, economic-social and demographic indices of the regions in 2018, according to which four from eight planning regions are expected to exceed the national average development index, and Skopje region is expected to maintain its value under this index. Single backslide in these projections is observed under the demographic index in terms of target values for Skopje and Pelagonija region. Regions are expected to achieve progress under all other parameters (see Table 25).

Table 25. Indicators from indices used to establish development of planning regions

Development Index				
Planning region	**2008**	**2012 (planned)**	**2012 (realized)**	**2018**
Skopje	1.48	1.62	1.51	1.51
Southeast	0.89	1.03	0.97	1.10
Pelagonija	0.73	0.80	0.91	1.05
Southwest	0.72	0.74	0.81	0.92
Polog	0.72	0.75	0.82	0.93
Vardar	0.69	0.75	0.73	0.85
East	0.67	0.72	0.96	1.08
Northeast	0.56	0.60	0.63	0.80
Economic-Social Index				
Planning region	**2008**	**2012 (planned)**	**2012 (realized)**	**2018**
Skopje	1.86	1.95	1.48	1.50
Southeast	1.38	1.45	1.29	1.35
Pelagonija	0.79	0.87	1.09	1.15
Southwest	0.5	0.58	0.98	1.05
Polog	0.18	0.20	0.50	0.70
Vardar	0.63	0.72	0.70	0.84
East	0.95	1.05	1.36	1.40

[35] "Funds enlisted in the table and allocated from budgets of institutions different than MLSG will be used in compliance with the procedures of relevant institutions. However, having in mind the size of these funds, their importance for regional development in Macedonia, as well as the need for their utilization as driver of balanced development, they need to be presented in summary, i.e. their utilization to be monitored as input parameters in the overall system on financial support for regional development in Macedonia (irrespective of the fact whether they are funds allocated at MLSG or another institution)". Action Plan for Implementing the Strategy on Regional Development 2016-2018, "Official Gazette of the Republic of Macedonia" no. 123/2016, p. 22

Northeast	0.33	0.38	0.27	0.40
Demographic Index				
Planning region	**2008**	**2012 (planned)**	**2012 (realized)**	**2018**
Skopje	1.25	1.29	1.53	1.50
Southeast	0.58	0.62	0.72	0.80
Pelagonija	0.69	0.74	0.80	0.65
Southwest	0.86	0.90	0.70	0.70
Polog	1.05	1.08	1.07	1.10
Vardar	0.73	0.78	0.76	0.85
East	0.5	0.54	0.65	0.70
Northeast	0.7	0.75	0.90	1.05

Conclusions

Why We Need Policy on Balanced Regional Development

✓ More than 50% of population growth in the state comes from Skopje region, with Skopje and Polog region accounting for approximately 90% of national population growth. Other regions have modest population growth rates, whereas Pelagonija and East region are characterized by population decline.

✓ Increased population figures, as parameter, show that in the last 10 years, only portion of regions could be considered advanced, those being: Skopje – with more than 30,000 new inhabitants, Polog – with more than 10,000 new inhabitants, Northeast and Southeast – with minimum population growth of 1,500 to 2,500 inhabitants.

✓ Comparison of data for the period 2005-2015 shows that shares of young population are decreasing, with the most dramatic examples recorded in Polog, Southeast and Southwest region, where differences of young population as shares in total population has changed by 5 to 6%, whereas the biggest growth in elderly population was observed in Skopje, Southeast, East and Vardar region, ranging from 2 to 3%.

✓ Highest decrease of socially endangered population is observed in Pelagonija, where the number of social allowance beneficiaries has been reduced by almost three times in the period 2006-2014. In the period of 8 years, Northeast, Vardar and Southeast region halved their respective numbers of social allowance beneficiaries, while Southwest and East region have almost twice as less social allowance beneficiaries compared to their respective 2006 figures.

✓ Although marked by modest decrease, distribution of unemployment per regions leads to the conclusion that major differences remain and should serve as basis for various measures and activities in different regions. In some regions, for example the Southeast, 2015 unemployment rate is higher by almost 16% and in Southwest region it is higher by 7% from the national average.

✓ Skopje region (which is the place of residence for 1/3 of population in the state) generates high 43% of GDP. Therefore, this region is dominant in terms of regional GDP per capita, as this rate is two and half times higher than GDP per capita in Northeast as the least developed region. Two more regions (Southeast and Vardar) have higher GDP per capita compared to the average rate. This is strong signal about the need for comprehensive and in-depth policy on balanced regional development that could yield positive results in the medium term.

✓ Skopje region accounts for unchanged share of around 37-38% in total number of active business entities, with Pelagonija accounting for around 11-12% and Southwest accounting for around 10%. Slightly more prominent progress is noted in Polog region, whose share is growing from 9.6% in 2008 to 10.8% in 2015, and has climbed to second place before Southwest region in terms of this indicator on regional development.

