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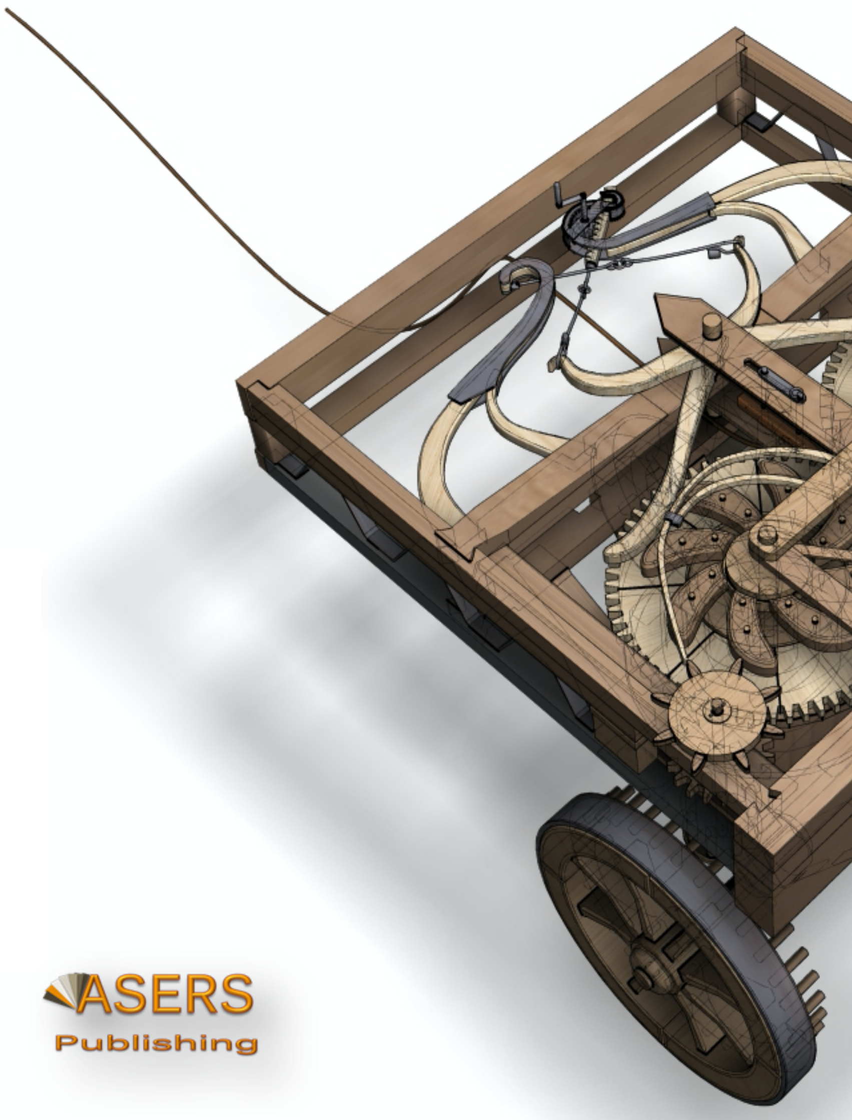

Biannually
Volume VII
Issue 2(14)
Winter 2016

ISSN 2068 – 7710
Journal **DOI**
http://dx.doi.org/10.14505/tpref

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Volume VII
Issue 2(14)
Winter, 2016

ASERS Publishing
http://www.asers.eu/asers-publishing
ISSN 2068 – 7710
Journal's Issue **DOI**:
http://dx.doi.org/10.14505/tpref.v7.2(14).00

Contents:

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Volume VIII, Issue 1(15), Summer 2017

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http://dx.doi.org/10.14505/tpref.v7.2(14).02DOI:
http://dx.doi.org/10.14505/tpref.v7.2(14).01

TESTING NON-LINEAR DYNAMICS, LONG MEMORY AND CHAOTIC BEHAVIOUR OF ENERGY COMMODITIES

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Suggested Citation:

Gencer, M. Ünal G. (2016). Testing non-linear dynamics, long memory and chaotic behaviour of energy commodities, *Theoretical and Practical Research in Economic Field*, (Volume VII, Winter 2016), 2(14): 85-97. DOI:10.14505/tpref.v7.2(14).01. Available from: http://www.asers.eu/publishing/index.php/tpref

Article's History:

Received Octomber, 2016; *Revised* November, 2016; *Accepted* December, 2016.

Abstract:

This paper contains a set of tests for nonlinearities in energy commodity prices. The tests comprise both standart diagnostic tests for revealing nonlinearities. The latter test procedures make use of models in chaos theory, so-called long-memory models and some asymmetric adjustment models. Empirical tests are carried out with daily data for crude oil, heating oil, gasoline and natural gas time series covering the period 2010-2015. Test result showed that there are strong nonlinearities in the data. The test for chaos, however, is weak or no existing. The evidence on long memory (in terms of rescaled range and fractional differencing) is somewhat stronger although not very compelling.

Keywords: Energy commodities, Lyapunov exponents, Correlation dimension, chaos, long memory

JEL Classification: C1, C6, C40, C150, G170

1. *Introduction*

There have been many efforts to exploit models that could explain the changes in the energy commodities price and forecast it accurately in spot and exchange trade markets. These models can be grouped into three categories: structural, linear and nonlinear time series models (Bacon 1991, Desbarats 1989). The structural models have been able to provide fairly reasonable explanations on the factors underlying the demand and supply movements, but they have not been usually successful in forecasting oil prices (Pindyck 1999). The linear and nonlinear time series models, such as ARMA and ARCH type models, have been able to do a better job in forecasting oil prices (Abosedra and Laopodis 1997, Morana 2001, Sadorsky 2002). However, if the underlying data generating process of the oil prices is nonlinear and chaotic, using the linear or nonlinear parametric ARCH-type models with changing means and variances will not be appealing.

Various daily financial time series present empirical evidence of the existence of chaotic structures, which are also found in many different financial markets in different economic sectors or economies (Alvarez-Ramirez *et al.* 2008, Alvarez-Ramirez *et al.* 2002, He and Chen 2010, He *et al.* 2007, 2009, He and Zheng 2008). The study of petroleum prices is largely based on the main stream literature of financial markets, whose benchmark assumptions are that returns of stock prices follow a Gaussian normal distribution and that price behavior obeys 'random-walk' hypothesis (RWH), which was first introduced by Bachelier (Bachelier 1900), since then RWH has been adopted as the essence of many asset pricing models. Another important context on this domain is the efficient market hypothesis (EMH) proposed by Fama which states that stock prices already reflect all market information available in evaluating their values (Fama 1970). However, RWH and EMH have been widely criticized in current financial literature (Alvarez-Ramirez *et al.* 2008, Alvarez-Ramirez *et al.* 2002, He and Chen 2010, He *et al.* 2007, 2009, He and Zheng 2008). Many important results in current literature suggest that returns in financial markets have fundamentally different properties that contradict or reject RWH and EMH. These ubiquitous properties identified are: fat tails (Gopikrishnan *et al.* 2001), long-term correlation (Alvarez-Ramirez *et al.* 2008), volatility clustering (oh *et al.* 2008), fractals/multifractals (He and Chen 2010, He *et al.* 2007, 2009, He and Zheng 2008) and chaos (Adrangi *et al.* 2001), etc. The long memory feature was also confirmed to exist in oil markets in Mostafaei and Sakhabakhsh 2011, Prado 2011, Wang *et al.* 2011, Choi and Hammoudeh 2009 study.

The goals of this paper are to (a) investigate the long memory property; (b) explore the non-linearity; and (c) detect the chaotic behavior of energy commodities.

The paper is organized as follows: in Section 2, we describe the statistical properties of the data and present the unit root and random walk test outputs. In section 3, we analyze the long memory property by ACF, GPH and Hurst Exponent tests. In Section 4, we examine the methods of testing nonlinearity and fractality, including the BDS and the Barnett and Wolff. In Section 5, we provide a series of tests to examine the chaos in energy commodities, including the largest Lyapunov exponent, correlation dimension and short-term prediction tests. Finally, in Section 6, we present a discussion of the results and some concluding remarks.

2. Statistical Overview of the Data

The data used are daily U.S. dollar based spot West Texas Intermediate (WTI) crude oil, New York Harbor Conventional Gasoline, New York Harbor No. 2 Heating Oil and Henry Hub Natural Gasprices as reported by the U.S. Energy Information Administration. The sample covers the period Dec 31, 2009 to Dec 31, 2015 for a total of 1,565 (1,564) energy price (returns) observations.

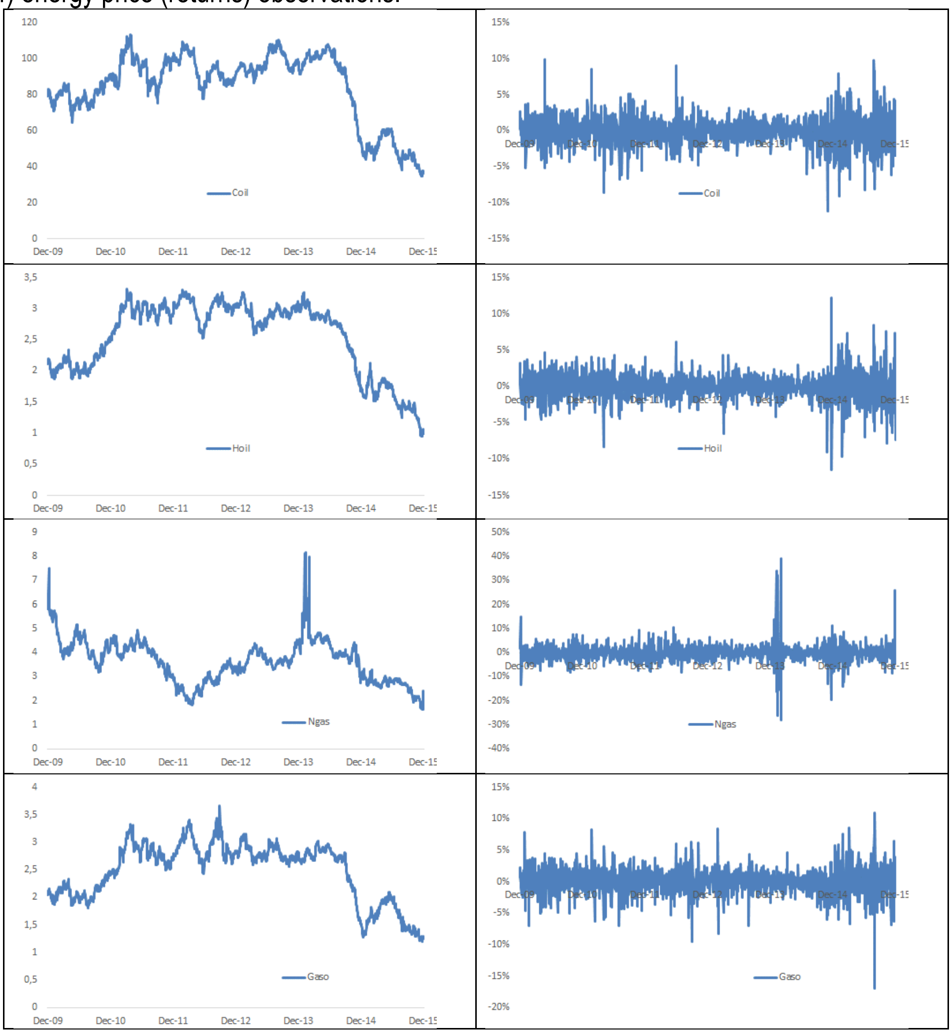

Figure 1. Price and Logarithmic Return Series

Units of measurement are 1 barrel for crude oil, 1 gallon for gasoline and heating oil and 1 million of British thermal units (Mmbtu) for natural gas. Figure 1 provides graphs of the evolution of energy spot price and returns series, respectively, over the sample period.

Energy price fluctuations are not the same in terms of their sizes and duration. This indicates that a dynamic nonlinear structure may exist in the data suggesting the use of nonlinear models, which are able to capture these irregularities.

Price do not exhibit a global trend in the period. Despite many changes, the price has always shown a tendency towards its mean.

Table 1. Descriptive Statistics

	COIL_R	HOIL_R	NGAS_R	GASO_R
Mean	-0,0005	-0,0005	-0,0006	-0,0003
Std. Dev.	0,0195	0,0176	0,0357	0,0204
Skewness	-0,0579	-0,1494	1,1877	-0,2945
Kurtosis	6,4941	8,4719	29,2512	8,1328
Jarque-Bera	796,48	1957,05	45275,61	1739,49
Probability	0,00	0,00	0,00	0,00
Box- Ljung Q(10)	22,08	21,09	31,20	17,21
Probability	0,01	0,01	0,02	0,02
Box- Ljung Q^2(10)	467,01	478,11	899,31	291,31
Probability	0,01	0,02	0,03	0,03
Engle ARCH	25,18	24,29	34,30	27,41
Probability	0,01	0,01	0,02	0,02

As seen in the table 1, the return series of energy commodity prices have standard deviation of 0.02 on average in the sample period suggesting that it has been highly volatile.

The normality test indicated strong deviation from the skewness and kurtosis from a normal distribution (skewness=0 and kurtosis=3) for all the spot prices. The Jacque-Bera statistics also rejected the hypothesis of normal distribution but has wide tails at the 1% significance level.

Based on the Ljung-Box statistics (10 lags), the null hypothesis of "No serial correlation" is rejected. Similarly, McLeod-Lee statistics reject the null hypothesis of "No serial correlation in squared series" and confirm Heteroskedasticity in return series suggesting that there exists some sort of nonlinear relationship in the squared series. This conclusion is also approved by Engle's ARCH test as well.

a. Unit Root

According to unit root tests of ADF and PP, the return series is stationary but KPSS and ERS tests unit root test show the series are nonstationary. Thus, such conditions might have been caused by the long memory feature in this series. For this reason, tests for checking the existence of this feature will be focused upon in the next part.

Table 2. Unit Root Tests

Tests	Coil - t stat.	Hoil - t stat.	Ngas - t stat.	Gaso - t stat.	t - Critical	Result
ADF	-41,8669	-42,2717	-35,9824	-41,8235	-1,9416	Stationary
PP	-41,8660	-42,2632	-35,9795	-41,8156	-1,9416	Stationary
KPSS	0,0378	0,0482	0,0444	0,0232	0,1000	Non-Stationary
ERS	2,1159	2,3148	1,8892	2,1112	0,4630	Non-Stationary

b. Variance Ratio

Variance ratio test is based on the fact that if a time series follows a random walk, in a finite sample the increments in the variance are linear in the observation interval. That is, the variance of difference data should be proportional to the sample interval.

The random walk hypothesis requires that the variance ratio for all the chosen aggregation intervals, q, be equal to one. If variance ratio is less than one than the series is said to be mean reverting and if variance ratio is greater than one than the series is said to be persistent.

As shown in Table 3, the martingale hypothesis – in the return series- is strongly rejected. So, it can be concluded that the generating process of the data is not random walk.

Table 3. Variance Ratio Tests

	Prob.	Stat.	Variance Ratio
Coil	1.5E-31	-11.69	0.46
Ngas	4.9E-04	-3.49	0.67
Hoil	7.8E-19	-8.86	0.47
Gaso	1.4E-14	-7.69	0.49

3. Long Memory

Long memory property is a sign of strong correlation between far-distance observations in a given time series. Hurst (1951), first, noticed that some time series have this property. However, in mid 1980s, after suggestion of critical concepts like unit root and cointegration, econometricians realized some other types of nonstationarity and partial stationarities which are frequently found in economic and financial time series (Granger and Joyeux 1980).

Long memory (or long-term dependence) is a special form of non-linear dynamics where a time series has non-linear dependence in its first and second moments and between distant observations, and a predictable component that increases its forecast ability (Thupayagale 2010). It also means that a time series displays slow decay in its autocorrelation functions (Belkhouja and Boutahary 2011). Consequently, "the presence of long memory implies that energy prices (actually returns) tend to be highly volatile, with price changes that often partially cancel out, although the original shock takes a long time to work through the system" (Arouri *et al.* 2011). The existence of long memory also invalidates the weak-form efficiency of the energy markets because the energy price returns can be predictable (Elder and Serletis 2008).

Generally, econometricians, used first-order differencing in their empirical analyses due to its ease of use (in order to avoid the problems of spurious regression in non-stationary data and the difficulty of fractional differencing). Undoubtedly, this replacement (of first-order differencing with fractional differencing) leads to over - or under- differencing and consequently loss of some of the information in the time series (Huang 2010). On the other hand, considering the fact that majority of the financial and economic time series are non-stationary and of the Differencing Stationary Process (DSP1) kind, in order to eliminate the problems related to over differencing and to obtain stationary data and get rid of the problems related to spurious regression, we can use Fractional Integration.

Diagnosing the long memory process is the most important step. Auto Correlation Functions (ACF) as a graphical test and spectral density test or Geweke and Porter-Hudak (GPH) test as the frequently used numeric tests are two main groups of tests that diagnose the long memory feature. In addition to these test, we also performed Hurst exponent and unit root tests.

a.ACF

Estimating the long memory parameter (d) is the milestone of modeling long memory property. ACF is commonly used methods for this purpose. Figure 2 depicts the ACF of the price, logarithmic returns and squared logarithmic return of the commodity energy prices. As clearly shown, in price series graph decreases very smoothly, a typical shape for time series that are non-stationary and have the long memory property. If such a series does not have the long memory property, it is expected that after first differencing, the series would become stationary in which we can see in logarithmic return ACF results dying out quickly. However, the autocorrelation functions represented in the squared returns of the daily energy prices show a typical feature of a long memory volatility property, which is very slow decays at the hyperbolic rate. This finding is in line with the findings of Martens and Zein (2004) and Brunetti and Gilbert (2000) who studied the volatility process of the crude oil futures data.

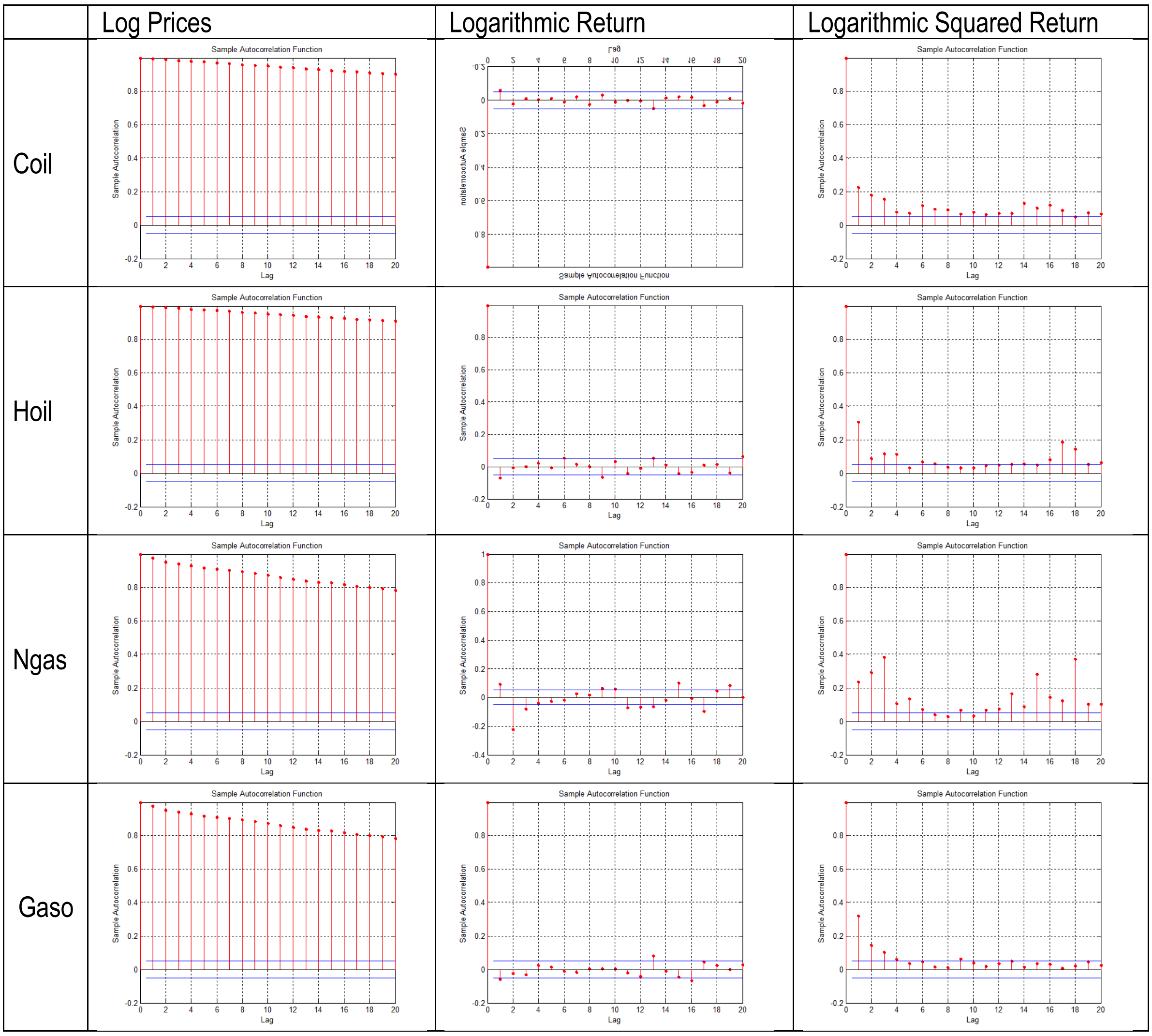

Figure 2. Auto Correlation Functions

b. GPH Test

Models considering long memory property are very sensitive to the estimation of long memory parameter as well as the pattern of damping of auto-correlation functions. In this section, the long memory parameter (d) was estimated using GPH approach. This method, invented by Gewek, Porter-Hudak (1987), is based on frequency domain analysis. GPH method applies a special regression technique called Log-Period Gram which allows us to distinguish between long-term and short-term trends. The slope of regression line calculated by this technique is exactly equal to long memory parameter. Table 4 reports the estimated long memory parameter for both the price, logarithmic return and logarithmic squared return series.

Table 4. GPH Test

Price	Coil	Hoil	Ngas	Gaso
D	1.08	0.98	0.68	1.09
Prob.	0.00	0.00	0.00	0.00
- Long Memory exist in price series				
Log Return	**Coil**	**Hoil**	**Ngas**	**Gaso**
D	0.07	0.00	-0.01	0.11
Prob.	0.56	0.99	0.94	0.38

- Long Memory doesn't exist in logarithmic return series				
Log Return Square	**Coil**	**Hoil**	**Ngas**	**Gaso**
D	0.39	0.40	0.38	0.24
Prob.	0.00	0.00	0.00	0.05
- Long Memory exist in logarithmic squared return series				

The results are in line with the ACF outputs.

c.Hurst Exponent

As is well known, systems with different Hurst exponents exhibit different dynamical behaviors: when 0 ≤ H (τ) < 0.5 the system has antipersistence features; when H (τ) =0.5 the time series is uncorrelated and indicates a Gaussian or gamma white-noise process. Stochastic processes with H (τ) =0.5 are also referred to as fractional Brownian motions. The price behaviors exhibit so-called random walks, while the system's memory is a Markov chain. When 0.5 <H (τ) ≤ 1 the systems under study are persistent and characterized by long-term memory that affects all time scales. The time series is initially value-dependent and has chaotic characteristics, thus it is hard to predict future trends. The system has long-term memory of historical information.

Table 5. Hurst Exponent

Series	Coil	Hoil	Ngas	Gaso
Daily Price	0.85	0.86	0.82	0.85
Daily Log. Return	0.53	0.52	0.53	0.52
Daily Squared Log. Return	0.64	0.63	0.62	0.61
Monthly Log. Return	0.76	0.76	0.76	0.77

We observed that there exists 0.5 <H (T) ≤ 1 for all price series. That is, the energy price systems are persistent and auto-correlated and exhibit long-term memory features.

The behaviors of daily returns exhibit distinct persistence and inherent long-term memory. Although the dynamic behaviors of daily returns of energy commodities are close to a Brownian motion, a long term memory mechanism emerges in the two systems as time scales are increased *i.e.* monthly. The Hurst exponents of daily returns of energy commodities are approximately 0.5, which implies the existence of noise in the systems. However, when time scales increased Hurst exponents of greater than 0.7, which implies that much less noise affects the system dynamics in long-term transaction behaviors. The Hurst exponents of daily squared returns of energy commodities are greater than 0.5, which implies the existence of long-memory in the volatility as well.

As a result, the evidence of long memory in returns is inconclusive by different long-memory tests, ACF, GPH and Hurst, but conclusive in squared returns.

4. Non-Linearity

a.BDS Test

This test was developed by Brock, Dechert and Scheinkman (1987). The main concept behind the BDS test is the correlation integral, which is a measure of the frequency with which temporal patterns are repeated in the data.

The BDS test is designed to evaluate hidden patterns of systematic forecastable nonstationary in time series. The test constructed to have high power against deterministic chaos, but it was discovered that it could be used to test other forms of nonlinearities as well.

BDS test makes it possible to distinguish between a nonlinear and a chaotic process. The results of BDS test are presented in Table 6.

It is evident from the results of the BDS test that the data strongly reject the null hypothesis of iid for all energy prices as the BDS statistic "w" was significant for all embedding dimensions tested. Also, it appears that the evidence of non-linearity was stronger for larger embedding dimensions, as the BDS statistic "w" increased with larger embedding dimensions.

The rejection of the BDS test in energy commodities suggests that the market contains nonlinear dynamics. This is consistent with the observations that energy commodity markets are subject to frequent shocks constantly and major extreme events are prevalent in these markets.

Table 6. BDS Test

	Coil		Hoil		Hoil		Gaso	
Dimension	BDS Stat.	Prob.	BDS Stat.	Prob.	BDS Stat.	Prob.	BDS Stat.	Prob.
2	9.17	0.0E+00	11.27	0.0E+00	7.82	5.1E-15	4.33	1.5E-05
3	11.13	0.0E+00	11.95	0.0E+00	9.09	0.0E+00	7.06	1.6E-12
4	12.49	0.0E+00	12.08	0.0E+00	10.35	0.0E+00	8.41	0.0E+00
5	13.46	0.0E+00	11.94	0.0E+00	11.10	0.0E+00	9.27	0.0E+00

b. Barnett and Wolff Test

BDS test were based on the correlation integral. To acquire more confidence in our results at this point, another test proposed by Barnett and Wolff (2005) is applied, based on high order spectral analysis, namely the third order moment. We set the test parameters in this analysis as recommended by Barnett and Wolff (2005): the embedding dimension we use is 5, the number of bootstrap replications is 1000, and the test significance level is 5%.

The results in Table 7 rejects the null hypothesis of linearity, confirm that the generating forces of the energy spot prices and returns are non-linear.

Table 7. Barnett and Wolff Test

	Price	Return
Coil	0.026	0.000
Ngas	0.130	0.000
Hoil	0.036	0.000
Gaso	0.038	0.000

5. Detecting Chaos

There are a variety of tests available for detecting chaos. Since the nulls and the alternatives in the existing tests are not the same, one cannot decide as to which test higher power (Barnett and Serletis 2000) has. This makes the distinction between deterministic and stochastic process by applying a single statistical test difficult. Therefore, in order to avoid misleading results and conclusions, we carry out a group of tests available within a nonlinear framework. The tests applied are Lyapunov exponent, correlation dimension and short-term forecast tests.

The BDS tests for iid versus a general nonlinearity in the series. The BDS test by alone does not provide clear evidence for the presence of chaos, even when the null of iid or linearity has been rejected. The correlation dimension is a non-statistical test, which uses integral correlation to test for chaos. The Lyapunov exponent test, however, can be considered as a more direct test for chaos, since it is based on one of the main characteristics of the chaotic series, namely, the extreme sensitivity to the initial condition.

a.Maximal Lyapunov Exponent

Lyapunov exponents (LE) is a quantitative measure of the sensitivity of a time series to the changes in the initial condition. It measures the rate of convergence (divergence) of two initially close points along their trajectory over time. In Lyapunov method, this rate is measured by an exponential function. The value of these exponents can be used to investigate the Local Stability of linear or nonlinear systems. In this test, positive values of exponents indicate exponential divergence of the series, high sensitivity to initial conditions and therefore, chaotic process. On the other hand, negative values of exponents indicate exponential convergence. Finally, when Lyapunov exponents are zero, one may argue that there is no converging or diverging trajectory in the data; *i.e.* the series follows a fixed process.

In this study we applied 3 different algorithms to calculate LE's.

i. Rosenstein:

In the method by Rosenstein et al. for each m-dimensional vector ξ in the reconstructed phase space its nearest neighbor ξ* is determined. Rosenstein et al. calculate C (k) ≤ log |f k (ξ) −f k (ξ*)| > where the angles denote averaging over all ξ = ξ1, ξ2 . . . The function C (k) shows roughly three different regimes. An initial regime of flat increase, a subsequent interval with exponential behavior, and finally a plateau (because the separation cannot go beyond the extension of the attractor). The maximal Lyapunov exponent is determined by the slope of C (k) in the usually quite short range of exponential behavior. Positive values of Rosenstein LE's are referring to the chaotic behavior of energy prices.

Table 8. Rosenstein Lyapunov Exponents

Coil - LE	m = 2	m = 3	m = 4	m = 5	Hoil - LE	m = 2	m = 3	m = 4	m = 5
Tau = 1	0,05	0,12	0,15	0,17	Tau = 1	0,10	0,18	0,22	0,22
Tau = 2	0,03	0,60	0,06	0,06	Tau = 2	0,33	0,90	0,09	0,11
Tau = 3	0,05	0,63	0,03	0,03	Tau = 3	0,34	0,93	0,04	0,05
Tau = 4	0,05	0,12	0,02	0,02	Tau = 4	0,24	0,03	0,03	0,03
Tau = 5	0,21	0,59	0,01	0,01	Tau = 5	0,35	0,02	0,02	0,02
Ngas - LE	**m = 2**	**m = 3**	**m = 4**	**m = 5**	**Gaso - LE**	**m = 2**	**m = 3**	**m = 4**	**m = 5**
Tau = 1	0,13	0,46	0,42	0,09	Tau = 1	0,18	0,31	0,26	0,22
Tau = 2	1,10	0,22	0,16	0,07	Tau = 2	0,63	0,14	0,10	0,11
Tau = 3	1,13	0,57	0,08	0,05	Tau = 3	0,20	0,08	0,05	0,04
Tau = 4	1,12	0,07	0,00	0,00	Tau = 4	0,20	0,05	0,03	0,02
Tau = 5	N/A	N/A	N/A	N/A	Tau = 5	0,20	0,03	0,02	0,02

ii. Taylor expansion (Discrete Volterra expansion):

We use a Volterra expansion model to approximate the Jacobian matrix to estimate the LE. Table 9 shows the estimation of the largest Lyapunov exponents for each of the embedding dimensions included in this analysis. As it's seen, the largest λ was positive for all embedding dimension we tested. The direct interpretation of the Lyapunov Exponent results is that the energy commodity spot return series is non-linear deterministic of low dimensions' dynamics, *i.e.,* chaos is governing energy commodity returns.

However, to avoid misleading conclusions, the effect of the noise in the data on the LE results cannot be ignored, and can explain the positive values of λ, as Abhyanker, Copeland and Wong (1997) pointed out.

That's why we applied chaos testing study in order to conclude that the series are chaotic or not?

Table 9. Taylor Expansion Lyapunov Exponent

LE	m = 2	m = 3	m = 4	m = 5
Coil	0,09	0,19	0,20	0,20
Hoil	0,10	0,23	0,24	0,25
Ngas	0,07	0,16	0,17	0,18
Gaso	0,08	0,17	0,17	0,17

iii. Minimum RMSE neural network

After estimation of network weights and finding network with minimum BIC, derivatives are calculated. Sum of logarithm of QR decomposition on Jacobian matrix for observations gives spectrum of Lyapunov Exponents.

Table10. Lyapunov Exponents

LE	Tau = 1	Tau = 2	Tau = 3	Tau = 4	Tau = 5
Coil	-1,15	-1,27	-1,56	-1,71	-2,07
Hoil	-1,90	-1,92	-2,06	-2,10	-2,88
Ngas	-0,83	-0,86	-1,14	-1,27	-5,53
Gaso	-1,10	-1,13	-1,17	-1,18	-1,31

Although the LE's found in Rosenstein and Volterra expansion models are positive, LE's calculated by minimum RMS neural network model are negative.

Based on these results, we could have concluded that chaotic behavior prevails. This might have been the artifact of the embedding method as it has been expounded in Dechert and Gencay (2000). This is to say that the largest Lyapunov exponent may not be preserved under Takens' embedding theorem.

b. Chaos Testing by Lyapunov Exponent

Chaotic dynamics are closely related in appearance to stochastic dynamics, and the BDS test cannot separate them. Hence, we need a practical test to detect chaos even when the data are noisy.

The chaos test starts with estimating the largest Lyapunov exponent in a noisy time series, and decides whether the data are chaotic or stochastic based on a confidence level α

It tests the positivity of the dominant (or largest) Lyapunov exponent λ at a specified confidence level. The test hypothesis H are: null hypothesis H0: $\lambda > 0$, which indicates the presence of chaos; and alternative hypothesis H1: $\lambda < 0$, which indicates the absence of chaos.

Table 11. Chaos Testing

	LE	Prob.	Chaotic
Coil	-0,39	0,00	No
Hoil	-0,36	0,00	No
Ngas	-0,38	0,00	No
Gaso	-0,42	0,00	No

Results show that energy commodities don't prevail the chaotic behavior.

c. Correlation Dimension

To quantify the degree of complexity of the system that has generated the energy price series, we calculate estimates of the correlation dimension (cd) over the range of embedding dimensions m= 1, 2… 20.

If the data are purely stochastic, the correlation dimension will equal m for all m. If the data are deterministic, the slope estimates will "saturate" or stabilize at some m, not rising any more as m is further increased. This saturation value of the slope is the correlation dimension estimate for the unobserved underlying process that generated the data.

In this study we applied the Grassberger and Procaccia's (GP) algorithm to calculate the correlation dimension. However, in interpreting the evidence it should be kept in mind that the GP algorithm may produce estimates with substantial upward bias for attractors and with downward bias for random noise (see Ramsey and Yuan 1990, Ramsey *et al.* 1990).

Correlation dimension estimates are reported in Figure3.The results suggest that the correlation dimension estimates generally increase with embedding dimension but they are below the theoretical values for a completely random process. However, the levels of dimension estimates do not reach a plateau suggesting absence of saturation even though their rate of change with respect to embedding dimension is less than one. Based on this evidence, there is lack of support for a strange attractor in the energy price series. Even if a strange attractor exists, it is not of low dimensionality. Moreover, a correlation dimension greater than about five implies essentially random data.

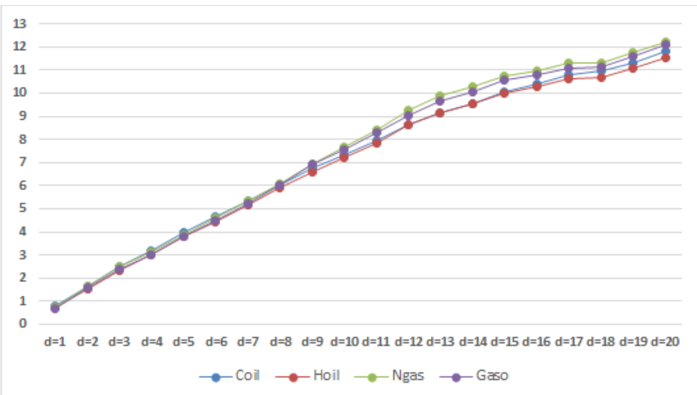

Figure 3. Correlation Dimension

There are two concerns about the calculation of cd for a time series. First, as Harrison *et al.* (1999) note, the presence of noise may result in an increase in cd as m increases. Therefore, using cd as a test for chaos in a noisy data may be misleading. Second, as Scheinkman and Lebaron (1989) point out, while an increase in m would not affect the estimate of cd after a certain point for infinite series, it would for finite data. This is particularly important in economic and financial series where the number of observations is very limited compared with empirical sciences.

While the correlation dimension measure is therefore potentially very useful in testing chaos, the sampling properties of the correlation dimension are unknown. As Barnett *et al.* (1997, 306) put it "if the only source of stochasticity is observational noise in the data, and if that noise is slight, then it is possible to filter the noise out of the data and use the correlation dimension test deterministically. If the correlation dimension is very large as in the case of high-dimensional chaos, it will be very difficult to estimate it without an enormous amount of data. In this regard, Ruelle (1990) argues that a chaotic series can only be distinguished if it has a correlation dimension well below $2\log_{10} N$, where N is the size of the data set, suggesting that with economic time series the correlation dimension can only distinguish low-dimensional chaos from high-dimensional stochastic processes - see also Grassberger and Procaccia (1983) for more details.

Therefore, the test results based on cd in finite data cannot be conclusive.

d. Short-Term Prediction

If we have a chaotic time series, we should expect to see predictor error starting out small for a small prediction interval and increasing as the prediction interval increases. No such relationship between predictor error and prediction interval will exist for a randomly-generated time series because the predictor error will always be large.

For a chaotic system and for a given short prediction interval, we should also see predictor errors decrease to a value near zero as d is increased to the correct minimal embedding dimension dE, then increase as d is increased beyond dE. This will not occur for a randomly-generated time series, because the predictor error will be large no matter what embedding dimension is used (Casdagli 1989).

Prediction is called the "inverse problem" in dynamical systems. That is, given a sequence of iterates from a time series, we want to construct a nonlinear map that gives rise to them. Such a map would be a candidate for a predictive model. The consideration of a nonlinear map is essential, since linear maps do not produce chaotic time series.

Several methods exist for predicting time series. The primary references for our study are Casdagli (1989) and Casdagli *et al.* (1992). The methods fall into the categories of global function representation and local function approximation. These functions can be one of the followings: Polynomials, Rational Functions, Wavelets, Neural nets, and Radial basis functions.

In this paper, we demonstrate the method of local fitting, using the ideas contained in Farmer/Sidorowich (1987) and Casdagli *et al.* (1992) as our guide.

We use the normalized mean-square-error (E) as a measure of how accurate our predictions are. In order to test our predictions, we will need to treat part of the data we have as the training data set (the part we assume we know) and the other part as the test data set, against which we plan to test our predictions.

Predictions are perfect if E = 0, while E = 1 indicates that the accuracy of predictions is no better than a constant predictor equal to the average of previous time series values. A value for E which is greater than one indicates even worse performance than the average of previous time series values.

Different choices of k (number of nearest neighbors), N (number of data points in the training data set), τ (delay), and the multi-step prediction code allows for different choices of T (prediction interval) is used for the prediction study and the following one is presented as an example.

Number of nearest neighbors; 5, number of data points in the training data set 1.464, delay; 1, prediction interval; 100, embedding dimension; three.

Of the delay parameters τ used, the value of one seemed to result in the most accurate predictions.

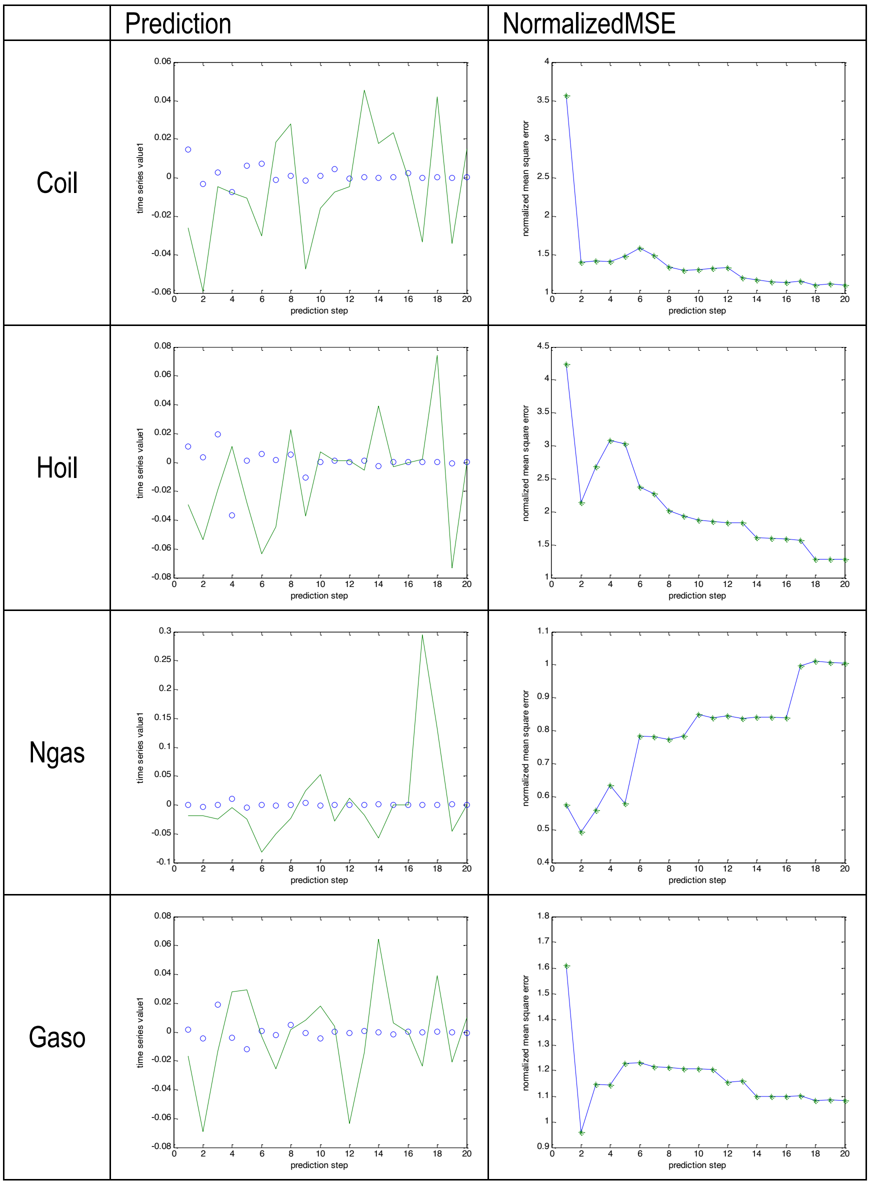

Figure 4. Short Term Prediction

Prediction and normalized errors results for energy commodities are as follows:

E > 1 indicates worse performance of prediction means that the series cannot be low-dimensional chaotic but stochastic.