✓ Comparison of respective figures on employees provides the conclusion that, in 2015, Southeast, Pelagonija and Polog region had lower number of companies employing more than 250 people, while Southeast and Pelagonija had lower number of companies employing 50 to 249 people.

✓ Analysis of regional shares in total equity investments raises concerns, as it shows that more than half of investments are made in the most developed region and there are almost no investments made in the least developed region (Northeast).

✓ Value of construction works shows that although in 2007 Polog region was deemed the main "construction site" in the state, in 2015 this region is the second to last in terms of the value of construction works, demonstrating better performance only than Southeast region, which is continuously ranked at the bottom of this list. In the last 9 years, lowest construction activity was observed in Northeast region which, with the exception of 2015, is ranked last according to the value of construction works performed.

✓ In the last 3 years, local self-government units in Skopje region, including the City of Skopje, accounted for 37.8% of total revenue generated by LSGUs in Macedonia. Lowest revenue was generated by municipalities in Vardar (7%) and Northeast (7.1%) region, followed by municipalities in Southeast (8.3%), East (8.6%), Southwest (9.3%), Pelagonija (10.6%) and Polog (11.2%).

✓ Most recent data on regional distribution of GDP, published by the State Statistical Office and calculated for 2013 based on their methodology, indicate a level of 29.0% and represents a decrease by 0.1% compared to 2012 figures. Decrease in the value of this share implies more balanced distribution of development among the regions.

Efficiency of The Policy on Balanced Regional Development

✓ For the purpose of stimulating balanced regional development, funds in the amount of at least 1% of GDP should be allocated from the Budget of Republic of Macedonia and disbursed through the Ministry of Local Self-Government. Later, on the basis of the classification of regions according to their development, the Ministry on Local Self-Government and the Bureau for Regional Development distribute these funds to individual regions according to their relevant share. Nevertheless, with every new budget, budget adjustment and final balance of the state budget, and every new Action Plan for Implementing the Strategy on Regional Development, contrary to the legal framework in place, the Government applies different methodology to calculate the shares of GDP invested in balanced regional development.

✓ Start of implementation of the law and policy on balanced regional development was faced with series of delays in application of the main components, which significantly impacted efficiency of the overall process and policy.

✓ Major disparity in development among planning regions in Macedonia is observed when comparing the developmental, economic-social and demographic indices, which showed visible differences between Skopje, on one side, and all other regions, on the other side, especially in the economic-social sphere. Especially worrying are tendencies observed in the least developed region, which according to most recent data is marked by high unemployment of 43.2%, highest number of social allowance beneficiaries per capita and share of around 2% in total equity investments in the state.

✓ Numerous independent reports on balanced regional development provide data according to which annual funds allocated by the Government for this purpose have not reached desirable 1% of GDP, notably because government reports operate with data ranging from around 1% to 3.25% of GDP, but these shares concern only planned funds, not realized funds, as confirmed by specific data in the final balance sheets of the state budget. Difference between these two approaches concerns the fact whether funds allocated from the Budget of Republic of Macedonia are actually awarded according to the methodology on distribution of funds for regional development and on the basis of development of individual planning regions (as stipulated in the Law on Balanced Regional Development) or on the basis of total allocations for regional development (representing continuation of the policy on development of underdeveloped areas, which existed prior to adoption of the Law on Balanced Regional Development).

✓ Every new Action Plan for Implementing the Strategy on Regional Development expands the number of government ministries and agencies whose budgets include a component on regional development, *but there are no available data that distribution of these funds is pursued in compliance with the priorities for balanced regional development, i.e. according to development of individual planning regions* (except for the fact that these funds are allocated in the budgets of several ministries and agencies and labelled “regional” and that they are not managed by the Ministry of Local Self-Government, which represents certain effort on the part of the state to support regional development).

✓ By applying this approach, policy makers have restored, in one segment, the process on development of underdeveloped regions to the level that existed prior to adoption of the Law and Strategy on Balanced Regional Development. It was this methodology on development of regions according to their underdeveloped status that implied specific difference compared to the previous policy, for which the policy maker claimed was “monocentric”.

✓ Information system is still not in place for updated and reliable data in relation to development of individual regions, which depends on the percentile distribution of funds per region from the Budget of Republic of Macedonia intended for balanced regional development.

✓ Action Plans for Implementing the Strategy on Regional Development do not present the shares according to the methodology stipulated in the Law and Strategy on Balanced Regional Development which, in the period 2008-2015, ranged from 0.002% to 0.3% of GDP.

✓ Analysis of available data provides the conclusion that funds anticipated for implementation of the policy on balanced regional development are gradually disappearing in the process on public finance management, from "allocation", "approval" and "disbursement" of funds from the Budget of Republic of Macedonia to their realization through the Ministry of Local Self-Government. Hence, for example, the average realization rate of funds planned and allocated from the Budget of RM and intended for realization through MLSG accounted for only 45% in the period 2009-2012, while the average realization rate of funds planned under the Action Plan 2009-2012 was even lower (29%). Nevertheless, comparison of indicators on paid/used funds from the Budget of RM and realized through the Ministry and the Bureau and funds planned under the Action Plan shows that in the period 2013-2015 this share has increased and accounted for 53%, when compared to its level of 29% in the period 2009-2012.

✓ On the account of insufficient amount of funds allocated from the Budget of RM and intended for regional development, as well as the trend on reduction of planned funds by means of budget adjustments, effectiveness in implementation of the policy on balanced regional development is not satisfactory.

✓ Data presented in action plans show small progress towards attainment of targets defined, however, there is still significant space for improvement in the fields of unemployment, difference in GDP per capita compared to the EU and among regions, population growth per regions and life expectancy.

✓ Certain parameters used in the past to analyse regional development show negative tendencies under the policy on balanced regional development (primarily they concern data on the population's ageing and total amount of equity investments), while other data provide small spark of hope for mitigation of consequences arising from long years of investments dominantly focused in the capital (data on active business entities and construction works performed).

General recommendations

According to data from official state institutions presented above, the policy on balanced regional development is facing serious challenges and is yet to engage in serious efforts in the future. Small progress has been achieved, as some regions are marked by positive trends in terms of economic parameters, but performance audit findings of the State Audit Office indicate that "there is insufficient efficiency in performance of the Programme on Balanced Regional Development". Evidence on the unfavourable situation is found in data whereby more than half of investments continue to be generated in the most developed region, population growth in most regions is negative, while the three least developed regions (Northeast, Polog and Southwest) are facing high unemployment and have not improved their respective contribution shares in GDP in the last years compared with other regions in the state.

The Government interprets the legal solution on annual allocation of 1% of GDP from the Budget of Republic of Macedonia to stimulate balanced regional development as accumulation of budget allocations from different ministries and agencies that have certain links to regional development, but not specifically with balanced regional development. In order to make regional development truly balanced, funding practices must adhere to the existing methodology or new methodology should be proposed for distribution of budget funds, according to which funds in the amount of 1% of GDP (or 3.25% as enlisted by the Government in one of its documents) will be awarded according to the relevant shares calculated on the basis of development in individual planning regions.

Within the shortest deadline possible, the Council on Balanced Regional Development or the Ministry of Local Self-Government should establish a transparent reporting system on all parameters related to regional development, thus increasing interest and awareness of citizens for this important policy for the state's future. Even the Strategy on Regional Development 2009-2019 refers to information system that would allow adherent compliance with the principles of transparency and accountability in strategy implementation, and would ensure access to public information related to strategy implementation.

Having in mind that policy on balanced regional development is in compliance with strategic goal of the Republic of Macedonia in terms of its aspiration to join the EU, as well as in line with guides on decentralization development anticipated in the Constitution of Republic of Macedonia, authors of this analysis believe that consensus is needed among main political actors for further and more committed implementation of the Law on Balanced Regional Development. Having in mind that this law and strategy had been adopted almost one decade ago and there is no visible progress in terms of awareness in society and in terms of political will, it seems as if the law does not exist. In spite of greater will and commitment demonstrated by management structures at relevant institutions and bodies toward capacity building for regional development, the possibility for establishment of special ministry of regional development should be reconsidered, following the example in Bulgaria (Ministry of Regional Development and Public Works), Croatia (Ministry of Regional Development and

EU Funds), Czech Republic (Ministry of Regional Development), Romania (Ministry of Regional Development and Public Administration), Poland (Ministry of Regional Development, which was renamed in 2013 as the Ministry of Infrastructure and Development), Latvia (Ministry of Environmental Protection and Regional Development), Ukraine (Ministry of Regional Development and Construction), Moldavia (ministry of Regional Development and Construction), Georgia (Ministry of Regional Development and Infrastructure) or Norway (Ministry of Local Government and Regional Development).

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AGGREGATION WITH TWO-MEMBER HOUSEHOLDS AND HOME PRODUCTION

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Abstract:

This note explores the problem of family labor supply decision in an economy with two-member households, joint home production, and fixed cost of joint labor supply. Even though the labor supply decisions are not indivisible per se, the presence of such fixed cost and partners with unequal labor productivity create non-convexities. The note shows how lotteries as in Rogerson (1988) can again be used to convexify consumption sets, and we perform aggregation over individual preferences. The main result demonstrated in the paper is that aggregate preferences of males do not differ from individual level ones. However, for females, the disutility of non-market work at the aggregate becomes separable from market work, but keeps its original (logarithmic) form, while the female labor elasticity of the market hours' supply increases from unity to infinity.