Please note that, since the series is not chaotic, results don't change for the higher embedding dimensions, that's why 3 is set for all predictions.

Conclusion

The limitations and inadequacies of the traditional linear stochastic framework with respect to explaining the dynamics of the behavior of energy commodity markets has led the recent research to new nonlinear approaches. Among them, chaotic models have attracted increasing attention since they have been shown to exhibit interesting theoretical and empirical features that could help to gain a better insight of the underlying of energy commodity markets mechanisms. However, in order to answer the question of whether low-dimensional chaos may offer a useful way to model the market phenomena, the more fundamental issue of determining whether chaotic behavior can indeed be observed in energy commodity time series has to be addressed first.

This research starts with a statistical analysis where both series are found to exhibit serial autocorrelation and deviations from normality with fat tails in their distributions.

The empirical analyses presented in this paper have given strong and unambiguous support to the existence of nonlinearities in the examined time series.

This result was compatible with a chaotic explanation so we proceeded further with the long-memory tests. Tests reveal long-term memory and fractal structure of the series analyzed, even in the presence of noise.

To shed more lights on the underlying data generating process of the energy commodity prices, we carried out various tests for deterministic chaos. The tests included correlation dimension, and Lyapunov Exponent.

The energy price series exhibited non-saturating correlation dimension, which in addition was not significantly different than the dimension of random series having the same distributional characteristics.

The largest Lyapunov exponent estimation was found to verify the possibility of a chaotic component in the energy commodity series. However, we show that this method, at least with the algorithm that we have employed (the most commonly used in the literature), is not by its own a reliable method to detect chaoticity since it is unable to distinguish between alternative specifications such as chaotic, Gaussian random, and fractal random sequences.

In a final step, the existence of a chaotic component in the energy commodity series was further investigated through nonlinear forecasting techniques based on phase space reconstruction. The results from these applications were compatible with our previous findings; the fitting precision is very low and the model has the very poor forecast effect which results in not showing chaotic behavior.

The results of this research, although they cannot be generalized beyond the specific data sets examined, result in considerable doubt on the importance of chaos theory in explaining the seemingly erratic behavior of the energy commodity markets. We argue that mature markets are highly unlikely to exhibit chaotic components and the empirical findings supporting this view are most probably due to the inadequate testing framework used and to improper application of the methods employed.

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Volume VII, Issue 2(14), Winter 2016

DOI:

A THEORY OF DEFLATION: CAN EXPECTATIONS BE INFLUENCED BY A CENTRAL BANK?

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Suggested Citation:

Harashima T. (2016). A Theory Of Deflation: Can Expectations be influenced by a Central Bank? *Theoretical and Practical Research in Economic Field*, (Volume VII, Winter 2016), 2(14):98-144. DOI:10.14505/tpref.v7.2(14).02. Available from: http://www.asers.eu/publishing/index.php/tpref

Article's History:

Received June, 2016; *Revised* August, 2016; *Accepted September*, 2016.

Abstract

This paper examines how to reverse deflation to inflation. Once deflation takes root, it is not easy to reverse because of the zero lower bound in nominal interest rates. My model indicates that there are two steady states where both inflation/deflation (i.e., changes in prices) and real activity (i.e., quantities) remain unchanged: that is, there are inflationary and deflationary steady states. The model indicates that, to switch a deflationary steady state to an inflationary steady state, a central bank needs to influence the time preference rates of the government and the representative household. It is not easy, however, to do so, and the best way of switching deflation to inflation may be to wait for a lucky event (i.e., an exogenous shock).

Keywords: Deflation; The zero lower bound; Monetary policies; Quantitative easing; Time preference

JEL Classification: E31, E52, E58

1. Introduction

Reversing deflation to inflation has been an important policy issue, especially since the 1990s, because some economies have experienced deflation (although temporary) and faced the zero lower bound (ZLB) in nominal interest rates. Although inflation is now tamed in most developed countries, deflation remains a concern. For example, deflation has haunted Japan since the 1990s, although the Bank of Japan has repeatedly tried to reverse the course of deflation, even by using unconventional monetary policies. Once deflation takes root, however, it is not easy to reverse because of ZLB. If nominal interest rates are stuck at or near zero, conventional monetary policies (*i.e.,* manipulations of nominal interest rates by a central bank) are little effective.

An alternative tool to reverse deflation is needed. An important prospective alternative tool is to influence households' expectations (*e.g.*, Eggertsson and Woodford 2003, Bernanke and Reinhart 2004, Bernanke *et al.* 2004, Blinder *et al.* 2008). However, theoretically, the effectiveness of this measure is ambiguous. In this paper, I examine the feasibility and effectiveness of manipulating expectations on the basis of the inflation/deflation model shown by Harashima (2004b, 2008, 2015a), as well as conventional inflation models—in particular, new-Keynesian Phillips curve (NKPC) models.

The inflation/deflation model of Harashima (2004b, 2008, 2015a) is based on a micro-foundation of trend inflation. It indicates that trend inflation is generated by the difference between the rate of time preference (RTP) of the government and that of the representative household (RTP RH). In addition, both the RTP of the government and the RTP RH are intrinsically temporally variable. Hence, the expectations of both RTPs' future values must be generated by households. The feasibility and effectiveness of expectation manipulation therefore depends on whether a central bank can influence the RTP expectations of both the government and the RH.

Even if nominal interest rates are bounded by ZLB, an economy can be stable if the absolute value of the deflation rate equals the real rate of interest, because the Fisher equation is satisfied. If households generate expectations of the government's RTP and the RTP RH that are consistent with this deflation rate, the economy will be stable. To reverse deflation to inflation, therefore, households' expectations of deflation need to be changed. There are several possible ways to influence households' expectations: verbal intervention, quantitative easing (QE), renouncing central bank independence, raising the nominal interest rate, and depreciating the exchange rate. In addition, imposing taxes on money may be used to reverse deflation, although the main aim in that case is not to influence households' expectations but to recover the ability to manipulate nominal interest rates. In this paper, I examine and evaluate the feasibilities and effectiveness of these measures. The results of examinations indicate that it is not easy for a central bank to reverse deflation to inflation by influencing households' expectations. This conclusion suggests that as inflation shows signs of changing to deflation, it is important for a central bank to act quickly and drastically, for example, by increasing the target rate of inflation and lowering nominal interest rates far more than it usually would (*e.g.,* Williams 2009).

2. A mechanism of deflation

2.1. The law of motion for inflation/deflation

2.1.1. The law of motion

The model constructed by Harashima (2004b, 2008, 2015a) is used as the inflation/deflation model in this paper. The details of the model are explained in Appendix A. The difference between it and conventional inflation models—particularly NKPC models—is discussed in Section 2.1.5. The model indicates that the law of motion for inflation/deflation is described by

$$\int_{t-1}^{t}\int_{s}^{s+1}\pi_{\upsilon}\,d\upsilon\,ds = \pi_t + \theta_G - \theta_P \tag{1}$$

where π_t is the inflation/deflation rate at time t, θ_G is the RTP of government, and θ_P is RTP RH. θ_G and θ_P are not necessarily identical. Equation (1) is the same as equation (A19) in Appendix A. It indicates that inflation/deflation accelerates or decelerates as a result of the government and the representative household reconciling the contradiction in their heterogeneous RTPs. A solution of the integral in equation (1) for given θ_G and θ_P is

$$\pi_t = \pi_0 + 6(\theta_G - \theta_P)t^2 \tag{2}$$

Generally, the path of inflation/deflation that satisfies equation (1) for $0 \leq t$ is expressed as

$$\pi_t = \pi_0 + 6(\theta_G - \theta_P)\exp[z_t \ln(t)] \ ,$$

where z_t is a time-dependent variable. The stream of z_t varies depending on the boundary condition. However, if π_t satisfies equation (1) for $0 \leq t$, and $-\infty < \pi_t < \infty$ for $-1 < t \leq 1$, then

$$\lim_{t \to \infty} z_t = 2 \ .$$

The proof is shown by Harashima (2008). Any inflation/deflation path that satisfies equation (1) for $0 \leq t$ therefore asymptotically approaches the path of equation (2).

Equation (2) indicates a trend component in inflation/deflation. In addition to this trend element, actual inflation/deflation will be influenced by output gaps and various disturbances in the short run; thus, an aggregate supply equation (a Phillips curve) consists of those elements as well as the trend component. The model of inflation therefore consists of an aggregate supply equation (which consists of equation (2) as the trend component, output gaps, and various disturbances), an aggregate demand equation, and an instrument rule for the central bank's manipulation of nominal interest rates (*e.g.,* a Taylor rule). See Harashima (2008) for the detailed model structure.

If a central bank is sufficiently independent, it can force the government to change θ_G and achieve $\theta_G = \theta_P$. If $\theta_G = \theta_P$ is kept, inflation/deflation neither accelerates nor decelerates by equation (1). In other words, to stabilize inflation, an independent central bank will punish the government (*i.e.*, force it to change its preference θ_G) if θ_G deviates from θ_P.

2.1.2. Stationarity

Output gaps and disturbances are basically stationary processes. Therefore, if $\theta_G = \theta_P$ is kept, $\pi_t - \pi_0$ is basically a stationary process with a mean of 0; that is, inflation/deflation becomes a stationary process with mean π_0 because, if $\theta_G = \theta_P$, the trend component disappears (*i.e.*, $\pi_t = 0$ in equation (2)). For example, when $\theta_G = \theta_P$ begins to be kept, π_t is negative (*e.g.*, –0.8% or $\pi_0 = -0.008$). An average deflation rate of –0.8% then will continue because $\theta_G = \theta_P$ is kept. The deflation rate may frequently temporarily deviate from –0.8% because of various shocks, but it will soon return to –0.8%.

Trends or unit roots in inflation were clearly observed during the great inflation in the 1960s and 1970s, but they are not clear in the current periods of low inflation. The reason for this lack of clarity (in other words, stationarity) is that central banks currently are sufficiently independent and $\theta_G = \theta_P$ is always kept.

2.1.3. Necessity of generating an expected θ_G

All households behave (*i.e.,* choose their optimal paths) on the basis of the expectation of future inflation/deflation. The model shown in Section 2.1.1 indicates that, to expect future inflation/deflation, households must know the future value of the government's RTP (θ_G). There is, however, no guarantee that θ_G and RTP RH will be constant across time; rather, the RTPs of the government and households will be intrinsically temporally variable. However, households cannot directly know even the current value of θ_G, because households and the government are different entities and do not inherently know each other's preferences. Therefore, households must somehow generate expectations of the future values of θ_G by calculating them using a structural model of the government's RTP, but they first must construct such a model. A model of the government' RTP and the various problems that are created when generating the expected RTP of government are presented by Harashima (2015b) and also in Appendix B.

Equation (2) indicates that π_t depends on θ_G. Therefore, households need to generate an expected inflation/deflation by generating an expected θ_G. That is, the need to generate an expected inflation/deflation necessitates the expectation of θ_G. This means that, if a central bank can influence the expected θ_G, it can also influence the expected inflation/deflation.

2.1.4. Necessity of generating an expected θ_P

As Becker (1980) and Harashima (2014a, b) indicate, it is not possible to assume that the representative household is the same as the average household in dynamic models. Harashima (2014a, b) shows an alternative definition of the representative household such that the behavior of the representative household is defined as the collective behavior of all households under sustainable heterogeneity. The reason why this alternative definition is needed, and the nature of sustainable heterogeneity, are shown in detail in Appendix C. Unlike the case in which the representative household is assumed to be the average household, this alternatively defined representative household reaches a steady state in which all households satisfy all of their optimality conditions in dynamic models, even if the households are heterogeneous. In addition, the alternatively defined representative household has an RTP that is equal to the average RTP as shown in equations (C7) and (C8) in Appendix C. This alternatively defined representative household requires that each household must generate its expected RTP RH *ex ante* for it to behave optimally, as shown in Appendix C (see also Harashima 2014a, b).

Equation (2) indicates that π_t depends not only on θ_G but also on θ_P. Therefore, households need to generate their expected levels of inflation/deflation by generating not only an expected θ_G but also an expected θ_P. Similar to the case of θ_G, if a central bank can influence the expected θ_P, it can also influence the expected inflation/deflation.

2.1.5. Comparison with conventional inflation models

A typical hybrid NKPC (*e.g.* Galí *et al.* 2005), is

$$\pi_t = \alpha_{\pi+1}\pi_{t+1|t} + \alpha_{\pi-1}\pi_{t-1} + \alpha_x x_t + \varepsilon_t \qquad (3)$$

where x_t is the output gap and ε_t is an i.i.d. shock with a zero mean at time t, $\pi_{t+1|t}$ π_t is the rate of inflation at time $t + 1$ expected at time t, and $\alpha_{\pi+1}$, $\alpha_{\pi-1}$, and α_x are constant coefficients. Hybrid NKPC inflation models consist of an aggregate supply equation (a Phillips curve) such as that expressed in equation (3), an aggregate demand equation, and an instrument rule for the central bank's manipulation of nominal interest rates (*e.g.*, a Taylor rule).

An important difference between the model shown in Section 2.1.1 and the hybrid NKPC inflation model shown here is whether or not a mechanism that generates a trend or unit root is explicitly incorporated on the basis of a micro-foundation. In the model in Section 2.1.1, a trend or unit root is generated naturally if $\theta_G \neq \theta_P$ by the law of motion for inflation/deflation. In the hybrid NKPC model, however, an *ad hoc* inclusion of a backward-looking element (π_{t-1}) is needed to generate a trend or unit root.

Nevertheless, if the backward-looking element (π_{t-1}) is excluded from equation (3) (*i.e.*, the hybrid NKPC is reduced to a pure NKPC), inflation also shows stationarity when $\hat{\theta}_G = \hat{\theta}_P$, where $\hat{\theta}_G$ is the expected θ_G and $\hat{\theta}_P$ is the expected θ_P. Hence, if $\hat{\theta}_G = \hat{\theta}_P$, the performances of the model in Section 2.1.1 and the pure NKPC model will be almost the same. However, there is still an important difference between the two models. The expected inflation/deflation depends on the expectations of θ_G and θ_P in the model in Section 2.1.1, whereas the expected inflation/deflation depends on expected future disturbances in the pure NKPC model.

2.2. The zero lower bound (ZLB)

Assume that the central bank is sufficiently independent and capable. Therefore, the central bank always keeps $\hat{\theta}_G = \hat{\theta}_P$ and inflation/deflation is a stationary process with mean π_0. The assumption of a sufficiently independent central bank is natural in most developed countries at the present time.

Suppose that there is a downward shock on expected θ_P and then $\hat{\theta}_G > \hat{\theta}_P$ in equation (2). By the law of motion for inflation/deflation, inflation begins to accelerate (or deflation begins to decelerate). To restore $\hat{\theta}_G = \hat{\theta}_P$, the central bank forces the government to lower θ_G by increasing nominal interest rates, and the government has no choice but to lower θ_G. As a result, $\hat{\theta}_G = \hat{\theta}_P$ is soon attained. However, because of the shock on the expected θ_P and the consequent change of θ_G, π_0 changes. To achieve $\pi_0 = \pi^*$, where π^* is the central bank's target rate of inflation, therefore, the central bank needs to further manipulate nominal interest rates and θ_G. After achieving $\pi_0 = \pi^*$, the central bank keeps $\hat{\theta}_G = \hat{\theta}_P$.

Now suppose that there is an upward shock on expected θ_P, and then $\hat{\theta}_G < \hat{\theta}_P$ in equation (2). By the law of motion for inflation/deflation, inflation begins to decelerate (or deflation begins to accelerate). To restore $\hat{\theta}_G = \hat{\theta}_P$, the central bank forces the government to raise θ_G by decreasing nominal interest rates. The government has no choice but to raise θ_G, but the recovery of $\hat{\theta}_G = \hat{\theta}_P$ is not necessarily guaranteed, because the central bank cannot lower nominal interest rates below the ZLB.[1] If nominal interest rates are bounded by the ZLB, the central bank no longer has the power to force the government to raise θ_G by manipulating nominal interest rates. Unlike the case of a downward shock on the expected θ_P, the capability of the central bank is constrained by the ZLB. As a result, $\hat{\theta}_G = \hat{\theta}_P$ is not necessarily restored and $\hat{\theta}_G < \hat{\theta}_P$ may continue. In this case, inflation eventually changes to deflation and deflation accelerates.

2.3. Households' expectation of $\theta_G = \theta_P$

How do households think the government behaves when nominal interest rates are bounded by the ZLB? They may perceive that, because the central bank is now powerless, the government will freely choose θ_G, and that it will behave on the basis of its intrinsic θ_G. They may also think that θ_G is still influenced by the central bank in the sense that, when deflation eventually changes to inflation, the central bank will certainly and immediately resume control of θ_G by manipulating nominal interest rates. For example, if the government behaves on the basis of intrinsic θ_G, deflation soon changes to inflation because $\theta_G > \theta_P$ intrinsically. Therefore, manipulations of nominal interest rates become effective again and θ_G can be controlled by the central bank as it was before. This

[1] Technically, central banks can make nominal interest rates slightly negative as the Bank of Japan and the European Central Bank did. Of course, however, these slightly negative interest rates do not indicate that ZLB does not exist.

second view indicates that the ZLB does not mean that the government can freely act on its own intrinsic RTP forever, but that it can merely temporarily escape from the discipline of the central bank. Which view is correct?

2.3.1. Habituation

People dislike changing their own preferences. Hence, people and governments feel psychological pain and disutility if their preferences are forced to change. On the other hand, studies on habituation in psychology (*e.g.,* Thompson and Spencer 1966, Groves and Thompson 1970, Rankin *et al.* 2009) imply that once a preference is changed, the initial psychological pain will gradually subside as the person acclimates to the change: that is, the psychological pain will persist but dwindle. According to Rankin *et al.* (2009), one of the common characteristics of habituation is that repeated application of a stimulus results in a progressive decrease in some parameter of a response to an asymptotic level. Although the psychological pain or a feeling of wrongness may never disappear completely, the level of psychological pain will gradually recede as time passes. Huge initial psychological pains and later acclimation will have an important effect on the discipline of the central bank, because the government will decide its behavior on the basis of its own expectations.

2.3.2. Inhibitory effect

Suppose that a government is considering whether it would be better to behave on its intrinsic θ_G (*i.e.,* $\bar{\theta}_G$) or on the θ_G that is equal to θ_P when nominal interest rates are zero. Unlike in "usual" inflation periods, the government has a choice, because nominal interest rates are zero. If it chooses to behave on $\bar{\theta}_G$, it can relieve the persisting but dwindling psychological pain (disutility) caused by the last forced change in θ_G (*i.e.,* by the last punishment), but it will suffer great psychological pain in the near future because behaving on $\bar{\theta}_G$ indicates the reversal of deflation to inflation (because $\bar{\theta}_G > \theta_P$ by nature) in the near future and the consequent resumption of enforcement (or punishment) by the independent central bank to change θ_G from $\bar{\theta}_G$ to θ_P. The government can enjoy its intrinsic preference only for a short period and soon will have to endure great psychological pains again. On the other hand, if the government continues to behave on $\theta_G = \theta_P$, it will not suffer great psychological pains in the near future even though it has to continue feeling the relatively small and subsiding psychological pains caused by the last forced change in θ_G (*i.e.,* by the last punishment).

Which of the two options a government chooses depends on the level of the initial psychological pain and that of the dwindling psychological pain as time passes. If the initial psychological pain is far larger than the dwindling psychological pain, the government will not change to behaving on $\bar{\theta}_G$ and will keep $\theta_G = \theta_P$. Let $Pain_{s,t}$ be the disutility of a forced change in θ_G in period t where the last forced change in θ_G is undertaken in period s. A larger (positive) value of $Pain_{s,t}$ indicates a larger magnitude of disutility. It is assumed that $\frac{dPain_{s,t}}{dt} < 0$ for $t \geq s$ because the initial psychological pain gradually subsides.

Suppose that deflation sets in and nominal interest rates become zero in period v. Suppose also that, if the government changes θ_G from θ_P to $\bar{\theta}_G$ in period v, deflation reverses to inflation and the central bank resumes forcing the government to change θ_G from $\bar{\theta}_G$ to θ_P in period w where $s < v < w$. The expected disutility of the government generated in period v if it changes θ_G from θ_P to $\bar{\theta}_G$ in period v is therefore

$$-\int_v^w \exp\left[-\theta_{G,t}(t-v)\right] pain_{s,t}dt + \int_w^\infty \exp\left[-\theta_{G,t}(t-w)\right] pain_{w,t}dt$$
$$= -\int_v^w \exp\left[-\bar{\theta}_G(t-v)\right] pain_{s,t}dt + \int_w^\infty \exp\left[-\theta_P(t-w)\right] pain_{w,t}dt$$

where $\theta_{G,t} = \bar{\theta}_G$ for $v \leq t < w$ and $\theta_{G,t} = \theta_P$ for $w \leq t$, because the government enjoys its intrinsic preference during $v \leq t < w$. On the other hand, if the government does not change θ_G from θ_P to $\bar{\theta}_G$, its expected disutility generated in period v is

$$\int_v^{\infty} \exp\left[-\theta_{G,t}(t-v)\right] pain_{s,t} dt$$
$$= \int_v^{\infty} \exp\left[-\theta_P(t-v)\right] pain_{s,t} dt$$

where $\theta_{G,t} = \theta_P$ for any t ($\geq v$) because the government continues to obey the central bank.

If the disutility in the former case is larger than that in the latter case—that is, if

$$-\int_v^{w} \exp\left[-\overline{\theta}_G(t-v)\right] pain_{s,t} dt + \int_w^{\infty} \exp\left[-\theta_P(t-w)\right] pain_{w,t} dt$$
$$> \int_v^{\infty} \exp\left[-\theta_P(t-v)\right] pain_{s,t} dt \,, \qquad (4)$$

—then the government will not change θ_G from θ_P to $\overline{\theta}_G$ and thus $\theta_G = \theta_P$ is basically kept even although nominal interest rates are zero and the central bank cannot directly deter the government from changing θ_G from θ_P to $\overline{\theta}_G$. Because $\frac{dPain_{s,t}}{dt} < 0$, then $pain_{w,t} > pain_{s,t}$ for $t > v$, and therefore the probability that inequality (4) holds will not be low. If the value of $\left|\frac{dPain_t(s)}{dt}\right|$ is relatively large—that is, if the initial psychological pain soon subsides—then inequality (4) will usually hold and the change of θ_G from θ_P to $\overline{\theta}_G$ will be always inhibited. Because people will acclimate to the psychological pain, as noted in Section 2.3.1, it is likely that the inhibitory effect usually influences the government's behavior.

The independence of the central bank (in other words, presumable punishments by the central bank) therefore will possess an inhibitory effect, because even if the central bank cannot manipulate nominal interest rates and directly deter the government from changing θ_G from θ_P to $\overline{\theta}_G$ because of ZLB, the government can be still nearly completely under the control of the central bank.

2.3.3. $\hat{\theta}_G = \hat{\theta}_P$, *even during deflation*

If households firmly believe that the central bank is sufficiently independent and the inhibitory effect is important in controlling the government's behavior, households will expect that the government will continue to keep $\theta_G = \theta_P$, even if nominal interest rates are zero. Therefore, households will basically generate $\hat{\theta}_G = \hat{\theta}_P$ even during deflation if the central bank is sufficiently independent. In the early periods after an upward shock on $\hat{\theta}_P$, households may temporarily generate $\hat{\theta}_G < \hat{\theta}_P$, but they will soon return to $\hat{\theta}_G = \hat{\theta}_P$. This expectation of $\theta_G = \theta_P$ even during deflation is an important factor that makes controlling deflation difficult, as will be shown in later in this section.

2.4. Inflationary and deflationary steady states

2.4.1. Two steady states

A steady state in which both real activity (quantities) and inflation/deflation (changes in prices) stay unchanged requires two conditions, $\theta_P = \bar{r}$ and $\hat{\theta}_G = \hat{\theta}_P$, where $\bar{r}$ is the real rate of interest at steady state. As is well known, $\theta_P = \bar{r}$ is the condition for a steady state of quantities (*e.g.*, Fisher 1930). $\hat{\theta}_G = \hat{\theta}_P$ is the condition for a steady state of inflation/deflation according to the law of motion for inflation/deflation. As shown in Section 2.3, $\hat{\theta}_G = \hat{\theta}_P$ will be kept even during deflation if the central bank is sufficiently independent.

For both $\theta_P = \bar{r}$ and $\hat{\theta}_G = \hat{\theta}_P$ to be simultaneously satisfied, π_0 in equation (2) needs to take an appropriate value. Among the various possible values, the state that satisfies

(a) $\pi_0 = \pi^*$

is a steady state, which I call an inflationary steady state. Inflation is stabilized at the target rate of the central bank (π^*). However, if deflation is also considered, another π_0 that is consistent with both $\hat{\theta}_G = \hat{\theta}_P$ and $\theta_P = \bar{r}$ can exist, such that

(b) $\pi_0 = -\bar{r}$,

which I call a deflationary steady state. The two steady states are identical except for the inflation/deflation rate.

In addition to the nature that $\hat{\theta}_G = \hat{\theta}_P$ even during deflation, the existence of a deflationary steady state (b) is another important factor that makes controlling deflation difficult. A deflationary steady state (b) can be chosen only when the nominal interest rate is zero (in other worlds, during deflation) because, if nominal interest rates are above zero, the central bank can manipulate nominal interest rates to achieve state (a). On the other hand, $\theta_P = \bar{r}$ cannot necessarily be satisfied for any value of π_0. If, and only if, state (b) is chosen (*i.e.,* if and only if $\pi_0 = -\bar{r}$), the condition $\theta_P = \bar{r}$ is satisfied when the nominal interest rates are zero. Therefore, a deflationary steady state (b) can compete with an inflationary steady state (a) as the steady state once nominal interest rates are stuck at ZLB.

Note that the two steady states are also the only possible steady states in NKPC models. If nominal interest rates are positive, the central bank keeps inflation at the target rate, and therefore an inflationary steady state (a) will be always realized. If nominal interest rates are stuck at ZLB, the instrument rule for the central bank's manipulation of nominal interest rates is useless; thus, inflation depends mostly on households' expected inflation. If households do not wish for the economy to collapse (or reach a non-optimal state), $\theta_P = \bar{r}$ is indispensable and only the expected inflation that is consistent with state (a) or (b) can be generated.

2.4.2. The choice between inflationary and deflationary steady states

The value of π_0 is not given exogenously. It is determined by households in the process of them generating $\hat{\theta}_G$ and $\hat{\theta}_P$, and it varies depending on how and when households generate (or change) $\hat{\theta}_G$ after a shock on $\hat{\theta}_P$. How do households determine the value of π_0? Households are rational and will not select a future path that results in collapse of the economy (or a non-optimal state) due to $\theta_P \neq \bar{r}$. Hence, households will select the value of π_0 that is consistent with $\theta_P = \bar{r}$. In addition, as discussed in Section 2.3, households will basically always generate the expectation of $\theta_G = \theta_P$ under a sufficiently independent central bank. Therefore, households select the value of π_0 that satisfies both $\hat{\theta}_G = \hat{\theta}_P$ and $\theta_P = \bar{r}$. The only states where π_0 satisfies the both conditions are states (a) and (b); thus, households generate only $\hat{\theta}_G$ that is consistent with either state (a) or (b). In other words, households have to choose the value of π_0 from either π^* or $-\bar{r}$ in the process of generating $\hat{\theta}_G$ and $\hat{\theta}_P$.

Of course, the central bank prefers an inflationary steady state (a) and does not want a deflationary steady state (b) because π_0 at state (b) is not the target rate π^*. The central bank therefore will want to make households choose state (a). However, if nominal interest rates are stuck at ZLB, the central bank cannot force households to choose state (a) by manipulating nominal interest rates. It must therefore find other tools to force households to choose state (a). The question arises, however, of whether such a useful and effective tool exists.

3. Difficulty of ending deflation

3.1. Forward-looking information

To switch from state (b) to (a), households' expectation of θ_G or θ_P must be changed because $\pi_0(=-\bar{r})$ is otherwise not changed. The value of π_0 can be changed only in the process of generating $\hat{\theta}_G$ or $\hat{\theta}_P$. In addition, after a change in $\hat{\theta}_G$ or $\hat{\theta}_P$, the condition $\hat{\theta}_G = \hat{\theta}_P$ must be soon be restored because inflation/deflation otherwise accelerates. As discussed in Section 2.3, households will soon restore the condition $\hat{\theta}_G = \hat{\theta}_P$ because the central bank is sufficiently independent. Therefore, if the central bank can influence households' expectation of θ_G or θ_P and make them change $\hat{\theta}_G$ or $\hat{\theta}_P$, it may be able to force households to switch from state (b) to (a).

Households will change $\hat{\theta}_G$ or $\hat{\theta}_P$ if they obtain new important forward-looking information that is related to future θ_G or θ_P. Hence, the ability of the central bank to force a change in $\hat{\theta}_G$ or $\hat{\theta}_P$ depends on whether it can deliver meaningful new forward-looking information that is related to $\hat{\theta}_G$ or $\hat{\theta}_P$ and can make households believe this new information.

3.2. Verbal intervention

One way for the central bank to deliver forward-looking information is through verbal intervention. Forward-looking guidance on the future path of interest rates can be regarded as a kind of verbal intervention in a broad sense. If households change their expectations because of statements from the central bank, the central bank may successfully force the switch from state (b) to (a). Nevertheless, households are not so naïve as to literally believe all the statements of the central bank. The statements are therefore meaningless unless households believe that they contain true forward-looking information. If households suspect that the statements disseminated are deceptive or untrue, the verbal intervention is useless. To succeed, the central bank must be perceived as sincere, honest, and capable.

Can even a sincere, honest, and capable central bank deliver a statement that will make households change their expectations and switch from state (b) to (a)? The central bank can ask, or even beg, households to change their expectations, but it is most likely difficult to persuade households that a deflationary steady state (b) is very harmful to them, because $\theta_P = \bar{r}$ is satisfied at state (b) and the economy proceeds as "normally" as it does at state (a). Therefore, it will not be easy to make households change their expectations of θ_G or θ_P by verbal intervention alone. If the central bank delivers "false" or deceptive information about θ_G or θ_P, households may temporarily change their expectations, but eventually the justification and credibility of the central bank will be questioned and damaged. Verbal intervention therefore will not be sufficiently effective to force a switch from state (b) to (a).

Verbal intervention is also predicted to be ineffective by NKPC models for almost the same reasons. Unless a deceptive statement is delivered by the central banks, households will not feel the need to change their expectations.

3.3. Quantitative easing (QE)

In the Great Recession after the subprime mortgage crisis, some central banks that faced near-zero nominal interest rates adopted QE as a monetary policy to stimulate the economy by increasing the quantity of money in the economy. If QE is effective in changing households' expectation of future θ_G or θ_P, it can be used as a monetary policy tool when nominal interest rates are stuck at ZLB.

QE is adopted on the basis of a strict interpretation of the quantity theory of money such that inflation/deflation is fundamentally governed by the growth rate of the money supply, which is exogenously given. However, the law of motion for inflation/deflation shown in Section 2 indicates that the quantity of money is irrelevant to inflation/deflation as shown in equation (2). The quantity of money will be determined endogenously after the rate of inflation/deflation is determined: that is, the direction of causality is from the rate of inflation/deflation to the quantity of money. Hence, a change in the quantity of money cannot directly affect inflation/deflation. Therefore, it will be difficult, if not impossible, to directly affect the expectation of θ_G or θ_P through the use of QE.

Nevertheless, if the use of QE delivers meaningful forward-looking information about future θ_G or θ_P, it could have a possible indirect effect on households' expectations. By observing QE, households will perceive that the central bank wants them to change their expectations, but households determine their behavior on the basis of their own levels of optimality. If their optimality is not changed by QE, it is unlikely that households will change their expectations. Therefore, unless inflation/deflation and households' optimality are directly affected by QE, $\hat{\theta}_G$ and $\hat{\theta}_P$ will not be affected—even indirectly.

The quantity of money is not usually included in NKPC models (*e.g.,* Ugai 2007, Woodford 2012), so it is doubtful, even in these models, whether QE would be able to directly affect households' choices to switch from state (b) to (a). However, if QE influences households' inflation/deflation expectations, it could be effective. The manner in which household's inflation/deflation expectations are generated theoretically in NKPC models is unclear, however, when nominal interest rates are stuck at ZLB. Therefore, it is difficult to arrive at any clear theoretical conclusion about the effectiveness of QE on inflation/deflation channeled through households' expectations of inflation/deflation.

3.4. Renouncing independence

The reason why only steady states (a) and (b) can be chosen is that the condition $\hat{\theta}_G = \hat{\theta}_P$ must be satisfied: that is, the central bank must be sufficiently independent, as discussed in Section 2. Even during deflation, $\hat{\theta}_G = \hat{\theta}_P$ will hold because of the inhibitory effect resulting from the central bank's independence. Conversely, if the central bank is not independent, a deflationary steady state (b) will not be chosen because the inhibitory effect does not exist, and it is likely that $\theta_G > \theta_P$. Therefore, if the central bank renounces its independence, a switch from deflation to inflation may be possible. Adopting the measure of "helicopter money" as permanent and irreversible QE may be a kind of renouncement of independence. This, however, is an extraordinary and unconventional monetary policy.

Preferences are hard to control solely by oneself. As discussed in Appendix A, even though a government is fully rational and is not weak, foolish, or untruthful, it is still difficult for it to self-regulate its preferences. An independent central bank is therefore essential to control the government's preference (*i.e.,* θ_G), which it has difficulty controlling by itself. Hence, renouncing the central bank's independence indicates that the government will behave on the basis of its own intrinsic preferences—particularly on $\bar{\theta}_G$ —and $\theta_G > \theta_P$ will prevail.

To have any lasting effect, the central bank would have to renounce its authority truthfully and indefinitely. At the least, households would need to firmly believe that the central bank has done so. There is another very serious problem with this solution. The renouncement will be accompanied by high or hyperinflation. If the bank's independence is actually and indefinitely renounced, the deflationary steady state (b) will change, but not necessarily to the inflationary steady state (a) because the central bank is no longer independent. High or hyperinflation will be generated as a byproduct or side effect. Although the reversal of deflation may be successfully achieved, price stability will not. It seems unlikely, therefore, that either the central bank or households would support this measure.

A different conclusion may be drawn in NKPC models, but it is not theoretically clear how the independence of the central bank affects inflation/deflation in these models. In other words, the reason why the central bank and not the government should manipulate nominal interest rates according to a pre-determined instrument rule (*e.g.,* a Taylor rule) is theoretically ambiguous. In NKPC models, it may be implicitly assumed that there is some difference in preferences between the government and the central bank, but this difference is not explicitly modeled. Therefore, it is unclear what would happen if the central bank renounced its independence in NKPC models.

3.5. Raising nominal interest rates

There is another extraordinary and unconventional monetary policy by which a central bank may be able to force a switch from state (b) to state (a): increasing nominal interest rates. Conventionally, when a central bank wants to raise inflation, it decreases nominal interest rates. By this measure, the government has to raise θ_G as the central bank desires because it cannot otherwise achieve optimality and, as a result, inflation increases. However, the same logic can be applied even if the central bank increases nominal interest rates until the government raises θ_G sufficiently. Hence, if nominal interest rates are increased, inflation may be also increased.

However, increasing nominal interest rates is very risky because θ_P may also be affected. Increasing nominal interest rates will generate a temporary recession, and households may feel increased levels of future uncertainty and raise $\hat{\theta}_P$. If $\hat{\theta}_P$ increases as much as θ_G increases, the effect of the higher θ_G will be cancelled out and inflation/deflation will not change. If that occurs, the act of raising nominal interest rates will have created a recession without solving the underlying problem of deflation. Because of this risk, this unconventional monetary policy almost certainly will not actually be used.

This measure may be considered to be effective in NKPC models. If nominal interest rates are stuck at ZLB, the instrument rule for the central bank's manipulation of nominal interest rates is useless, and inflation depends primarily on households' expected inflation/deflation. Usually, θ_P is exogenously given and constant in NKPC models; thus, when nominal interest rates are raised by the central bank, the economy will collapse (or reach a non-optimal state) owing to the permanent condition $\theta_P \neq \bar{r}$ unless the households' inflation expectation is raised. If households strongly want to avoid an economic collapse, they will raise their expected inflation level so that this measure could possibly succeed. However, the conclusions will differ greatly depending on the assumptions of how inflation/deflation expectations are generated by households.

3.6. Depreciating the exchange rate

A sharp depreciation of the exchange rate raises prices of imported goods and services and may therefore temporarily cause deflation to change to inflation. If households change $\hat{\theta}_G$ or $\hat{\theta}_P$ and consequently π_0 because of this shock on the exchange rate, state (b) may be switched to state (a). Furthermore, if a government or central bank is able to deliberately depreciate the exchange rate sharply (*i.e.*, if it can freely manipulate the exchange rate), it can use this as a tool to switch from state (b) to (a).

This strategy has two problems. First, it is not certain that a shock on the exchange rate will always affect the expectations of θ_G or θ_P, because the exchange rate is irrelevant to θ_G or θ_P directly according to the law of motion for inflation/deflation. If a change in the exchange rate possesses some forward-looking information, $\hat{\theta}_G$ or $\hat{\theta}_P$ may be influenced indirectly by the change in the exchange rate, but that is not a theoretical certainty. Second, and more importantly, it is difficult for a government or central bank to freely manipulate the exchange rate. A change in exchange rates affects international trade and finance; thus, unilateral manipulation of exchange rates is problematic in the international community. At the least, this type of action will be fiercely condemned internationally. As a result, this strategy most likely will not be adoptable, at least not overtly.

Depreciating the exchange rate may be judged as effective in NKPC models (Svensson 2001, Coenen and Wieland 2003, 2004). Depreciated exchange rates and the ensuing temporary inflation may change households' inflation/deflation expectations. However, it is not clear why, or how, households change their expectations. In addition, the important problems related to the reaction of the international community remain the same. Therefore, in NKPC models, this strategy also seems unlikely to be used, at least not overtly.

3.7. Waiting for a lucky event

The expectations of θ_G and θ_P will of course be affected also by various exogenous shocks. There may be an exogenous shock that is large enough to make households switch from state (b) to (a). For example, if there is a large upward shock on the prices of imported goods (*e.g.*, due to a hike in oil prices or a sharp exogenous depreciation in the exchange rate), households may think that a switch from state (b) to state (a) is better for them because it may be easier for them to adapt to the shock in inflationary steady state (a) than in deflationary steady state (b). Other examples include a large upward shock on $\hat{\theta}_G$ and a large downward shock on $\hat{\theta}_P$. There are many other possible exogenous shocks that may affect the households' expectations and cause them to switch from state (b) to state (a). Nevertheless, these types of exogenous shock represent luck or randomness (given that exchange rates cannot be manipulated unilaterally by a government or central bank).

3.8. Imposing taxes on money

Several economists have proposed a tax on money to generate a negative rate of real interest (Fukao 2005, Buiter 2005). By using these taxes, nominal interest rates again become useful to achieve a target rate of inflation. This measure is therefore different from the previously discussed ones that are intended to influence the households' expectations. Because taxes are imposed by the government and not the central bank, the independence of the central bank is meaningless and the role of central bank becomes ambiguous. That said, once the deflation is reversed to inflation, the independent central bank again takes the initiative in controlling inflation.

This measure has the same problem as raising nominal interest rates. Imposing taxes on money may affect θ_P; in particular, it may increase θ_P. An increased θ_P leads to recession and the acceleration of deflation according to the law of motion for inflation/deflation. Imposing taxes on money therefore is very risky. If θ_P is not affected, this measure would be effective, but it seems likely that θ_P will be affected and any existing recession will be aggravated and deflation will accelerate. Because of these risks, this measure will almost certainly not actually be undertaken.

This measure may be considered to be effective in NKPC models. Because θ_P is usually exogenously given and constant in these models, inflation can be controlled by the monetary taxes without raising θ_P—that is, without the risk of aggravating an existing recession or accelerating deflation. Hence, this measure may be predicted to succeed with a high probability in examinations based NKPC models.

3.9. Difficulty in switching from state (b) to (a)

Examinations in this section have shown that it is difficult for a central bank to make households switch from a deflationary steady state (b) to an inflationary steady state (a). Verbal intervention and QE are basically ineffective. The extraordinary and unconventional monetary policy of renouncing central bank independence will have a large impact on the households' expectations and reverse deflation, but it will be accompanied by a serious negative side effect—that is, high or hyper-inflation. Another extraordinary and unconventional monetary policy—raising nominal interest rates—has a high risk of introducing or worsening a recession without reversing the ongoing deflation. Imposing taxes on money shares the same risks. It seems that the most effective policy to realize a switch from state (b) to (a) is to wait for an exogenous event—that is, to get lucky.