Keywords: family labor supply; home production; aggregation

JEL Classification: E1; J22; J46

1. Introduction

The standard real business cycle model, *e.g.* Kydland and Prescott (1982), does not capture well labor market behavior. After looking at micro evidence and finding that males and females differ in terms of their labor supply elasticities, Cho and Rogerson (1988) demonstrated that the existence of families, which earlier representative-agent models ignores, might be important at the aggregate level. After all, a typical household is not a one-member entity, but mostly comprises two members. The household usually pools resources together (to diversify risk) and makes joint decisions, aiming to maximize the utility of the family. This note explores the problem of family labor supply as in Cho and Rogerson (1988), where the partners have different productivities, and in case both partners work in the market sector, the family faces a certain fixed cost (*e.g.*, increased inconvenience due to the tight working schedules of both partners). The novelty in this note is the introduction of home production technology as well, where the latter also provides consumption flows. The note explores how the availability of home production interacts with the families' structure and whether this matters for aggregate preferences. After all, non-market production is likely to be important particularly for families, who enjoy it jointly, *e.g.* house cleaning, cooking, etc. In addition, the availability of joint home production technology provides an alternative use of time for each of the members in the household. This note shows that despite the fact that all labor decisions are continuous, the presence of family labor supply fixed cost introduces a non-convexity. Therefore, lotteries as in Rogerson (1988) can again be used to convexify households' consumption sets, and aggregation over individual preferences. The main result demonstrated in the paper is that aggregate preferences of males do not differ from individual level ones. However, for females, the disutility of non-market work at the aggregate becomes separable from market work, but keeps its original (logarithmic) form, while the elasticity of

the market hours supply increases from unity to infinity. This result is reminiscent to the one found in Vasilev (2016) in a model with home production and indivisible labor choice in the market sector, but without families.

2. Model Description

2.1. Household problem

The theoretical setup presented in this note is a one-period economy without physical capital. The representation is identical to an economy where the capital has been already chosen optimally. There is a unit measure of identical male individuals (indexed by subscript "1"), and a unit measure of identical female individuals (indexed by subscript "2"). In the paper, "households" and "families" will be used interchangeably. There is no formal "marriage" decision in the model, in the sense that each male is already matched with a female. For simplicity, it will be assumed there is no bargaining within the family, no possibility for divorce and thus no options outside marriage. The household's role is to pool income and to make joint decisions. The preferences of the partners in each family are also the same, and are defined over composite consumption (*c*) and leisure (*l*), and are represented by a separable utility function *u(c,l)* as follows

$$u = \ln(c) + \ln(l),$$

where

c =[a (c_m)^b+(1-a)(c_n)^b]^(1/b),

is, as in McGrattan, Rogerson and Wright (1997), a Constant Elasticity of Substitution (CES) aggregation of market- and non-market ("home") consumption, denoted by c_m and c_n, respectively. Parameters a and $1-a$, where $0 < a < 1$, denote the weights attached to different consumption categories in the aggregate consumption bundle, and parameter $b > 0$ measures the degree of substitutability between market and home production.

Each household member has a unit endowment of time, which can be either supplied in the market sector, used to produce non-market output, or enjoyed as leisure, hence

$$l = 1 - h_m - h_n.$$

Non-market output (y_n) is non-tradable and non-storable consumption good (c_n), and can be produced using labor as follows:

$$y_n = c_n = h_{1,n} + h_{2,n},$$

where each member of each household can supply any number of hours in the non-market sector. Home production is jointly enjoyed by the male and the female in the family, thus $c_n = c_{1,n} = c_{2,n}$. More specifically, non-market output is a public good from the family point of view, while market consumption is a private (and rivalrous) good.

The labor choice in the market sector is also continuous. The hourly wage rate in the market sector is w_1 for males, and w_2 for females, which are taken as given.[36] Finally, each household claims an equal share of the representative firm's profit, denoted by π.[37] The budget constraint that each household faces is then

$$c_{1,m} + c_{2,m} = w_1 h_{1,m} + w_2 h_{2,m} + \pi$$

Household's utility maximization problem is to

$$U = \ln c_1 + \ln c_2 + \ln(1 - h_{1,m} - h_{1,n}) + \ln(1 - h_{2,m} - h_{2,n}) - I(h_{1,m}, h_{2,m})\tau$$

where $I(h_{1,m}, h_{2,m})$ is an index functions, which takes a value of unity in case of a positive labor supply by both parents ($h_{1,m}h_{2,m} > 0$), and zero otherwise. $\tau > 0$ represents the size of the fixed cost incurred when both parents are working.

The problem is to max Eq. (6) subject to (2) - (5). The problem is thus one of choosing $\{(c_{1,m}, h_{1,m}, h_{1,n}), (c_{2,m}, h_{2,m}, h_{2,n})\}$ optimally by taking $\{w_1, w_2, \pi\}$ as given.[38]

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[36] In equilibrium, in order to be consistent with data, we will calibrate the model so that $w_1 > w_2$.

[37] This is a technical assumption, which guarantees positive consumption even for unemployed people.

[38] Note that by choosing $h_{1,n}$ and $h_{2,n}$, the household chooses optimally c_n as well.

2.1 Firms

There is a representative firm producing a homogeneous final consumption good using male and female labor. For simplicity, its price is normalized to unity. As in Cho and Rogerson (1988), it will be assumed that female hours are only an imperfect substitute to male hours, with degree of proportionality $0 < \nu < 1$. This modeling choice reflects difference in productivities as proxied by wages in data, e.g. Reder (1962). Finally, the production function features decreasing returns to scale and is given by

$$Y = F(H_{1,m} + \nu H_{2,m}), F'(.) > 0, F''(.) < 0, F'(0) = \infty, F'(\bar{h}_{1,m} + \nu \bar{h}_{2,m}) = 0,$$

where, as in Cho and Rogerson (1988), $\bar{h}_{1,m}$ and $\bar{h}_{2,m}$ are the optimal market labor supply decisions made by individual males and females, respectively. The last constraint means that the firm faces a capacity constraint: If the male and female in each household work full-time in the market sector, the marginal product of an hour worked is zero. Together with the decreasing returns to scale that would produce positive economic profit (and thus guarantee a positive level of market consumption) in equilibrium.

The firm acts competitively by taking the wage rates $\{w_1, w_2\}$ as given, and chooses hours $\{H_{1,m}, H_{2,m}\}$ employed to maximize profit.

3. Decentralized Competitive Equilibrium (DCE)

A Decentralized Competitive Equilibrium (DCE) is defined by consumption and labor allocations for males and females, wage rates, and aggregate profit $\Pi(=\pi)$ such that (i) all households maximize utility; (ii) the stand-in firm maximizes profit; (iii) all markets clear.

4. Characterizing the DCE

It will be shown that in the DCE, if it exists, only some of the females in the households will be employed, but everyone enjoys the same utility level of consumption.[39] From the firm's problem, the wage rates are

$$w_1 = F'(H_{1,m} + \nu H_{2,m}),$$
$$w_2 = \nu F'(H_{1,m} + \nu H_{2,m}).$$

Thus, $w_1 > w_2$ follows directly from the assumption that female hours are imperfect substitutes for male hours ($\nu \in (0,1)$) in the production of market consumption. Moreover, if there is a DCE in which not all females work, then it must be the case that not all females in the households would receive the same consumption bundle. Denote this mass of employed females by ϕ, and the proportion of unemployed females by $1 - \phi$. Then the total fixed cost resulting from family labor supply equals $\phi\tau$. Note that this setup is very similar to a setup with non-convexities, as in Rogerson (1988). More specifically, even though there is no indivisibility in the individual female labor supply decision sets, due to the presence of fixed costs when both partners work, the setup is isomorphic to a model where the females can only supply $h_{2,m} \in [0, \bar{h}_2]$.

Thus, in equilibrium, aggregate male and female hours are $H_{1,m} = \bar{h}_{1,m}$ and $H_{2,m} = \phi \bar{h}_{2,m}$, respectively, and using those expressions, we can compute equilibrium profit. Next, if ϕ is the equilibrium proportion of females employed, then total utility for females that work in the market sector should equal to the utility of females that do not supply any hours in the market sector. In particular, the females that work will have higher utility of consumption, while those who do not work enjoy higher utility of leisure. Let $c_{2,m}^{e}$ denote the market consumption of working females, and $c_{2,m}^{u}$ denote the market consumption of non-working females. Similarly, let c_2^{e} denote the composite consumption of working females, and c_2^{u} denote the composite consumption of non-working females, where $c_{2,m}^{e} > c_{2,m}^{u}$, and $c_2^{e} > c_2^{u}$. Females that work, supply $\bar{h}_{2,m}$ in the market, and females that do not work, supply zero hours in the market.

Next, from the symmetry of the production of the home production output, it follows that the choice for home hours will be symmetric, with $h_n = h_{1,n} = h_{2,n}$. Furthermore, $h_n = h_n(\phi)$, *i.e.*, non-market hours are a function of the proportion of females employed in the market sector. Using the Implicit Function Theorem on the optimality condition for home hours, we can show $\frac{dh_n}{d\phi} > 0$, which follows from the complementarity between the two types of consumption in the household's utility function. To show that the DCE actually exists, it is sufficient to

[39] As demonstrated below, this follows from the difference in productivity (wages). In a way, it can be also inferred from the model setup that the females also bear the fixed cost in case both partners in the family work in the market sector.