This conclusion suggests that, when inflation shows a sign of changing to deflation in the near future, it is extremely important for the central bank to make households continue to choose inflationary steady state (a). It will be easier for a central bank to make them do so before deflation sets in, for example, by increasing the target rate of inflation and lowering nominal interest rates far more than usual (*e.g.,* Williams 2009). If a central bank can successfully make households continue to choose state (a), the deflationary state can be averted altogether.

Verbal intervention and use of QE were also considered to be basically ineffective in the NKPC models, but the effects of renouncing central bank independence and raising nominal interest rates are unclear. On the other hand, imposing taxes on money may be effective, but this measure has never actually been tried.

Conclusion

Once deflation takes root, it is not easy to reverse because of the ZLB. If nominal interest rates are stuck at the ZLB, the central bank loses power to manipulate nominal interest rates. The manipulation of expectations is instead regarded as an important alternative tool. In this paper, the feasibility and effectiveness of the manipulation of expectations were examined in the inflation/deflation model shown by Harashima (2004b, 2008, 2015a) as well as in conventional inflation models—particularly new-Keynesian Phillips curve (NKPC) models.

There are only two steady states where both $\theta_P = \bar{r}$ and $\hat{\theta}_G = \hat{\theta}_P$ are satisfied: inflationary steady state (a), at which $\pi_0 = \pi^*$, and deflationary steady state (b), at which $\pi_0 = -\bar{r}$. Deflationary steady state (b) can be a steady state because the condition $\hat{\theta}_G = \hat{\theta}_P$ can hold owing to the inhibitory effect of the independent central bank, even if nominal interest rates are stuck at ZLB. To switch from state (b) to (a), the households' expectations need to be deliberately changed by the central bank. There are several possible ways to influence households' expectations, but there is no decisive measure to certainly change their expectations. Verbal intervention and QE are not effective. Renouncing the independence of the central bank may be effective but has very negative side effects, and raising nominal interest rates is also very risky. It is uncertain whether depreciating the exchange rate is effective, and it is practically infeasible for international political reasons. Imposing taxes on money is another measure that is very risky and not guaranteed to work. The best way to switch from state (b) to (a) may simply be to wait for a fortuitous exogenous event (*i.e.,* to be lucky). It may therefore be prudent for central banks to act drastically when inflation shows a sign of changing to deflation in the near future, for example, by increasing the target rate of inflation and lowering nominal interest rates far more than usual.

Even at deflationary steady state (b), however, the economy proceeds as normally as it does at inflationary steady state (a). Therefore, it may not be necessary for households to struggle to switch from deflationary steady state (b) to inflationary steady state (a), even though the central bank is very dissatisfied with deflationary steady state (b).

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Appendix A

A1. The law of motion for inflation/deflation

A1.1. The government

A1.1.1. The government budget constraint

The government budget constraint is

$$\dot{B}_t = B_t i_t + G_t - X_t - \vartheta_t \ ,$$

where B_t is the nominal obligation of the government to pay for its accumulated bonds, i_t is the nominal interest rate for government bonds, G_t is the nominal government expenditure, X_t is the nominal tax revenue, and ϑ_t is the nominal amount of seigniorage at time *t*. The tax is assumed to be lump sum, the government bonds are long term, and the returns on the bonds are realized only after the bonds are held during a unit period (*e.g.,* a year). The government bonds are redeemed in a unit period, and the government successively refinances the bonds by issuing new ones at each time *t*. Let [The picture can't be displayed.], [The picture can't be displayed.], [The picture can't be displayed.], and $\varphi_t = \frac{\vartheta_t}{P_t}$, where P_t is the price level at time *t*. Let also [The picture can't be displayed.] be the inflation rate at time *t*. By dividing by P_t, the budget constraint is transformed to $\frac{\dot{B}_t}{P_t} = b_t i_t + g_t - x_t - \varphi_t$, which is equivalent to

$$\dot{b}_t = b_t i_t + g_t - x_t - \varphi_t - b_t \pi_t = b_t (i_t - \pi_t) + g_t - x_t - \varphi_t \ . \quad \text{(A1)}$$

Because the returns on government bonds are realized only after holding the bonds during a unit period, investors buy the bonds if $\bar{i}_t \geq E_t \int_t^{t+1} (\pi_s + r_s) ds$ at time *t*, where $\bar{i}_t$ is the nominal interest rate for bonds bought at *t* and r_t is the real interest rate in markets at *t*. Hence, by arbitrage, $\bar{i}_t = E_t \int_t^{t+1} (\pi_s + r_s) ds$ and if r_t is constant such that $r_t = r$ (*i.e.*, if it is at steady state), then

$$\bar{i}_t = E_t \int_t^{t+1} \pi_s \, ds + r \ .$$

The nominal interest rate $\bar{i}_t = E_t \int_t^{t+1} \pi_s \, ds + r$ means that, during a sufficiently small period between *t* and *t* + *dt*, the government's obligation to pay for the bonds' return in the future increases not by [The picture can't be displayed.] but by [The picture can't be displayed.]. If π_t is constant, then $E_t \int_t^{t+1} \pi_s \, ds = \pi_t$ and $\bar{i}_t = \pi_t + r$, but if π_t is not constant, these equations do not necessarily hold.

Since bonds are redeemed in a unit period and successively refinanced, the bonds the government is holding at *t* have been issued between *t* - 1 and *t*. Hence, under perfect foresight, the average nominal interest rate for all government bonds at time *t* is the weighted sum of $\bar{i}_t$ such that

$$i_t = \int_{t-1}^{t} \bar{i}_s \left(\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv} \right) ds = \int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \left(\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv} \right) ds + r \ ,$$

where [The picture can't be displayed.] is the nominal value of bonds at time *t* that were issued at time *s*. If the weights $\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t}\, dv}$ between *t* - 1 and *t* are not so different from each other, then approximately $i_t = \int_{t-1}^{t}\int_{s}^{s+1} \pi_{\upsilon}\, d\upsilon\, ds + r$. To be precise, if the absolute values of π_s for [The picture can't be displayed.] are sufficiently smaller than unity, the differences among the weights are negligible and then approximately

$$i_t = \int_{t-1}^{t}\int_{s}^{s+1} \pi_{\upsilon}\, d\upsilon\, ds + r \tag{A2}$$

(see Harashima 2008). The average nominal interest rate for the total government bonds, therefore, develops by $i_t = \int_{t-1}^{t}\int_{s}^{s+1} \pi_{\upsilon}\, d\upsilon\, ds + r$. If π_t is constant, then $\int_{t-1}^{t}\int_{s}^{s+1} \pi_{\upsilon}\, d\upsilon\, ds = \pi_t$; thus, $i_t = \pi_t + r$. If π_t is not constant, however, the equations $\int_{t-1}^{t}\int_{s}^{s+1} \pi_{\upsilon}\, d\upsilon\, ds = \pi_t$ and $i_t = \pi_t + r$ do not necessarily hold.

A1.1.2. An economically Leviathan government

Under a proportional representation system, the government represents the median household whereas the representative household from an economic perspective represents the mean household.[2] Because of this difference, they usually have different preferences. To account for this essential difference, a Leviathan government is assumed in the model.[3] There are two extremely different views regarding government's behavior in the literature on political economy: the Leviathan view and the benevolent view (*e.g.*, Downs 1957, Brennan and Buchanan 1980, Alesina and Cukierman 1990). From an economic point of view, a benevolent government maximizes the expected economic utility of the representative household, but a Leviathan government does not. Whereas the expenditure of a benevolent government is a tool used to maximize the economic utility of the representative household, the expenditure of a Leviathan government is a tool used to achieve the government's own policy objectives.[4] For example, if a Leviathan government considers national security to be the most important political issue, defense spending will increase greatly, but if improving social welfare is the top political priority, spending on social welfare will increase dramatically, even though the increased expenditures may not necessarily increase the economic utility of the representative household.

Is it possible, however, for such a Leviathan government to hold office for a long period? Yes, because a government is generally chosen by the median of households under a proportional representation system (*e.g.,* Downs 1957), whereas the representative household usually presumed in the economics literature is the mean household. The economically representative household is not usually identical to the politically representative household, and a majority of people could support a Leviathan government even if they know that the government does not necessarily pursue only the economic objectives of the economically representative household. In other words, the Leviathan government argued here is an economically Leviathan government that maximizes the political utility of people, whereas the conventional economically benevolent government maximizes the economic utility of people. In addition, because the politically and economically representative households are different (the median and mean households, respectively), the preferences of future governments will also be similarly different from those of the mean representative household. In this sense, the current and future governments presented in the model can be seen as a combined government that goes on indefinitely; that is, the economically Leviathan government always represents the median representative household.

The Leviathan view generally requires the explicit inclusion of government expenditure, tax revenue, or related activities in the government's political utility function (*e.g.*, Edwards and Keen 1996). Because an economically Leviathan government derives political utility from expenditure for its political purposes, the larger

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[2] See the literature on the median voter theorem (*e.g.,* Downs 1957). Also see the literature on the delay in reforms (*e.g.,* Alesina and Drazen 1991; Cukierman *et al.* 1992).

[3] The most prominent reference to Leviathan governments is Brennan and Buchanan (1980).

[4] The government behavior assumed in the fiscal theory of the price level reflects an aspect of a Leviathan government. Christiano and Fitzgerald (2000) argue that non-Ricardian policies correspond to the type of policies in which governments are viewed as selecting policies and committing themselves to those policies in advance of prices being determined in markets.

the expenditure is, the happier the Leviathan government will be. But raising tax rates will provoke people's antipathy, which increases the probability of being replaced by the opposing party that also nearly represents the median household. Thus, the economically Leviathan government regards taxes as necessary costs to obtain freedom of expenditure for its own purposes. The government therefore will derive utility from expenditure and disutility from taxes. Expenditure and taxes in the political utility function of the government are analogous to consumption and labor hours in the economic utility function of the representative household. Consumption and labor hours are both control variables, and as such, the government's expenditure and tax revenue are also control variables. As a whole, the political utility function of economically Leviathan government can be expressed as $u_G(g_t, x_t)$.[5] In addition, it can be assumed on the basis of previously mentioned arguments that The picture can't be displayed. and The picture can't be displayed., and therefore that The picture can't be displayed. and The picture can't be displayed..[6] An economically Leviathan government therefore maximizes the expected sum of these utilities discounted by its time preference rate under the constraint of deficit financing.

A1.1.3. The optimization problem

The optimization problem of an economically Leviathan government is

$$Max\ E\int_0^\infty u_G(g_t, x_t)\exp(-\theta_G t)dt$$

subject to the budget constraint

$$\dot{b}_t = b_t(i_t - \pi_t) + g_t - x_t - \varphi_t\ , \qquad \text{(A3)}$$

where u_G is the constant relative risk aversion utility function of the government, θ_G is the government's rate of time preference, and E is the expectation operator. All variables are expressed in per capita terms, and population is assumed to be constant. The government maximizes its expected political utility considering the behavior of the economically representative household that is reflected in i_t in its budget constraint.

A1.2. Households

The economically representative household maximizes its expected economic utility. Sidrauski (1967)'s well-known money in the utility function model is used for the optimization problem. The representative household maximizes its expected utility

$$E\int_0^\infty u_P(c_t, m_t)\exp(-\theta_P t)dt$$

subject to the budget constraint

$$\dot{a}_t = (r_t a_t + w_t + \sigma_t) - [c_t + (\pi_t + r_t)m_t] - g_t\ ,$$

[5] It is possible to assume that governments are partially benevolent. In this case, the utility function of a government can be assumed to be The picture can't be displayed., where c_t is real consumption and l_t is the leisure hours of the representative household. However, if a lump-sum tax is imposed, the government's policies do not affect steady-state consumption and leisure hours. In this case, the utility function can be assumed to be The picture can't be displayed..

[6] Some may argue that it is more likely that The picture can't be displayed. and The picture can't be displayed.. However, the assumption used is not an important issue here because $-x_t\left[\frac{\partial u_G(g_t,x_t)}{\partial x_t}\right]^{-1}\frac{\partial^2 u_G(g_t,x_t)}{\partial x_t^{2}}\frac{\dot{x}_t}{x_t}=0$ at steady state. Thus, the results are not affected by which assumption is used.

where u_P and θ_P are the utility function and the time preference rate of the representative household, c_t is real consumption, w_t is real wage, σ_t is lump-sum real government transfers, m_t is real money, $a_t = k_t + m_t$, and k_t is real capital. It is assumed that $r_t = f'(k_t)$, $w_t = f(k_t) - k_t f'(k_t)$, The picture can't be displayed., The picture can't be displayed., The picture can't be displayed., and The picture can't be displayed., where $f(\cdot)$ is the production function. Government expenditure (g_t) is an exogenous variable for the representative household because it is an economically Leviathan government. It is also assumed that, although all households receive transfers from a government in equilibrium, when making decisions, each household takes the amount it receives as given, independent of its money holdings. Thus, the budget constraint means that the real output The picture can't be displayed. at any time is demanded for the real consumption c_t, the real investment The picture can't be displayed., and the real government expenditure g_t such that The picture can't be displayed.. The representative household maximizes its expected economic utility considering the behavior of government reflected in g_t in the budget constraint. In this discussion, a central bank is not assumed to be independent of the government; thus, the functions of the government and the central bank are not separated. This assumption can be relaxed, and the roles of the government and the central bank are explicitly separated in Section A2.

Note that the time preference rate of government (θ_G) is not necessarily identical to that of the representative household (θ_P) because the government and the representative household represent different households (*i.e.*, the median and mean households, respectively). In addition, the preferences will differ because (1) even though people want to choose a government that has the same time preference rate as the representative household, the rates may differ owing to errors in expectations (*e.g.,* Alesina and Cukierman 1990); and (2) current voters cannot bind the choices of future voters and, if current voters are aware of this possibility, they may vote more myopically as compared with their own rates of impatience in private economic activities (*e.g.,* Tabellini and Alesina 1990). Hence, it is highly likely that the time preference rates of a government and the representative household are heterogeneous. It should be also noted, however, that even though the rates of time preference are heterogeneous, an economically Leviathan government behaves based only on its own time preference rate, without hesitation.

A1.3. The simultaneous optimization

First, I examine the optimization problem of the representative household. Let Hamiltonian H_P be $H_P = u_P(c_t, m_t)\exp(-\theta_P t) + \lambda_{P,t}[r_t a_t + w_t + \sigma_t - c_t - (\pi_t + r_t)m_t - g_t]$, where $\lambda_{P,t}$ is a costate variable, c_t and m_t are control variables, and a_t is a state variable. The optimality conditions for the representative household are;

$$\frac{\partial u_P(c_t, m_t)}{\partial c_t}\exp(-\theta_P t) = \lambda_{P,t} \,, \tag{A4}$$

$$\frac{\partial u_P(c_t, m_t)}{\partial m_t}\exp(-\theta_P t) = \lambda_{P,t}(\pi_t + r_t) \,, \tag{A5}$$

The picture can't be displayed. , (A6)

$$\dot{a}_t = (ra_t + w_t + \sigma_t) - [c_t + (\pi_t + r_t)m_t - g_t] \,, \tag{A7}$$

The picture can't be displayed. . (A8)

By conditions (A4) and (A5), $\left[\frac{\partial u_P(c_t, m_t)}{\partial c_t}\right]^{-1} \frac{\partial u_P(c_t, m_t)}{\partial m_t} = \pi_t + r_t$, and by conditions (A4) and (A6),

$$-c_t\left[\frac{\partial u_P(c_t,m_t)}{\partial c_t}\right]^{-1}\frac{\partial^2 u_P(c_t,m_t)}{\partial c_t^2}\frac{\dot{c}_t}{c_t}+\theta_P=r_t\ . \quad \text{(A9)}$$

Hence,

$$\theta_P = r_t = r \quad \text{(A10)}$$

at steady state such that The picture can't be displayed. and The picture can't be displayed..

Next, I examine the optimization problem of the economically Leviathan government. Let Hamiltonian H_G be $H_G = u_G(g_t, x_t)\exp(-\theta_G t)+\lambda_{G,t}[b_t(i_t-\pi_t)+g_t-x_t-\varphi_t]$, where $\lambda_{G,t}$ is a costate variable. The optimality conditions for the government are;

$$\frac{\partial u_G(g_t,x_t)}{\partial g_t}\exp(-\theta_G t)=-\lambda_{G,t}\ , \quad \text{(A11)}$$

$$\frac{\partial u_G(g_t,x_t)}{\partial x_t}\exp(-\theta_G t)=\lambda_{G,t}\ , \quad \text{(A12)}$$

$$\dot{\lambda}_{G,t}=-\lambda_{G,t}(i_t-\pi_t)\ , \quad \text{(A13)}$$

$$\dot{b}_t=b_t(i_t-\pi_t)+g_t-x_t-\varphi_t\ , \quad \text{(A14)}$$

$$\lim_{t\to\infty}\lambda_{G,t}\, b_t=0\ . \quad \text{(A15)}$$

Combining conditions (A11), (A12), and (A13) and equation (A2) yields the following equations:

$$-g_t\left[\frac{\partial u_G(g_t,x_t)}{\partial g_t}\right]^{-1}\frac{\partial^2 u_G(g_t,x_t)}{\partial g_t^2}\frac{\dot{g}_t}{g_t}+\theta_G=i_t-\pi_t=r_t+\int_{t-1}^{t}\int_{s}^{s+1}\pi_\upsilon\, d\upsilon\ ds-\pi_t \quad \text{(A16)}$$

and

$$-\ x_t\left[\frac{\partial u_G(g_t,x_t)}{\partial x_t}\right]^{-1}\frac{\partial^2 u_G(g_t,x_t)}{\partial x_t^{\,2}}\frac{\dot{x}_t}{x_t}+\theta_G=i_t-\pi_t=r_t+\int_{t-1}^{t}\int_{s}^{s+1}\pi_\upsilon\, d\upsilon\ ds-\pi_t\ . \quad \text{(A17)}$$

Here, $g_t\left[\frac{\partial u_G(g_t,x_t)}{\partial g_t}\right]^{-1}\frac{\partial^2 u_G(g_t,x_t)}{\partial g_t^2}\frac{\dot{g}_t}{g_t}=0$ and $x_t\left[\frac{\partial u_G(g_t,x_t)}{\partial x_t}\right]^{-1}\frac{\partial^2 u_G(g_t,x_t)}{\partial x_t^{\,2}}\frac{\dot{x}_t}{x_t}=0$ at steady state such that The picture can't be displayed. and The picture can't be displayed.; thus,

$$\theta_G=r_t+\int_{t-1}^{t}\int_{s}^{s+1}\pi_\upsilon\, d\upsilon\, ds-\pi_t\ . \quad \text{(A18)}$$

Hence, by equation (A10),

$$\int_{t-1}^{t}\int_{s}^{s+1}\pi_\upsilon\, d\upsilon\, ds=\pi_t+\theta_G-\theta_P \quad \text{(A19)}$$

at steady state such that The picture can't be displayed., The picture can't be displayed., The picture can't be displayed., and The picture can't be displayed..[7]

Equation (A19) is a natural consequence of simultaneous optimization by the economically Leviathan government and the representative household. If the rates of time preference are heterogeneous between them, then

$$i_t - r = \int_{t-1}^{t}\int_{s}^{s+1} \pi_{\upsilon}\, d\upsilon\, ds \neq \pi_t \ .$$

This result might seem surprising because it has been naturally conjectured that $i_t = \pi_t + r$. However, this is a simple misunderstanding because π_t indicates the instantaneous rate of inflation at a point such that The picture can't be displayed., whereas $\int_{t-1}^{t}\int_{s}^{s+1} \pi_{\upsilon}\, d\upsilon\, ds$ roughly indicates the average inflation rate in a period. Equation (A19) indicates that π_t develops according to the integral equation $\pi_t = \int_{t-1}^{t}\int_{s}^{s+1} \pi_{\upsilon}\, d\upsilon\, ds - \theta_G + \theta_P$. If π_t is constant, the equations $i_t = \pi_t + r$ and $\int_{t-1}^{t}\int_{s}^{s+1} \pi_{\upsilon}\, d\upsilon\, ds = \pi_t$ are true. However, if π_t is not constant, the equations do not necessarily hold. Equation (A19) indicates that the equations $i_t = \pi_t + r$ and $\int_{t-1}^{t}\int_{s}^{s+1} \pi_{\upsilon}\, d\upsilon\, ds = \pi_t$ hold only in the case where $\theta_G = \theta_P$ (i.e., a homogeneous rate of time preference). It has been previously thought that a homogeneous rate of time preference naturally prevails; thus, the equation $i_t = \pi_t + r$ has not been questioned. As argued previously, however, a homogeneous rate of time preference is not usually guaranteed.

A1.4. The law of motion for trend inflation

Equation (A19) indicates that inflation accelerates or decelerates as a result of the government and the representative household reconciling the contradiction in heterogeneous rates of time preference. If π_t is constant, the equation $\pi_t = \int_{t-1}^{t}\int_{s}^{s+1} \pi_{\upsilon}\, d\upsilon\, ds$ holds; conversely, if $\pi_t \neq \int_{t-1}^{t}\int_{s}^{s+1} \pi_{\upsilon}\, d\upsilon\ \ ds$, then π_t is not constant. Without the acceleration or deceleration of inflation, therefore, equation (A19) cannot hold in an economy in which The picture can't be displayed.. In other words, it is not until The picture can't be displayed. that inflation can accelerate or decelerate. Heterogeneous time preferences (The picture can't be displayed.) bend the path of inflation and enables inflation to accelerate or decelerate. The difference of time preference rates $(\theta_G - \theta_P)$ at each time needs to be transformed to the accelerated or decelerated inflation rate π_t at each time.

Equation (A19) implies that inflation accelerates or decelerates nonlinearly in the case in which The picture can't be displayed.. For a sufficiently small period *dt*, π_{t+1+dt} is determined with π_s The picture can't be displayed. that satisfies $\int_{t-1}^{t}\int_{s}^{s+1} \pi_{\upsilon}\, d\upsilon\ ds - \pi_t = \theta_G - \theta_P$, so as to hold the equation $\int_{t}^{t+dt}\int_{s}^{s+1} \pi_{\upsilon}\, d\upsilon\, ds = \int_{t-1}^{t-1+dt}\int_{s}^{s+1} \pi_{\upsilon}\, d\upsilon\, ds + \pi_{t+dt} - \pi_t$. A solution of the integral equation (A19) for given θ_G and θ_P is

$$\pi_t = \pi_0 + 6(\theta_G - \theta_P)t^2 \ . \qquad \text{(A20)}$$

Generally, the path of inflation that satisfies equation (A19) for $0 \le t$ is expressed as

$$\pi_t = \pi_0 + 6(\theta_G - \theta_P)\exp[z_t \ln(t)] \ ,$$

where z_t is a time dependent variable. The stream of z_t is various depending on the boundary condition, *i.e.*, the past and present inflation during $-1 < t \le 0$ and the path of inflation during $0 < t \le 1$ that is set to make π_0 satisfy

[7] If and only if $\theta_G = -\dfrac{g_t - x_t - \varphi_t}{b_t}$ at steady state, then the transversality condition (A1-15) The picture can't be displayed. holds. The proof is shown in Harashima (2008).

equation (A19). However, z_t has the following important property. If π_t satisfies equation (A19) for $0 \le t$, and $-\infty < \pi_t < \infty$ for $-1 < t \le 1$, then

$$\lim_{t \to \infty} z_t = 2 \cdot$$

Proof is shown in Harashima (2008). Any inflation path that satisfies equation (A19) for $0 \le t$ therefore asymptotically approaches the path of equation (A20). The mechanism behind the law of motion for inflation (equation (A20)) is examined more in detail in Harashima (2008).

A2. The central bank

A central bank manipulates the nominal interest rate according to the following Taylor-type instrument rule in the model;

The picture can't be displayed. , (A21)

where π^* is the target rate of inflation and The picture can't be displayed., γ_π, and γ_x are constant coefficients. The picture can't be displayed. as is usually assumed.

In Section A1, central banks are not explicitly considered because they are not assumed to be independent of governments. However, in actuality, central banks are independent organizations in most countries even though some of them are not sufficiently independent. Furthermore, in the conventional inflation model, it is the central banks that control inflation and governments have no role in controlling inflation. Conventional inflation models show that the rate of inflation basically converges at the target rate of inflation set by a central bank. The target rate of inflation therefore is the key exogenous variable that determines the path of inflation in these models.

Both the government and the central bank can probably affect the development of inflation, but they would do so in different manners, as equation (A20) and conventional inflation models indicate. However, the objectives of the government and the central bank may not be the same. For example, if trend inflation is added to conventional models by replacing their aggregate supply equations with equation (A20), inflation cannot necessarily converge at the target rate of inflation because another key exogenous variable (θ_G) is included in the models. A government makes inflation develop consistently with the equation (A20), which implies that inflation will not necessarily converge at the target rate of inflation. Conversely, a central bank makes inflation converge at the target rate of inflation, which implies that inflation will not necessarily develop consistently with equation (A20). That is, unless either θ_G is adjusted to be consistent with the target rate of inflation or the target rate of inflation is adjusted to be consistent with θ_G, the path of inflation cannot necessarily be determined. Either θ_G or the target rate of inflation need be an endogenous variable. If a central bank dominates, the target rate of inflation remains as the key exogenous variable and θ_G should then be an endogenous variable. The reverse is also true.

A central bank will be regarded as truly independent if θ_G is forced to be adjusted to the one that is consistent with the target rate of inflation set by the central bank. For example, suppose that $\theta_G > \theta_P$ and a truly independent central bank manipulates the nominal interest rate according to the Taylor-type instrument rule (equation (A21)). Here,

$$i_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds + r = \theta_G + \pi_t \qquad \text{(A22)}$$

at steady state such that The picture can't be displayed., The picture can't be displayed., The picture can't be displayed., and The picture can't be displayed. by equations (A2), (A10), and (A19). If the accelerating inflation rate is higher than the target rate of inflation, the central bank can raise the nominal interest rate from $i_t = \theta_G + \pi_t$ (equation (A22)) to

$$i_t = \theta_G + \pi_t + \psi$$

by positive ψ by intervening in financial markets to lower the accelerating rate of inflation. In this case, the central bank keeps the initial target rate of inflation because it is truly independent. The government thus faces a rate of increase of real obligation that is higher than θ_G by the extra rate ψ.[8] If the government lowers θ_G so that $\theta_G < \theta_P$ and inflation stops accelerating, the central bank will accordingly reduce the extra rate ψ. If, however, the government does not accommodate θ_G to the target rate of inflation, the extra rate ψ will increase as time passes because of the gap between the accelerating inflation rate and the target rate of inflation widens by equation (A20) and γ_x in Taylor-type instrument rules is usually larger than unity, say 1.5. Because of the extra rate ψ, the government has no other way to achieve optimization unless it lowers θ_G to one that is consistent with the target rate of inflation. Once the government recognizes that the central bank is firmly determined to be independent and it is in vain to try to intervene in the central bank's decision makings, the government would not dare to attempt to raise θ_G again anymore.

Equation (A20) implies that a government allows inflation to accelerate because it acts to maximize its expected utility based only on its own preferences. A government is hardly the only entity that cannot easily control its own preferences even when these preferences may result in unfavorable consequences. It may not even be possible to manipulate one's own preferences at will. Thus, even though a government is fully rational and is not weak, foolish, or untruthful, it is difficult for it to self-regulate its preferences. Hence, an independent neutral organization is needed to help control θ_G. Delegating the authority to set and keep the target rate of inflation to an independent central bank is a way to control θ_G. The delegated independent central bank will control θ_G because it is not the central bank's preference to stabilize the price level—it is simply a duty delegated to it. An independent central bank is not the only possible choice. For example, pegging the local currency with a foreign currency can be seen as a kind of delegation to an independent neutral organization. In addition, the gold standard that prevailed before World War II can be also seen as a type of such delegation.

Note also that the delegation may not be viewed as bad from the Leviathan government's point of view because only its rate of time preference is changed, and the government can still pursue its political objectives. One criticism of the argument that central banks should be independent (*e.g.*, Blinder 1998) is that, since the time-inconsistency problem argued in Kydland and Prescott (1977) or Barro and Gordon (1983) is more acute with fiscal policy, why is it not also necessary to delegate fiscal policies? An economically Leviathan government, however, will never allow fiscal policies to be delegated to an independent neutral organization because the Leviathan government would then not be able to pursue its political objectives, which in a sense would mean the death of the Leviathan government. The median household that backs the Leviathan government, but at the same time dislikes high inflation, will therefore support the delegation of authority but only if it concerns monetary policy. The independent central bank will then be given the authority to control θ_G and oblige the government to change θ_G in order to meet the target rate of inflation.

Without such a delegation of authority, it is likely that generally $\theta_G > \theta_P$ because θ_G represents the median household whereas θ_P represents the mean household. Empirical studies indicate that the rate of time preference negatively correlates with permanent income (*e.g.,* Lawrance 1991), and the permanent income of the median household is usually lower than that of the mean household. If generally $\theta_G > \theta_P$, that suggests that inflation will tend to accelerate unless a central bank is independent. The independence of the central bank is therefore very important in keeping the path of inflation stable.

Note also that the forced adjustments of θ_G by an independent central bank are exogenous shocks to both the government and the representative household because they are planned solely by the central bank. When a shock on the expected θ_G is given, the government and the representative household must recalculate their optimal paths including the path of inflation by resetting θ_G, π_t, and φ.

[8] The extra rate ψ affects not only the behavior of government but also that of the representative household, in which the conventional inflation theory is particularly interested. In this sense, the central bank's instrument rule that concerns and simultaneously affects both behaviors of the government and the representative household is particularly important for price stability.

Appendix B

B1. Preference vs. rationality

The law of motion for inflation/deflation discussed in Section 2.1 indicates that, if the government behaves on the basis of its intrinsic RTP, inflation will accelerate. On the other hand, if people strongly dislike inflation acceleration, a government has to behave so as to not accelerate inflation; however, this conflicts with its own intrinsic preference.

B1.1. The conflict between preference and rationality

Behaving on the basis of its own intrinsic preferences does not mean that a government acts in a stupid, foolish, or irrational manner; rather, it behaves quite normally by naturally adhering to its intrinsic preferences. A fundamental question arises, however: Even if the government is acting quite normally, is this behavior rational? In economics, rationality usually means that, given the available information, optimal decisions are made to achieve an objective, and rational behavior is generally assumed. However, can rational behavior still prevail when a government cannot optimize its behavior to achieve its objective? This special situation emerges if the central bank is perfectly independent and is firmly determined to stabilize inflation and if, at the same time, the intrinsic time preference rate of government is unchangeable. In this situation, the economy will become severely destabilized because it is impossible to satisfy equation (1). Therefore, the government cannot achieve its objective (i.e., cannot maximize its expected utility) and can behave only irrationally. Conversely, if the government wants to optimize its objective and behave rationally, it must change its time preference. Clearly, trade-offs between rationality and time preference exist in some situations, and either rationality or time preference must be endogenized.

Nevertheless, it is highly unlikely that people will not optimize their behavior to meet their objectives (i.e., maximize utility) if they have complete knowledge of the optimal path. Hence, rationality should prevail over preferences, and time preference will be endogenized when a clash between rationality and time preference occurs. If time preference is endogenized, rational decisions become possible. Even though rationality should eventually prevail over preferences, governments will not easily change their own preferences. They will resist endogenizing them and search for options to escape from doing so—it is this stubborn nature that drives a government to deviate from the path specified by its central bank. Even though unfavorable consequences are expected if no change is made, it can be very difficult to change one's own preferences alone. Controlling preferences therefore usually requires the help of other people or institutions; this is one of the reasons why independent central banks were established to stabilize inflation.

If a central bank is not sufficiently independent, the government must change its RTP on its own so as to not accelerate inflation. A government must then rein in its preferences on its own. The RTP of government, therefore, is determined through the struggle between preference and rationality inside the government. If rationality prevails, inflation does not accelerate, but if preference prevails, inflation will accelerate.

B1.2. Two environments

Models are simplified representations of reality. Therefore, models can be classified by how far the chosen model simplifies reality. In particular, models are classified by whether they are based on the assumption that all agents are homogeneous (*i.e.,* a homogeneous environment) or on the assumption that agents are heterogeneous (*i.e.*, a heterogeneous environment).

In models based on a homogeneous environment, it is usually assumed that rationality always prevails over preference, because it has generally been regarded that there is no conflict between preference and rationality in a homogeneous environment. In general, the dominance of rationality in a homogeneous environment has been undoubted (*i.e.,* the rational expectation hypothesis has been accepted).

On the other hand, dominance of rationality in a heterogeneous environment is not necessarily guaranteed because, unlike in a homogeneous environment, serious contradictions between preference and rationality arise in a heterogeneous environment. For example, Becker (1980) showed that, if the RTPs of households are heterogeneous, the most patient household will eventually own all the capital in an economy and the other households cannot achieve optimality. That is, all households except the most patient household cannot behave rationally in the sense that rational households behave in such a way to achieve optimality, if they adhere to their own intrinsic RTPs. Harashima (2004b, 2008, 2015a) showed another case. If a government adheres to its own intrinsic RTP that is higher than the RTP of the representative household, inflation accelerates. If people dislike inflation acceleration and thereby the government has to behave under the condition that it does not accelerate inflation, there is no path that satisfies all optimality conditions for the government as long as it adheres to its own intrinsic RTP. In a heterogeneous environment, therefore, conflicts between preference and rationality can occur.

B1.3. Necessary intelligence

The struggle between preference and rationality is dealt with in the human brain. To resolve conflicts, humans need particular powers or functions—that is, different types of intelligence.

B1.3.1. Sustainability in a union or society

Properly dealing with the struggle between preference and rationality is essential for humans because humans do not live alone—they are social and live in groups. However, the struggle has the potential to destroy a society. In a heterogeneous environment, if preference prevails over rationality, there is no guarantee that a political union or society is sustainable because some members of society cannot achieve optimality. In theory, this problem does not exist in a homogeneous environment, because the conflict basically does not exist and competitive equilibria are optimal for all people. On the other hand, in a heterogeneous environment, competitive equilibria are not necessarily optimal for all people because people have heterogeneous preferences, as discussed in Section B1.2. Many of the people who cannot achieve optimality will strongly oppose the government or other people, and it is likely that the political union or society will collapse, possibly violently.

A political union or society is formed and maintained because it provides benefits to its members. Behaviors that support a union or society are important for humans to survive. The type of potential vulnerability in heterogeneous environments that is discussed above indicates that various types of intelligence are essential to properly manage the struggle between preference and rationality.

B1.3.1.1. Calculations

In a heterogeneous environment, relationships among people are far more complicated than in a homogeneous environment because people do not all behave in the same way in a heterogeneous environment. Humans must possess the intelligence to cope with these complicated relationships. They need to be able to calculate the outcomes of various activities in a heterogeneous group of people, evaluate the outcomes, and select the best action to take among many options in their brains.

The number of calculations required to reach an optimal solution is far larger in a heterogeneous environment than in a homogeneous one because the number of types of people that must be considered and the number of interconnections among heterogeneous people are far greater in a heterogeneous environment. If each person's brain can cope with this extremely large number of calculations, people can behave rationally (*i.e.*, always take the best actions that are calculated to be optimal, that is, the ones that are consistent with the model) even in a heterogeneous environment. If this does not occur, rationality may not prevail over preference.

B1.3.1.2. Evaluation

After a variety of potential outcomes are calculated, many options are evaluated on the basis of the results of calculations to select the optimal option. Therefore, people must have the intelligence to evaluate options. The optimal future path is more complicated in a heterogeneous environment than in a homogeneous environment, because households act differently. The intelligence needed for evaluation allows people to accurately identify the optimal future path by comparing and evaluating various aspects of many different complicated paths.

B1.3.1.3. Self-control

In addition, another type of intelligence is required—that which allows people to align their preferences so as to follow the optimal option. Even if an optimal option is appropriately calculated and evaluated, the optimal option cannot be implemented if people's preferences are not properly controlled. That is, people must exercise self-control. This type of intelligence applies to other activities as well—for example, when a person is on a diet. Children often have difficulty exercising self-control because this type of intelligence is not yet fully developed in childhood. In addition, it seems highly likely that it is also not necessarily sufficiently developed in many adults, and even adults will often lose the battle when forced to choose an option that is against their own preferences.

B1.3.2. Intelligence needed when the three types of subordinate intelligence are deficient

It remains unclear whether humans are sufficiently equipped with the necessary types of intelligence to deal with the calculation, evaluation, and self-control aspects of decision-making in a heterogeneous environment. For example, the capacity of a human's brain may be insufficient to process the extremely large number of calculations necessary in a heterogeneous environment. If this first type of intelligence is insufficient, it will be even more difficult to evaluate which option is appropriate to prevent disrupting the political union or society. Furthermore, even if the intelligence needed for calculations is sufficient, actions taken will not be optimal if the evaluation process is biased or poor.

If any part of the three subordinate intelligences is deficient, however, humans still have alternative methods to employ. For example, they can use approximations. The number of calculations needed will be significantly reduced if an appropriate approximation method is used. The intelligence needed for approximation is likely basically different from the three types of subordinate intelligence, although there may be partial overlap between them. For appropriate approximations, the concept of "fluid intelligence" will be particularly important.

B1.3.3. Fluid intelligence

In psychology and psychometrics, many types of intelligence have been considered, including fluid intelligence, crystallized intelligence, short-term memory, long-term storage and retrieval, reading and writing ability, and visual processing. Among these, the importance of the difference between fluid intelligence and crystallized intelligence has been particularly emphasized. According to Cattell (1963, 1971), fluid intelligence is the ability to solve novel problems by thinking logically without only depending on knowledge previously acquired. This type of intelligence signifies the ability to deal with new situations without relying on knowledge gained at school or through experience. With the help of fluid intelligence, people can flexibly adapt their thinking to new kinds of problems or situations. By contrast, crystallized intelligence is the capacity to acquire and use previously obtained knowledge.

Fluid intelligence is essential when people make approximate calculations and need to judge which approximation is the best among many choices. These judgments are very difficult because we do not know the true values. Therefore, judgments must be made after comprehensive consideration of various choices. Such judgments represent "something new" in the sense that they will not necessarily be judged as best in future periods and under different circumstances. People need to make new judgments in any future period. That is, we must solve an "unknown problem" on each occasion to make the best approximation. Thus, these judgments are innovations that are made by using a person's fluid intelligence. Fluid intelligence is therefore essential in a heterogeneous environment.

These types of judgments are similar to decisions made in politics. Political conditions change from moment to moment. Yesterday's optimal political decision may be a non-optimal political decision today. Furthermore, nobody knows for certain whether today's political decision is truly optimal. Historians examine whether past political decisions were optimal, but there are many political decisions over which even historians cannot reach consensus about their optimality.

B1.4. The degree of rationality in a heterogeneous environment

B1.4.1. The item response theory

Fluid intelligence can be modeled on the basis of the item response theory, which is used widely in psychometric studies (*e.g.,* Lord and Novick 1968; van der Linden and Hambleton 1997). In particular, the item response function is used to describe the relationship between abilities and item responses.

A typical item response function is

$$\tilde{p}(\tilde{\mu}) = \tilde{c} + \frac{1-\tilde{c}}{1+e^{-\tilde{a}(\tilde{\mu}-\tilde{b})}} ,$$

where $\tilde{p}$ is the probability of a correct response (*e.g.,* answer) to an item (*e.g.,* test or question), $\tilde{\mu}$ ($\infty > \tilde{\mu} > -\infty$) is a parameter that indicates an individual's ability, $\tilde{a}$ (> 0) is a parameter that characterizes the slope of the function, $\tilde{b}$ ($\infty \geq \tilde{b} \geq -\infty$) is a parameter that represents the difficulty of an item, and $\tilde{c}$ ($1 \geq \tilde{c} \geq 0$) is a parameter that indicates the probability that an item can be answered correctly by chance.

B1.4.2. The probability of dominance of rationality

How frequently rationality prevails over preference can be modeled with an item response function. Let *FI* be the degree of fluid intelligence in a person. Larger values of *FI* indicate stronger fluid intelligence in the sense that a person more correctly grasps (approximates) a situation by using fluid intelligence. Let also p_{HE} be the probability that rationality prevails over preference in a heterogeneous environment. On the basis of the item response theory, p_{HE} can be modeled as a function of *FI* such that

$$p_{HE}(FI) = \hat{c} + \frac{1-\hat{c}}{1+e^{-\hat{a}(FI-\hat{b})}} \quad , \tag{B1}$$

where $\hat{a}$ (> 0) is a parameter that characterizes the slope of the function, $\hat{b}$ ($\infty \geq \hat{b} \geq -\infty$) is a parameter that represents the difficulty and complexity of a situation, and $\hat{c}$ ($1 \geq \hat{c} \geq 0$) is a parameter that indicates the probability that rationality prevails over preference by exogenous factors. If *FI* is sufficiently large, rationality almost always prevails over preference in a heterogeneous environment, but if it is very small, preference almost always prevails over rationality.

An important implication of equation (B1) is that the rational expectation hypothesis is not necessarily acceptable in a heterogeneous environment. If *FI* is small (*i.e.,* fluid intelligence is weak), preference will often prevail over rationality and thus the rational expectation hypothesis cannot be unconditionally accepted.

B1.5. Fluid intelligence of government

According to the median voter theorem (*e.g.,* Downs 1957), a government behaves just as the median voter prefers in a one-person one-vote democratic political system. This theorem suggests that the fluid intelligence of government is equal to that of the median voter. On the other hand, the top-level positions in government are usually occupied by the best and brightest in a country, and they will almost certainly have stronger fluid intelligence than the median voter. However, does that mean these officials will make decisions that are different from those of the median voter? If they do so, they will be forced to step down in the next election according to the median voter theorem. Only politicians who make the same decisions as the median voter will be able to occupy top-level positions. Hence, it is likely that the fluid intelligence of government is practically equal to that of the median voter when dealing with issues in which preference and rationality conflict.