show the existence of a fixed point $\phi \in (0,1)$ by analyzing a non-linear equation using that in equilibrium utility has to be the same for all households. First, it is trivial to show that an equilibrium where all females working ($\phi = 1$) is not possible, since by assumption then it follows that $w_1 = F'(\bar{h}_{1,m} + \nu\bar{h}_{2,m}) = 0$. Similarly, no females working ($\phi = 0$) is not an equilibrium outcome either, since the firm would offer a very high wage for the first unit of female labor, and by taking a full-time job a worker could increase her utility. Finally, since the equation equalizing female utility independent of employment status is monotone in ϕ, as the utility function is a sum of monotone continuous functions, and from the assumptions imposed on the production function. Then from the intermediate value theorem it directly follows that there exists a unique $\phi \in (0,1)$ that is consistent with equilibrium.[40] Given the indivisibility of the female labor supply in the market sector, the equilibrium allocation obtained above contains non-convexity, and is therefore not Pareto optimal, as demonstrated in Rogerson (1988). More specifically, a Social Planner (SP) could make everyone better off by using employment lottery and choosing the fraction ϕ of females to work and give every female the same consumption $c_2 = \phi c_{2,m}^e + (1-\phi)c_{2,m}^u$, independent of the employment status. Note that such an allocation is both feasible, and provides a higher level of total utility. Showing feasibility is trivial, it follows directly from the presence of unit mass of the females in the model.

Next, it will be shown that the new allocation, which is independent of household's employment status, makes households better off since it generates higher utility on average.[41] This follows from the convexity of the CES aggregation and the concavity of the logarithmic function. Thus, the SP is indeed giving in expected utility terms an allocation that is an improvement over the initial equilibrium allocation.

Then, as a result of pooling resources within the family, another round of Pareto-improving redistribution can be implemented by giving everyone (male or female) the same composite consumption[42]

$$\hat{c} = \frac{1}{2}[c_1 + c_2]$$

In other words, the family in the model acts as an insurance mechanism that can achieve complete insurance across the members of the family. Such redistribution is again feasible and makes everyone better off.[43] Aggregate preferences now look as follows

$$U = 2\ln[\hat{c}] + \ln[1 - \bar{h}_{1,m} - h_n(phi)] + \phi\ln[1 - \bar{h}_{2,m} - h_n(\phi)] + (1-\phi)\ln[1 - h_n] - \phi\tau$$

Note that since the fraction of females employed is

$\phi = H_{2,m}/\bar{h}_{2,m}$

By letting

$$A = \frac{\tau}{\bar{h}_{2,m}} > 0,$$

the aggregate utility function simplifies to

$$U = 2\ln[\hat{c}] + \ln[1 - \bar{h}_{1,m} - h_n(\phi)] + \ln[1 - h_n(\phi)] - AH_{2,m} + H_{2,m}\frac{1}{\ln[1 - h_n(\phi)]}$$

Finally, letting

$$B = A + \frac{1}{\ln[1 - h_n(\phi)]} > 0,$$

and also using that aggregate hours supplied by males

[40]More precisely, there are a lot of equilibria: in each equilibrium the names of the females working are different, but in every equilibrium the same fraction of females ϕ that works is the same.

[41]For now, the fixed cost can be ignored, as it is present in both cases.

[42]This means giving everyone the same market consumption, as the non-market consumption is the same from the very beginning.

[43]Note that the redistribution is identical to the one where we equalize consumption within each family, and then equalize consumption across families.

$$H_{1,m} = \bar{h}_{1,m},$$

results in an aggregate utility of the form

$$U = 2\ln C + \ln[1 - H_{1,m} - h_n(\phi)] + \ln[1 - h_n(\phi)] - BH_{2,m}.$$

The aggregate utility function above is of an interesting and novel form: On the aggregate, when there is a continuum of a two-member household with fixed cost of family labor supply and home production, the representative agent obtained from the aggregation procedure features different preferences of work. More specifically, the disutility of labor when it comes to male hours is still logarithmic (so there is no difference between the individual labor supply elasticity for males and the same elasticity for males at the aggregate level), but in the female market sector is now linear, while the disutility of home labor is logarithmic as in the individual utility function. The split of the two types of labor (market and non-market) for females, but not males, is a novel result that is driven by the fact that market labor for females behaves as if it were indivisible, while non-market hours were divisible

Conclusion

Our model with two-member households and home production can generate large aggregate labor supply elasticities while being consistent with the micro evidence (see Heckman and MaCurdy 1980, MaCurdy 1981, Killingsworth 1983, Kydland 1984, and the references therein.) that most individuals have a very low labor supply elasticity. In addition, the interaction with home production produces different elasticity of market vs. non-market labor supply for females only. This heterogeneity in aggregate preferences for labor between males and females is an interesting finding as well, and due to the fixed cost of joint labor supply, despite the homogeneity of preferences at the individual level. Lastly, given that the population in the model has a combined mass of two, the scaling factor in front of the utility from aggregate consumption will affect the degree of substitution between consumption and hours.

As a possible venue for future research, we plan to extend the model to a dynamic and stochastic setting, and feed the derived aggregate utility function above in a sophisticated Real-Business-Cycle model with physical capital, distortionary taxation, government spending, and home production to investigate the effect of those preferences for the transmission of technology and fiscal shocks. However, such investigations are beyond the scope of this note.