B1.6. The nature of $\hat{c}$

The value of $\hat{c}$ is affected by exogenous factors. For example, if the central bank is sufficiently independent and capable, $\hat{c}$ becomes unity—that is, the central bank makes rationality always prevail over preference with regard to the RTP of government. The government is always forced to change its RTP as the central bank orders. It is likely that many institutions or mechanisms work to raise the value of $\hat{c}$. For example, constitutions, laws, treaties, and many government and international organizations will raise the value of $\hat{c}$ by urging governments to maintain rationality. Such institutions and mechanisms have probably been adopted in many societies, because experience has taught us that they help ensure that rationality prevails over preference in a heterogeneous environment. As new institutions or mechanisms were invented and adopted, the probability that rationality prevails over preference may have gradually increased (by increasing the value of $\hat{c}$) through time. Therefore, it is likely that, as civilization has progressed, $\hat{c}$ has increased, and rationality more frequently prevails over preference in a heterogeneous environment.

B2. A model of government RTP

B2.1. Determinants of θ_G

The value of θ_G will usually be equal to the RTP of the median voter, as discussed in Section B1.5. However, in some cases, other elements will also affect the value of θ_G. The determinants of θ_G will be basically classified into the following two elements.

B2.1.1. Preference element

In this paper, I call the determinant that is equal to that of the median voter's RTP the "preference element." This element usually determines the main body of θ_G. Let $\theta_{G,pre}$ be the preference element component of θ_G, and $\theta_{P,med}$ be the intrinsic RTP of the median voter. As discussed in Section B1.5, the intrinsic $\theta_{G,pre}$ is basically equal to the intrinsic $\theta_{P,med}$ in a one-person one-vote democratic political system. Therefore, in the following sections, I assume that $\theta_{G,pre} = \theta_{P,med}$.

B2.1.2. Political element

The determinant that is peculiar to the government's RTP is the "political element." Let $\theta_{G,pol}$ be the political element component of θ_G. If a political system is maintained and stable forever, the political element will be nil, and θ_G will be determined only by the preference element. However, if a political system is unstable, the political element component is not zero, and it increases as the political system becomes more unstable. Although rare, it is possible for a political system to collapse. There are many historical examples of the collapse of a political system. These have been often observed, for example, after a defeat in a large-scale war or after a revolution. The political element is of great significance when a political system is on the brink of collapse. Faced with an impending collapse of the system, the incumbent government will do anything possible to survive the crisis. From the government's perspective, the far future is meaningless—survival is the primary objective. It imposes taxes and increases expenditures so as to avoid immediate collapse. As a result, its actions become increasingly myopic and impatient in the sense that it does not concern itself with future economic conditions. This behavior indicates an increase in $\theta_{G,pol}$.

For most democratic countries, the probability of an imminent collapse of the political system will be negligible, and we may assume that $\theta_{G,pol}$ is zero in those countries, but the political element is very important in politically unstable countries.

B2.2. The model

Section B1 indicates that p_{HE} needs to be expected to generate an expected $\theta_{G,pre}$. Let $p_{HE,G}$ be the p_{HE} of the government and $p_{HE,P}$ be the p_{HE} of the median voter. Because basically $\theta_{G,pre}$ = $\theta_{P,med}$ as discussed in Section B2.1, $p_{HE,G}$ = $p_{HE,P}$ generally, and thereby it is reasonable to assume that $p_{HE,G}$ = $p_{HE,P}$. Therefore, in a one-person one-vote political system,

$$p_{HE,G} = p_{HE,P} = \hat{c} + \frac{1-\hat{c}}{1+e^{-\hat{a}\left(FI_{P,med}-\hat{b}\right)}} \, , \tag{B2}$$

where $FI_{P,med}$ is the FI of the median voter. Equation (B2) indicates that the smaller $FI_{P,med}$ is, the smaller $p_{HE,G}$ is and the higher the probability of inflation acceleration.

Suppose that the central bank is not independent of the government. Thereby, the government has to control its RTP by itself, that is, without being forced to so by the central bank. (The case for an independent central bank is discussed in Section B2.4.) Suppose also for simplicity that the probability that a political system is on the brink of collapse is p_{inst} and $\theta_{G,pol}$ takes a unique positive value, and the probability of a stable political system is then 1– p_{inst} and $\theta_{G,pol}$ = 0. The model of the government's RTP that is used to generate the expected RTP of government is therefore

$$\begin{aligned}\theta_G &= p_{HE,G}\theta_P + \left(1-p_{HE,G}\right)\left(p_{inst}\theta_{G,pol}+\theta_{G,pre}\right) \\ &= \left(\hat{c} + \frac{1-\hat{c}}{1+e^{-\hat{a}\left(FI_{P,med}-\hat{b}\right)}}\right)\theta_P + \left[1-\left(\hat{c} + \frac{1-\hat{c}}{1+e^{-\hat{a}\left(FI_{P,med}-\hat{b}\right)}}\right)\right]\left(p_{inst}\theta_{G,pol}+\theta_{G,pre}\right).\end{aligned} \tag{B3}$$

Equation (B3) indicates that the RTP of government is equal to θ_P when rationality prevails over preference with the probability $p_{HE,G}$. When preference prevails over rationality with the probability 1 – $p_{HE,G}$, the RTP of government is equal to the intrinsic RTP of government. The intrinsic RTP of government consists of $\theta_{G,pol}$ with the probability p_{inst} and $\theta_{G,pre}$.

Because $\theta_{G,pre}$ = $\theta_{P,med}$ (as assumed in Section B2.1.1), then by equation (B3),

$$\begin{aligned}\theta_G &= p_{HE,G}\theta_P + \left(1-p_{HE,G}\right)\left(p_{inst}\theta_{G,pol}+\theta_{P,med}\right) \\ &= \left(\hat{c} + \frac{1-\hat{c}}{1+e^{-\hat{a}\left(FI_{P,med}-\hat{b}\right)}}\right)\theta_P + \left[1-\left(\hat{c} + \frac{1-\hat{c}}{1+e^{-\hat{a}\left(FI_{P,med}-\hat{b}\right)}}\right)\right]\left(p_{inst}\theta_{G,pol}+\theta_{P,med}\right).\end{aligned} \tag{B4}$$

In most democratic countries, the probability of the occurrence of extreme political instability is very low. For those countries, therefore, the model is reduced to a simpler form by assuming p_{inst} = 0 such that

$$\theta_G = p_{HE,G}\theta_P + (1 - p_{HE,G})\theta_{P,med}$$
$$= \left(\hat{c} + \frac{1-\hat{c}}{1+e^{-\hat{a}(FI_{P,med}-\hat{b})}}\right)\theta_P + \left[1 - \left(\hat{c} + \frac{1-\hat{c}}{1+e^{-\tilde{a}(FI_{P,med}-\hat{b})}}\right)\right]\theta_{P,med} \,. \qquad \text{(B5)}$$

Note that if the people and the government have sufficiently strong fluid intelligences, an independent central bank may not be necessary. However, p_{HE} will not be unity even in a country whose people have the highest p_{HE} in the world. Therefore, it is possible for $\theta_G > \theta_P$ in some period in any country; thus, an independent central bank is still important for all countries.

B2.3. Generating an expected θ_G by using heuristics

B2.3.1. Difficulty in expecting θ_G

Specifying the functional form of the structural model of θ_G is only half of the problem of generating an expected θ_G. Although we have the functional form of the model, as shown in equation (B5), we still cannot generate an expected θ_G unless we specify appropriate values of the parameters $\hat{a}$, $\hat{b}$ and $\hat{c}$. Furthermore, to generate the expected θ_G, we must also know the expected values of θ_P, $\theta_{P,med}$, and $FI_{P,med}$.

We may roughly specify the parameter values of $\hat{a}$, $\hat{b}$ and $\hat{c}$ through the results of some type of social experiment, or we may use the estimates derived from other kinds of model concerning fluid intelligence. By substituting these values for the parameter values in the structural model of θ_G, the model could be calibrated. However, expectations based on these estimates will most likely be rather inaccurate and therefore problematic in terms of decision-making on future actions.

A far more serious problem is obtaining the expected future values of θ_P, $\theta_{P,med}$, and $FI_{P,med}$. It is not certain whether the values of θ_P and $\theta_{P,med}$ are constant across time; in fact, many researchers have posited that it is much more likely that they are temporally variable (*e.g.,* Uzawa 1968, Epstein and Hynes 1983, Lucas and Stokey 1984, Parkin 1988, Obstfeld 1990, Becker and Mulligan 1997). Therefore, there is no guarantee that the future values of θ_P and $\theta_{P,med}$ will equal past ones, so the past values cannot be used as substitutes for the expected future values of θ_P and $\theta_{P,med}$. Hence, to generate the expected future values of θ_P and $\theta_{P,med}$, we have to calculate them on the basis of structural models of θ_P and $\theta_{P,med}$. Even if we knew the functional forms of these structural models, we would still need to determine the parameter values for the models. To determine them, however, we would need to obtain a sufficiently large amount of data on the past values of θ_P and $\theta_{P,med}$—that is, the intrinsic RTPs of the representative household and the median voter. Although a household knows its own RTP, it cannot directly observe θ_P and $\theta_{P,med}$ in the same way that it can obtain data on aggregate consumption, investment, production, inflation, trade, and other indicators at relatively low cost. Without data on the past values of θ_P and $\theta_{P,med}$, we cannot estimate the parameter values. Therefore, we cannot generate the expected future values of θ_P and $\theta_{P,med}$ on the basis of their structural models.

Past data on the real interest rate may be used as a substitute for past θ_P because θ_P is basically equal to the real interest rate at steady state (Fisher 1930). However, during a transition period after θ_P changes, θ_P is not equal to the rate of real interest. Therefore, unless θ_P is constant across time, this substitution does not seem to be sufficiently useful. In addition, if $\theta_{P,med}$ is constant across time, we may approximate the value of $\theta_{P,med}$ on the basis of historical economic and political (election) data. However, as stated in the previous paragraph, it is not known whether θ_P and $\theta_{P,med}$ are constant across time.

Note that, if we assume that RTP is identical for all households, an expected θ_P and $\theta_{P,med}$ are no longer necessary because the RTP of any household is equal to both θ_P and $\theta_{P,med}$. This assumption is very problematic, however, because it is not merely expedient for the sake of simplicity. It is also a critical requirement to eliminate the need for generating an expected θ_P and $\theta_{P,med}$. Therefore, any rationale for assuming identical RTPs should be validated; that is, it should be demonstrated that identical RTPs do exist and are universally observed. In any case, RTP is unquestionably not identical among households. Therefore, households must generate the expected values of θ_P and $\theta_{P,med}$.

B2.3.2. Expectations based on beliefs

Faced with the difficulty of generating expected values of θ_P and $\theta_{P,med}$ and knowing the parameter values in the model of θ_G, households may have to use the concept of bounded rationality to make decisions. One of a few alternatives available for a household to use is its "beliefs" in θ_P and $\theta_{P,med}$ as well as in $\hat{a}$, $\hat{b}$, $\hat{c}$, and $FI_{P,med}$. The use of beliefs does not mean that households deviate from rationality; rather, it is the most rational option in an environment where insufficient information is available.

Belief is merely that, however—belief. There is no guarantee that the value a household believes to be true is actually the correct value. Therefore, it may often change, but it will be changed only if forward-looking information becomes available. In some cases, a household will change its belief when new data are obtained, but in other cases the household will not, depending on how it interprets the new information. This is particularly true when the household believes that it has extracted forward-looking information about θ_P and $\theta_{P,med}$ from the newly obtained data.

B2.3.3. Heuristics

When households interpret the information extracted from new data, they may use heuristic methods such as a simplified linear reduced form model of θ_G. Studies of the use of heuristics and bounded rationality in this context would be useful for better understanding the interpretation mechanism. Heuristic methods will be implemented through the use of fluid intelligence. Hence, the value of $FI_{P,med}$ will also be important in improving the accuracy of expectations generated on the basis of heuristics.

There may be many possible simplified linear reduced form models of θ_G that could be used as heuristic methods, although most of them may be *ad hoc*. Even though such reduced form models are far less credible than a structural model, they may be utilized as a heuristic method of interpretation. Although simplified linear reduced form models may often result in misleading conclusions, they may sometimes provide useful information.

B2.4. Independent central bank

B2.4.1. Generating expected $\theta_G - \theta_P$ through the actions of a central bank

A heuristic way of generating an expected θ_G is to use information about $\theta_G - \theta_P$. The model of inflation acceleration presented in Section 2 indicates that inflation acceleration and deceleration are governed by the value of $\theta_G - \theta_P$. Therefore, what people really need to know is not the expected θ_G but the expected $\theta_G - \theta_P$. If the central bank is sufficiently independent, θ_G is determined by the central bank. In this case, people do not need to know the RTP of the government, but rather the responses of the central bank to $\theta_G - \theta_P$. If an easy method exists to know the response of the central bank to $\theta_G - \theta_P$, households will not have to generate expected θ_G; they need only observe the decisions of the central bank.

Of course, people cannot directly observe the value of $\theta_G - \theta_P$, but they can observe the response of the central bank to $\theta_G - \theta_P$. An independent central bank will raise interest rates if it judges that $\theta_G - \theta_P > 0$. Households can then adjust their expectations accordingly.

B2.4.2. Guaranteed $\theta_G = \theta_P$

If the central bank is sufficiently independent and capable, and successfully controls θ_G, then it is not even necessary for households to generate an expected value for $\theta_G - \theta_P$ because, in this case, θ_G will also equal θ_P. As discussed in Section B1.6, if the central bank is sufficiently independent and capable, then $\hat{c} = 1$ in equation (B2) and thereby, by equations (B4) a nd (B5), $\theta_G = \theta_P$. The central bank ensures that rationality always prevails over preference with regard to the RTP of government. If the independence of the central bank is very credible, households will always expect that $\theta_G = \theta_P$ at all times in the future.

Appendix C

C1. The representative household in dynamic models

C1.1. The assumption of the representative household

The concept of the representative household is a necessity in macroeconomic studies. It is used as a matter of course, but its theoretical foundation is fragile. The representative household has been used given the assumption that all households are identical or that there exists one specific individual household, the actions of which are always average among households (I call such a household "the average household" in Appendix C). The assumption that all households are identical seems to be too strict; therefore, it is usually assumed explicitly or implicitly that the representative household is the average household. However, the average household can exist only under very strict conditions. Antonelli (1886) showed that the existence of an average household requires that all households have homothetic and homogeneous utility functions. This type of utility function is not usually assumed in macroeconomic studies because it is very restrictive and unrealistic. If more general utility functions are assumed, however, the assumption of the representative household as the average household is inconsistent with the assumptions underlying the utility functions.

Nevertheless, the assumption of the representative household has been widely used, probably because it has been believed that the representative household can be interpreted as an approximation of the average household. Particularly in static models, the representative household can be seen to approximate the average household. However, in dynamic models, it is hard to accept the representative household as an approximation of the average household because, if RTPs of households are heterogeneous, there is no steady state where all of the optimality conditions of the heterogeneous households are satisfied (Becker 1980). Therefore, macroeconomic studies using dynamic models are fallacious if the representative household is assumed to approximate the average household.

C1.2. The representative household in static models

Static models are usually used to analyze comparative statics. If the average household is represented by one specific unique household for any static state, there will be no problem in assuming the representative household as an approximation of the average household. Even though the average household is not always represented by one specific unique household in some states, if the average household is always represented by a household in a set of households that are very similar in preferences and other features, then the representative household assumption can be used to approximate the average household.

Suppose, for simplicity, that households are heterogeneous such that they are identical except for a particular preference. Because of the heterogeneous preference, household consumption varies. However, levels of consumption will not be distributed randomly because the distribution of consumption will correspond to the distribution of the preference. The consumption of a household that has a very different preference from the average will be very different from the average household consumption. Conversely, it is likely that the consumption of a household that has the average preference will nearly have the average consumption. In addition, the order of the degree of consumption will be almost unchanged for any static state because the order of the degree of the preference does not change for the given state.

If the order of consumption is unchanged for any given static state, it is likely that the household with consumption that is closest to the average consumption will also always be a household belonging to a group of households that have very similar preferences. Hence, it is possible to argue that, approximately, one specific unique household's consumption is always average for any static state. Of course, it is possible to show evidence that is counter to this argument, particularly in some special situations, but it is likely that this conjecture is usually true in normal situations, and the assumption that the representative household approximates the average household is acceptable in static models.

C1.3. The representative household in dynamic models

In dynamic models, however, the story is more complicated. In particular, heterogeneous RTPs pose a serious problem. This problem is easily understood in a dynamic model with exogenous technology (*i.e.,* a Ramsey growth model). Suppose that households are heterogeneous in RTP, degree of risk aversion (ε), and productivity of the labor they provide. Suppose also for simplicity that there are many "economies" in a country, and an economy consists of a household and a firm. The household provides labor to the firm in the particular economy, and the firm's level of technology (A) varies depending on the productivity of labor that the household in its economy provides. Economies trade with each other: that is, the entire economy of a country consists of many individual small economies that trade with each other.

A household maximizes its expected utility, $E\int_0^{\infty} u(c_t)\exp(-\theta t)dt$, subject to $\dot{k}_t = f(k_t) - c_t$, where $u(\bullet)$ is the utility function; $f(\bullet)$ is the production function; θ is RTP; E is the expectation operator; $y_t = \frac{Y_t}{L_t}$, $k_t = \frac{K_t}{L_t}$, and $c_t = \frac{C_t}{L_t}$; Y_t (≥ 0) is output, K_t (≥ 0) is capital input, L_t (≥ 0) is labor input, and C_t (≥ 0) is consumption in period t. The optimal consumption path of this Ramsey-type growth model is

$$\frac{\dot{c}_t}{c_t} = \varepsilon^{-1}\left(\frac{\partial y_t}{\partial k_t} - \theta\right),$$

and at steady state,

$$\frac{\partial y_t}{\partial k_t} = \theta \ . \qquad \text{(C1)}$$

Therefore, at steady state, the heterogeneity in the degree of risk aversion (ε) is irrelevant, and the heterogeneity in productivity does not result in permanent trade imbalances among economies because $\frac{\partial y_t}{\partial k_t}$ in all economies is kept equal by market arbitrage. Hence, heterogeneity in the degree of risk aversion and productivity does not matter at steady state. Therefore, the same logic as that used for static models can be applied. Approximately, one specific unique household's consumption is always average for any time in dynamic models, even if the degree of risk aversion and the productivity are heterogeneous. Thus, the assumption of the representative household is also acceptable in dynamic models even if the degree of risk aversion and the productivity are heterogeneous.

However, equation (C1) clearly indicates that heterogeneity in RTP is problematic. As Becker (1980) shows, if RTP is heterogeneous, the household that has the lowest RTP will eventually possess all capital. With heterogeneous RTPs, there is no steady state where all households achieve all of their optimality conditions. In addition, the household with consumption that is average at present has a very different RTP from the household with consumption that is average in the distant future. The consumption of a household that has the average RTP will initially be almost average, but in the future the household with the lowest RTP will be the one with consumption that is almost average. That is, the consumption path of the household that presently has average consumption is notably different from that of the household with average consumption in the future. Therefore, any individual household cannot be almost average in any period and thus cannot even approximate the average household. As a result, even if the representative household is assumed in a dynamic model, its discounted expected utility $E\int_0^{\infty} u(c_t)\exp(-\theta t)dt$ is meaningless, and analyses based on it are fallacious.

If we assume that RTP is identical for all households, the above problem is solved. However, this solution is still problematic because that assumption is not merely expedient for the sake of simplicity; rather, it is a critical requirement to allow for an assumed representative household. Therefore, the rationale for identical RTPs should be validated; that is, it should be demonstrated that identical RTPs are actually and universally observed. RTP is, however, unquestionably not identical among households. Hence, it is difficult to accept the representative household assumption in dynamic models based on the assumption of identical RTP.

The conclusion that the representative household assumption in dynamic models is meaningless and leads to fallacious results is very important, because a huge number of studies have used the representative household assumption in dynamic models. To solve this severe problem, an alternative interpretation or definition of the representative household is needed.

Note that in an endogenous growth model the situation is even more complicated. Because a heterogeneous degree of risk aversion also matters, the assumption of the representative household is more difficult to accept, so an alternative interpretation or definition is even more important when endogenous growth models are used.

C2. Sustainable heterogeneity

C2.1. The model

Suppose that two heterogeneous economies—economy 1 and economy 2—are identical except for their RTPs. Households within each economy are assumed to be identical for simplicity. The population growth rate is zero. The economies are fully open to each other, and goods, services, and capital are freely transacted between them, but labor is immobilized in each economy.

Each economy can be interpreted as representing either a country (the international interpretation) or a group of identical households in a country (the national interpretation). Because the economies are fully open, they are integrated through trade and form a combined economy. The combined economy is the world economy in the international interpretation and the national economy in the national interpretation. In the following discussion, a model based on the international interpretation is called an international model and that based on the national interpretation is called a national model. Usually, the concept of the balance of payments is used only for the international transactions. However, because both national and international interpretations are possible, this concept and terminology are also used for the national models in Appendix C.

RTP of household in economy 1 is θ_1 and that in economy 2 is θ_2, and $\theta_1 < \theta_2$. The production function in economy 1 is $y_{1,t} = A^{\alpha} f(k_{1,t})$ and that in economy 2 is $y_{2,t} = A^{\alpha} f(k_{2,t})$, where $y_{i,t}$ and $k_{i,t}$ are, respectively, output and capital per capita in economy i in period t for i = 1, 2; A is technology; and α $(0 < \alpha < 1)$ is a constant. The population of each economy is $\frac{L}{2}$; thus, the total for both is L, which is sufficiently large. Firms operate in both economies. The current account balance in economy 1 is τ_t and that in economy 2 is $-\tau_t$. The production functions are specified as

$$y_{i,t} = A^{\alpha} k_{i,t}^{1-\alpha} ;$$

thus, $Y_{i,t} = K_{i,t}^{1-\alpha} (AL)^{\alpha}$ $(i = 1,2)$. Because A is given exogenously, this model is an exogenous technology model (Ramsey growth model). The examination of sustainable heterogeneity based on an endogenous growth model is shown in Harashima (2014a).

Because both economies are fully open, returns on investments in each economy are kept equal through arbitration, such that

$$\frac{\partial y_{1,t}}{\partial k_{1,t}} = \frac{\partial y_{2,t}}{\partial k_{2,t}} . \tag{C2}$$

Because equation (C2) always holds through arbitration, equations $k_{1,t} = k_{2,t}$, $\dot{k}_{1,t} = \dot{k}_{2,t}$, $y_{1,t} = y_{2,t}$, and $\dot{y}_{1,t} = \dot{y}_{2,t}$ also hold.

The accumulated current account balance $\int_0^t \tau_s ds$ mirrors capital flows between the two economies. The economy with current account surpluses invests them in the other economy. Because $\frac{\partial y_{1,t}}{\partial k_{1,t}} \left(= \frac{\partial y_{2,t}}{\partial k_{2,t}} \right)$ are returns on investments, $\frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds$ and $\frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$ represent income receipts or payments on the assets that an economy owns in the other economy. Hence,

$$\tau_t - \frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$$

is the balance on goods and services of economy 1, and

$$\frac{\partial y_{1,t}}{\partial k_{1,t}}\int_0^t \tau_s ds - \tau_t$$

is that of economy 2. Because the current account balance mirrors capital flows between the economies, the balance is a function of capital in both economies, such that

$$\tau_t = \kappa(k_{1,t}, k_{2,t}) .$$

The government (or an international supranational organization) intervenes in the activities of economies 1 and 2 by transferring money from economy 1 to economy 2. The amount of transfer in period *t* is g_t, and it is assumed that g_t depends on capital inputs, such that

$$g_t = \bar{g}k_{1,t} ,$$

where $\bar{g}$ is a constant. Because $k_{1,t} = k_{2,t}$ and $\dot{k}_{1,t} = \dot{k}_{2,t}$,

$$g_t = \bar{g}k_{1,t} = \bar{g}k_{2,t} .$$

Each household in economy 1 therefore maximizes its expected utility

$$E\int_0^\infty u_1(c_{1,t})\exp(-\theta_1 t)dt ,$$

subject to

$$\dot{k}_{1,t} = A^\alpha k_{1,t}^{1-\alpha} - c_{1,t} + (1-\alpha)A^\alpha k_{1,t}^{-\alpha}\int_0^t \tau_s ds - \tau_t - \bar{g}k_{1,t} , \tag{C3}$$

and each household in economy 2 maximizes its expected utility

$$E\int_0^\infty u_2(c_{2,t})\exp(-\theta_2 t)dt ,$$

subject to

$$\dot{k}_{2,t} = A^\alpha k_{2,t}^{1-\alpha} - c_{2,t} - (1-\alpha)A^\alpha k_{2,t}^{-\alpha}\int_0^t \tau_s ds + \tau_t + \bar{g}k_{2,t} , \tag{C4}$$

where $u_{i,t}$ and $c_{i,t}$, respectively, are the utility function and per capita consumption in economy *i* in period *t* for *i* = 1, 2; and *E* is the expectation operator. Equations (C3) and (C4) implicitly assume that each economy does not have foreign assets or debt in period *t* = 0.

C2.2. Sustainable heterogeneity without government intervention

Heterogeneity is defined as being sustainable if all of the optimality conditions of all heterogeneous households are satisfied indefinitely. First, the natures of the model when the government does not intervene (*i.e.*, $\bar{g} = 0$) are examined. The growth rate of consumption in economy 1 is

$$\frac{\dot{c}_{1,t}}{c_{1,t}} = \varepsilon^{-1}\left\{(1-\alpha)A^\alpha k_{1,t}^{-\alpha} + (1-\alpha)A^\alpha k_{1,t}^{-\alpha}\frac{\partial\int_0^t \tau_s ds}{\partial k_{1,t}} - \alpha(1-\alpha)A^\alpha k_{1,t}^{-\alpha-1}\int_0^t \tau_s ds - \frac{\partial \tau_t}{\partial k_{1,t}} - \theta_1\right\} .$$

Hence,

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\varepsilon^{-1}\lim_{t\to\infty}\left\{(1-\alpha)A^{\alpha}k_{1,t}^{-\alpha}+(1-\alpha)A^{\alpha}k_{1,t}^{-\alpha}\frac{\partial\int_0^t\tau_s ds}{\partial k_{1,t}}-\alpha(1-\alpha)A^{\alpha}k_{1,t}^{-\alpha-1}\int_0^t\tau_s ds-\frac{\partial\tau_t}{\partial k_{1,t}}-\theta_1\right\}=0$$

and thereby

$$\lim_{t\to\infty}(1-\alpha)A^{\alpha}k_{1,t}^{-\alpha}\left[1+(1-\alpha)\Psi\right]-\Xi-\theta_1=0,$$

where $\Xi=\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}}=\lim_{t\to\infty}\frac{\tau_t}{k_{2,t}}$ and $\Psi=\lim_{t\to\infty}\frac{\int_0^t\tau_s ds}{k_{1,t}}=\lim_{t\to\infty}\frac{\int_0^t\tau_s ds}{k_{2,t}}$. $\lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}=\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=0$, and Ψ is constant at steady state because $k_{1,t}$ and τ_t are constant; thus, $\Xi=\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}}$ is constant at steady state. For Ψ to be constant at steady state, it is necessary that $\lim_{t\to\infty}\tau_t=0$ and thus $\Xi=0$. Therefore,

$$\lim_{t\to\infty}(1-\alpha)A^{\alpha}k_{1,t}^{-\alpha}\left[1+(1-\alpha)\Psi\right]-\theta_1=0, \qquad \text{(C5)}$$

and

$$\lim_{t\to\infty}(1-\alpha)A^{\alpha}k_{2,t}^{-\alpha}\left[1-(1-\alpha)\Psi\right]-\theta_2=0$$

because

$$\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\varepsilon^{-1}\lim_{t\to\infty}\left\{(1-\alpha)A^{\alpha}k_{2,t}^{-\alpha}-(1-\alpha)A^{\alpha}k_{2,t}^{-\alpha}\frac{\partial\int_0^t\tau_s ds}{\partial k_{2,t}}+\alpha(1-\alpha)A^{\alpha}k_{2,t}^{-\alpha-1}\int_0^t\tau_s ds+\frac{\partial\tau_t}{\partial k_{2,t}}-\theta_2\right\}=0$$

Because $\lim_{t\to\infty}(1-\alpha)A^{\alpha}k_{1,t}^{-\alpha}\left[1+(1-\alpha)\Psi\right]=\theta_1$, $\lim_{t\to\infty}(1-\alpha)A^{\alpha}k_{2,t}^{-\alpha}\left[1-(1-\alpha)\Psi\right]=\theta_2$, and $\frac{\partial y_{1,t}}{\partial k_{1.t}}=\frac{\partial y_{2,t}}{\partial k_{2.t}}=A^{\alpha}k_{1,t}^{-\alpha}=A^{\alpha}k_{2,t}^{-\alpha}$, then

$$\Psi=\frac{\theta_1-\theta_2}{2(1-\alpha)\lim_{t\to\infty}\frac{\partial y_{1,t}}{\partial k_{1.t}}}. \qquad \text{(C6)}$$

By equations (C5) and (C6),

$$\lim_{t\to\infty}\frac{\partial y_{1,t}}{\partial k_{1,t}}+\lim_{t\to\infty}\frac{\partial y_{1,t}}{\partial k_{1,t}}(1-\alpha)\Psi=\theta_1;$$

thus,

$$\lim_{t\to\infty}\frac{\partial y_{1,t}}{\partial k_{1,t}}=\frac{\theta_1+\theta_2}{2}=\lim_{t\to\infty}\frac{\partial y_{2,t}}{\partial k_{2,t}}\ . \tag{C7}$$

If equation (C7) holds, all of the optimality conditions of both economies are indefinitely satisfied. The state indicated by equation (C7) is called the "multilateral steady state" or "multilateral state" in the following discussion. By procedures similar to those used for the endogenous growth model in Harashima (2014a), the condition of the multilateral steady state for *H* economies that are identical except for their RTPs is shown as

$$\lim_{t\to\infty}\frac{\partial y_{i,t}}{\partial k_{i,t}}=\frac{\sum_{q=1}^{H}\theta_q}{H} \tag{C8}$$

for any *i*, where *i* = 1, 2, … , *H*.

Because

$$\Psi=\frac{\theta_1-\theta_2}{2(1-\alpha)\lim_{t\to\infty}\frac{\partial y_{1,t}}{\partial k_{1,t}}}=\frac{\theta_1-\theta_2}{(1-\alpha)(\theta_1+\theta_2)}<0$$

by equation (C7), then by $\lim_{t\to\infty}\frac{\int_0^t\tau_s ds}{k_{1,t}}=\Psi<0$,

$$\lim_{t\to\infty}\int_0^t\tau_s ds<0\ ;$$

that is, economy 1 possesses accumulated debts owed to economy 2 at steady state, and economy 1 has to export goods and services to economy 2 by

$$\left|(1-\alpha)A^{\alpha}k_{1,t}^{-\alpha}\int_0^t\tau_s ds\right|$$

in every period to pay the debts. Nevertheless, because $\lim_{t\to\infty}\tau_t=0$ and $\Xi=0$, the debts do not explode but stabilize at steady state. Because of the debts, the consumption of economy 1 is smaller than that of economy 2 at steady state under the condition of sustainable heterogeneity.

Note that many empirical studies conclude that RTP is negatively correlated with income (*e.g.,* Lawrance 1991, Samwick 1998, Ventura 2003). Suppose that, in addition to the heterogeneity in RTP ($\theta_1<\theta_2$), the productivity of economy 1 is higher than that of economy 2. At steady state, the consumption of economy 1 would be larger than that of economy 2 as a result of the heterogeneity in productivity. However, as a result of the heterogeneity in RTP, the consumption of economy 1 is smaller than that of economy 2 at steady state under sustainable heterogeneity. Which effect prevails will depend on differences in the degrees of heterogeneity. For example, if the difference in productivity is relatively large whereas that in RTP is relatively small, the effect of the productivity difference will prevail and the consumption of economy 1 will be larger than that of economy 2 at steady state under sustainable heterogeneity.

C3. An alternative definition of the representative household

C3.1. The definition

Section C2 indicates that, when sustainable heterogeneity is achieved, all heterogeneous households are connected (in the sense that all households behave by considering other households' optimality) and appear to be behaving collectively as a combined supra-household that unites all households, as equations (C7) and (C8) indicate. The supra-household is unique and its behavior is time-consistent. Its actions always and consistently represent those of all households. Considering these natures of households under sustainable heterogeneity, I present the following alternative definition of the representative household: "the behavior of the representative household is defined as the collective behavior of all households under sustainable heterogeneity."

Even if households are heterogeneous, they can be represented by a representative household as defined above. Unlike the representative household defined as the average household, the collective representative household reaches a steady state where all households satisfy all of their optimality conditions in dynamic models. In addition, this representative household has a RTP that is equal to the average RTP as shown in equations (C7) and (C8).[9] Hence, we can assume not only a representative household but also that its RTP is the average rate of all households.

C3.2. Universality of sustainable heterogeneity

An important point, however, is that this alternatively defined representative household can be used in dynamic models only if sustainable heterogeneity is achieved, but this condition is not necessarily always naturally satisfied. Sustainable heterogeneity is achieved only if households with lower RTPs behave multilaterally or the government appropriately intervenes. Therefore, the representative household assumption is not necessarily naturally acceptable in dynamic models unless it is confirmed that sustainable heterogeneity is usually achieved in an economy.

Notwithstanding this flaw, the representative household assumption has been widely used in many macroeconomic studies that use dynamic models. Furthermore, these studies have been little criticized for using the inappropriate representative household assumption. In addition, in most economies, the dire state that Becker (1980) predicts has not been observed even though RTPs of households are unquestionably heterogeneous. These facts conversely indicate that sustainable heterogeneity—probably with government interventions—has been usually and universally achieved across economies and time periods. In a sense, these facts are indirect evidence that sustainable heterogeneity usually prevails in economies.

Note that because the representative household's behavior in dynamic models is represented by the collective behavior of all households under sustainable heterogeneity, RH's RTP is not intrinsically known to households, but they do need to have an expected rate. Each household intrinsically knows its own preferences, but it does not intrinsically know the collective preference of all households. Therefore, in dynamic models, it must be assumed that all households do not *ex ante* know RH's RTP, but households estimate it from information on the behaviors of other households and the government.

C4. Need for an expected RTP RH

C4.1. The behavior of household

Achieving sustainable heterogeneity affects the behavior of the individual household because sustainable heterogeneity indicates that each household must consider the other households' optimality (as well as the behavior of the government, if necessary). This feature does not mean that households behave cooperatively with other households. Each household behaves non-cooperatively based on its own RTP, but at the same time, it behaves considering whether the other households' optimality conditions are achieved or not. This consideration affects the actions a household takes in that it affects the choice of a household's initial consumption.

Sustainable heterogeneity indicates that a household's future path of consumption has to be consistent with the future path of sustainable heterogeneity. Thereby, a household sets its initial consumption such that it will proceed on the path that is consistent with the path of sustainable heterogeneity and eventually reach a steady state.

C4.2. Deviation from sustainable heterogeneity

C4.2.1. Political elements

What happens if a household deviates from sustainable heterogeneity? A deviation means that a household sets its initial consumption at a level that is not consistent with sustainable heterogeneity. For less advantaged households (*i.e.,* households with higher RTPs), the only way to satisfy all of their optimality conditions is to set their initial consumption consistent with sustainable heterogeneity. Therefore, they will not take the initiative to deviate. In contrast, the most advantaged households (*i.e.,* those with the lowest RTP) can

[9] If sustainable heterogeneity is achieved with the help of the government's intervention, the time preference rate of the representative household will not be exactly equal to the average rate of time preference.

satisfy all of their optimality conditions even if they set initial consumption independent of sustainable heterogeneity. The incentive for the most advantaged household to select a multilateral path will be weak because the growth rate of the most advantaged household on the multilateral path is lower than that on the unilateral path.

When economy 1 selects the unilateral path, does economy 2 quietly accept the unfavorable consequences shown in Becker (1980)? From an economic perspective, the optimal response of economy 2 is the one shown in Harashima (2010): economy 2 should behave as a follower and accept the unfavorable consequences. However, if other factors—particularly political ones—are taken into account, the response of economy 2 will be different. Faced with a situation in which all the optimality conditions cannot be satisfied, it is highly likely that economy 2 would politically protest and resist economy 1. It should be emphasized economy 2 is not responsible for its own non-optimality, which is a result of economy 1's unilateral behavior in a heterogeneous population. Economy 2 may overlook the non-optimality if it is temporary, but it will not if it is permanent. As shown in Harashima (2010), the non-optimality is permanent, it is quite likely that economy 2 will seriously resist economy 1 politically.

If economy 1 could achieve its optimality only on the unilateral path, economy 1 would counter the resistance of economy 2, but this is not the case. Because of this, economy 2's demand does not necessarily appear to be unreasonable or selfish. Faced with the protest and resistance by economy 2, economy 1 may compromise or cooperate with economy 2 and select the multilateral path.

C4.2.2. Resistance

The main objective of economy 2 is to force economy 1 to select the multilateral path and to establish sustainable heterogeneity. This objective may be achieved through cooperative measures, non-violent civil disobedience (*e.g.,* trade restrictions), or other more violent means.

Restricting or abolishing trade between the two economies will cost economy 1 because it necessitates a restructuring of the division of labor, and the restructuring will not be confined to a small scale. Large-scale adjustments will develop that involve all levels of divided labor, because they are all correlated with each other. For example, if an important industry had previously existed only in one economy, owing to a division of labor, and trade between the two economies was no longer permitted, the other economy would have to establish this industry while also maintaining other industries. As a result, economy 1 would incur non-negligible costs. More developed economies have more complicated and sophisticated divisions of labor, and restructuring costs from the disruption of trade will be much higher in developed economies. In addition, more resources will need to be allocated to the generation of technology because technology will also no longer be traded. Finally, all of the conventional benefits of trade will be lost. Trade is beneficial because of the heterogeneous endowment of resources, as the Heckscher-Ohlin theorem shows. Because goods and services are assumed to be uniform in the models presented in Appendix C, the benefits of trade are implicit in the models. However, in the real word, resources such as oil and other raw materials are unevenly distributed, so a disruption or restriction of trade will substantially damage economic activities on both national and international levels.

The damage done by trade restrictions has an upper limit, however, because the restructuring of the division of labor, additional resource allocation to innovation, and loss of trade benefits are all finite. Therefore, in some cases, particularly if economies are not sufficiently developed and division of labor is not complex, the damage caused will be relatively small. Hence, a disruption of trade (non-violent civil disobedience in the national models) may not be sufficiently effective as a means of resistance under some these conditions.

In some cases, harassment, sabotage, intimidation, and violence may be used, whether legal or illegal. In extreme cases, war or revolution could ensue. In such cases, economy 1 will be substantially damaged in many ways and be unable to achieve optimality. The resistance and resulting damages will continue until sustainability is established.

In any case, the objective of economy 2's resistance conversely implies that establishing sustainability eliminates the risk and cost of political and social instability. The resistance of economy 2 will lower the desire of economy 1 to select the unilateral path.

C4.2.3. United economies

An important countermeasure to the fragility of sustainable heterogeneity for less advantaged economies is the formation of a union of economies. If economies other than economy 1 are united by commonly selecting the multilateral path within them, their power to resist economy 1 will be substantially enhanced. Consider the multi-economy model shown in Harashima (2010). If the economies do not form a union, the power to resist the unilateral actions of economy 1 is divided and limited to the power of each individual economy. However, if the economies are united, the power to resist economy 1 increases. If a sufficient number of economies unite, the multilateral path will almost certainly be selected by economy 1.

To maintain the union, any economy in the union should have the explicit and resolved intention of selecting the multilateral path within the union, even if it is relatively more advantaged within the union. To demand that relatively more advantaged economies select the multilateral path, less advantaged economies themselves must also select the multilateral path in any case. Otherwise, less advantaged economies will be divided and ruled by more advantaged economies. For all heterogeneous people to happily coexist, all of them should behave multilaterally. At the same time, Harashima (2010) indicates that the more advantaged an economy is, the more modestly it should behave, *i.e.,* the more it should restrain itself from accumulating extra capitals.

In general, therefore, the most advantaged (the lowest RTP) household will be forced to set its initial consumption consistent with sustainable heterogeneity.

C4.3. Need for an expected RTP RH

Because all households need to set their initial consumption consistent with sustainable heterogeneity to achieve it, households must calculate the path of sustainable heterogeneity before setting their initial consumption levels. To calculate this level, each household first must know the value of RTP RH. However, although a household naturally knows the value of its own RTP, it does not intrinsically know the value of RTP RH. To know this, a household would have to know the values of all of the other households' RTPs. Hence, the expected value of RTP RH must somehow be generated utilizing all other relevant available information. The necessity of an expected RTP RH is critically important because RTP plays a crucial role as the discount factor in dynamic models.

Note that, if we assume that RTP is identical for all households, an expected RTP RH is no longer needed because any household's own RTP is equal to the RTP RH. This solution is still problematic, however, because the assumption is not merely expedient for the sake of simplicity; rather, it is a critical requirement to eliminate the need for an expected RTP RH. Therefore, any rationale for assuming identical RTPs should be validated; that is, it should be demonstrated that identical RTPs do exist and are universally observed. However, RTP is unquestionably not identical among households. Therefore, households must use expected values of RTP RH.

C5. The RTP model

C5.1. Need to know the structural model

If RTP RH is a constant parameter, as has been long and widely assumed, the need for an expected RTP RH would not be a serious problem. The historical mean of an unchanging RTP RH could be estimated relatively precisely based on long-term data of various economic indicators even if the structural model remained unknown. The RTP RH could be specified as the RTP that is most consistent with long-term trends of the indicators.

Although RTP has been treated as a constant parameter in many studies, this feature has not been demonstrated either empirically or theoretically. Rather, the assumption is merely expedient for the sake of simplicity. There is another practical reason for this treatment: models with a permanently constant RTP exhibit excellent tractability (see Samuelson 1937). However, some have argued that it is natural to view RTP as temporally variable, and the concept of a temporally varying RTP has a long history (*e.g.,* Böhm-Bawerk 1889; Fisher 1930). More recently, Lawrance (1991) and Becker and Mulligan (1997) showed that people do not inherit permanently constant RTPs by nature and that economic and social factors affect the formation of RTPs. Their arguments indicate that many incidents can affect and change RTP. Models of endogenous RTP have been presented, the most familiar of which is Uzawa's (1968) model.

If the RTP RH is temporally variable, its future stream must be expected by households, and a rational expectation is a model-consistent expectation. To generate rational expectations of RTP RH, therefore, the structural model of the RTP RH (*i.e.,* equations that fundamentally describe how it is endogenously formed) needs to be known.

C5.2. Endogenous RTP models

C5.2.1. Uzawa's (1968) model

The most well-known endogenous RTP model is that of Uzawa (1968). It has been applied in many analyses (*e.g.,* Epstein and Hynes 1983, Lucas and Stokey 1984, Epstein 1987, Obstfeld 1990). However, Uzawa's model has not necessarily been regarded as a realistic expression of the endogeneity of RTP because it has a serious drawback in that impatience increases as income, consumption, and utility increase. The basic structure of Uzawa's model is

$$\theta_t = \theta^*[u(c_t)],$$

$$0 < \frac{d\theta_t}{du(c_t)},$$

in which RTP in period t (θ_t) is temporally variable and an increasing function of present utility $u(c_t)$ where c_t is consumption in period t. The condition $0 < \frac{d\theta_t}{du(c_t)}$ is necessary for the model to be stable. This property is quite controversial and difficult to accept *a priori* because many empirical studies have indicated that RTP is negatively correlated with permanent income (*e.g.,* Lawrance 1991); thus, many economists are critical of Uzawa's model. Epstein (1987), however, discussed the plausibility of increasing impatience and offered some counter-arguments. However, his view is in the minority, and most economists support arguments in favor of a decreasing RTP, such that $\frac{d\theta_t}{du(c_t)} < 0$. Hence, although Uzawa's model attracted some attention, the analysis of the endogeneity of RTP has progressed very little. Although Uzawa's model may be flawed, it does not mean that the conjecture that RTP is influenced by future income, consumption, and utility is fallacious. Rather, it means that an appropriate model in which RTP is negatively correlated with income, consumption, and utility has not been presented.

C5.2.2. Size effect on impatience

The problem of $0 < \frac{d\theta_t}{du(c_t)}$ in Uzawa's model arises because distant future levels of consumption have little influence on factors that form RTP; that is, RTP is formed only with the information on present consumption, and it must be revised every period in accordance with consumption growth. However, there is no *a priori* reason why information on distant future activities should be far less important than the information on the present and near future activities. Fisher (1930) argued that

> Our first step, then, is to show how a person's impatience depends on the *size* of his income, assuming the other three conditions to remain constant; for, evidently, it is possible that two incomes may have the same time shape, composition and risk, and yet differ in size, one being, say, twice the other in every period of time.
>
> In general, it may be said that, other things being equal, the smaller the income, the higher the preference for the present over the future income. It is true of course that a permanently small income implies a keen appreciation of wants as well as of immediate wants. … But it increases the want for immediate income *even more* than it increases the want for future income. (p. 72)

According to Fisher's (1930) view, a force that influences RTP is a psychological response derived from the perception of the "size of the entire income or utility stream." This view indicates that it is necessary to probe how people perceive the size of the entire income or utility stream.

Little effort has been directed toward probing the nature of the size of the utility or income stream on RTP, although numerous psychological experiments have been performed with regard to the anomalies of the expected utility model with a constant RTP (*e.g.,* Frederick *et al.* 2002). Analyses using endogenous RTP models so far have merely introduced the *a priori* assumption of endogeneity of RTP without explaining the reasoning for doing so in detail. Hence, even now, Fisher's (1930) insights are very useful for the examination of the size effect. An important point in Fisher's quote is that the size of the infinite utility stream is perceived as "permanently" high or low. The size difference among the utility streams may be perceived as a permanently continuing difference of utilities among different utility streams. Anticipation of a permanently higher utility may enhance an emotional sense of well-being because people feel they are in a long-lasting secure situation, which will generate a positive psychological response and make people more patient. If that is true, distant future utilities should be taken into account equally with present utility. Otherwise, it is impossible to distinguish whether the difference of utilities will continue permanently.

From this point of view, the specification that only the present utility influences the formation of RTP, as is the case of Uzawa's model, is inadequate. Instead, a simple measure of the size where present and future utilities are summed with equal weight will be a more appropriate measure of the size of a utility stream.[10]

C5.3. Model of RTP[11]

C5.3.1. The model

The representative household solves the maximization problem as shown in Section C1.3. Taking the arguments in Section C5.2 into account, the "size" of the infinite utility stream can be defined as follows.

Definition 1: The size of the utility stream *W* for a given technology *A* is

$$W = \lim_{T \to \infty} E \int_0^T \rho(t) u(c_t) dt \ ,$$

where *E* is the expectation operator, and

$$\rho(t) = \frac{1}{T} \text{ if } 0 \leq t \leq T$$

$$\rho(t) = 0 \text{ otherwise.}$$

$\rho(t)$ indicates weights and has the same value in any period. Thus, the weights for the evaluation of future utilities are distributed evenly over time, as discussed in Section C5.2.

To this point, technology *A* has been assumed to be constant. If *A* is temporally variable (A_t) and grows at a constant rate and the economy is on a balanced growth path such that A_t, y_t, k_t, and c_t grow at the same rate, then the definition of *W* needs to be modified because any stream of c_t and $u(c_t)$ grows to infinity. It is then impossible to distinguish the sizes of the utility stream by simply summing up c_t as $T \to \infty$ as shown in Definition 1. Because balanced growth is possible only when technological progress is Harrod neutral, I assume a Harrod neutral production function such that

$$y_t = \omega A_t^{\varpi} k_t^{1-\varpi} \ ,$$

where $\varpi (0 < \varpi < 1)$ and $\omega (0 < \omega)$ are constants. To distinguish the sizes of utility stream, the following value is set as the standard stream of utility,

$$u(\tilde{c} e^{\psi t}) \ ,$$

where $\tilde{c} (0 < \tilde{c})$ is a constant and $\psi (0 < \psi)$ is a constant rate of growth. Streams of utility can be compared with this standard stream. If a constant relative risk aversion utility function is assumed, a stream of utility can be compared with the standard stream of utility as follows:

$$\frac{u(c_t)}{u(\tilde{c} e^{\psi t})} = \frac{c_t^{1-\gamma}}{(\tilde{c} e^{\psi t})^{1-\gamma}} = \frac{1-\gamma}{\tilde{c}^{1-\gamma}} u\left(\frac{c_t}{e^{\psi t}}\right) .$$

[10] Das (2003) showed another stable endogenous time preference model with decreasing impatience. Her model is stable, although the rate of time preference is decreasing because endogenous impatience is almost constant. In this sense, the situation her model describes is very special.

[11] The idea of this type of endogenous time preference model was originally presented in Harashima (2004a).

By using this ratio, a given stream of utility can be distinguished from the standard stream of utility. That is, the size of a utility stream *W* for a given stream of technology A_t that grows at the same rate ψ as y_t, k_t, and c_t can be alternatively defined as

$$W = \lim_{T\to\infty} E\int_0^T \rho(t) u\left(\frac{c_t}{e^{\psi t}}\right) dt \ .$$

Clearly, if ψ = 0, then the size (*W*) degenerates into the one shown in Definition 1.

If there is a steady state such that

$$\lim_{t\to\infty} E\left[u(c_t)\right] = E\left[u(c^*)\right],$$

or for the case of expected balanced growth,

$$\lim_{t\to\infty} E\left[u\left(\frac{c_t}{e^{\psi t}}\right)\right] = E\left[u(c^*)\right],$$

where c^* is a constant and indicates steady-state consumption, then

$$W = E\left[u(c^*)\right]$$

for the following reason. Because $\lim_{t\to\infty} E\left[u(c_t)\right] = E\left[u(c^*)\right]$(or $\lim_{t\to\infty} E\left[u\left(\frac{c_t}{e^{\psi t}}\right)\right] = E\left[u(c^*)\right]$), then

$$\lim_{T\to\infty} \int_0^T \rho(t)\left\{E\left[u(c^*)\right] - E\left[u(c_t)\right]\right\} dt = E\left[u(c^*)\right] - W$$

(or $\lim_{T\to\infty} \int_0^T \rho(t)\left\{E\left[u(c^*)\right] - E\left[u\left(\frac{c_t}{e^{\psi t}}\right)\right]\right\} dt = E\left[u(c^*)\right] - W$) .

In addition,

$$\lim_{T\to\infty} \int_0^T \rho(t)\left\{E\left[u(c^*)\right] - E\left[u(c_t)\right]\right\} dt = 0$$

(or $\lim_{T\to\infty} \int_0^T \rho(t)\left\{E\left[u(c^*)\right] - E\left[u\left(\frac{c_t}{e^{\psi t}}\right)\right]\right\} dt = 0$) .

Hence, $W = E\left[u(c^*)\right]$; that is, RTP is determined by steady-state consumption (c^*).

The RTP model presented in Appendix C is constructed on the basis of this measure of *W*. An essential property that must be incorporated into the model is that RTP is sensitive to, and a function of, *W* such that

$$\theta = \theta^{**}(W),$$

where $\theta^{**}(W)$ is monotonically continuous and continuously differentiable. Because *W* is a sum of utilities, this property simply reflects the core idea of an endogenous RTP. However, this property is new in the sense that RTP is sensitive not only to the present utility but also to the entire stream of utility, that is, the size of the utility stream represented by the utility of steady-state consumption. This property is intuitively acceptable because it is likely that people set their principles or parameters for their behaviors considering the final consequences of their behavior (*i.e.*, the steady state; see, *e.g.,* Barsky and Sims 2012).

Another essential property that must be incorporated into the model is

$$\frac{d\theta}{dW} < 0 .$$

Because $W = E[u(c^*)]$ and $0 < \frac{du(c_t)}{dc_t}$, RTP is inversely proportionate to *c**. This property is consistent with the findings in many empirical studies, which have shown that RTP is negatively correlated with permanent income (*e.g.,* Lawrance 1991).

In summary, the basic structure of the model is:

$$\theta = \theta^{**}(W) = \theta^{**}\{E[u(c^*)]\} ,$$
$$\frac{d\theta}{dW} = \frac{d\theta}{dE[u(c^*)]} < 0 . \quad \text{(C9)}$$

This model is deceptively similar to Uzawa's endogenous RTP model and simply replaces c_t with *c** and $0 < \frac{d\theta_t}{du(c_t)}$ with $\frac{d\theta}{dE[u(c^*)]} < 0$. However, the two models are completely different because of the opposite characteristics of $0 < \frac{d\theta_t}{du(c_t)}$ and $\frac{d\theta}{dE[u(c^*)]} < 0$.

C5.3.2. Nature of the model

The model can be regarded as successful only if it exhibits stability. In Uzawa's model, the economy becomes unstable if $0 < \frac{d\theta_t}{du(c_t)}$ is replaced with $\frac{d\theta_t}{du(c_t)} < 0$. In this section, I examine the stability of the model.

C5.3.2.1. Equilibrium RTP

In Ramsey-type models, such as shown in Section C1.3, if a constant RTP is given, the value of the marginal product of capital (*i.e.,* the value of the real interest rate) converges to that of the given RTP as the economy approaches the steady state. Hence, when a RTP is specified at a certain value, the corresponding expected steady-state consumption is uniquely determined. Given fixed values of other exogenous parameters, any predetermined RTP has unique values of expected consumption and utility at steady state. There is a one-to-one correspondence between the expected utilities at steady state and the RTPs; therefore, the expected utility at steady state can be expressed as a function of RTP. Let c_x^* be a set of steady-state consumption levels, given a set of RTPs (θ_X) and other fixed exogenous parameters. The concept of $\theta \rightarrow W$ discussed above can be described as

$$g(\theta) = E[u(c^*)](= W) , \quad \text{(C10)}$$

where $c^* \in c_x^*$ and $\theta \in \theta_x$. On the other hand, RTP is a continuous function of steady-state consumption as shown in equation (C9) such that $\theta = \theta^{**}(W) = \theta^{**}\{E[u(c^*)]\}$. The reverse function is

$$h(\theta) = E[u(c^*)](= W) . \quad \text{(C11)}$$

The equilibrium RTP is determined by the point of intersection of the two functions, $g(\theta)$ and $h(\theta)$, as shown in Figure C1. Figure C2 shows the special but conventionally assumed case for $h(\theta)$ in which θ is not sensitive to *W*, and RTP is constant. There exists a point of intersection because both $g(\theta)$ and $h(\theta)$ are monotonically continuous for $\theta > 0$. $h(\theta)$ is monotonically continuous because $\theta^{**}(W)$ is monotonically continuous. $g(\theta)$ is monotonically continuous because, as a result of utility maximization, $c^* = f(k^*)$ and

$\theta = \dfrac{df(k^*)}{dk^*}$, where k^* is capital input per capita at steady state such that $k^* = \lim_{t\to\infty}(k_t)$. Because $f(k^*)$ and $\dfrac{df(k^*)}{dk^*}$ are monotonically continuous for $k^* > 0$, c^* is a monotonically continuous function of θ for $\theta > 0$. Here, because u is monotonically continuous, then $E[u(c^*)] = g(\theta)$ is also monotonically continuous for $\theta > 0$.

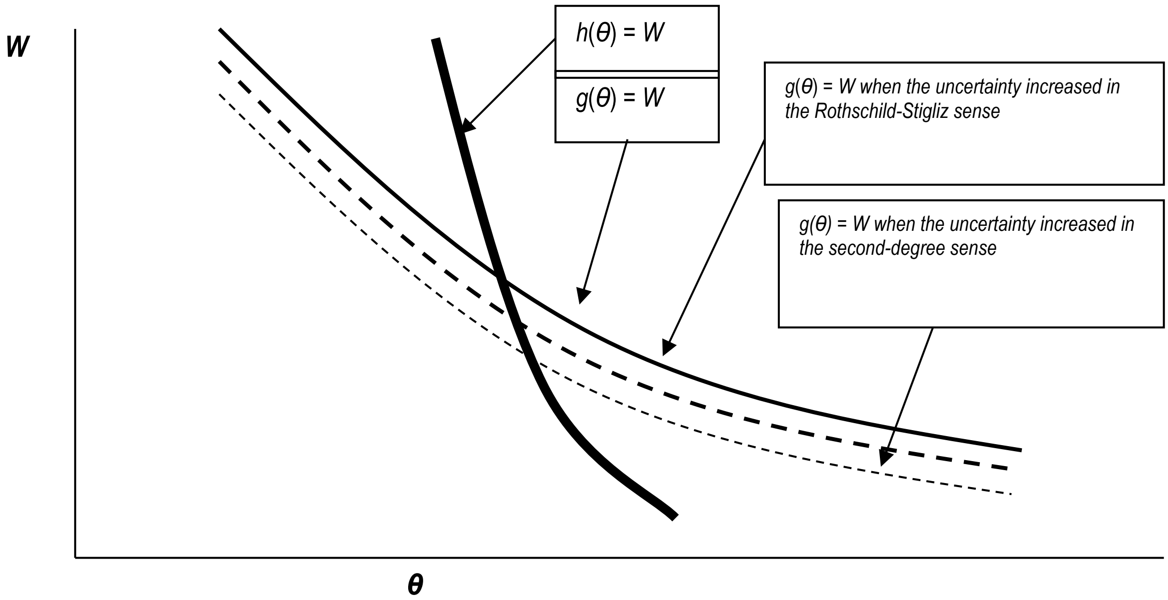

Figure C1. Endogenous time preference

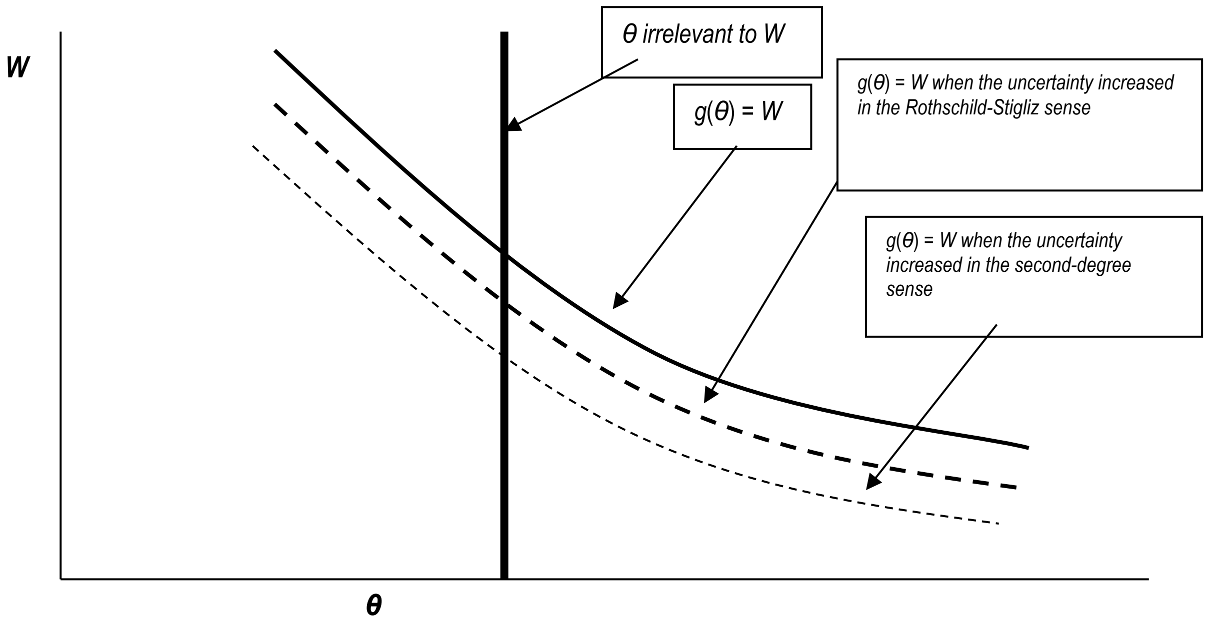

Figure C2. Permanently constant time preference

The function $g(\theta)=E[u(c^*)]=W$ is a decreasing function of θ because higher RTP results in lower steady state consumption. The function $h(\theta)=E[u(c^*)]=W$ is also a decreasing function of θ because $\frac{d\theta}{dW}<0$. Thus, both $g(\theta)$ and $h(\theta)$ are decreasing, but the slope of $h(\theta)$ is steeper than that of $g(\theta)$ as shown in Figure C1. This is true because $g(\theta)=W$ is the consequence of a Ramsey-type model as shown in Section C1.3; thus, if $\theta\rightarrow\infty$, then $g(\theta)=W\rightarrow 0$ because $\theta=i_t\rightarrow\infty$ and $k_t\rightarrow 0$, and if $\theta\rightarrow 0$, then $g(\theta)=W\rightarrow\infty$ because $\theta=i_t\rightarrow 0$ and $k_t\rightarrow\infty$. The function $h(\theta)=W$ indicates the endogeneity of RTP, and because RTP is usually neither zero nor infinity, then even if $h(\theta)=W\rightarrow 0$, $\theta<\infty$, and $h(\theta)=W\rightarrow\infty$, $0<\theta$. Hence, the locus $h(\theta)=W$ cuts the locus $g(\theta)=W$ downward from the top, as shown in Figure C1. Hence, the locus $h(\theta)=W$ is more vertical than $g(\theta)=W$, and thereby a permanently constant RTP, as shown in Figure C2, has probably been used as an approximation of the locus $h(\theta)=W$ for simplicity.

C5.3.2.2. Stability of the model

RTP is constant unless a shock that changes the expected *c** occurs because *W* does not depend on *t* but on the expected *c**. Thus, the same RTP and steady state continue until such a shock hits the economy. Therefore, the endogeneity of RTP only matters when a shock occurs. This constancy is the key for the stability of the model. Once the RTP corresponding to the intersection (Figure 1) is determined, it is constant and the economy converges at a unique steady state unless a shock that changes the expected *c** occurs. The shock is exogenous to the model, and the economy does not explode endogenously but stabilizes at the steady state. Hence, the property $\frac{d\theta}{dW}<0$ in the model, which is consistent with empirical findings, does not cause instability.

The model is therefore acceptable as a model of endogenous RTP. Furthermore, because RTP is endogenously determined, the assumption of irrationality is not necessary for the determination of RTP. Nevertheless, a shock on RTP can be initiated by a shock on the expected *c**; thus, even if the so-called animal spirits are directly irrelevant to determination of RTP, they may be relevant in the generation of shocks on the expected *c**.

C6. Frequent RTP shocks

C6.1. Difficulty in knowing RTP RH

To estimate the parameter values of equation (C11) in the structural model of RTP RH, it is necessary to obtain a sufficiently large amount of data on the value of RTP RH. To obtain these data, a household must know the RTPs of all the other households. Although a household knows its own RTP, it has almost no information about the RTPs of all the other households much less time-series data on each household's RTP. Because of the lack of available data, a household cannot estimate the parameter values in equation (C11) in the structural model of RTP RH even if it knows the functional forms of equations in the structural model.

We can easily generate data on aggregate consumption, investment, production, inflation, trade, and other factors at a relatively low cost, but we cannot directly observe the value of RTP RH. Nonetheless, many estimates of RTP have been reported, but they are not based on a structural model of RTP. Most are the results of experimental studies or indirect estimates based on other models (*e.g.,* Ramsey growth models) on the assumption that RTP is constant. Experiments can give us some information on the RTPs of test subjects, but we should not naively use these estimates as the RTP RH in the calculation of the future path of economy because they vary widely according to the experimental environments. Furthermore, most of the indirect estimates were calculated on the assumption that RTP is constant, which as discussed previously, is most likely not the case. The basic problem is that no credible estimation method of RTP RH has been established.

C6.2. Expectations based on beliefs

The lack of observable data on RTP RH will significantly hinder households from generating rational expectations of the future path of economy. How do households rationally expect their future streams of consumption and production and calculate their optimal paths without information on RTP RH, which is indispensable as the discount factor? The historical mean of RTP RH estimated by long-term data is not consistent with a rational expectation of the future stream because RTP is not constant. Without a reliable method for estimating the parameters of the structural model, it is impossible for households to generate rational expectations of the future path of the economy.

An alternative way of estimating expected values of RTP RH is needed, but even if an alternative method is utilized, households still have to behave as rationally as possible even in an environment of significantly incomplete information. In this situation, household may have to use the concept of bounded rationality to make decisions. It is possible that the only alternative for a household is to use its "belief" about the RTP RH. The use of a belief does not mean that households deviate from rationality; rather, it is the most rational behavior they can use in an environment where insufficient information is available.

Such a belief is defined in Appendix C as the range of values of RTP RH within which a household believes that the true RTP RH exists. Households utilize the belief in place of equation (C11). More specifically, suppose that household *i* ($i \in N$) believes that the RTP RH in the future is situated in the range λ_i, where the subjective probability density at any point on λ_i is identical (i.e., its distribution shape is uniform). Because households have no information about the shape of the distribution, they assume that it is uniform. This supposition means that household *i* believes that λ_i is stationary. Let $\bar{\lambda}_i$ be the mean of λ_i. Suppose that household *i* calculates its optimal future path on the belief that the mean of future values of RTP RH is $\bar{\lambda}_i$. By equation (C10), *W* can be calculated based on $\bar{\lambda}_i$, and the expected future path of economy can be calculated.

Households can equally access all relevant information. Therefore, if the belief of a household is very different from those of the majority, the household will soon perceive that its belief is different, through observing the behavior of majority. The household will change its belief to the almost same as those of the majority because otherwise it cannot achieve optimality as expected on the assumption that sustainable heterogeneity is achieved. Hence, it is likely that households' beliefs become similar, and thereby, it is assumed for simplicity that households' beliefs are identical.

Note that households do not cooperatively and collectively expect the future path of economy (*i.e.,* the representative household's future path), but each household independently and individually generates its own expectations based on its belief in RTP RH. The household thereby creates its own expected future path considering the expected representative household's future path. The aggregates are the sum of all household's independent and individual activities, but if sustainable heterogeneity is achieved, the aggregates appear to be the same as the results of the representative household's activities.

C6.3. Refining beliefs

A household knows that its expectation is based on its beliefs and not the structural model. Therefore, it will always want to refine the belief, that is, raise the probability that the belief is the correct value, by exploiting all currently available relevant information. Let a set of currently available economic indicators be $\boldsymbol{I}_t$ (*e.g.,* the observed data on consumption, production, inventory, etc.). These data may provide some useful information on the past RTP RH, and a household may refine its belief based on this information. These data and equation (C10) can be used to generate estimates of past values of RTP RH. However, $\boldsymbol{I}_t$ includes noise, and data in $\boldsymbol{I}_t$ will usually be somewhat inconsistent between the elements of $\boldsymbol{I}_t$. In addition, because equation (C10) indicates the steady state values that are achieved after a long-period transition, the short-term past data included in $\boldsymbol{I}_t$ are basically insufficient to obtain a credible estimate. Therefore, the estimate of the past values of RTP RH based on $\boldsymbol{I}_t$ and equation (C10) will usually have a large confidence interval. Let $\bar{\mu}_I$ be the estimated past RTP RH and μ_I be its confidence interval of, for example, 95%. Because households can equally access all relevant information, assume for simplicity that μ_I and $\bar{\mu}_I$ are identical for all households.

Although a household knows that $\bar{\mu}_I$ is not a credible estimate, has a large confidence interval, and is merely an estimate (usually a point estimate) of a past value, it will strive to utilize the information derived from $\bar{\mu}_I$ to refine its beliefs in the future value of RTP RH. Usually $\bar{\mu}_I$ will not be equal to $\bar{\lambda}_i$, but the ranges of λ_i and μ_I may partly overlap. Household *i* may utilize the information from this partial overlap to refine its belief (*i.e.,* information of how λ_i is different from μ_I). $\bar{\mu}_I \neq \bar{\lambda}_i$ indicates that the belief $\bar{\lambda}_i$ is wrong, $\bar{\mu}_I$ is wrong, both are wrong, or both are right if the true past RTP RH is $\bar{\mu}_I$ but the true future RTP RH is $\bar{\lambda}_i$. The belief $\bar{\lambda}_i$ may be wrong because the RTP RH will change in the near future, and $\bar{\mu}_I$ may be wrong because the RTP RH changed during the period in which the data were obtained. In addition, a household knows that μ_I is the result of all households' activities based on their beliefs, not on the true value of RTP RH. These uncertainties arise because households cannot know the parameters of the structural model. Without using the structural model, household *i* cannot judge whether $\bar{\lambda}_i$ is wrong, $\bar{\mu}_I$ is wrong, both are wrong, or both are right. As a result, household *i* will not easily adjust its belief from $\bar{\lambda}_i$ to $\bar{\mu}_I$.

However, it is still likely that information about the difference between λ_i and μ_I can be used to refine the belief. To extract the useful information, the following rules may be used:

Rule 1: if $\bar{\mu}_I$ is included in λ_i, the belief is not adjusted; otherwise, the belief is adjusted from $\bar{\lambda}_i$ to $\bar{\mu}_I$.

Rule 2: if $\bar{\lambda}_i$ is included in μ_I, the belief is not adjusted; otherwise, the belief is adjusted from $\bar{\lambda}_i$ to $\bar{\mu}_I$.

Rule 3: if λ_i and μ_I overlap at or above a specified ratio, the belief is not adjusted; otherwise, the belief is adjusted from $\bar{\lambda}_i$ to $\bar{\mu}_I$.

The above rules may be seen as a type of adaptive expectation because μ_I indicates the past RTP RH. However, in the situation where the parameters of the structural model of the RTP RH are unknown, it may be seen as rational to utilize the information contained in μ_I by adopting one of these rules.

C6.4. Changing beliefs

However, it does not seem likely that a household will refine its belief following one of the rules shown above because the rules are basically backward looking and will not be adopted as a tool for refining the belief if a household is convinced that the RTP RH is temporally variable. The belief will only be changed if forward-looking information is available, that is, when a household becomes aware of information about the future RTP RH in μ_I. For example, the difference between λ_i and μ_I may reflect an unexpected and large positive technology shock that occurred after the formation of belief λ_i. Because the effects of the technology shock will persist for long periods in the future, household *i* will most likely change its belief. In this case, a household will not simply refine its belief from $\bar{\lambda}_i$ to $\bar{\mu}_I$; it will change to another value that is formed as an entirely new belief.

Whether a household changes its belief or not, therefore, will depend not simply on μ_I but on the information the household can extract from μ_I about the future path of the economy. Hence, in some cases, a household will change its belief when new values of μ_I are obtained, but in other cases, it will not, depending on how the household interprets the information contained in μ_I.

C6.5. Heuristics

When a household interprets μ_I, it may also use heuristic methods, for example, a simplified linear reduced form model of RTP RH. Studies of the use of heuristics and bounded rationality in this context would be useful for better understanding the interpretation mechanism of μ_I. There are many possible simplified linear reduced form models of RH's RTP that could be used as heuristic methods although most of them may be ad hoc. Even though such reduced form models are far less credible than a structural model, they may be utilized as a heuristic method of interpreting μ_I by households. Although these types of models may often result in misleading conclusions, they may sometimes provide useful information. For example, if a linear correlation between RTP RH and a financial indicator exists, even if it is weak or temporary, changes in the financial indicator may contain useful information about changes in the RTP RH. Therefore, if a household believes that this correlation exists, it will use this information to interpret μ_I.

C6.6. Frequent RTP shocks

Households must have expected values of RTP RH for sustainable heterogeneity, but as previously discussed, the expectations are not based on the structural model but rather on a belief that is not guaranteed to generate the correct value. In addition, the belief can be influenced by heuristic considerations. These features indicate that the expected values of RTP RH will fluctuate more frequently than the intrinsic RTP RH.

Households' expectations of RTP RH will change when the intrinsic RTP RH shifts, for example, when new information about shocks on the factors that determine equation (C10) becomes available. For a given θ, $E\left[u\left(c^*\right)\right]$ changes if the expectation of future productivity changes. Productivity at the macro level will be influenced by scientific technology, financial technology, social infrastructure, and other factors. If expectations about these factors in the future changes, the expected future productivity and $E[u(c^*)]$ will also change. In addition, even if intrinsic RTP RH does not change, the expected RTP RH will change if a household's belief is altered because of new information contained in μ_I. Hence, the expected RTP RH can change independently of intrinsic changes in RTP RH. Therefore, even if intrinsic changes in RTP RH occur infrequently, changes in the expected RTP RH may occur more frequently.

A household's expected RTP RH can potentially change every time new information on μ_I becomes available if it contains the information that makes beliefs change. Information concerning factors that affect the expected RTP RH will become available frequently, and at least some of the information may be both very important and unexpected. In addition, there will be many disturbances in the fundamental factors that affect equation (C10), and many of these disturbances will also cause μ_I to change. As discussed previously, a household may interpret these changes in μ_I as a change in the true RTP RH. Therefore, it is likely that households' expected RTP RH change more frequently than the intrinsic RTP RH, and thereby, that time preference shocks also occur more frequently than previously thought.

Even a small piece of additional information about the belief can significantly change the path of the economy. For example, if many households believe a rumor (whether it is true or not) related to information about the interpretation of μ_I and respond similarly to it, their expectations will be changed in the same direction by the rumor. If all households respond similarly to an untrue rumor and change their expectations equally to an untrue value, the economy will proceed based on the incorrect expectation of RTP RH. The $\bar{\mu}_I$ that is observed a few periods later will follow these wrongly expected values of RTP RH. Upon obtaining new data of $\bar{\mu}_I$ that are consistent with these wrongly expected values, households will judge that their (incorrect) changes were in fact correct. As a result, the incorrect expectations become self-fulfilling. This spurious situation may reach an impasse at some point in the future because the expectations are based not on a structural model but on the (incorrect) beliefs. Households will not anticipate the impasse until the economy reaches it because they believe that the wrongly expected RTP RH (*i.e.,* the currently held belief) is true.

DOI: http://dx.doi.org/10.14505/tpref.v7.2(14).03

THE EQUIVALENCE OF BERTRAND EQUILIBRIUM IN A DIFFERENTIATED DUOPOLY AND COURNOT EQUILIBRIUM IN A DIFFERENTIATED OLIGOPOLY

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Suggested Citation:

Lee, D.J., Han, S., Ono, Y., Oh, J. (2016). The equivalence of Bertrand equilibrium in a differentiated duopoly and Cournot equilibrium in a differentiated oligopoly, *Theoretical and Practical Research in Economic Field*, (Volume VII, Winter 2016), 2(14): 145-154. DOI:10.14505/tpref.v7.2(14).03. Available from: http://www.asers.eu/publishing/index.php/tpref.

Article's History:

Received September, 2016; *Revised* Octomber, 2016; *Accepted* November, 2016.

Abstract

This paper examines whether Bertrand equilibrium in a differentiated duopoly can be duplicated with Cournot competition or not. We show that the degree of product differentiation plays an important role in the duality between those equilibria. Our main claims are two. One is that there exists a unique duality condition that satisfies the equivalence between Bertrand and Cournot equilibrium irrespectively of market structures. The other is that the number of firms of Cournot competition that satisfies Bertrand equilibrium increases with the degree of product differentiation.

Keywords: Bertrand equilibrium, Cournot equilibrium, product differentiation, equivalence

JEL Classification: D21, D43, L13

1. Introduction

It is well known, from the viewpoint of social welfare, that price competition is more efficient than quantity competition. Suppose a duopoly situation where firms produce a homogeneous product and marginal costs are constant and equal for both firms. In a price competition, the price equals the marginal cost, while, in a quantity competition, the price is above it. Even though both products are differentiated, the results are the same: the equilibrium price under Cournot competition is higher than it under Bertrand competition except a perfect differentiation.

Since Dixit (1979) has first proposed a differentiated duopoly, many studies have been produced an array of extensions and generalization of Singh and Vives (1984)[1]. The literature on the relationship between Cournot

[1] For a horizontal market, see Cheng (1985), Dastidar (1997), Qiu (1997), Hackner(2000), and Amir and Jin (2001), and for a vertical market, Correa-Lopez (2007), Arya et al. (2008), Mukherjee *et al.* (2012), Alipranti *et al.* (2014), and Lee and Choi (forthcoming).

and Bertrand competition has been concerned with two main issues. One stream focuses on the comparison between Bertrand and Cournot competition in both a horizontal and a vertical market. The other is to analyze it in a mixed oligopoly.

Even though many studies have focused on the comparison between competition modes, the duality between Cournot and Bertrand competition has received little attention. So, we consider whether Bertrand equilibrium can be duplicated by Cournot competition in a horizontal (or vertical) oligopoly or not. We show that the degree of product differentiation plays an important role in the duality between those models in a differentiated duopoly. Our main conclusions are two. First, we find a unique duality condition that satisfies the equivalence between Bertrand and Cournot equilibrium irrespectively of market structures. Second, the number of firms, in Cournot competition, that satisfies Bertrand equilibrium increases with the degree of product differentiation.

The paper is organized as follows. In section 2, we set up the model. Section 3 analyzes whether Bertrand equilibrium can be duplicated with Cournot competition in a horizontal market. Section 4 examines it in a vertical structure. Finally, we conclude the concluding remarks.

2. The Model

We consider an economy with a duopolistic sector, consisting of two firms produce a differentiated good. Both firms operate under constant returns to scale. Each firm's unit cost of production equals c exogenously. We analyze whether Bertrand equilibrium can be duplicated with Cournot competition in a differentiated industry. The demand structure of our model is adapted from Dixit (1979). A representative consumer maximizes $U(q_I, q_j) - \sum p_i q_i, i, j = 1,2 \; and \; i \neq j$, where q_i is the quantity, and p_i is its price. U is assumed to be quadratic and strictly concave $U(q_i, q_j) = a(q_i + q_j) - (q_i^2 + 2dq_iq_j + q_j^2)/2$, where $d \in [0,1], i, j = 1,2$, and $i \neq j$. This utility function gives rise to a linear demand structure. The Inverse and direct demands are as follows:

$$p_i = a - q_i - dq_j \;\; and \;\; q_i = \frac{a(1-d) - p_i + dp_j}{1-d^2}; \; i, j = 1, 2, and \; i \neq j, \qquad (1)$$

The parameter *d* of the demand function expresses the degree of product differentiation, ranging from zero when goods are independent to one when the goods are perfect substitutes.

3. Horizontal Oligopoly

In this section, we analyze whether Bertrand equilibrium can be duplicated with Cournot competition in a horizontal oligopolistic market. As a benchmark, we show that Bertrand equilibrium differs from Cournot equilibrium. In Cournot competition both firms choose quantities, in Bertrand competition, prices. In both cases, the equilibrium concept is non-cooperative Nash equilibrium. In Cournot competition, firm *i* chooses q_i so as to maximize its profit, taking as a given q_j, while, in Bertrand competition, it chooses p_i so as to maximize its profit, taking as a given p_j as follows:

$$\max_{q_i} \pi_i = (a - c - q_i - dq_j)q_i,$$

$$\max_{p_i} \pi_i = (p_i - c)\left(\frac{a(1-d) - p_i + dp_j}{1-d^2}\right).$$

It is straightforward to compute Bertrand and Cournot equilibrium. We obtain the equilibrium quantities and prices under Bertrand and Cournot competition, respectively, as follows:

$$q^B = \frac{a-c}{2+d-d^2}, \qquad (2.1)$$

$$p^B = c + \frac{(1-d)(a-c)}{2-d}, \qquad (2.2)$$

$$q^C = \frac{a-c}{2+d}, \tag{2.3}$$

$$p^C = c + \frac{a-c}{2+d}, \tag{2.4}$$

where the superscript '*B*' and '*C*' denotes Bertrand and Cournot competition, respectively.

From Eq. (2.1), Eq. (2.2), Eq. (2.3), and Eq. (2.4), we obtain $p^C - p^B = d^2(a-c)/(4-d^2) \geq 0$ (and similary $q^B - q^C = d^2(a-c)/(4+4d-d^2-d^3) \geq 0$, which are nonnegative. Quantities are lower and prices are higher in Cournot than in Bertrand competition.

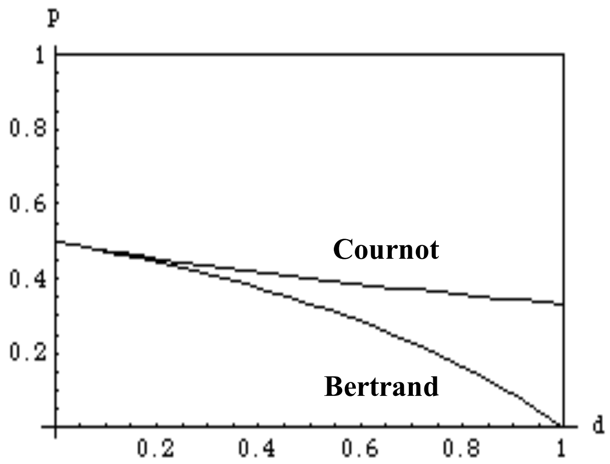

Figure 1. Equilibrium Prices between Bertrand and Cournot when $\mathbf{a-c=1}$ and $\mathbf{c=0}$

From now on, we show that Bertrand equilibrium can be duplicated with Cournot competition. Consider an economy in which two types of firm, $T \in \{i, j\}$, producing a differentiated good at a constant marginal (=average) cost *c*. Each type consists of *n* number of firms. The inverse demand is given by:

$$p^i = a - Q_K^i - dQ_K^j i,j = 1,2, i \neq j, K = 1, \dots, n, \tag{3}$$

where $Q_K^i = q_1^i + \cdots + q_n^i \text{and} Q_K^j = q_1^j + \cdots + q_n^j$.

Firm 1 in type *i* sets its quantity q_1^i so as to maximize its profit for given rival firms' outputs $(Q_K^i - 1 = q_2^i + \cdots + q_n^i \; and \; Q_K^j = q_1^j + \cdots + q_n^j)$ as follows:

$$\max_{q_1^i} \pi_1^i = (p^i - c)q_1^i = (a - c - Q_K^i - dQ_K^j)q_1^i.$$

Differentiating the maximization problem with respect to q_1^i, we obtain the reaction function as follows:

$$\frac{\partial \pi_1^i}{\partial q_1^i} = a - c - Q_K^i - dQ_K^j - q_1^i = 0. \tag{4}$$

On the other hand, firm 1 in type *j* sets its quantity q_1^j so as to maximize its profit for given rival firms' outputs $(Q_K^i = q_1^i + \cdots + q_n^i \; and \; Q_K^j - 1 = q_2^j + \cdots + q_n^j)$ as follows:

$$\max_{q_1^j} \pi_1^j = (p^j - c)q_1^j = (a - c - Q_K^j - dQ_K^i)q_1^j.$$

Differentiating the maximization problem with respect to q_1^j, we obtain the reaction function as follows:

$$\frac{\partial \pi_1^j}{\partial q_1^j} = a - c - Q_K^j - dQ_K^i - q_1^j = 0. \tag{5}$$

Summing Eq. (4) and Eq. (5) and solving them, we obtain the equilibrium quantity as follows:

$$q_K^i = q_K^j = \frac{a-c}{(1+d)n+1}, \tag{6.1}$$

$$Q_K^i = Q_K^j = \frac{(a-c)n}{(1+d)n+1}. \tag{6.2}$$

Comparing Eq. (2.1) and Eq. (2.3) to Eq. (6.2) and Eq. (3), we obtain the following result:

$$n = \frac{1}{1-d^2}. \tag{7}$$

We summarized the results in Proposition 1.