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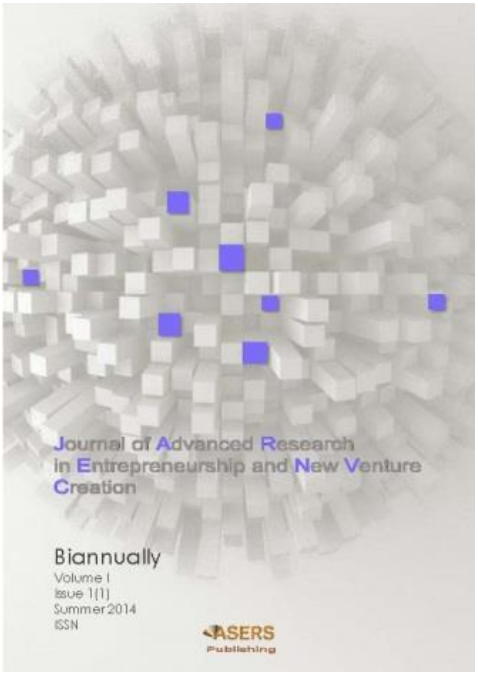

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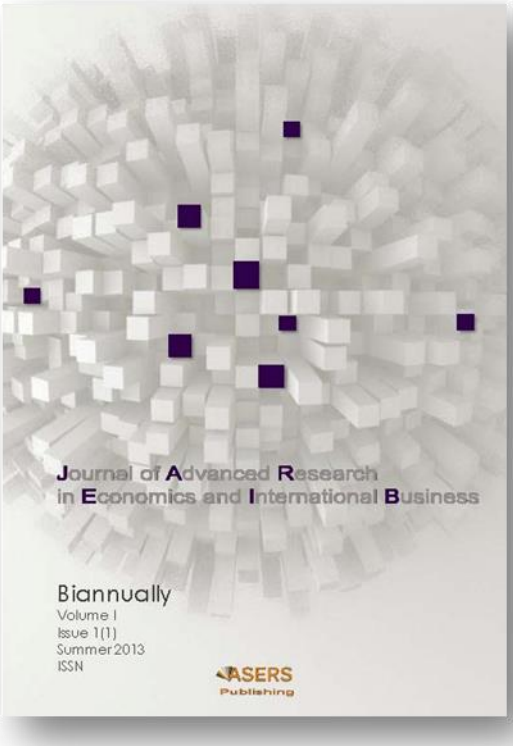

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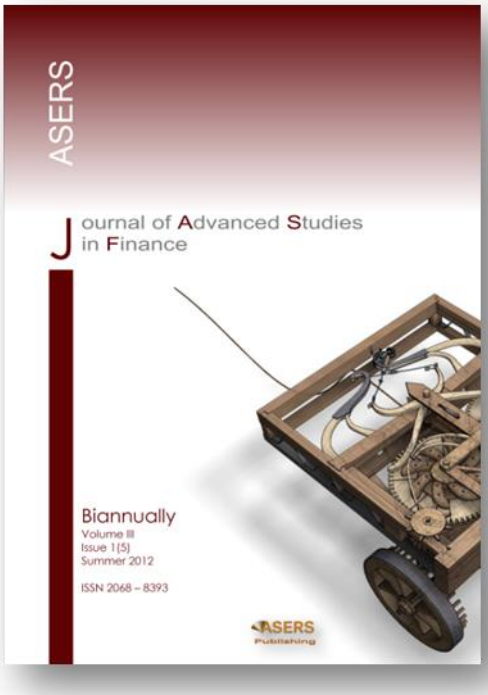

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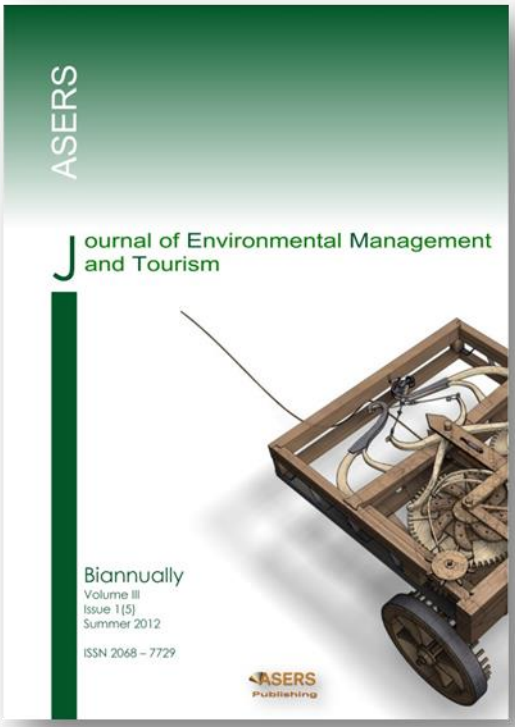