Proposition 1. If the number of firms in each type is equivalent to $1/(1-d^2)$, Bertrand equilibrium can be perfectly duplicated with Cournot competition.

By differentiating Eq. (7) with respect to d, we have the following result.

Lemma 1. The number of firms, in Cournot competition, that satisfies Bertrand equilibrium increases with the degree of product differentiation.

4. Vertical Oligopoly

We extend our model to vertical structures (monopolistic and bilateral duopoly). The timing of the games is as follows. At stage one, each upstream firm sets its input price (w in a monopolistic duopoly and (w_i, w_j) in a bilateral duopoly). At stage two, each downstream firm sets the quantity.

As a benchmark, we first consider a monopolistic duopoly in which an upstream firm produces an input and sells it at one price to two downstream firms. In Cournot competition, at stage two, downstream firm *i* sets its quantity q_i so as to maximize its profit for given rival's quantity q_j and input price w and in Bertrand competition, downstream firm *i* sets its price p_i so as to maximize its profit for a given rival's output price p_j and input price w. Therefore, its maximization problems are as follows:

$$\max_{q_i} \pi_i = (p_i - w)q_i = (a - w - q_i - dq_j)q_i,$$

$$\max_{p_i} \pi_i = (p_i - w)\left(\frac{a(1-d) - p_i + dp_j}{1-d^2}\right).$$

We obtain the equilibrium quantities and prices under Bertrand and Cournot competition, respectively, as follows:

$$q_1^B = \frac{a-w}{2+d-d^2}, \tag{8.1}$$

$$p_1^B = \frac{(1-d)a \mp w}{2-d}, \quad (8.2)$$

$$q_1^C = \frac{a-w}{2+d}, \quad (8.3)$$

$$p_1^C = \frac{a+(1+d)w}{2+d}. \quad (8.4)$$

At stage one, the upstream firm sets the input price w so as to maximize its profit. Its maximization problems under Cournot and Bertrand competition are, respectively, as follow:

$$\max_w \Pi = (w-c)(q_1+q_2) = \frac{2(w-c)(a-w)}{2+d},$$

$$\max_w \Pi = (w-c)(q_1+q_2) = \frac{2(w-c)(a-w)}{2+d-d^2}.$$

We obtain the equilibrium input prices, quantities, and prices under Bertrand and Cournot competition, respectively, as follows:

$$w^B = w^C = \frac{a+c}{2}, \quad (9.1)$$

$$q_i^B = \frac{a-c}{2(2+d-d^2)}, \quad (9.2)$$

$$p_i^B = c + \frac{(a-c)(3-2d)}{2(2-d)}, \quad (9.3)$$

$$q_i^C = \frac{a-c}{2(2+d)}, \quad (9.4)$$

$$p_i^C = c + \frac{(a-c)(3+d)}{2(2+d)}. \quad (9.5)$$

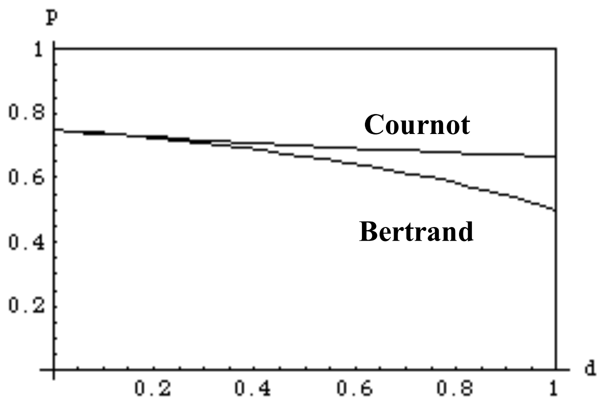

Figure 2. Equilibrium Prices in a Monopolistic Duopoly when $a-c=1$ and $c=0$

As a benchmark, we also consider a bilateral duopoly in which each upstream firm produces an input and sells it to its own downstream firm. In Cournot competition, at stage two, downstream firm *i* sets its quantity q_i so as to maximize its profit for given rival's quantity q_j and input price w_i. On the other hand, in Bertrand competition, downstream firm *i* sets its price p_i so as to maximize its profit for given rival's output price p_j and input price w_i. Therefore, its maximization problems are, respectively, as follows:

$$\max_{q_i} \pi_i = (p_i - w_i)q_i = (a - w_i - q_i - dq_j),$$

$$\max_{p_i} \pi_i = (p_i - w_i)q_i = (p_i - w_i)\left(\frac{a(1-d) - p_i + dp_j}{1-d^2}\right).$$

We obtain the equilibrium price and quantity under Bertrand and Cournot competition, respectively, as follows:

$$q_i^B = \frac{a(2-d-d^2) - (2-d^2)w_i + dw_j}{4-5d^2+d^4}, \tag{10.1}$$

$$p_i^B = \frac{a(2-d-d^2) + 2w_i + dw_j}{4-5d^2+d^4}, \tag{10.2}$$

$$q_i^C = \frac{a(2-d) - 2w_i + dw_j}{4-d^2}, \tag{10.3}$$

$$p_i^C = \frac{a(2-d) + (2-d^2)w_i + dw_j}{4-d^2}. \tag{10.4}$$

At stage one, each upstream firm sets the input prices (w_i, w_j) so as to maximize its profit. Upstream firm *i*'s maximization problems are, respectively, as follow:

$$\max_{w_i} \Pi_i = (w_i - c)q_i = (w_i - c)\left(\frac{a(2-d) - 2w_i + dw_j}{4-d^2}\right),$$

$$\max_{w_i} \Pi_i = (w_i - c)(q_1 + q_2) = (w_i - c)\left(\frac{a(2-d-d^2) - (2-d^2)w_i + dw_j}{4-5d^2+d^4}\right).$$

Finally, we obtain the equilibrium input prices, quantities, and prices under Bertrand and Cournot competition, respectively, as follows:

$$w_i^B = w_j^B = c + \frac{(2-d-d^2)(a-c)}{4-d-2d^2}, \tag{11.1}$$

$$w_i^C = w_j^C = c + \frac{(2-d)(a-c)}{4-d}, \tag{11.2}$$

$$q_i^B = q_j^B = \frac{(2-d^2)(a-c)}{(2+d-d^2)(4-d-2d^2)}, \tag{11.3}$$

$$p_i^B = p_j^B = c + \frac{2(1-d)(3-d^2)(a-c)}{(2-d)(4-d-d^2)}, \tag{11.4}$$

$$q_i^C = q_j^C = \frac{2(a-c)}{(2+d)(4-d)}, \tag{11.5}$$

$$p_i^C = p_j^C = c + \frac{(6-d^2)(a-c)}{(2+d)(4-d)}. \tag{11.6}$$

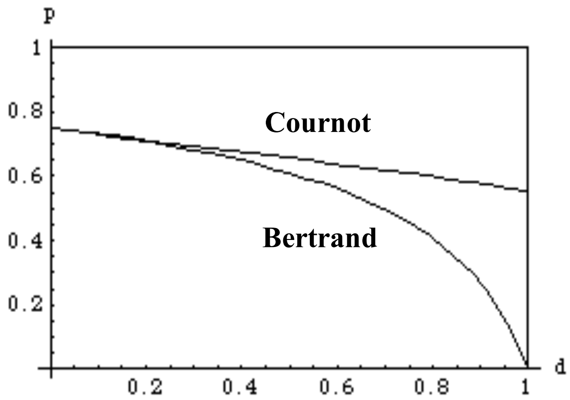

Figure 3. Equilibrium Prices in a Bilateral Duopoly when $a-c=1$ and $c=0$

Next, we analyze the equivalence between Bertrand and Cournot equilibrium in vertical structures. We first consider a monopolistic duopoly in which an upstream firm produces an input and sells it at one price to two types of downstream firms, $M \in \{i,j\}$, producing a differentiated good. Each type consists of *n* number of downstream firms. The upstream firm has a constant marginal (=average) cost *c*. For simplicity, one unit of the final product needs exactly one unit of the input and the cost of transforming the input into the final product is normalized to zero. The timing of the games is as follows. At stage one, the upstream firm sets the input price. At stage two, each downstream firm chooses the output.

At stage 2, under Eq. (3), downstream firm 1 in type *i* sets its quantity q_1^i so as to maximize its profit for given rival firms' quantities ($Q_{K-1}^i = q_2^i + \cdots + q_n^i \; and \; Q_K^j = q_1^j + \cdots + q_n^j$) and input price w as follows:

$$\max_{q_1^i} \pi_1^i = (p^i - w)q_1^i = (a - w - Q_K^i - dQ_K^j)q_1^i.$$

Differentiating the maximization problem with respect to q_1^i, we obtain the reaction function as follows:

$$\frac{\partial \pi_1^i}{\partial q_1^i} = a - w - Q_K^i - dQ_K^j - q_1^i = 0. \tag{12}$$

On the other hand, downstream firm 1 in type *j* sets its output q_1^j so as to maximize its profit for given rival firms' outputs ($Q_K^i = q_1^i + \cdots + q_n^i \; and \; Q_{K-1}^j = q_2^j + \cdots + q_n^j$) as follows:

$$\max_{q_1^j} \pi_1^j = (p^j - w)q_1^j = (a - w - Q_K^j - dQ_K^i)q_1^j.$$

Differentiating the maximization with respect to q_1^j, we obtain the reaction function as follows:

$$\frac{\partial \pi_1^j}{\partial q_1^j} = a - w - Q_K^j - dQ_K^i - q_1^j = 0. \tag{13}$$

Summing (12) and Eq. (13) and solving them, we obtain the equilibrium quantities as follows:

$$q_K^i = q_K^j = \frac{a-w}{n+dn+1}, \tag{14.1}$$

$$Q_K^i = Q_K^j = \frac{(a-w)n}{n+dn+1}. \tag{14.2}$$

At stage one, the upstream firm sets the input price w so as to maximize its profit. Its maximization problem is as follows:

$$\max_{w} \Pi = (w-c)(Q_K^i + Q_K^j) = 2(w-c)\left(\frac{(2-w)n}{n+dn+1}\right).$$

Differentiating the maximization with respect to w, we obtain the reaction function as follows:

$$\frac{\partial \Pi}{\partial w} = \frac{2n(a+c-2w)}{n+dn+1} = 0. \tag{15}$$

Solving (15), we obtain the equilibrium input price as follows:

$$w = \frac{a+c}{2}. \tag{16.1}$$

Substituting (16.1) into (14.1) and (14.2), we obtain the equilibrium quantity for each downstream firm and for industry, respectively, as follows:

$$q_K^i = q_K^j = \frac{a-c}{2(n+dn+1)}, \tag{16.2}$$

$$Q_K^i = Q_K^j = \frac{(a+c)n}{2(n+dn+1)}. \tag{16.3}$$

Comparing (9.3) to (16.3), we obtain the following result:

$$n = \frac{1}{1-d^2}. \tag{17}$$

Proposition 2. If the number of downstream firms of each type is equivalent to $1/(1-d^2)$, If the number of firms in each type is equivalent to $1/(1-d^2)$, Bertrand equilibrium in a monopolistic duopoly can be perfectly duplicated with Cournot competition.

Finally, we turn to a bilateral duopolistic market in which each upstream firm, $U \in \{i,j\}$, produces an input at a constant marginal (=average) cost *c*. Each upstream firm sells its input to its *n* number of downstream firms.

At stage 2, under (3), downstream firm 1 of type *i* sets its quantity q_1^i so as to maximize its profit for given rival firms' quantities ($Q_K^i - 1 = q_2^i + \cdots + q_n^i$ *and* $Q_K^j = q_1^j + \cdots + q_n^j$) and input price w_i as follows:

$$\max_{q_1^i} \pi_1^i = (p^i - w_i)q_1^i = (a - w_i - Q_K^i - dQ_K^j)q_1^i.$$

Differentiating the maximization problem with respect to q_1^i, we obtain the reaction function as follows:

$$\frac{\partial \pi_1^i}{\partial q_1^i} = a - w_i - Q_K^i - dQ_K^j - q_1^i = 0. \tag{18}$$

On the other hand, downstream firm 1 of type *j* sets its quantity q_1^j so as to maximize its profit for given rival firms' quantitiess ($Q_K^i = q_1^i + \cdots + q_n^i$ *and* $Q_{K-1}^j = q_2^j + \cdots + q_n^j$) and input price w_j as follows:

$$\max_{q_1^j} \pi_1^j = (p^j - w_j)q_1^j = (a - w_j - Q_K^j - dQ_K^i)q_1^j.$$

Differentiating the maximization problem with respect to q_1^j, we obtain the reaction function as follows:

$$\frac{\partial \pi_1^j}{\partial q_1^j} = a - w_j - Q_K^j - dQ_K^i - q_1^j = 0. \tag{19}$$

Summing (18) and (19) and solving them, we obtain the equilibrium quantities for each downstream firm and for industry, respectively, as follows:

$$q_K^i = q_K^j = \frac{(n(1-d)+1)a - (n+1)w_i - dnw_j}{n^2(1-d^2)+2n+1}, \tag{20.1}$$

$$Q_K^i = Q_K^j = \frac{n(n(1-d)+1)a - (n+1)w_i - dnw_j}{n^2(1-d^2)+2n+1}. \tag{20.2}$$

At stage one, the upstream firm *i* sets the input price w_i so as to maximize its profit for a given rival firm's input price w_j. Its maximization problem is as follows:

$$\max_{w_i} \Pi_i = (w_i - c)Q_K^i = (w_i - c)\left(\frac{n(n(1-d)+1)a - (n+1)w_i - dnw_j}{n^2(1-d^2)+2n+1}\right).$$

Differentiating the maximization with respect to w_i, we obtain the reaction function as follows:

$$\frac{\partial \Pi_i}{\partial w_i} = \frac{n(a+c) - dna + dnw_j}{2(n+1)} = 0. \tag{21}$$

Solving (21), we obtain the following result:

$$w_i = w_j = \frac{(n+1)(a+c) - dna}{2(n+1) - dn}. \tag{22.1}$$

Finally, we obtain the equilibrium quantities for each downstream firm and for industry, respectively, as follows:

$$q_K^i = q_K^j = \frac{(n+1)(a-c)}{(2(n+1)-dn)(n+dn+1)}, \tag{22.2}$$

$$Q_K^i = Q_K^j = \frac{n(n+1)(a-c)}{(2(n+1)-dn)(n+dn+1)}. \tag{22.3}$$

Comparing (22.3) to (11.4), we obtain the following result:

$$n = \frac{1}{1-d^2}. \tag{23}$$

Proposition 3. If the number of downstream firms in each type is equivalent to $1/(1-d^2)$, Bertrand equilibrium in a bilateral duopoly can be perfectly duplicated with Cournot competition.

Conclusion

This paper examines the relation between Bertrand equilibrium and Cournot equilibrium in a differentiated oligopoly. We consider whether Bertrand equilibrium can be duplicated by Cournot competition in a (or vertical) duopoly or not. We show that the degree of product differentiation plays an important role in the equivalence between Bertrand and Cournot equilibria in differentiated duopoly. Our main conclusions are two. First, we find a unique duality condition that satisfies the equivalence between Bertrand and Cournot equilibrium irrespectively of market structures. Second, the number of firms, in Cournot competition, that satisfies Bertrand equilibrium increases with the degree of product differentiation.

Acknowledgments

This research is supported by Japan Society for the Promotion of Science, Grant Numbers 15H03396, 15K03749, 26780262.

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DOI: http://dx.doi.org/10.14505/tpref.v7.2(14).04

DISCRETE TIME OR CONTINUOUS TIME, THAT IS THE QUESTION: THE CASE OF SAMUELSON'S MULTIPLIER-ACCELERATOR MODEL

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Suggested Citation:

Luo, Y., Luo, M. (2016). Discrete time or continuous time, that is the question: the case of Samuelson's Multiplier-Accelerator model, *Theoretical and Practical Research in Economic Field*, (Volume VII, Winter 2016), 2(14):155-159. DOI:10.14505/tpref.v7.2(14).04. Available from: http://www.asers.eu/publishing/index.php/tpref.

Article's History:

Received November, 2016; *Revised* November, 2016; *Accepted* December, 2016.

Abstract:

A basic problem in economic dynamics is the choice of continuous-time or discrete-time in mathematical modeling. In this paper, we study the continuous-time Samuelson's multiplier-accelerator model and compare this continuous-time model with its classical discrete-time model. We find that although time scales do not affect the probability of return to equilibrium solution in Samuelson's multiplier-accelerator model, but time scales have an influence on the perfect symmetry of periodic motion.

Keywords: Samuelson's multiplier-accelerator model, time scales, perfect symmetry, probability, return to equilibrium

JEL Classification: B41, C02, E1, E2, E32, H5

1. Introduction

Measurement cannot be separated from theory. Theoretical thinking is better through a mathematical representation. The choice of mathematical representations depends on the essential features in empirical observation and theoretical perspective. A mathematical representation should be powerful enough to display stylized features to be explained and simple enough to manage its mathematical solution to be solved.

The choice of the kind of 'time' (continuous or discrete) to be used in the construction of dynamic models is a moot question. We know that such a choice implies the use of different analytical tools: differential equations in continuous time and difference equations in discrete time. In mathematical economics and econometrics, many models are in the forms of difference equations in discrete-time without much justification. There were only a few economists using continuous-time models.

Economists like use discrete-time models more than continuous-time model in economic modeling because, on the one hand, economic data are reported in terms of discrete-time such as annual data、 seasonal data and monthly data, on the other hand, discrete-time model is easy to run regression. However, compared with discrete-time model, continuous-time models have different behavioral solutions and different stability conditions in nature.

In this paper, we study the continuous-time Samuelson's multiplier-accelerator model and compare this continuous-time model with its classical discrete-time model. We find that although time scales do not affect the probability of return to equilibrium solution in Samuelson's multiplier-accelerator model, but time scales have an influence on the perfect symmetry of periodic motion.

The structure of the rest of the paper is as follows. In part 2, we give discrete-time Samuelson's multiplier-accelerator model. In part 3, we give continuous-time Samuelson's multiplier-accelerator model. In part 4 we give the probability of return to equilibrium solution in Samuelson's multiplier-accelerator model. In part 5 we give a conclusion.

2. Discrete time Samuelson's multiplier-accelerator model

The original version of the Samuelson's multiplier-accelerator is in discrete time (Samuelson 1939):

$$C_t = aY_{t-1} \quad (1)$$

$$I_t = b\left(C_t - C_{t-1}\right) \quad (2)$$

$$Y_t = C_t + I_t + G \quad (3)$$

where C is consumption, I is investment, G is government expenditure, Y is income, and 0<a<1, b>0.

We have a second-order difference equation

$$Y_t - a\left(1+b\right)Y_{t-1} + abY_{t-2} = G \quad (4)$$

The model has five types of solutions:

(1) Monotonically converging regime A and its borderline;
(2) Damped oscillation regime B;
(3) Explosive oscillation regime C;
(4) Monotonically diverging regime D and its borderline;
(5) Periodic oscillation curve PQ'.

We should notice that the periodic oscillation occurs only at the borderline between B and C regime, *i.e.* curve PQ'. Patterns in the parameter space are shown in Figure 1.

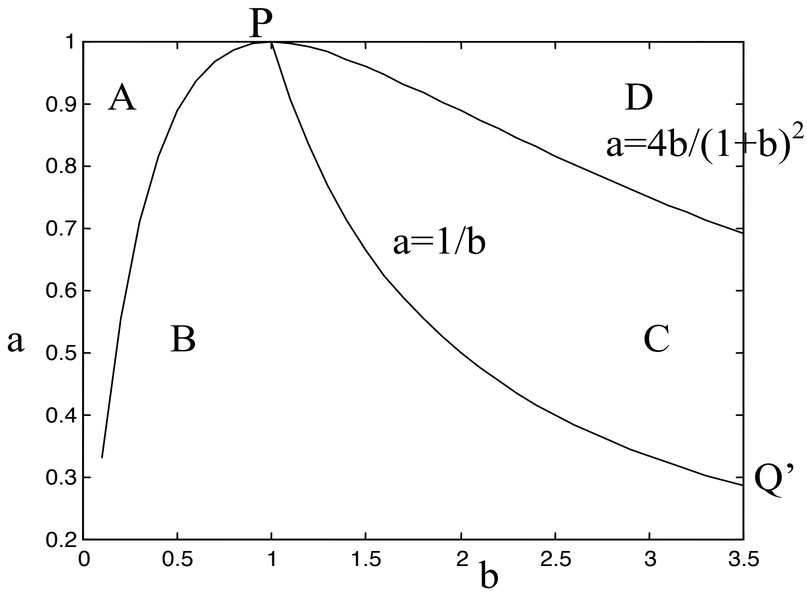

Figure 1. Discrete-time Samuelson's multiplier-accelerator diagram.
A and its borderline stands for monotonically converging; B, damped oscillation; PQ', periodic oscillation; C, explosive oscillation; D and its borderline, monotonically diverging.

3. Continuous time Samuelson's multiplier-accelerator model

We discuss the continuous time version of the above Samuelson's model to demonstrate the relation between discrete time and continuous time linear models.

We simply replace the difference by the derivative in the above Samuelson's model. We have

$$C(t) = aY(t-1) \;= a\left[Y(t) - \left(Y(t) - Y(t-1)\right)\right] \;= a[Y(t) - Y'(t)] \quad (5)$$

$$I(t)=b[C(t)-C(t-1)]=ba\left[\left(Y(t)-Y^{/}(t)\right)\ -\left(Y(t-1)-Y^{/}(t-1)\right)\right]\ =ba\left[Y^{/}(t)-Y^{//}\ (t)\right] \quad (6)$$

In (5), Y(t)-Y'(t) stands for the correction of current income, so the consumption at time t, C(t), can be regarded as linear function of the correction of the income at time t, Y(t). In (6), Y'(t)-Y''(t) stands for the correction of the change of current income, so the investment at time t, I(t), can be regard as linear function of the correction of the change of the income at time t, Y(t).

As a result, we have continuous time Samuelson's multiplier-accelerator model

$$C(t)=\ a[Y(t)-Y^{/}(t)] \quad (5)$$

$$I(t)=\ ba[Y^{/}(t)-Y^{//}(t)] \quad (6)$$

$$Y(t)=C(t)+I(t)+G \quad (7)$$

We have a second-order differential equation

$$Y^{//}(t)+\frac{1-b}{b}Y^{/}(t)+\frac{1-a}{a*b}Y(t)=G/(a*b) \quad (8)$$

In (8), taking $Y^{//}(t)=Y^{/}(t)=\ 0$ we have equilibrium solution

$$Y(t)=\ G/(1-a) \quad (9)$$

The correspondent homogeneous equation of (8) is

$$Y^{//}(t)+\frac{1-b}{b}Y^{/}(t)+\frac{1-a}{a*b}Y(t)=0 \quad (10)$$

The characteristic equation is

$$\lambda^2+\frac{1-b}{b}\lambda+\frac{1-a}{a*b}=0 \quad (11)$$

two roots of the characteristic equation are

$$\lambda_1=\left(-b_1+\sqrt{\Delta}\right)/2$$

$$\lambda_2=\left(-b_1-\sqrt{\Delta}\right)/2$$

where $b_1=\frac{1-b}{b}$b1, $\Delta=-b_1^2-4(1-a)/a*b$

A. When Δ>0, equation (11) have two different real roots and the general solutions of (8) are

$$Y(t)=C_1exp(\lambda_1t)\ +C_2exp(\lambda_2t)+G/(1-a)$$

If $b_1>0$, *i.e.*1> b, λ_1, λ_2 are all negative roots and the solutions are monotonically converging.

If $b_1<0$, *i.e.*1< b, λ_1, λ_2 are all positive roots and the solutions are monotonically diverging.

If $b_1=0$, Δ= -4(1-a)/(ba)<0 (for 0<a<1, b>0), so $b_1=0$ is impossible.

B. When Δ=0, equation (11) have two same real roots and the general solutions of (8) are

$$Y(t)=(C_1+C_2t\ )exp(-t\ b_1/2)+G/(1-a)$$

If $b_1>0$, *i.e.* 1> b, the solutions are monotonically converging.

If $b_1<0$, *i.e.* 1< b, the solutions are monotonically diverging.

If $b_1=0$, *i.e.* 1= b, a= 1, but 0<a<1, so $b_1=0$ is impossible.

When $\Delta<0, \lambda_1, \lambda_2=\alpha \pm i\beta$, Where $a=-b_1/2, b=\sqrt{-\Delta}/2$.

The general solutions of (8) are

$Y(t)=(C_1 cos(\beta t)\ +C_2 sin(\beta t))exp(\alpha t)+G/(1-a)$

If $\alpha>0$, *i.e.* $b_1<0$, *i.e.* 1 < b, the solutions are explosive oscillation

If $\alpha<0$, *i.e.* $b_1>0$, *i.e.* 1> b, the solutions are damped oscillation,

We have a periodic solution only when b = 1.

Similarly, this continuous-time model also has four dynamic regimes. Its pattern regimes are shown in Figure 2. Compared with the discrete-time Samuelson's model, the only difference is the changing of the periodic border.

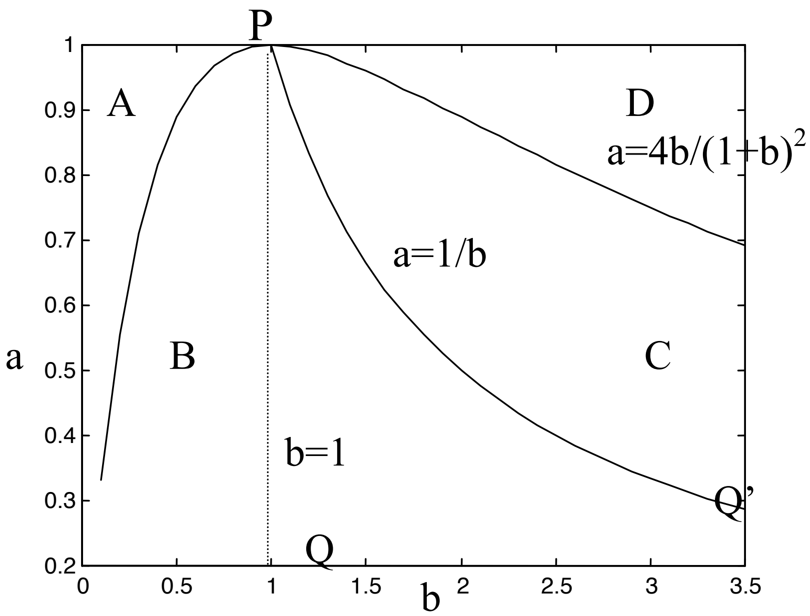

Figure 2. Continuous-time Samuelson's multiplier-accelerator diagram
The periodic boundary shifts from PQ' to PQ

4. The probability of return to equilibrium solution in Samuelson's multiplier-accelerator model

In the above analysis, we can find that the equilibrium solution of Samuelson's multiplier-accelerator model is the same as that of G/(1-a) for both discrete time and continuous time.

In the following, we discuss the probability of return to equilibrium solution in Samuelson's multiplier-accelerator model when the multiplier a and the accelerator b are in the range 0 <a <1 and 0 <b <= T. This probability is equal to the ratio of the area of the region returning to equilibrium solution to that of the whole region (0 <a <1 and 0 <b <= T).

From Figure 1 and Figure 2, we can find that the probability of return to equilibrium solution in Samuelson's multiplier-accelerator model is the same for both discrete time and continuous time, which is as follows.

When 0 <T <= 1, Prob = 100%.

When T>1, $Prob = \{4\left[ln\left(T+1\right) +1/\left(T+1\right)\right] - 4ln2-1\}/T.$

When T⟶+∞, Prob = 0.

Note that when T>1, 1/T<Prob<1.

It can be seen that the probability of return to equilibrium solution in Samuelson's multiplier-accelerator model is a continuous function of T and monotonically decreasing (see Table 1).

Table 1. The probability of return to equilibrium solution in Samuelson's multiplier-accelerator model

T	1	1.7	15	2785	2860
Prob	100.00%	98.94%	50.45%	1.00%	0.98%

Where T is the length of the range of the accelerator b, and Prob is the probability of return to equilibrium solution. When 0 <T <= 1, Prob = 100%. When T>1, Prob = {4[ln(T+1) +1/(T+1)] - 4ln2 -1} / T. When T⟶+∞, Prob = 0.

Conclusion

In this paper, we study the continuous-time Samuelson's multiplier-accelerator model and compare this continuous-time model with its classical discrete-time model. We find that although time scales do not affect the probability of return to equilibrium solution in Samuelson's multiplier-accelerator model, but time scales have an influence on the perfect symmetry of periodic motion.

The probability of return to equilibrium solution in Samuelson's multiplier-accelerator model is getting smaller with the increase of the range of the accelerator b (the length of the range is equal to T) when the multiplier a is greater than 0 and less than 1, regardless of discrete time or continuous time.

When 0<T<=1, the probability of return to equilibrium solution in Samuelson's multiplier-accelerator model is 100%. The perfect symmetry of periodic motion of the continuous-time Samuelson's multiplier-accelerator model occurs at b = 1 (of course T = 1 at this time), while the perfect symmetry of the periodic motion of the discrete-time Samuelson's multiplier-accelerator model does not appear.

When T>1, the probability of return to equilibrium solution in Samuelson's multiplier-accelerator model is greater than 1/T and less than 1. The perfect symmetry of periodic motion of the continuous-time Samuelson's multiplier-accelerator model occurs at b = 1 and the perfect symmetry of the periodic motion of the discrete-time Samuelson's multiplier-accelerator model occurs at a = 1/b.

When T tends to +∞, the probability of return to equilibrium solution in Samuelson's multiplier-accelerator model is astonishing zero.

Therefore, although the difference equations have large similarity with the differential equations, when abstracting economic phenomenon into mathematical models, the choice between the difference equation and the differential equation is not just a matter of convenience. We should carefully examine its empirical and theoretical foundation.

Ackowledgement

We would like to thank Ping Chen and Laura Ungureanu for helpful suggestions.

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DOI: http://dx.doi.org/10.14505/tpref.v7.2(14).05

FOREIGN DIRECT INVESTMENT IN LATIN AMERICA: THE CASE OF PERU

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Suggested Citation:

Metaxas, T., Kechagia, P. (2016). Foreign Direct Investment in Latin America: The Case of Peru, *Theoretical and Practical Research in Economic Field*, (Volume VII, Winter 2016), 2(14): 160-172. DOI:10.14505/tpref.v7.2(14).05. Available from: http://www.asers.eu/publishing/index.php/tpref

Article's History:

Received July, 2016; *Revised* August, 2016; *Accepted* September, 2016.

Abstract

The majority of the Latin American countries proceeded to political, financial and social reforms so as to improve their attractiveness and to absorb more foreign capitals in order to achieve economic growth. Among the developing countries of the region, Peru managed to attract significant amounts of foreign direct investment mostly because of the abundance of natural resources. Nevertheless, Peru did not manage to become the top foreign capitals destination among the Latin American countries for several reasons. It is observed that the foreign investors' dominant policy in FDI, the fact that the majority of the infrastructure realized in urban regions, the investment conditions, the external debt, the private sector and the ineffective use of natural resources held back the country's attractiveness to foreign investors.

Keywords: Peru, Latin America, Foreign Direct Investment, foreign capital

JEL Classification: F21, O16, R11, O54

1. Introduction

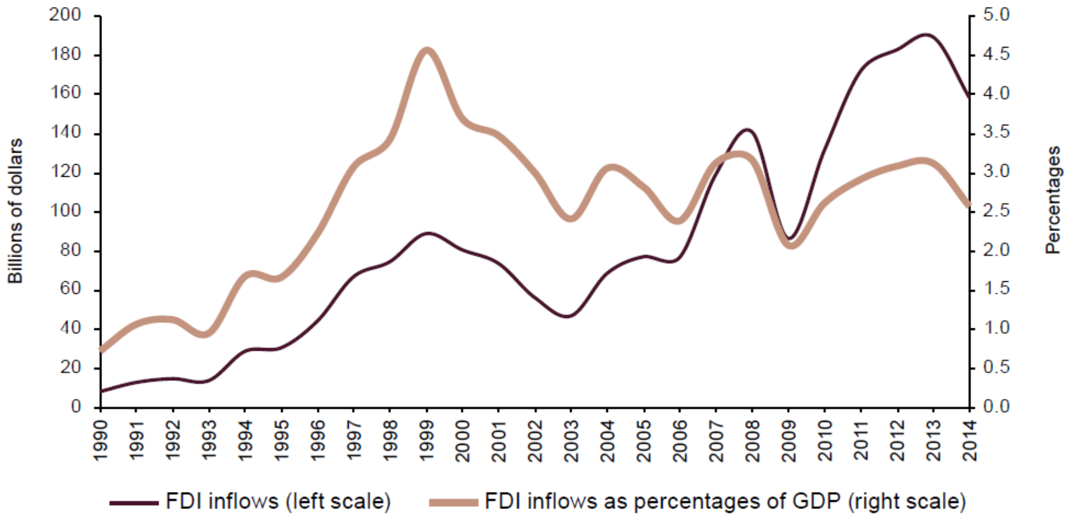
The Latin American and the Caribbean countries have been a centre of attraction of Foreign Direct Investment (FDI) over the past decades. Among the countries of the region it is observed (Olapido 2013, ECLAC 2015) that Brazil, Mexico and Chile received most of the inward FDI in the region, despite the fact that other countries, such as Peru proceeded so significant economic, social and political reforms in order to increase their attractiveness to foreign investors. Nevertheless, Peru, despite the fact that managed to attract significant amount of FDI inflows since the 90s, never became the top FDI destination in the region. As presented below, Peru ranks 4th until 2010 in FDI inflows (Olapido 2013), while it ranks 5th until 2014 (ECLAC 2015). Chavez and Dupuy (2010) also argued that during the past decades Peru has displaced from small foreign capital inflows player to significant FDI recipient. The purpose of the essay is the investigation of the reasons that held back the country's further attractiveness towards foreign investors and its evolution into the greatest FDI destination among the Latin American and Caribbean countries. We aim at examining the reasons that prevented Peru from becoming the highest ranked country in FDI inflows in the region, considering the significant political and financial reforms applied by the Peruvian government.

2. Foreign Direct Investment in the Latin American and the Caribbean countries

A growing number of developing countries attract FDI inflows, which enhance their financial growth, the macroeconomic stability, the governmental policy and infrastructure (Metaxas and Kechagia 2016). Over the past years the Latin American and the Caribbean countries have attracted a significant amount of FDI inflow, taking into consideration the economic problems observed worldwide because of the financial crisis (Olapido 2013). As a result, the countries of the region increased the Gross Domestic Product (GDP), achieved economic growth and reduced poverty rates. Moreover, during the period 1980 – 2010 there has been observed a positive relation between FDI and economic growth in a sample of six Latin American countries (Anaya and Alvaro 2012). In addition, FDI inflows from foreign investors in Latin America were not affected significantly because of the recent financial crisis (Leither and Stehrer 2013).

Thus, the characteristics of the Latin American countries that attract foreign capital have been studied. It has been investigated the relation among the FDI inflows in Latin America, the financial freedom and the economic development during the period 1970 – 1990 and it has been observed that the foreign capital inflow in the region is positively associated to the financial freedom of the host countries, while it has also been noticed that FDI inflow contributed to economic development for the countries that receive foreign capitals. Nevertheless, it is important for the host countries to afford human capital and to be economical stable and opened to international markets (Bengoa and Sanchez – Robles 2003).

The following figure presents the FDI flows in Latin America and in the Caribbean from 1990 to 2014. It is observed that the FDI flows in the region are characterized by a stable increase, except the year 2010, that the flows remained stable to the levels of 2009. In addition, we observe that from 2007 until today the scale of FDI inflows was higher than the scale of FDI inflows as percentages of GDP.

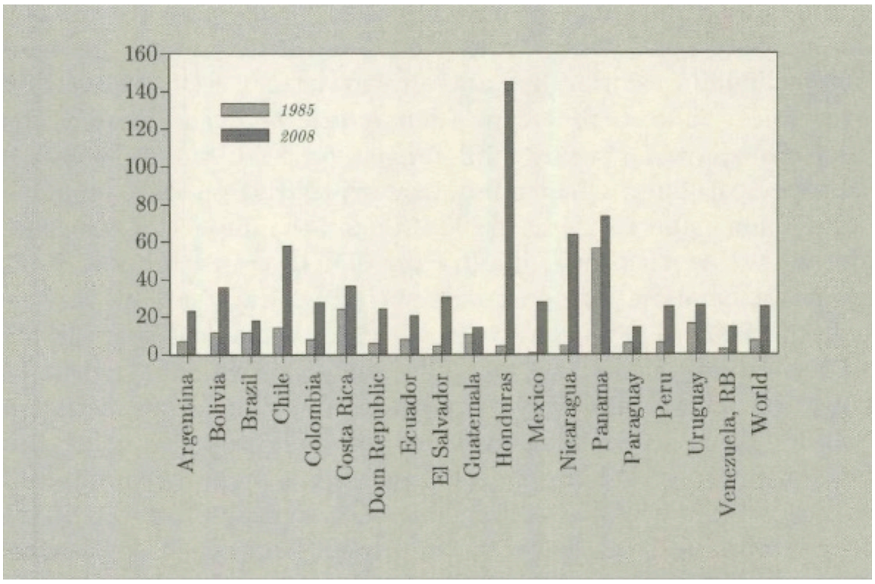


Source: ECLAC (2015)

Figure 1. Latin America and the Caribbean: FDI inflows (1990 – 2014) (Billions dollars and GDP percentage)

Moreover, during the time period from 1990 to 2010 it has been observed that the Latin American countries that received more foreign capitals were opened to international trade and they were characterized by a stable political and economic environment. On the contrary, the countries of the region that were not stable faced severe difficulties on attracting FDI inflow. Therefore, in order to increase stability, it is suggested that these countries apply policies of privatization and sovereign guarantees (Sanchez – Martin *et al.* 2014).

As presented in Figure 2, in 2008 most of the Latin American countries presented higher inward FDI stock as a percentage of GDP compared to 1985 and to other developing countries. In some cases, this ratio was higher than the world average (Subasat and Bellos 2013).



Source: Subasat and Bellos (2013)

Figure 2. Inward FDI stock as a percentage of GDP (1985 – 2006)

As for the remittances, it has been investigated whether they are positively related to FDI inflow. Thus, a study realized in a sample of 35 countries in three different regions (Latin America, Asia – Pacific and Africa) concluded that remittances do not influence the cross - country variation regarding foreign capital inflow (Basnet and Upadhyaya 2014).

Moreover, it is argued that the US multinational companies will continue investing their capitals in the Latin American countries since they are already aware of the characteristics of the specifics region from previous investments. In addition, capital inflows in Latin America have increased significantly over the past decade and they are expected to increase even further since multinational companies seek to expand new markets. In particular, the US investors aim at developing a free trade market in the Latin American countries through performing FDI and developing free trade agreements (Arbelaez and Ruiz 2013). Similarly, the US free trade market is expected to improve the financial systems of the Latin American countries, as well as their creditability and their investment environment (Armijo 2013).

Apart from the US investors, the Latin American countries have also been influenced significantly by the Chinese investors. In particular, US investors increased the amount of FDI flows performed in Latin America over the past two decades. Thus, China invested its capitals in certain Latin American industries, such as natural resources and infrastructures (Kotschwar 2014). However, Zegarra (2013) argued that the railroads in certain Latin American countries, including Peru which is studied in the following section, could be further improved so as to reduce the transportation costs.

Nevertheless, it is argued that the economic reforms realized by the Latin American countries could not necessarily attract foreign capital. Thus, the macroeconomic and the governmental measures taken by the countries of the region are not always associated with higher FDI inflows. It is possible that reforms such as privatization, capital liberalization or introduction of new taxation policies do not necessarily attract foreign investors' interest. On the contrary, it is suggested that trade liberalization and lower expropriation risks are more likely to attract foreign capital in the studied countries (Biglaiser and ReRouen 2006).

Furthermore, the taxation system plays a crucial role in attracting FDI. Thus, providing tax incentives, improving the taxation policy and lowering taxation coefficients will attract greater amount of foreign capital in the Latin American countries (Van Parys 2012). In addition, the macroeconomic stability of the Latin American countries also influences positively the foreign investors. It is observed that for the studied period from 1990 to 2005 there was a positive relation between FDI and economic development in Latin America (Adbelmalki *et al.* 2012). Furthermore, it has been observed a positive relation between FDI inflow and reform of intellectual property rights in the Latin American countries (Khoury and Peng 2011).

Another factor that should be taken into consideration when investing in the Latin American countries is the governmental policy. Thus, it is argued that good governance is taken into consideration by the countries that invest their capitals abroad. However, it is supported that poor governance is likely to attract FDI in both the transition economies and the Latin American countries. Therefore, it is suggested that apart from good governance several incentives should be provided to the investors so as to attract FDI, such as an effective bureaucracy and legal system (Subasat and Bellos 2013). Similarly, it is argued that there is a positive relation between FDI inflow and institutional quality in a sample of 19 Latin American countries (Fukumi and Nishkima 2010).

3. Foreign Direct Investment in Peru

Nowadays Peru is a fast – growing developing economy, which is characterized by medium per capita GDP, high human development index and macroeconomic stability. In addition, the country is listed among the countries that received most of the FDI inflow in the region of Latin America and Caribbean, as shown in Table 1, according to ECLAC (2012) report, as presented in Olapido (2013).

Table 1. Latin America and the Caribbean: FDI income by receiving country and GDP growth rate, 1980 – 2010 (millions of dollars and percentages)

Country	1980-1989		1990-1999		2000-2008		2009		2010	
	FDI	GDP	FDI	GDP	FDI	GDP	FDI	GDP	FDI	GDP
Brazil	25,438	2.99	61,369	1.70	182,052	3,72	25,948	-0,6	48,461	7.5
Mexico	8,590	2,29	44,821	3,38	203,398	2,82	15,206	-6,2	17,725	5,5
Chile	12,440	4,39	26,062	4,39	66,603	6,38	12,874	-1,7	15,095	5.2
Peru	**1,109**	**0,36**	**4,837**	**3,24**	**17,461**	**5,61**	**5.575,9**	**0,8**	**7328.0**	**8.8**
Colombia	2,092	3,40	8,830	2,86	32,861	4,41	7.137,2	1,7	6759.9	4.0
Argentina	4,323	-0,7	29,124	4,52	61,227	3,87	4.071,1	0,9	6193.0	9.2
Dominican Republic	352.2	3,79	129,2	4,89	5,148	5.33	2165.4	3.5	2625.8	7.8
Guatemala	1,108	0,97	2,273	4,07	4,200	3,75	573,7	0,5	678,3	2,8
Bolivia	574.1	-0,4	1,941	3,99	5,413	3.72	425.7	3.4	650.8	4.1
Trinidad & Tobago	1,881	-1,3	4,006	2,74	10,938	7,42	709,1	-3,5	549.4	0.2
Bahamas	566,1	4,04	1,026	1,64	5,087	1,68	664,0	-5,4	499,1	0,9
Ecuador	976.2	2,27	3,578	1,84	8,935	5,01	319,0	0,4	164.1	3.6
Barbados	128,4	2,23	224,4	1,87	675,3	1,2	159,7	-5,3	162,8	0,7
Haiti	109,1	0,01	127,0	-0,1	200,4	0,5	37,4	2,9	150,4	-5,1
El Salvador	179.7	−1.9	581.3	4.89	3,947	2.55	430.6	-3.1	89.0	1.4
Costa Rica	781.1	2,29	1,539	4,2	5.48	5,600	4.64	1322.6	-1.3	1412.0
Venezuela	1,932	-0,2	13,146	2,46	41,617	4,78	-3,105	-3,2	-1,404	1.5

Source: Olapido (2013), authors' calculations

Thus, it is noticed that Peru achieved high GDP growth rate during the studied period. Olapido (2013) argued that this GDP growth rate is positively influenced by the high domestic demand and the convenient external financing circumstances for the country.

In addition, it is suggested that the Peruvian economy has managed to attract FDI inflow because of the neoliberal regime implied and because of the export – oriented policy. Moreover, it is argued that over the past two decades several financial and political transformations have taken place, while the privatizations have increased significantly. Thus these transformations have led to economic growth and the country attracted significant amount of FDI inflow. In addition, the country applied neoliberal market strategies and therefore it managed to attract foreign capital inflow. Also, Peru achieved to incorporate to foreign markets and to increase significantly the development rates (Bury 2005).

It is also suggested that the national culture influences positively the amount of the FDI received. Thus, Rauch *et al.* (2013) argued that the Peruvian companies are innovative and that their owners have various cultural orientations which enable them to develop worldwide relationships. As presented in table 2, in several Latin American and Caribbean countries FDI inflows decreased during 2004 – 2014. Furthermore, it is observed that FDI inflows in the region were highly concentrated and therefore the majority of the foreign capitals were absorbed by solely five countries. When regarding to the countries that received more FDI, it is observed that Brazil managed to increase significantly the amount of foreign capitals invested in the country. Finally, we observe that smaller economies were able to attract proportionally larger amounts of foreign capitals, taking into consideration the size of the host country.

Table 2. FDI inflows in Latin America and the Caribbean by country and region
(Millions of dollars)

Economy	2004-2007	2008	2009	2010	2011	2012	2013	2014
South America	**50.074**	**95.388**	**59.194**	**95.113**	**133.487**	**146.901**	**128.322**	**119.502**
Brazil	21.655	45.058	25.949	48.506	66.660	65.272	63.996	62.495
Mexico	25.734	28.574	17.644	15.962	23.560	18.998	44.627	22.795
Chile	9.174	16.604	13.392	15.510	23.309	28.457	19.264	22.002
Colombia	7.247	10.565	8.035	6.430	14.648	15.039	16.199	16.054
Peru	**3.284**	**6.924**	**6.431**	**8.455**	**7.665**	**11.918**	**9.298**	**7.607**
Uruguay	1.001	2.106	1.529	2.289	2.504	2.536	3.032	2.755
Ecuador	449	1.058	308	163	644	585	731	774
Bolivia	111	513	423	643	859	1.060	1.750	648
Venezuela	1.713	2.627	-983	1.574	5.740	5.973	2.680	320
Paraguay	95	209	95	210	619	738	72	236
Central America	**4.891**	**7.406**	**4.442**	**5.863**	**8.504**	**8.864**	**10.680**	**10.480**
Panama	1.578	2.402	1.259	2.363	3.132	2.980	4.654	4.719
Costa Rica	1.255	2.078	1.347	1.466	2.176	2.332	2.677	2.106
Guatemala	535	754	600	806	1.026	1.244	1.295	1.396
Honduras	686	1.006	509	969	1.014	1.059	1.060	1.144
Nicaragua	290	627	434	490	936	768	816	840
El Salvador	547	539	294	-230	218	482	140	275
The Caribbean	**4.818**	**9.616**	**5.281**	**4.809**	**6.637**	**8.284**	**6.322**	**6.027**
Total	**85.517**	**140.984**	**86.561**	**131.746**	**172.190**	**183.047**	**189.951**	**158.803**

Source: ECLAC (2015), authors' calculations

When regarding to the case of Peru, we observed that from the year 2012 the FDI inflows reduced significantly. Nevertheless, it should be highlighted that the previous years the country received significant amounts of foreign capitals, taking into consideration that size of the Peruvian economy.

The characteristics of the multinational companies that choose to invest their capitals in Peru, as well as in other Latin American countries for the time period from 1988 to 1999 have also been investigated. Such characteristics mostly referred to the foreign countries macroeconomic and governmental policies. It is observed that the multinational companies studied for the specific time period applied a dominant strategy when investing in Peru. Thus, it is suggested that Peru should attract capitals from countries with minimum institutional differences, while it is argued that the Peruvian institutional policy reforms so as to attract more FDI (Trevino and Mixon 2004).

FDI in Peru for the time period 1979 – 1992 performed by the Japanese multinational companies were investigated by Tuman and Emmert (1999). The study conducted in 20 Latin American countries, among which Peru. The researchers observed that both the political and the financial situation in Peru were taken into consideration by the Japanese multinational companies so as to invest their capitals in the country, among which the market size, the financial adjustment strategies and the politically stable environment.

Furthermore, from 1990 the Peruvian economy intensified the efforts to attract foreign capital. Such efforts mostly focused on fighting against the populist Peruvian system, on reducing the foreign debt and on developing a stable political and financial environment. Hence, from the early 1990s the Peruvian government encouraged the price deregulation, adopted financial policies so as to reduce inflation and increased privatization. Therefore, the Peruvian economy became opened to foreign markets and achieved higher FDI inflow (Rojas 2001).

Moreover, it has been investigated whether FDI lead to financial development, focusing on the regions of Latin America and Caribbean. The study referred to 16 countries of the particular regions and on a 30-year time period. The research concluded that Peru is listed among the countries in which FDI lead to financial development. Additionally, the study concluded that there is bidirectional causality between FDI and economic growth in Peru (Olapido 2013).

Also, it is argued that the poverty rates in Peru have decreased by more than 10% over the past decades because of the foreign capital inflow (Castro *et al.* 2012). The foreign capital inflow in Peru has also affected positively the productivity rates and contributed to long term growth (Alfaro *et al.* 2008). Moreover, the improvement of the investment conditions and its relation to Peru's international integration has been discussed. Thus, it is argued that Peru should further improve its investment climate so as to achieve effective global integration (Dollar *et al.* 2006).

In addition, the FDI inflows in Peru by sector have been studied. In particular, Chavez and Dupuy (2010) investigated the inward FDI stock in Peru by sector and industry. They argued that during the studied period (2000 – 2009) the majority of the FDI stock was absorbed by the services sector. Thus, as presented in Table 3, the services industries attracted more than the half inward FDI stock, while significant amounts of FDI stock also attracted the energy, the manufacturing and the oil and mining sectors.

Table 3. Distribution of Peruvian inward FDI stock by sector and industry in 2000 and 2009 (US$ million)

Sector / industry	2000	2009
All sectors / industries	12.306	18.840
Primary	2.004	4.529
Mining, quarrying, petroleum	1.953	4.320
Mining, quarrying	1.855	3.964
Petroleum	98	356
Agriculture, forestry, fishing	51	209
Secondary	1.554	2.842
Manufacturing	1.554	2.842
Services	7.211	8.866
Communications	4.588	3.675
Finance	1.683	2.872
Construction and housing	60	718
Tourism	58	64
Transport	28	295
Other services	794	1.242
Energy	1.537	2.603

Source: Proinversion, Peruvian Investment Promotion Agency in Chavez and Dupuy (2010).

The sectoral FDI inflow in Peru has also been investigated by Gonzalez – Vicente (2012), who focused on the Chinese mining companies. The study focused on the criteria based on which the Chinese mining companies choose to invest or not on the Latin American countries, among which Peru. The researcher argued that the Chinese mining companies base their investment decisions on political and geostrategic criteria. The study is based on qualitative and quantitative data for a ten – year time period and on the use of case studies. The research concludes that the Chinese mining companies choose to invest on liberal economies, while they take into consideration the market risks and the opportunities before proceeding to a foreign investment. The case of the mining companies that choose to invest their capitals in Peru has also been investigated by Ericsson and Larsson (2012). The researchers suggested that Peru is the fifth larger recipient worldwide of foreign capital inflow when regarding to the Chinese mining companies in 2010, while it ranks sixth when regarding to the mining companies worldwide, as shown in Table 4.

Table 4. Top 10 Countries for Mining Investment, 2011

		Investment ($ billion)	Share (%)	Rank in 2010
1	Australia	99	15	1
2	Canada	92	14	2
3	Chile	54	8	4
4	Brazil	46	7	3
5	Russia	46	7	6
6	Peru	44	6	5
7	USA	32	5	8
8	South Africa	25	4	7
9	Philippines	17	3	9
10	Guinea	16	2	11
	Total	471	71	

Source: Ericsson and Larsson (2012)

It is notable that in 2013 Peru managed to attract higher inflows of foreign capital compared to previous years. It is estimated that the mining investment in the country increased by 13%, reaching to almost $50 billion, as presented in Table 5. Thus, the percentage increase was higher compared to other countries, among which Brazil and South Africa (Larsson and Ericsson 2014).

Table 5. Top 10 Mining Investments, 2013

		Investment ($ billion)	Share (%)	Rank in 2010
1	Canada	117	15	2
2	Australia	100	13	1
3	Russia	74	9	5
4	Chile	69	9	3
5	Brazil	57	7	4
6	Peru	49	6	6
7	USA	45	6	7
8	South Africa	25	3	8
9	Mexico	18	2	11
10	Philippines	17	2	10
	Total	571	72	

Source: Larsson and Ericsson (2014)

As shown in Table 6, Peru is listed among the countries that the Chinese mining companies choose to invest their capitals worldwide.

Table 6. Top Destinations for Chinese Mining FDI 2000 – 2010 (by number of controlled projects)

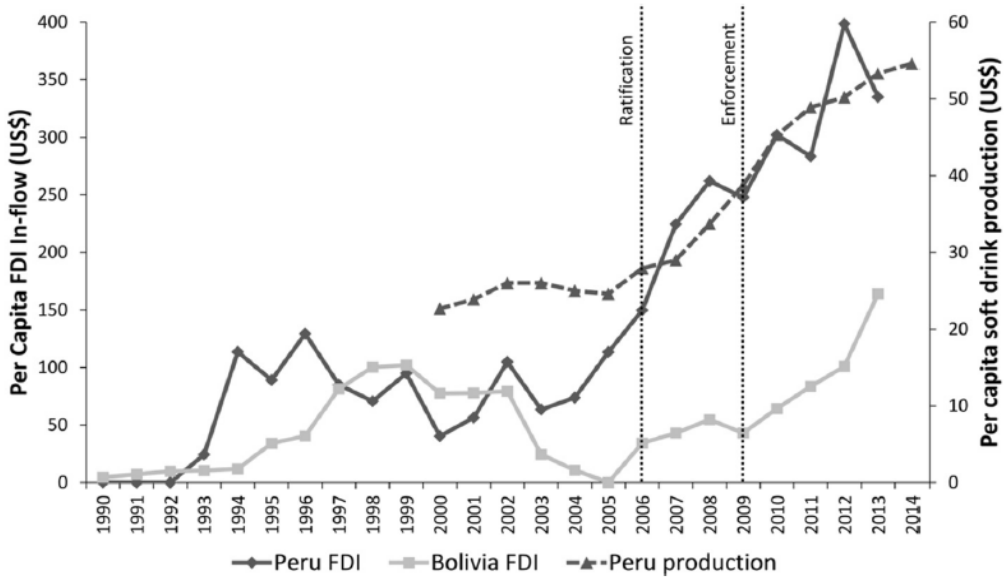
Country		Chinese projects	% Chinese projects	2000 – 2010 non – Chinese FDI M&Q (% world total)	Country's total mining projects
1	Australia	37	33,04	15,30	1,046
2	Canada	13	11,61	13,20	540
3	Tajikistan	7	6,25	0,10	27
4	Peru	6	5,36	3,30	188
5	Ecuador	4	3,57	0,10	29
5	Zimbabwe	4	3,57	1,60	59
7	Laos	3	2,68	0,30	8
7	Myanmar	3	2,68	0,00	12
Total	112	100.00			6,643

Source: Gonzalez – Vicente (2012)

Hence, it is observed that Peru is listed among the top destinations of foreign capital when regarding to the Chinese mining companies. Nevertheless, according to the study of Irwin and Gallagher (2013) the foreign capitals invested in Peru by the Chinese mining companies could possibly affect negatively the employment rates and the environmental conditions.

Moreover, the internationalization procedure of the Peruvian economy was investigated by Rivas and Mayorga (2011), who focused on the Peruvian restaurants. It is argued that the Peruvian restaurants increased their competitiveness worldwide by becoming multinational companies, which along of the economic recovery of the country attract foreign capital, while at the same time the expansion to foreign economies was facilitated.

Another study performed by Baker et al (2016) focused on the impact of FDI and trade liberalization on the Peruvian and Bolivian food industry. The study concluded that in Peru the free trade agreements influenced positively the foods and beverages sector, it diversified the products and thus the country managed to attract more foreign capitals. On the contrary, the Bolivian government did not proceed to free trade agreements and consequently did not attract more FDI in the specific sector, as presented in figure 3. It is observed that FDI inflows in Peru rose significantly during the period 1990 – 2013, while from 2004 the rate of increase was higher. In addition, it is argued that from 2006, that is to say the year that Peru ratified the free trade agreements, the per capita soft drink production rose significantly as well.



Source: Baker et al (2016)

Figure 3. Trends in per capita FDI and per capita soft – drink production in Peru and Bolivia

Apart from the mining industry and the food industry, the Peruvian economy also managed to attract foreign capital form the Chinese petroleum industry. Thus, the Chinese economy introduced a foreign strategy through investment its capitals in Peru, focusing on the oil and gas industry (Xu 2010). Similarly, the attraction of foreign capitals by the Peruvian petroleum industries has contributed to the reduction of the social inequalities in the country, while at the same time it has been supported the environment protection (Moser 2001).

Moreover, Peru managed to attract foreign capital so as to develop the hydroelectric industry. It is argued that the Peruvian government aimed at achieving sustainable development and social benefits through improving the hydroelectric infrastructure. Thus, the business climate has been improved so as to attract foreign capitals in the sector, while social and financial benefits were provided to the foreign firms (Cole and Roberts 2011). The FDI inflow attracted by the Peruvian infrastructure industry has also been studied. It is suggested that the foreign capital received contributed on improving the Peruvian infrastructure and therefore on the country's development. Moreover, it is observed that telecommunications, airports and electricity in Peru have been improved significantly; nevertheless, most of these improvements are observed in the urban regions (Urrunaga and Aparicio 2013).

However, it has investigated whether the FDI inflow could lead to financial instability for the Peruvian economy (Agudelo and Castano 2011). It is suggested that the foreign capital inflows in developing countries, such as Peru, result to economic instability, mostly during economic crises. Thus, for the time period 1999 – 2008 it is observed that there is no relation between the FDI inflow and the financial stability using time series models.

4. Overall Assessment

The Latin American and the Caribbean countries absorb an increasing amount of foreign capitals. In particular, in the specific region the foreign capitals enhanced the economic development, while the amount of them was not influenced significantly because of the recent financial crisis. The countries of the region that received the majority of the foreign capitals have certain common characteristics, including the efforts to achieve financial freedom and stability, the openness to the international trade and the measures that led to political stability. In addition, FDI inflows enabled the host economies so as to improve their infrastructure, to offer tax motives to the investors and to achieve macroeconomic stability. Moreover, host countries improve their governmental policy via receiving FDI inflows. In the studied economies it is observed that economic growth is achieved when receiving foreign inflows, which then lead to the reduction of the inequalities in the host country. The present paper focused on the case of Peru, which managed to attract significant amount of inward FDI; nevertheless, despite the political, social and financial reforms made, the country did not manage to become the top FDI destination among the Latin American and the Caribbean countries.

The Peruvian government proceeded to successful financial and political reforms over the past decades so as to improve the country's attractiveness towards foreign investors and thus to absorb more FDI inflows. Nowadays, the Peruvian economy enables the financial freedom and the openness to foreign markets and international trade. As a result Peru achieved sustained economic growth and improved the investment environment. Our study concludes that FDI in Peru are influenced by the factors presented in Table 7.

Table 7. Factors that influence FDI inflows in Peru

Factors	Peru
Type of regime	Neoliberal, open and transparent regime
Political and financial conditions	Stable political and economic environment
Ownership	Increased privatizations
Economic development	Achieved great economic growth
Socioeconomic indicators	Reduced the poverty rates, increased GDP

Therefore, we observe that the factors that attracted more FDI inflows in Peru regarded the country's type of regime, the political and financial situation, the ownership regime, the level of development and the socioeconomic indicators. When regarding to the type of regime, we conclude that the Peruvian government followed a neoliberal, open and transparent policy. The governmental policy of Peru is based on deregulation and liberalization in order to achieve goods and services mobility worldwide, to improve the productive abilities via attracting FDI and to achieve economic growth. Thus, the Peruvian economy managed to integrate successfully in the global market and to attract significant flows of foreign capitals.

In addition, the neoliberal Peruvian policies contributed to the country's independence from foreign economies, to the improvement of the social conditions and to the development of the key industries. Therefore, the Peruvian financial and social issues were not completely controlled by the state and the system applied could be characterized as self – regulating. In other words, the more liberalized Peruvian regime aimed at the economic development through promoting free trade, openness to foreign capitals and market orientation. The Peruvian government also protected the property rights of the foreign investors and guaranteed complete freedom on the foreign capital movement.

However, the freedom provided by the governmental regime was limited to financial and social issues, but even included free technological transfer. These liberated policies promoted the improvement of the Peruvian residents' living standards and consequently economic growth. It should though mentioned that the liberalization of the governmental policies could lead to instability since public control on financial issues could loosen and automatic stabilizers could be rendered ineffective. Thus, the Peruvian economy could be more vulnerable to economic crises, while social conflicts could arise.

In summary, we observe that Peru, despite the political, financial and legal reforms performed, never managed to rank first among the Latin American and the Caribbean countries. Therefore, we argue that various reasons held back the country's' attractiveness towards foreign investors, as presented in Table 8. Firstly, we observe that foreign investors that chose to invest their capitals in Peru applied a dominant investment policy. Therefore, the circumstances under which Peru absorb FDI played a significant role in the country's attractiveness. We argue that the multinational companies that used dominant investment strategies when investing in Peru influenced both the type and the quality of the foreign capital absorbed. As for the dominant policy applied by multinational companies and foreign investors, we suggest the investment of foreign capitals into high – value – added project so as to improve the local investment conditions, to adapt recent technological methods and to improve the educational level. Furthermore, we suggest the orientation of local companies towards R&D activities and the protection of the intellectual property rights. In addition, local enterprises should improve their capability in adapting modern technology imported from multinational firms.

Secondly, it is observed that infrastructure, including telecommunications, electricity and airports, realized mostly in the urban regions. Therefore, limited foreign capitals were invested in rural regions and a result this could affect negatively the living conditions of the habitants in the rural regions and increase the income inequality among them. We thus observe that the FDI distribution was not absolutely effective among the regions and it is possible that this inequality affected negatively the development of the rural areas. Moreover, foreign investors would be discouraged to invest in these areas because of the lack of infrastructure and the low level of development.

Thirdly, we argue that the investment climate of Peru could further be improved. The Peruvian government made efforts so as to ameliorate the investment conditions and to strengthen the country's relations with the trading economies. Hence, it is suggested that Peru should apply investment strategies so as to attract foreign investors and to make easier to export. Moreover, reforms in certain business sectors, such as the private sector, could include the protection of the intellectual property rights and the securing of licences. In order to ameliorate the investment climate obstacles for foreign investors should be abolished so that the Peruvian economy becomes even more opened. Thus, the attractiveness of Peru towards foreign investors would be higher. In addition it is suggested that the improvement of the investment environment could promote the domestic investments as well and therefore enhance the achievement of economic growth and the global integration.

Fourthly, we observed that despite the fact that the Peruvian government managed to reduce the external debt, a more effective foreign debt management policy should be applied. We argue that the efforts to reduce external debt should be intensified so that the country gains better access to the capital markets. Therefore, we suggest that the Peruvian government should avoid signing conditional loans from international organizations, as well as to avoid borrowing from other countries in order to avoid high economic deficits in the following years. Furthermore, the reduction of the foreign debt would boost the country's economic development, which would therefore attract more foreign capitals. In addition, Peru should satisfy its debt obligations and the country should proceed to negotiations and collaboration with its creditors.

Fifthly, we argue that the Peruvian private sector could further be developed so as to attract more foreign investors. The country managed to promote the trade liberalization, to improve the infrastructure and to promote a social policy through reducing the poverty rates and increasing the GDP. Thus, taking into account the fact that Peru increased its privatizations it is suggested that the country should develop a competitive private sector. It is important that privatizations take place in every Peruvian region, despite the low productivity rates observed in some of these regions. In addition, it is important to provide financial and technical facilities to the multinational companies that choose to invest their capitals in the country. Moreover, privatizations could further promote the productivity, the innovation, the entrepreneurship and the competitiveness.

Sixthly, Peru is abundant in natural resources since the country attracted foreign inflows in the petroleum, the oil and the gas industry, while it also improved the hydroelectric sector; nevertheless, we argue that the efforts to protect the available natural resources were limited, despite the fact that the country attracted significant amounts of inward FDI in sectors such as mining and quarrying. Hence, it is suggested that an effective management system should be applied for the Peruvian natural resources. The extraction on non – renewable materials should be managed, while at the same time the amount of raw materials exported should be controlled. In addition, the benefits deriving from the extraction of the natural resources should be distributed equally in the Peruvian region so as to avoid social inequalities and conflicts. Moreover, the climate changes should be considered so as to develop effective industrial development policies. The Peruvian trade could then be further developed and the country's competitiveness could be improved in actions, such as recycling, environment protection etc., were encouraged.

Table 8. The factors that prevented the attractiveness of Peru

Source		Deficit or problem
Foreign investors	Dominant policy	Improvement of the local conditions
Peruvian government	Infrastructures improved mostly in urban regions	More effective regional allocation of FDI inflows
Peruvian government	Further improvement of the investment environment	Promotion of an investment – friendly policy environment
Peruvian economy	External debt	Effective foreign debt management strategy
Peruvian business industry	Private sector	Increased privatizations
Peruvian government	Protection of the natural resources	Effective management system in natural resources

It should be noted that the present paper is subjected to certain limitations. The first limitation refers to the fact that the study focuses on the case of a single country and therefore it is difficult to generalize the findings. However, the case of Peru is chosen based on its ranking in total FDI inflows over the past years. Secondly, the study is limited on the Latin American and the Caribbean countries. As a result, we have not conducted a comparative analysis towards countries of other geographical regions, such as Asia, that proceed to various reforms but never managed to become the top FDI destination. In summary, we argue that the above mentioned limitation do not influence the study's findings.

Conclusions

Significant flows of foreign capitals are directed towards the Latin American and the Caribbean countries annually over the past decades. Nevertheless, the recipient countries faced severe obstacles and weaknesses in exploiting FDI for their benefit. Therefore, the countries of the region had to improve the microeconomic and macroeconomic stability, to improve the infrastructure, to absorb new technology methods and to improve the legal and political framework. Similarly, Peru proceeded to such reforms so as to attract more foreign capitals, to reduce the poverty rated, to improve the infrastructure and to achieve economic growth via increasing the GDP. Peru, in order to increase its attractiveness, made efforts so as to achieve macroeconomic stability, to control the foreign debt, to render its regime more democratic through fighting against populism and to become a politically stable country. We observe that these efforts, along with the abundance in natural resources and the neoliberal and export – oriented policy applied, increased the country's' attractiveness to foreign investors.

However, we argue that the efforts, so that the country increases further its attractiveness, should be intensified. We observed that efforts should be made in improving the local conditions, in allocating more effectively the regional FDI inflows, in promoting an investment – friendly policy strategy, in managing more effectively the external debt, in increasing privatizations and in managing more effectively the natural resources. Therefore, we conclude that efforts towards these directions could render Peru the top FDI destination among the Latin American and the Caribbean countries. In summary, the country should offer more generous financial, social and governmental incentives to the foreign investors. The further improvement of the political and macroeconomic conditions would improve its attractiveness. However, measures should be taken so as to protect the environment and to manage the resource reserves. Furthermore, in order to improve the political stability increased privatization is suggested. Hence, more financial and political measures are suggested so as to improve its rank among the region's top FDI destinations.

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DOI: http://dx.doi.org/10.14505/tpref.v7.2(14).06

AGGREGATION WITH SEQUENTIAL NON-CONVEX PUBLIC - AND PRIVATE - SECTOR LABOR SUPPLY DECISIONS

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Suggested Citation:

Vasilev, A. (2016). Aggregation with sequential non-convex public- and private-sector labor supply decisions, *Theoretical and Practical Research in Economic Field*, (Volume VII, Winter), 2(14):173-177. DOI:10.14505/tpref.v7.2(14).06. Available from: http://www.asers.eu/publishing/index.php/tpref.

Article's History:

Received August, 2016; *Revised* September, 2016; *Accepted* September, 2016.

Abstract:

This paper explores the problem of non-convex labor supply decisions in an economy with both private and public sector jobs. In contrast to Vasilev (2015a), the sectoral labor choice is made in a sequential manner. Still, the micro-founded representation obtained from explicit aggregation over homogeneous individuals again features different disutility of labor across the two sectors. Thus, there is little merit in the timing of the sectoral non-convex labor choice.

Keywords: indivisible labor; public employment; sequential lottery; aggregation

JEL Classification: J22, J45

1. Introduction

This paper explores the problem of non-convex labor supply decisions in an economy with both private and public sector jobs. In contrast to Vasilev (2015a), the sectoral labor choice is made in a sequential manner.

2. Model Setup

The theoretical setup follows to a great extent Vasilev (2015a), except for the timing of the sectoral labor supply decisions. The economy is static, there is no physical capital, and agents face a sequential non-convex decision in a two-sector economy. Since the focus is on a one- period world, the model abstracts away from technological progress, population growth and uncertainty. There is a large number of identical one-member households, indexed by *i* and distributed uniformly on the [0, 1] interval. In the exposition below, we will use small case letters to denote individual variables and suppress the index *i* to save on notation.

2.1 Households

Each household maximizes the following utility function

$$Max\left\{c,h^p,h^g\right\}\left\{ln[c^\eta+S^\eta]^{(1/\eta)}\right\}+\alpha ln(1-h^p-\bar{h}^g) \quad (2.1)$$

where c, S, h^p, h^g denotes private consumption, consumption of the public good, hours worked in the private sector, and hours worked in the government sector. The parameter $\alpha > 1$ measures the relative weight of leisure in the utility function. Total consumption is a Constant Elasticity of Substitution (CES) aggregation of private consumption and consumption of government services, where $\eta > 0$ measures the degree of substitutability between private and public consumption. Each household is endowed with 1 unit of time that can be allocated to work in the private sector, work in the government sector, or leisure

$$h^p + h^g + l = 1 \tag{2.2}$$

Labor supply in each sector is assumed to be discrete $h^p \in \{0, \overline{h}^p\}, h^g \{0, \overline{h}^g\}$. In contrast to Vasilev (2015a), within the period, each household decides first to look for a job in the private sector, and if unsuccessful, will search for work in the public sector. The wage rate per hour worked in the private and public sectors is w^p and w^g, respectively.

In addition to labor income income, households hold shares in the private firm and receives an equal profit share π. Income is subject to a (equal) lump-sum tax t, where $t = T$, with T *denoting* aggregate tax revenue. Therefore, each household's budget constraint is

$$c^j \leq w^j h^j + \pi - t, j = p, g \tag{2.3}$$

Households act competitively by taking the wage rates $\{w^p, w^g\}$, aggregate outcomes $\{C, S, H^p, H^g\}$ and lump-sum taxes $\{T\}$ as given. Each household chooses $\{c^j, h^p, h^g\}$ to maximize (2.1) s.t. (2.2)-(2.3).

2.2 Firms

There is a representative firm in the private sector producing a homogeneous final consumption good, which uses labor as an only input. The production function is given by

$$Y = F(H^p), F' > 0, F'' < 0, F'(\overline{H}^p) = 0, \tag{2.4}$$

where the last assumption is imposed to proxy a capacity constraint. The firm acts competitively by taking the hourly wage rate $\{wp\}$, aggregate outcomes $\{C, S, H^g\}$ and policy variable $\{T\}$ as given. Accordingly, $\{H^p\}$ is chosen to maximize static aggregate profit:

$$\max_{H^p \geq 0} F(H^p) - w^p H^p, \tag{2.5}$$

Given the assumption imposed on the production function, in equilibrium, the firm will realize positive economic profit.

2.3 Government

The government hires employees to provide public services and levies lum-sum taxes on households to finance the government wage bill. The technology of the public good provision uses labor H^g as an input, which is remunerated at a non-competitive wage rate $w^g = \gamma w^p$. Parameter $\gamma \geq 1$ will measure the fixed gross mark-up of government sector wage rate over the private sector one. Such a mark-up is a stylized fact for the major EU economies, *e.g.* Vasilev (2015b). The production function of public services is as follows:

$$S = S(H^g),\ S' > 0,\ S'' < 0,\ S'(\overline{H}^g) = 0, \tag{2.6}$$

where the last assumption guarantees that not everyone will work in the production of the public good. In addition, the public good is a pure non-market output, thus it will not appear in the government budget constraint.

The government runs a balanced budget: The public sector wage bill is financed by levying a lump-sum tax T on all households

$$w^g H^g = T. \tag{2.7}$$

In terms of fiscal instruments available at the government's disposal, the government takes total public sector hours, H^g, as given, and sets the public sector wage rate, w^g, as a fixed gross mark-up above the competitive wage rate. In a sense, the government faces a supply curve for labor in the public sector and determines the demand for government employees. Lump-sum taxes will be then residually chosen to guarantee that the budget is balanced.

3. Decentralized Competitive Equilibrium

Given the choice of T, a DCE is defined by allocations $\{c, h^p, h^g, S\}$, wage rates $\{w^p, w^g\}$, and firm's profit π s.t (i) all households maximize utility; (ii) the private firm maximizes profit; (iii) the government budget constrain is balanced; (iv) all markets clear.

4. Characterizing the DCE

Given the restrictions imposed on the production functions in the private and public sector goods, it follows that not everyone will be employed in the private sector in the first stage. Therefore, everyone doing the same - working or not working in the first stage - is not an equilibrium.

Proof: Case (1): For any positive and finite wage in the private sector, *i.e.* $0 < w^p < \infty$, both sectors will want to hire a bit of labor. Hence, $h^p = 0$ cannot be an equilibrium because firm will have a positive labor demand for any finite wage, and households will have zero consumption, $c = 0$, which is ruled out as an optimal choice from the monotonicity of the logarithmic utility.

Case (2): $h^p = \bar{h}^p$ only if $w^p = 0$, which follows from the assumptions on both production technologies. At such wage rates both the firm and government will want to hire everyone, but no household will want to supply any labor. Thus having everyone working is not optimal either. *QED*

Denote the proportion of households employed in the private sector by q. The rest of the households, $1 - q$, will go to the public sector to search for a job. Again, everyone working or not working in the second stage is not an equilibrium outcome. Thus, in the second stage, there will be a proportion λ of those who remained unemployed after stage one, or $(1-q)\lambda$ of all households, that will be employed in the public sector. The proportion λ will be chosen optimally to equate the utility of those who will be employed in the public sector, and those who will not. (This is a sort of rationing scheme, or a lottery. We assume that the government cannot hire all the people who do not work in the private sector.) To achieve this, and to guarantee that this sequential choice is time-consistent, we have to solve the game backwards. In this way households who decide not to work in the private sector cannot end up getting less utility. Alternatively, the government knows q from the first stage, when it chooses $\lambda(1-q)$in the second stage that will equate the utilities of the all the three groups of individuals. In game-theoretic language, q is a best (Nash) response to λ and λ is a best (Nash) response to q.

Hence, if there is a DCE, it must be that in equilibrium not everyone will get the same private consumption. Still, everyone consumes the same level of public good, as it is assumed to be non-excludable and non-rivalrous. The households that work will have higher utility of private consumption, while those who do not work will enjoy more utility from leisure. Lastly, every household belonging to the same type will enjoy the same level of total utility.

Therefore, we will consider an equilibrium in which q of the people are employed in the private sector, and λ of the people who go to public sector are employed in the second stage, $0 < q + (1-q)\lambda < 1$. Thus, $H^p = q\bar{h}^p, \ and \ H^g = \lambda(1-q)\bar{h}^g$.

From the firm's optimization problem, we obtain the expression for the competitive hourly wage

$$F'(q\bar{h}^p) = w^p \tag{2.8}$$

Hence, there will be positive economic profits amounting to

$$\pi = \Pi = F(q\bar{h}^p) - F'(q\bar{h}^p)q\bar{h}^p > 0, \tag{2.9}$$

which follow from the assumption that the production function features decreasing returns to scale. Next, equilibrium government output is

$$S = S(\lambda(1-q)\overline{h}^{g}), \tag{2.10}$$

and lump-sum tax revenue equals

$$T = w^{g}\lambda(1-q)\overline{h}^{g} = \gamma w^{p}\lambda(1-q)\overline{h}^{g} = \gamma F'(q\overline{h}^{p})\lambda(1-q)\overline{h}^{g}. \tag{2.11}$$

Now we will show the existence of a unique pair (*q, λ*) ∈ (0, 1) × (0, 1) by analyzing a system of two non-linear equations. Those equations use the equality of utility of those who work and those who do not in the same sector. Households in the private sector are indifferent between working or not working:

$$\begin{aligned} &ln[(F'(q\overline{h}^{p})\overline{h}^{p} + F(q\overline{h}^{p}) - F'(q\overline{h}^{p})q\overline{h}^{p} - T)^{\eta} + (S(\lambda(1-q)\overline{h}^{g}))^{\eta}]^{\frac{1}{\eta}} + \alpha ln(1-\overline{h}^{p}) = \\ &= ln[(F(q\overline{h}^{p}) - F'(q\overline{h}^{p})q\overline{h}^{p} - T)^{\eta} + (S(\lambda(1-q)\overline{h}^{g}))^{\eta}]^{\frac{1}{\eta}} + \alpha ln(1) \end{aligned} \tag{2.12}$$

Similarly, in the second stage, unemployed households are indifferent between working or not in the public sector

$$\begin{aligned} &ln[(w^{g}\overline{h}^{g} + F(q\overline{h}^{p}) - F'(q\overline{h}^{p})q\overline{h}^{p} - T)^{\eta} + (S(\lambda(1-q)\overline{h}^{g}))^{\eta}]^{\frac{1}{\eta}} + \alpha ln\,(1-\overline{h}^{g}) = \\ &= ln[(F(q\overline{h}^{p}) - F'(q\overline{h}^{p})q\overline{h}^{p} - T)^{\eta} + (S(\lambda(1-q)\overline{h}^{g}))^{\eta}]^{\frac{1}{\eta}} + \alpha ln(1) \end{aligned} \tag{2.13}$$

Substitute out the public sector wage rate with its equivalent expression from the government budget constraint

$$w^{g}(q) = \gamma w^{p}(q) = \gamma F'(q\overline{h}^{p}). \tag{2.14}$$

Then do the same for the lump-sum taxes to obtain

$$T(q,\ \lambda) = w^{g}\lambda(1-q)\overline{h}^{g} = \gamma F'(q\overline{h}^{p})\lambda(1-q)\overline{h}^{g}. \tag{2.15}$$

Next, proving existence and uniqueness of optimal (*q, λ*) ∈ (0, 1) × (0, 1) follows trivially from the Brower's Fixed Point and the assumptions on the functional forms of utility and production functions. Note that there are a lot of equilibria (in terms of the "names" of the people working), all of them with the same fraction of population *q* working in the private sector, w and *λ*(1 − *q*) working in the public sector. Let c^j, c_n, *j* = *p, g* denotes the private consumption of individuals that work in each sector, and those who do not. Note that those who do not work have not been selected to work in the private sector during the first stage, and then have remained unemployed after the second stage.

Because of the presence of the public good and the non-convexities, the First Welfare Theorem does not hold, so this equilibrium is not PO. Therefore, there exists an alternative allocation that a SP could choose that can make everyone better off. More specifically, the Social Planner (SP) can improve upon the initial equilibrium allocation by giving each household a consumption level independent of the fact whether they worked or not. For example, giving everyone

$$c = qc^{p} + (1-q)\lambda c^{g} + (1-q)(1-\lambda)c_{n} \tag{2.16}$$

is Pareto improving, as the new consumption allocation is feasible, and gives a higher utility in expected terms, hence perfect insurance is achieved.

After equalizing private consumption across states, aggregate utility function becomes

$$U = ln[c^{\eta} + s^{\eta}]^{\frac{1}{\eta}} + q\alpha ln(1-\overline{h}^{p}) + (1-q)\lambda\alpha ln(1-\overline{h}^{g})$$

Notice that $H^p = g\bar{h}^p$, then $q = H^p / \bar{h}^p$. Similarly, $H^g = \lambda(1-q)\bar{h}_g$, then $\lambda(1-q) = H^g / \bar{h}_g$. Substitute those expressions into the utility function to obtain

$$U = ln[c^\eta + s^\eta]^{(1/\eta)} - AH^p - BH^g$$

where A and B are functions of model parameters. As it turns out, the assumption of sequentiality does not change the form of the aggregate utility function obtained in Vasilev (2015a).

Conclusions

This paper explores the problem of non-convex labor supply decisions in an economy with both private and public sector jobs. In contrast to Vasilev (2015a), the sectoral labor choice is made in a sequential manner. Still, the micro-founded representation obtained from explicit aggregation over homogeneous individuals again features different disutility of labor across the two sectors. Thus, there is little merit in the timing of the sectoral non-convex labor choice.

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DOI: http://dx.doi.org/10.14505/tpref.v7.2(14).07

THE ECONOMIC POWER OF VETO PLAYERS – THE CONNECTION BETWEEN FISCAL POLICIES, AND POLITICAL SYSTEMS

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Suggested Citation:

Wasniewski, K. (2016). The economic power of veto players – the connection between fiscal policies, and political systems, *Theoretical and Practical Research in Economic Field*, (Volume VII, Winter), 2(14):178-204. DOI:10.14505/tpref.v7.2(14).07. Available from: http://www.asers.eu/publishing/index.php/tpref.

Article's History:

Received June, 2016; *Revised* July, 2016; *Accepted* July, 2016.

Abstract

The present paper explores the correlation between political systems, and fiscal policies, focusing on those changes in fiscal policy that may be induced by modifications in the partisan structure of political systems, as compared to cross-sectional differences between constitutional systems. The government's fiscal stance is studied chiefly as capital balances, rather than current flows. The theoretical model defines the possible appropriation of liquid assets in the public sector, starting from the basic fiscal equation. Empirical research consists of both a quantitative, econometric part, and qualitative case studies. Quantitative research allows concluding that political systems do differ as for the amount of liquid capital held by the public sector. Three broad clusters of countries are defined, regarding their political systems, and these clusters display a significant disparity as for their observable fiscal stance. Case studies sampled from those clusters lead to conclude that the amount of liquid assets held by the public sector changes in close correlation to political polarization. The main path open for future research is the question whether fiscal variables can indicate pre-emptively the emergence of political veto players, even before their official appearance in the partisan, or the constitutional structure.

Keywords: fiscal policy, political systems, institutional economics

JEL Classification: H3, H6, H11

1. Introduction

In the evaluation of fiscal policies, economists frequently use the basic distinction into the cyclical factors, and the structural ones. The latter are understood as the broadly spoken institutional context of the economy, with the political system seen as a distinct, and significant, institutional factor. Still, actual fiscal policies seem to be largely irrational, as governments tend to be de facto restrictive when fiscal expansion would be advisable, and vice versa; political factors appear as a distortion to the optimal fiscal action. The present paper attempts to study that distortion starting from the pork barrel theory, as introduced by Barry Weingast *et al.* (1981), where public expenditures are made of financial flows appropriated by the most influential social groups represented in the political system. In other words, the partisan structure of the political system is supposed to impact significantly the actual fiscal policy, which, in turn, is seen as a game of claims on capital.