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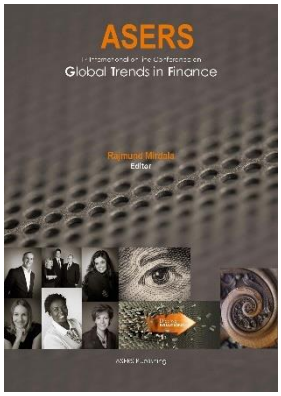

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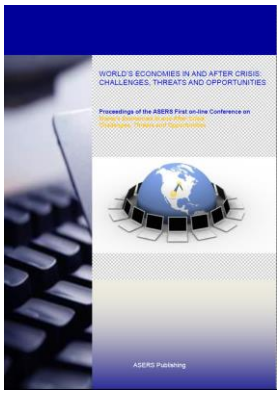

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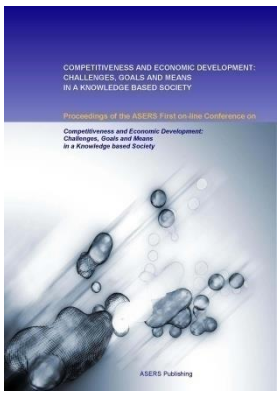

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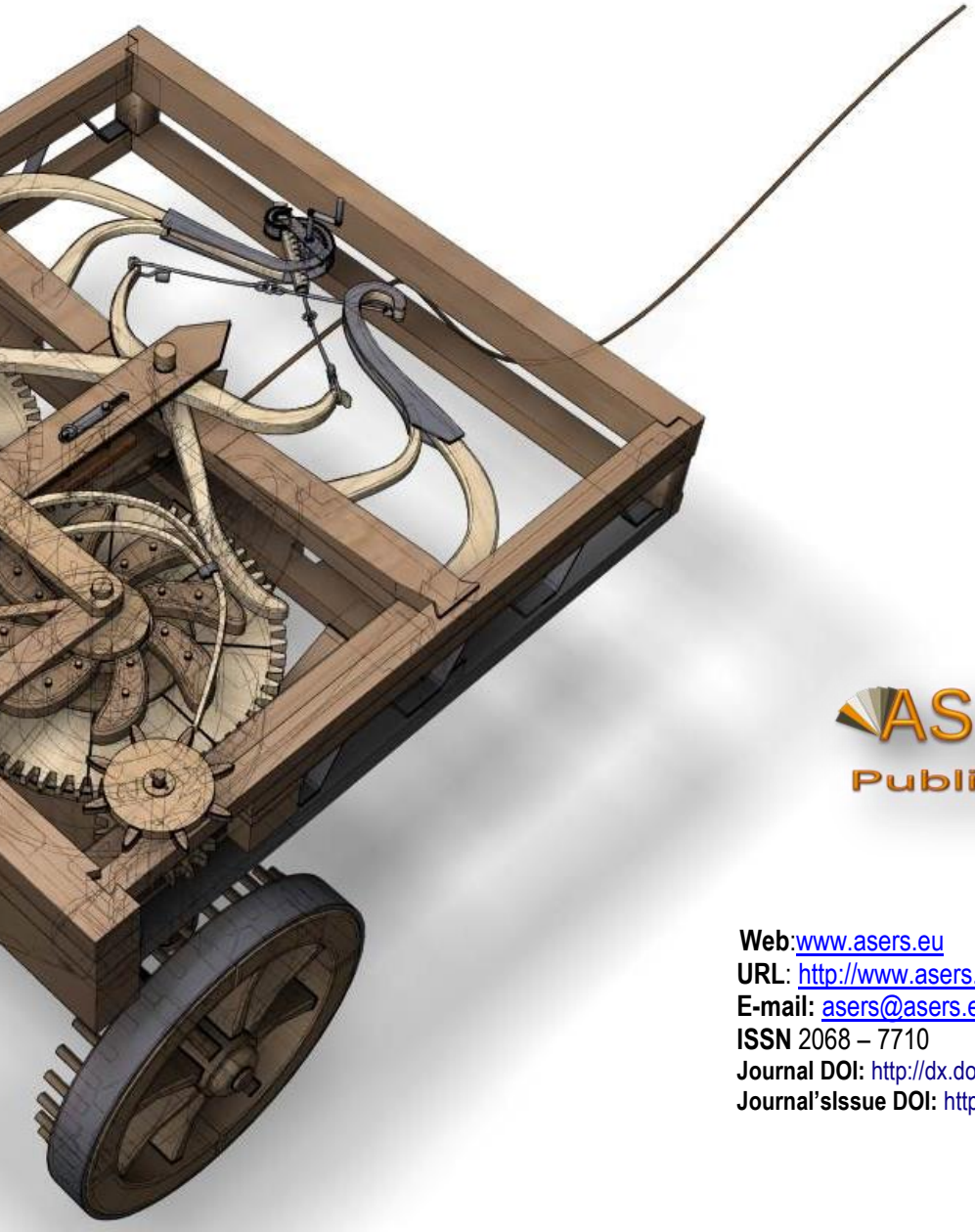

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