2. Theoretical background

For any given economy, it is possible to draw an economically optimal path of fiscal action (Arestis and Sawyer 2003, Arestis 2009). Yet, most governments diverge significantly, in their actual fiscal policies, from that optimum (Barro 1990, King and Rebelo 1990, Turnovsky 2000, Afonso and Claeys 2007). On the other hand, econometric analysis can easily demonstrate that the variance of fiscal aggregates is not fully explained by the variance of cyclical macroeconomic variables, *e.g.* unemployment, inflation, current account balance etc. The resulting, residual values of fiscal aggregates are usually called 'structural', as opposed to cyclical. Thus, the actual fiscal stance of probably any government in the world contains a component, which is relatively autonomous vis-a-vis the economic cycle, and seems sub-optimally rational in the context of fiscal optimization. That component is commonly associated with the political system in place. James Buchanan was probably one of the first economists to show a systemic approach to the interaction between fiscal policy and politics, with a focus on the descriptive side of the problem (Buchanan 1976). In 1989, Roubini and Sachs introduced the concept of structural debt to GDP ratio, as opposed to the cyclically determined one (Roubini and Sachs 1989). They made a connection between that structural tendency to indebtedness, on the one hand, and the characteristics of the political system in place - more specifically to the relative strength of the government in wage bargaining. The more dispersed political power, the lesser that strength, and the greater proneness to the accumulation of public debt. In empirical terms, that theoretical construct was used to explain increasing, but unevenly increasing public debt in the developed countries over the 1970s and the 1980s. This path of research, attempting to make a theory about that interaction, has been developing steadily over the last three decades, yet staying more or less in the shadow of the cyclically-oriented school in the research about fiscal policy (Schick 1998a, 1998b). The main outcome of studies at the frontier of politics and macroeconomics is that the fiscal adaptation to cyclical economic factors is always limited in its breadth and scope by structural attributes of the political system. The works of Alberto Alesina and Roberto Perotti, among others, attempt to delve into the fine details of politics at the level of public finance (Alesina and Perotti 1995a, 1995b, 1996).

The present paper attempts a contribution to constructing theoretical tools for the prediction of actual fiscal policies, according to the characteristics of the political system in place, and according to the future possible changes in that system. In other words, some kind of fiscal function of the political system looms at the horizon. The first, and probably the most important question is whether at all, and for what reasons should we consider the characteristics of political systems as structural, regarding fiscal policies, and oppose them to cyclical factors like inflation, exchange rates or unemployment. In other words, are politics really more "structural" than, for example, the patterns observable in the labour market or in the stock exchange? After all, and especially in the case of developed countries, the long terms trends of inflation or unemployment are frequently more predictable than public policies and political institutions.

Three intuitive distinctions seem to underlie the general, theoretical dichotomy between the cyclical and the structural factors of fiscal policies. Firstly, macroeconomic changes impact the actual tax revenues. The same tax base yields different tax revenue, and creates various pressures for public borrowing, according to the rate of inflation, economic growth and unemployment. Secondly, it is possible to demonstrate econometrically that in the overall variance of the actual fiscal balance there is a part clearly attributable to the variance of macroeconomic factors. The remaining residual is called "structural balance". Thirdly, there is the "exogenous <> endogenous" dichotomy. We tend to consider political factors as structural just because we see them as inherent to the given, national social system. On the other hand, factors that we use to call cyclical (inflation, unemployment etc.) are perceived as at least partly exogenous and imposed by global economic trends.

That last distinction (*i.e.* endogenous <> exogenous) seems to be the soundest approach, yet there is a theoretical doubt to elucidate. If political factors were considered to be strictly endogenous, there would be no point in any cross-sectional comparison, or in any congruent measurement of political characteristics (Almond 1956). There is need for some kind of common denominator(s) in political systems. That common denominator is to be found both in the classics, and in the most recent developments. As for the classics, there is an old claim, postulated by Francois Quesnay, Adam Smith, David Ricardo, and Jean Baptiste Say: public spending is rather a form of circulating capital than a financial flow strictly spoken. The assumption that governments manage capital balances rather than flows seem to be constantly present in the discourse about public policy. The French historical school (Braudel 1981), as well as to its newest developments in economics (Piketty 2013) claim very much the same. Strangely enough, that assumption can be used for quite disparate argumentations, as, for example, those about the burden created by public debt. Both the advocates of debt seen as a burden (Meade 1958, Modigliani 1961, Diamond 1965), and the partisans of the so-called Ricardian equivalence (Sraffa 1951; Barro 1990, 1989a, 1974, 1979, 1986, 1987, 1989b) implicitly assume that public borrowing creates some sort of additional liquidity in the capital balances of the economy. As far as public debt is concerned, it is interesting to point out that borrowing is essentially a legal tool for facilitating the transfer of capital. Once more, we return to the classics, and to Adam Smith's intuition that massive public borrowing takes place when there is a wealthy class of private owners with substantial financial surpluses to invest. In other words, governments borrow mostly because they can, not because they have to. Over 80% of the global, gross public debt is owed by 6 highly developed economies: United States, Japan, Germany, France, United Kingdom, and Italy. One possible interpretation of this stylized fact is that the biggest economies absorb the major part of global macroeconomic shocks, and thus accumulated the greatest mass of public debt. Still, another explanation is possible. In the times of economic shocks, investors are risk averse; they turn massively away from risky investment in productive assets and shift their investment towards much less risky sovereign debt. Hence, massive accumulation of public debt occurs not only as a symptom of financial distress on the part of the governments, but also as a shift in investment strategies played in financial markets.

The second theoretical cornerstone to consider is the issue of rational policy. From the point of view of an economist, governments are largely irrational in their fiscal action. Macroeconomic optimization of public policies assumes that fiscal policy consists most of all in managing the current macroeconomic forces that impact the fiscal performance. Basic fiscal aggregates – spending, tax revenues and debt – are supposed to be the cumulative effect of a series of current, short term adaptations, on the part of the government, to the macroeconomic situation at hand. In this view, budgetary deficit happens or increases when the government fails to match its spending to tax revenues. An implicit assumption here is that the null fiscal balance, with spending equal to tax revenues, is the fiscal equilibrium. Any departing from that heavenly state is a symptom of disequilibrium. Consequently, public borrowing is a compensatory action on the part of the government, to stay financially liquid when in financial distress.

In political sciences, this approach to fiscal policy is somehow reflected in the so-called "pork barrel theory" (Weingast *et al.* 1981), which states that the relative disparity of interests among social groups represented in the political system impacts government expenditure. Just as more different pigs mean the necessity of a bigger barrel to feed them, a greater diversity of social interests represented in the political system means more money necessary to satisfy them. That relationship has its source in the phenomenon of political bargaining between various agents inside the government. The "*I-do-something-for-your-voters-if-you-do-something-for-my-voters*" pattern of policy making is widely known, and the pork barrel theory associates it with fiscal policies. Once more, just as in the macroeconomic optimization of fiscal policies, we come to the implied assumption that there is a hypothetically efficient fiscal policy (probably associated with null budgetary deficit). Both the cyclical economic factors, and the political bargaining may deviate the actual fiscal policy from that state of equilibrium.

Thus, we face those two possible approaches to fiscal policy. It can be viewed as the management of current flows, with the null fiscal balance being the state of equilibrium. Conversely (or complementarily), fiscal policy may be considered as public management of capital balances. Here we come back to the connection between political systems, and fiscal policies. Roubini and Sachs argue that political systems differ in their inherent aptitude to deal with macroeconomic shocks: countries with more dispersed political power tend to be feebler in their response to such shocks than countries with noticeably more concentrated political power. Conversely, the present paper argues that political systems differ in their fiscal stances mostly because there is a close match between political systems, and the amount of capital those systems need to work. In that respect, the research follows the theoretical path set by Barry Weingast and his claim that political systems work to a large extent as a game of claims on capital, played between public and private agents (Weingast 1981, 1995). The present paper claims that each individual political system rests on a specific amount of legitimation conferred to public rulers through a long-term, discursive process, as well as on a specific appropriation of capital by the public sector. Jurgen Habermas's theory of politics (Habermas 1975, 1979, 1996), and Nancy Fraser's concept of "strong publics" (Fraser 1990) are an additional theoretical reference in that respect. Appropriation of capital is understood as such a natural possession of capital, which through its duration and autonomy from external claims allows the appropriation of capital rent.

Two interesting intellectual streams have been emerging recently, and they seem quite promising for the purposes of the present research: the theory of veto players, and the initiative known as the Database of Political Institutions. The theory of veto players, as introduced by George Tsebelis (2002), characterises the current state of any political system through its relative capacity for policy change, or, conversely, its tendency to preserve the status quo. Any status quo has its counterpart in the so-called winset, which is the set of its realistically taken alternatives. The success of any policy, whether in favour of the status quo, or of its winset, stems from the number and strength of veto players, who support it. Veto players are "*individual or collective actors whose agreement is necessary for a change of the status quo*". The current status quo rests on the unanimity core, or the set of actions that meet common agreement of all the veto players. As they all agree about the policies contained in the unanimity core, none of them supports any change to it. The more veto players are there in the political system, the larger is the unanimity core, and the least is the likelihood of policy change. In other words, more veto players give more inertia to current policies. Veto players can emerge or disappear through a twofold mechanism of institutional change. Firstly, they can be institutional veto players, designated as such through the legal rules of the constitutional order. Secondly, they can emerge as partisan veto players, i.e. political forces that form within the space given to political rivalry by the given set of constitutional rules. Ideological distances between individual veto players are just as important for the formation of policies, as the constitutional division of powers. For example, the ideological distance between the president in office, and the majoritarian party in the parliament may influence the policy making just as strongly, as would do the constitutional partition of powers between the president, and the parliament. A substantial body of research claims that the partisan fragmentation within a given constitutional framework has significant impact upon public policies (Weingast 1981, Mukherjee 2003, Bawn and Rosenbluth 2006, Eslava and Nupia 2010).

The theory of veto players seems to be one of the theoretical pillars of the Database of Political Institutions, (DPI), as published by the World Bank (Beck *et al.* 2001, Keefer 2012). The DPI attempts a long-expected classification of political systems. The main axes of discrimination seem to be internal diversity, and stability. The former refers to variously measured number of distinct veto players, both constitutional and partisan. As for structural stability, it is the opposite of democratic competition in the system.

Constitutional orders change slowly, and not very frequently. On the other hand, the partisan structure of political systems may change much faster. An interesting question appears in that context, namely whether at all and to what extent can a change in the partisan structure within a given constitutional order impact the pattern of capital appropriation in public agents, and what overall changes in fiscal policy can such a change provoke. This is the predictive issue that the present paper attempts to explore.

The theoretical model of the issues studies aims at showing, how exactly does the process of capital appropriation in the public sector take place. The starting point is the basic fiscal equation, namely: *revenues* + *borrowing* = *expenditures*. That basic equation is modified for the purposes of the present research, by adding a second component on the right side, namely capital accumulation. Equation (1) formalizes this modified view, where T stands for revenues (which are usually and for the major part made of tax revenues), ΔD means current borrowing or the change in the overall gross public debt, E represents gross expenditures of the government, and ΔC is the change in capital held by the government.

$$T + \Delta D = E + \Delta C \qquad (1)$$

Equation (1) follows the stylized fact that most governments, whatever their current fiscal flows, hold some capital goods other than the strictly spoken financial means necessary to finance current expenses. The right side of the equation represents the numerical outcome of a structure at work. The structure in question is made of four types of public entities, namely: budgetary units, public executive agencies, public targeted funds, and public-private partnerships. Budgetary units are the building blocks of the strictly spoken administrative structure in the public sector. They are fully financed through the current budget of the government, and fully accountable within one fiscal year. They use capital only for financing current expenditures, and their appropriation of capital is based on the "*use it of lose it*" rule within the real budgetary cycle. The latter means that the next year budget is drafted during the second quarter of the preceding fiscal year, and voted in the fourth quarter. Hence, the full cycle of capital appropriation in budgetary units is actually rather two fiscal years than one. The institution of consolidation in current public accounts can create an additional, shorter cycle of capital appropriation in budgetary units. This institution consists in the right, conferred usually to the Minister of Finance or other organ in charge of Treasury matters, to consolidate all the temporarily available, financial balances of budgetary units on one account, and to use that account for short-term, financial placements (*e.g*. overnight deposits).

Public executive agencies follow specific missions ascribed by specific laws distinct from the budget, and from the regulations of fiscal governance. These laws form the legal basis of their existence. The mission of executive agencies usually consists in carrying out long-term tasks connected to large non-wage expenditures. The distribution of targeted subsidies, or the maintenance of strategic reserves of food or medicines are good examples. Public executive agencies have more fiscal autonomy than budgetary units: they receive subsidies from the current budget, but these subsidies usually do not make the full financial basis of their expenditures. In the same manner, those agencies can retain their current financial surpluses over many fiscal years. In other words, the financial link of executive agencies with the current fiscal flows is fluid and changing from one budgetary cycle to another. The cycle of capital appropriation in executive agencies is essentially equal to their actual lifecycle as separate units.

Targeted public funds are separate public entities in charge of managing specific masses of capital paired with specific public missions to carry out. Just as executive agencies, targeted funds have a separate legal basis of their own. Their specificity consists in quite a strict distinction in their accounts: all the current costs of governance should be covered out of the financial rent of the capital managed, and the possible budgetary subsidies should serve only to back up the financial disbursements directly linked to the mission of the given fund. The distinction between executive agencies and targeted funds may be fluid: some agencies are de facto funds, and some funds are actually agencies. The central assumption to retain is that both appropriate capital quite independently from the current budgetary cycle.

Public-private partnerships are joint ventures, through which private agents are commissioned to carry out specific public missions, in exchange of subsidies, direct payments or specific rights. One of the most obvious examples are contract-based healthcare systems, in which private providers of healthcare services are commissioned to fulfil the constitutional mission of the state to provide for citizens' health. Subtler schemes are possible, of course. Private agents may provide, with their own financial means, for the creation of some infrastructure commissioned by the government, and their payment is the right to exploit said infrastructure.

The point of all that structural specification is to demonstrate that the broad category of fiscal flows that we use to call "public expenditures" (mostly for the sake of convenience) is actually a financial compound. It covers both the expenditures strictly spoken (*i.e.* current payments for goods and services), and capital outlays that accrue to many different pockets of capital appropriated by public agents in many different ways. Capital accruals have different cycles, ranging from the ultra-short (days or weeks) cycle of consolidated accounting in budgetary units, passing through the mid-range cycle of appropriation in executive agencies and public-private partnerships, up to the frequently many-decade long cycle of capital appropriation in targeted public funds. Each of those pockets of capital makes a unit of economic power, in the hands of some public agents. Each accrual to from such a capital pocket means a shift up or down in the actual economic power of those agents. Equation (2) formalizes this step in theoretical development, with the acronyms BU, EA, TF, and PP standing, respectively for: budgetary units, executive agencies, targeted funds, and public-private partnerships. Equation (3) goes further in this path and states that the total stream of financial inflows to public treasury, through current revenues and current borrowing, is congruent with the sum of the strictly spoken public expenditures, and capital accruals in the public sector.

$$E_{BU}+\Delta C_{BU}+E_{EA}+\Delta C_{EA}+E_{TF}+\Delta C_{TF}+E_{PP}+\Delta C_{PP}=E\quad+\Delta C \tag{2}$$

$$T\quad+\Delta D\quad=E_{BU}+\Delta C_{BU}+E_{EA}+\Delta C_{EA}+E_{TF}+\Delta C_{TF}+E_{PP}+\Delta C_{PP} \tag{3}$$

Following the distinction marked in equations (2) and (3), two basic models of budgetary accounting are possible: the cash-based on the one hand, and the accrual based one on the other hand. The cash-based model largely prevails in the world, and allows public agents to record officially their fiscal flows only when, and just when cash is spent or received, respectively as for public expenditures and public revenues. Conversely, the accrual-based model compels public agents to record fiscal flows when the corresponding economic events take place; in other words, it assumes that the emergence of liabilities or claims on the part of public agents is equivalent to actual cash flows. The cash-based system, still applied in the fiscal practice of most countries, allows public agents to keep some bills unpaid until the creditors become impatient in legal terms, thus to create and illusion of good fiscal performance in the meantime, and to pump up gross public indebtedness. Similarly, that system allows leaving some tax claims without actual enforcement, thus creating a discreetly governed system of unofficial tax crediting for the chosen ones. In terms of the theoretical model of the present research, the cash-based system is the very foundation of all the small, semi-hidden pockets of capital present in the public sector. It also encourages the phenomenon known as budgetary slack, which consists in deliberate, financial underperformance on the part of public agents, in order to obtain or to retain more capital than they actually need to carry out the mission assigned (Jensen 2003). On the other hand, the accrual-based system creates the obligation to consider liabilities and claims as actual flows, even if cash does not change hands. By the same means, it shaves off most of those little pockets of capital.

As for the left side of the fiscal equation, the present model assumes a deep, qualitative distinction between current public revenues (mostly taxation) and borrowing, understood as accruals. Public revenues are based on unilateral, valid claims on the part of the government. In a balance sheet, those claims are located among the liquid assets held by the public sector. Conversely, borrowing is a form of capital transfer, made on a contractual base. Public debt is a liability, and thus is to be found on the passive side of the balance sheet. Public debt of any given country is a mass of capital that has been transferred to the government, from the private sector. Even if the past borrowing had been spent long ago in cash flow terms, liabilities remain. That can be seen as liquidity transfer from the public sector to the private one: when governments borrow, and spend the capital borrowed, they pay for goods and services supplied by the private sector, but keep on endorsing the liability resulting from borrowing. It is important to remember, among others, that a substantial part of public spending, namely the wages of public officers, are technically paid to private persons employed at the corresponding jobs. Thus, the payroll of the public sector is a cash transfer to the private sector, too.

Summing up, the theoretical model applied in the empirical research presented further, assumes that the fiscal stance of any government represents two different types of financial occurrences: current flows and capital accruals. They can be partly independent from each other, and studied separately.

Any veto player in the political system derives their actual political power from two factors, namely political legitimation, and actual economic power conveyed by the natural possession of capital. Veto players in the political system temporarily appropriate each capital accrual in the public sector. For the purposes of the present research, it is further assumed that said appropriation is significantly additive, *i.e.* the more veto players in the system, the more capital they need to support their political legitimation.

3. Quantitative empirical research

The basic idea behind quantitative empirical research introduced in this chapter was to verify the assumption that political systems differ with respect to liquid capital balances held by the public sector, and that changes in the partisan structure of the political systems are correlated to changes in those capital balances.

Table 1. Structure of the sample used in empirical research, by country and number of year - observations

Country	Number of year - observations	*Country*	Number of year - observations	*Country*	Number of year - observations
Algeria	12	Hungary	8	Nigeria	13
Australia	24	Iceland	31	Norway	33
Austria	25	Ireland	33	Pakistan	11
Bahrain	23	Iran	17	Panama	10
Belgium	33	Israel	13	Peru	13
Belize	11	Italy	25	Poland	18
Bolivia	13	Japan	33	Portugal	16
Bośnia and Herzegovina	15	Jordan	25	Qatar	23
Brazil	13	Kazachstan	11	Saudi Arabia	14
Bulgaria	13	Kenya	15	Solomon Islands	10
Canada	33	Korea	12	South African Republic	13
Cape Verde	11	Latvia	13	Spain	28
Chile	20	Liban	13	Swaziland	12
Columbia	14	Lesotho	8	Sweden	20
Denmark	18	Liberia	13	Switzerland	30
Egypt	11	Libya	23	Syria	21
Estonia	17	Lithuania	13	Trinidad and Tobago	13
Ethiopia	21	Malawi	8	Turkey	11
FYR Macedonia	14	Maledives	16	Ukraine	15
Fiji	21	Mali	13	United Arab Emirates	14
Finland	33	Mexico	15	United Kingdom	33
France	30	Morocco	17	United States	12
Germany	22	Namibia	7	Uruguay	10
Ghana	12	Netherlands	18	Yemen	14
Greece	20	New Zealand	28	Zambia	8
Guyana	6	Niger	18		

Source: author's

The general empirical basis for this research was the Database of Political Institutions, (DPI), as published by the World Bank (Beck *et al.* 2001, Keefer 2012). The DPI was the background, against which fiscal, and macroeconomic data was projected, mostly on the grounds of the World Economic Outlook database (WEO), as published by the International Monetary Fund in April 2015. A sample of countries has been selected so as to cover a broad range of cases, besides just the developed economies. The sample consisted of 77 countries, the list of which, in a structured form, is given in Table 1. Due to the limitations of that database, the general span of observation covered the years 1980 - 2012.

The first, somehow preliminary step of empirical research was to establish a classification of political systems, according to the previously introduced, theoretical distinctions. The classification of political systems for the purposes of the present research starts with the constitutional approach, and follows into the partisan one. In order to represent the basic constitutional structure of political systems, two variables have been selected in the rich structure of the Database of Political Systems. The first is the type of political system according to the presence and powers of the president, coded in the DPI as 'system'. The second is the type of electoral competitiveness in parliamentary elections, covered by the variables 'plurality', and 'proportionality' in the DPI.

The distinction between presidential systems, and the parliamentary ones takes into account two main categories of veto players: institutional, and partisan. In other words, veto players can emerge and change their relative impact upon the system following to patterns: the regulatory, constitutional definition of their role, and the discretionary freedom of political action offered by that role. In presidential systems, the president is a strong veto player, and tends to concentrate power in their hands. Conversely, parliamentary systems are based on diversified and dispersed political power, without that one central veto player in the presidential seat. Systems with assembly – elected presidents are an interesting hybrid of the two, probably prone to balance towards the concentration, or the dispersion of political power, following the personal talents of the president in place.

The general assumption is that regimes with a strong component of electoral plurality favour "winner-takes-it-all" elections. This, in turn, promotes the interests of big, strong political parties, making them strong veto players, and reduces the veto playing positions of small parties. In other words, plural electoral regimes tend to reduce the overall number of partisan veto players, but they confer important impact to the players who manage to enter the scene. On the other hand, proportionality in elections allows a broader representation of small political parties and non-partisan representatives in the legislative body. That creates more veto players with more disparate political power.

As for the partisan structure of the political system, the most general variable in the DPI seems to be political polarization, more specifically: POLARIZ, and POLARIZ_STRICT. They are compound variables, based partly competitiveness in the appointment of legislators and executives of the government (variables: LIEC and EIEC), and partly upon the maximum difference between the chief executive's party's value (EXECRLC) and the values of the three largest government parties and the largest opposition party. The latter valuation is made on the grounds of the basic distinction of economic programs into: conservative, Christian democratic, communist, socialist, social democratic, and centrist. We have POLARIZ = 0, when the democratic competitiveness is below a critical level, as well as when the ruling party does not focus on economic issues or when there is no clear information. Otherwise, and according to doctrinal differences, the variable can take the value of 1 or 2.

The detailed composition of the sample of observations studied is given in Table 2. A few general comments on that structure seem pertinent before passing to properly spoken quantitative analysis. Just as in the full contents of the DPI, the sample studied is dominated by three big clusters: plural electoral regimes in presidential systems with no observable polarization (N = 233 observations), plural electoral regimes in parliamentary systems, with no observable polarization as well (N = 205), and strongly polarized, parliamentary systems with proportional elections (N = 213). This clustering suggests that plural electoral regimes favour the formation of partisan structures around groups of interest rather that around ideological stances. That appears as a logical consequence of the "winner-takes-it-all" principle in plural elections, which favours big electoral funds and robust campaigns, and clearly discourages ideological discords. Those observable clusters are an indication for further quantitative analysis, to consider those three clusters as three dominant types of political systems.

Constitutional orders with assembly – elected presidents are very feebly represented in the sample: 57 observations across both electoral regimes and all the possible cases of partisan polarization. Hence, this category can be treated rather as an exception that an important case. Still, an interesting pattern emerges systems with assembly – elected presidents, namely the absence of moderately polarized partisan structures. This particular class of political systems displays either no polarization at all, or a very pronounced one.

Table 2. Structure of the sample regarding political systems, following the constitutional, and the partisan partition

	Political system		
Electoral regime	***Presidential***	***Assembly – Elected President***	***Parliamentary***
Plural elections	***POLARIZ = 0>>N = 233*** *Bahrain 2003 – 2012; Bolivia 2006 – 2012; Chile 2002 – 2009; Egypt 2006 – 2011; Ghana 2005 – 2001; Islamic Republic of Iran 1996 – 2012; Jordan 1990 – 2009; Kazakhstan 2009 – 2007; Kenya 1998 – 2007; Korea 2005 – 2012; Lithuania 2000, 2004; Malawi 2005 – 2012; Maldives 1997 – 2009; Mali 2000 – 2002; Mexico 1998 – 2000; Morocco 1996 – 2012; Nigeria 2000 – 2012; Pakistan 2003 – 2008; Panama 2003 – 2012; Poland 1998 - 2007; Swaziland 2007 – 2012; Syria 1990 – 2010; United States 2001 – 2010; Yemen 2000 – 2012; Zambia 2005 – 2011* ***POLARIZ = 1>>N = 35*** *Bolivia 2003 – 2005; Brazil 2007 – 2012; Chile 1993, 2010 – 2012; Korea 2001 – 2004; Mexico 2001 – 2006, 2010 – 2012; Niger 1995, 1996; Poland 2011 – 2012; Ukraine 1998 – 99, 2000 – 2002* ***POLARIZ = 2>>N = 33*** *Bolivia 2000 – 2002; Brazil 2000 – 2006; Chile 1994 – 2001; Ghana 2001 – 2004; Maldives 2010 – 2012; Mexico 2007 – 2009; Poland 2008 – 2010; United States 2011, 2012*	***POLARIZ = 0>>N = 25*** *Egypt 2004 – 2005; Greece 1980 – 1986; Lebanon 2000 – 2012; Yemen 1999* ***POLARIZ = 1>>*** ***No records*** ***POLARIZ = 2>>N = 4*** *Pakistan 2009 - 2012*	***POLARIZ = 0>>N = 205*** *Australia 1989 – 2010; Belize 2002 – 2012; Canada 1981 – 2004, 2012; Ethiopia 1996 – 2000, 2006 – 2012; Fiji 2000 – 2001; France 1983 – 1986, 2003 – 2012; Greece 1987 – 1999; Hungary 2007 – 2012; Italy 1995 – 96, 2002 – 2008; Japan 1981 – 83, 1987 – 2012; Lesotho 2000 – 2002, 2008 – 2012; New Zealand 1985 – 1994; Spain 1985 – 93, 2001 – 2004, 2012; Trinidad and Tobago 2001 – 2012; United Kingdom 1980 – 2010;* ***POLARIZ = 1>>N = 12*** *Hungary 2005 – 06; Italy 1994 , 1997 – 2001; Latvia 2007 – 2010* ***POLARIZ = 2>>N = 104*** *Australia 1999 – 2012; Canada 1980, 2005 – 2011; France 1987 – 2002; Germany 1991 – 2012; Greece 1993; Italy 2009 – 2012; Japan 1980, 1984 – 86, 1994 – 96; New Zealand 1995 – 2012; 1994 – 2011; Trinidad and Tobago 2000; United Kingdom 2011 - 2012*
Proportional elections	***POLARIZ = 0>>N = 59*** *Algeria 2003 – 2007; Cape Verde 2002 – 2011; Kazakhstan 2008 – 2012; Liberia 2000 – 2003; Namibia 2006 – 2012; Niger 2010 – 2011; Peru 2000 – 2012; Poland 1998 – 2006; Ukraine 2011 – 2012; Uruguay 2005 – 2012* ***POLARIZ = 1>>N = 17*** *Algeria 2001, 2002, 2008 – 2012; Cape Verde 2012; Israel 2001; Peru 2001 – 2006; Poland 1996 – 97* ***POLARIZ = 2>>N = 7*** *Colombia 1999 – 2002; Israel 2000; Uruguay 2003 - 2004*	***POLARIZ = 0>>N = 25*** *Bulgaria 2000 – 2001; Estonia 1996 – 1999; Guyana 2007 – 2012; South Africa 2000 – 2012;* ***POLARIZ = 1*** ***No records*** ***POLARIZ = 2*** ***N = 2*** *Estonia 2000 - 2001*	***POLARIZ = 0>>N = 29*** *Bulgaria 2010 – 2012; Ireland 1980 – 81, 2007; FYR Macedonia 2003 – 2011; Portugal 2006 – 2009; Turkey 2003 – 2012* ***POLARIZ = 1>>N = 62*** *Finland 1991 – 95, 2003 - 2011; Iceland 1988 – 91, 2000 – 2007; Ireland 1988 – 94, 1998 - 2012; Israel 2002 – 2003; Italy 1988 – 1993; Latvia 2000 – 2006* ***POLARIZ = 2>>N = 213*** *Austria 1988 – 2012; Belgium 1980 – 2012; Denmark 1995 – 2012; Finland 1980 – 90, 1996 – 2002, 2012; Iceland 1982 – 99, 2008 – 09; Ireland 1982 – 87, 1995 – 97; Israel 2004 – 2012; Netherlands 1995 – 2012; Norway 1980 – 2012; Portugal 1997 – 2012; Sweden 1993 – 2012; Turkey 2002*

Source: Database of Political Institutions

Thirdly, and this seems the most important for predictive purposes, countries studied tend to stay quite firmly within one pattern of constitutional order, over the period of observation, yet they frequently move between various cases of partisan polarization. Thus, it confirms one of the theoretical intuitions expressed in the introduction, namely that predicting the fiscal function of partisan political structures might have a greater practical value that the prediction of outcomes brought by constitutional changes.

The next step of empirical investigation was to follow the disparities of typical fiscal aggregates across political systems. Fiscal aggregates have been divided into two categories, namely current and capital, following the basic intuitions of the present research. The structural fiscal balance, gross public revenues, and gross public expenditures are classified as current aggregates, *i.e.* rather flows than balances. Conversely, gross and net public debt is considered as capital measures (balances rather than flows). The differential between gross and net debt, namely the amount of financial assets held by the public sector, is included in this category too. The author is aware of the conceptual risk connected to that variable. Those financial assets include, for a large part, those held by central banks as monetary reserves. Thus, this could be a monetary variable rather that a fiscal one. Yet, the amount of those financial assets in public hands is not exclusively monetary, in the first place, and, secondly, it impacts significantly the fiscal, borrowing capacity of the government. Hence, this is a variable at the fringe of fiscal policy, and the rest of the economy.

As for gross public revenues, parliamentary systems are clearly tax-greedier than the presidential ones (Table 3). They also seem much more sensitive to political polarization: any increase in that respect is connected to significantly higher public revenues. Gross public expenditures follow a similar pattern, and yet, within each constitutional order, they seem much more sensitive than revenues to shifts in political polarization. The observation of structural fiscal balances seems to indicate that the shift from no observable polarization to moderate one has more impact than a further deepening of polarization from moderate to high.

Table 3. Average values of current fiscal aggregates, % of the GDP

Gross public revenues			
	Political system		
Electoral regime	***Presidential***	***Assembly – Elected President***	***Parliamentary***
Plural elections	POLARIZ = 0>>26,827; POLARIZ =1>>26,918; POLARIZ = 2>> 26,736	POLARIZ = 0>>24,471; POLARIZ =2>>13,579	POLARIZ = 0>>36,904; POLARIZ =1>>41,829; POLARIZ = 2>>40,563
Proportional elections	POLARIZ = 0>>29,308; POLARIZ =1>>31,545; POLARIZ = 2>>28,757	POLARIZ = 0>>29,818; POLARIZ = 2>>35,298	POLARIZ = 0>>34,338; POLARIZ =1>>42,733; POLARIZ = 2>>48,972
Gross public expenditures			
	Political system		
Electoral regime	***Presidential***	***Assembly – Elected President***	***Parliamentary***
Plural elections	POLARIZ = 0>>29,052; POLARIZ = 1 28,707; POLARIZ = 2>>30,884	POLARIZ = 0>>34,103; POLARIZ = 2>>20,113	POLARIZ = 0>>40,072; POLARIZ = 1>>46,832; POLARIZ = 2>>42,453
Proportional elections	POLARIZ = 0>>30,156; POLARIZ = 1 33,067; POLARIZ = 2>>32,029	POLARIZ = 0>>31,55; POLARIZ = 2>>34,134	POLARIZ = 0>>37,476; POLARIZ = 1>>45,796; POLARIZ = 2>>49,926
Structural fiscal balance			
	Political system		
Electoral regime	***Presidential***	***Assembly – Elected President***	***Parliamentary***
Plural elections	POLARIZ = 0>>-2,651; POLARIZ = 1>>-2,155; POLARIZ = 2>>-4,247	POLARIZ = 0>>-16,676; POLARIZ = 2>>n.a.	POLARIZ = 0>>-3,643; POLARIZ = 1>>-4,305; POLARIZ = 2>>-1,726
Proportional elections	POLARIZ = 0>>-1,913; POLARIZ = 1; -2,676; POLARIZ = 2>>-2,036	POLARIZ = 0>>-2,607; POLARIZ = 2>>n.a.	POLARIZ = 0>>-4,108; POLARIZ = 1>>-3,159; POLARIZ = 2>>-3,089

Source: author's

Variables referring to capital accruals in the public sector display a significantly greater disparity across political systems than current flows do. In other words, the empirically observable differences between political systems as for their patterns of capital appropriation are noticeably more pronounced than differences referring to current fiscal management. It proves that the central assumption of the present paper is a material, real distinction worth further research (Table 4).

Table 4. Average values of capital fiscal aggregates, % of the GDP

Gross public debt			
	Political system		
Electoral regime	***Presidential***	***Assembly – Elected President***	***Parliamentary***
Plural elections	POLARIZ = 0>>55,186; POLARIZ = 1>>48,927; POLARIZ = 2>>55,383	POLARIZ = 0>>112,071; POLARIZ = 2>>60,978	POLARIZ = 0>>70,739; POLARIZ = 1>>75,684; POLARIZ = 2>>53,648
Proportional elections	POLARIZ = 0>>90,934; POLARIZ = 1>>39,432; POLARIZ = 2>>63,105	POLARIZ = 0>>40,553; POLARIZ = 2>>4,954	POLARIZ = 0>>42,491; POLARIZ = 1>>53,612; POLARIZ = 2>>62,025
Net public debt			
	Political system		
Electoral regime	***Presidential***	***Assembly – Elected President***	***Parliamentary***
Plural elections	POLARIZ = 0>>46,036; POLARIZ = 1>>31,829; POLARIZ = 2>>41,441	n.a.	POLARIZ = 0>>47,574; POLARIZ = 1>>65,77; POLARIZ = 2>>35,039
Proportional elections	POLARIZ = 0>>70,994; POLARIZ = 1>>18,32; POLARIZ = 2>>56,36	n.a.	POLARIZ = 0>>34,581; POLARIZ = 1>>18,165; POLARIZ = 2>>15,254
Financial assets held by the government (gross debt minus net debt)			
	Political system		
Electoral regime	***Presidential***	***Assembly – Elected President***	***Parliamentary***
Plural elections	POLARIZ = 0>>9,151; POLARIZ = 1>>17,098; POLARIZ = 2>>13,942	n.a.	POLARIZ = 0>>23,165; POLARIZ = 1>>9,915; POLARIZ = 2>>18,609
Proportional elections	POLARIZ = 0>>19,94; POLARIZ = 1>>21,112; POLARIZ = 2>>6,745	n.a.	POLARIZ = 0>>7,91; POLARIZ = 1>>35,447; POLARIZ = 2>>46,771

Source: author's

Following the observable clustering of political systems in the sample studies, three "big" types are defined for the purposes of further empirical investigation. They are:

a) **Cluster #1**: Presidential systems with plural elections, and no observable political polarization: structural balance -2,651% of GDP, gross public indebtedness 55,186% of the GDP, financial assets held by the public sector 9,151% of the GDP

b) **Cluster #2**: Parliamentary systems with plural elections, and no observable political polarization: structural balance -3,643% of GDP, gross public indebtedness 70,739% of the GDP, financial assets held by the public sector 23,165% of the GDP

c) **Cluster #3**: Parliamentary systems with proportional elections and high political polarization: structural balance -3,089% of GDP, gross public indebtedness 62,025% of the GDP, financial assets held by the public sector 46,771% of the GDP

The definition of those 3 clusters shows even more sharply the explanatory power of capital appropriation as a characteristic of political systems. The interesting, general observation is that cluster #1, which hosts the least veto players in the system, seems to be the most frugal in fiscal terms, both with respect to current fiscal management, and to capital appropriation. Any shift from this cluster, thus any addition of veto players, through constitutional rules or partisan polarization, is clearly associated to more profusion in fiscal stances. Considering constitutional and partisan distinctions as an overall indicator of the number of veto players in the system, we can roughly consider clusters #1, and #3 as the opposite poles of the scale, with cluster #2 found somewhere in the middle. Cluster #1 has probably the least veto players, cluster #3 has the most of them, and cluster #2 is a medium case. Following this intuition, the presence of more veto players in the political system is associated most of all to a much greater tendency of the public sector to accumulate liquid financial assets.

The next step in empirical research was to assess the impact of fiscal policies upon selected socio-economic phenomena, in different political systems. The phenomena in question are: the accumulation of private savings, the allocation of said savings in productive assets (gross investment), the structure of the labor market as represented with the rate of vulnerable employment, and primary completion rate in the educational system. Five explanatory variables have been selected to be included in a linear, multiple regression model, namely: the structural fiscal balance, gross public debt, the amount of financial assets held by the government, gross public revenues, and gross public expenditures. Pre-emptive, econometric tests showed that gross public debt and net public debt are mutually redundant in this regression. One of these two had to be selected, and the choice was gross public debt. It shows the total amount of capital transferred to the public sector of a country with the help of obligatory contracts. Each of the explained variables has been regressed on the explanatory ones through the Ordinary Least Squares method. Standardized values of all the variables have been used, to provide for non-stationary trends. As for all the outcome variables, a constant residual was assumed to exist. The software used for statistical computations was Wizard for Mac OS. For each outcome variable, the results of regression in the general sample were compared with the three clusters identified in the previous subchapters. Detailed results of regression tests are presented in tables 5 – 8, after their general presentation to be found in the paragraphs here below.

In the general sample (N = 721), the formation of gross national savings (as defined by the International Monetary Fund) in the private sector is under a significant influence of fiscal variables. The constant residual is negative: without the crowding out effect of fiscal policies, savings would display a downward trend. There is a clear, and mostly positive link between the expansion of fiscal aggregates and the accumulation of private wealth. Two interesting phenomena are to notice. Firstly, among all the fiscal aggregates taken into account the structural balance is the only one to display rather a weak link to the formation of private savings. Secondly, gross public revenues are positively linked to private savings, whilst public expenditures display a negative link. In other words, taxation seems to be positively linked to private wealth, whilst public expenditures are rather a substitute to private accumulation.

As the same regression is run in sub-samples defined according to the previously adopted classification of political systems, one central observation is to note: the strength of the econometric connection increases, both at the level of the overall R^2 accuracy, and as for the t Student significance in particular variables. The link between fiscal policies and the formation of private savings seem to be system–specific. Interestingly, the overall explanatory power of fiscal variables grows as the cluster studied moves towards a greater number of veto players. A pattern emerges: the more veto players in the political system, the greater the impact of fiscal policy upon the formation of savings.

As the present research is very much oriented on capital appropriation in the public sector, gross public debt and financial assets held by the public sector deserve a closer look as explanatory factors. In the general sample, both have positive impact upon private savings. The influence of gross public debt seems relatively weak, while the accumulation of financial assets by public agents is a strongly positive and significant factor. When split into the three clusters, the same regression shows really disparate results. In the clusters #1 and #2, gross public debt seems to be rather a disturbing factor regarding private savings, whilst in the cluster #3 it is strongly and positively correlated. The accumulation of financial assets in the public sector changes its impact upon private savings from cluster to cluster as well.

As we pass form the formation of private capital to its allocation in productive assets through investment, the first salient observation is the generally lower explanatory power of fiscal variables. Just as in the case of private savings, that explanatory power grows as we plunge into particular clusters of political systems. Once more, a system – specific response to fiscal policy is to notice. In clusters #1 and #2, fiscal variables seem to be mostly a disturbing factor to private investment, whilst in the cluster #3 the relationship seems to be more stable. In other words, the more veto players in the political system, the more predictable the impact of fiscal policies upon private investment.

The primary completion rate is probably the most “social” and the least “economic” among the four outcome variables studied in this subchapter. It is also the least explained by fiscal variables in the general sample. With an R^2 equal to 0,041, there is hardly any connection. Still, when going into specific clusters, the correlation significantly gains in robustness, and each cluster displays a different pattern of correlation. Just as in the case of private savings, as we move from cluster #1 to #3, thus as we add veto players in the system, the explanatory power of fiscal factors grows.

The rate of vulnerable employment is astride the social and the purely economic outcomes of fiscal policies. In the general sample, the explanatory power of fiscal variables is pretty strong. Differently from the previous outcomes under scrutiny, transferring the analysis to specific clusters does not unequivocally increase that explanatory power. Only the cluster #3 displays stronger correlation that the general sample. Still, one can notice the same phenomenon of the explanatory power gaining in strength, as more veto players are present in the system.

Table 5. Results of regression tests as for the explanation of gross national savings, as defined by IMF

Explained variable: std(Gross national savings, as defined by IMF) SAMPLE: GENERAL N = 721 R^2 = 0,323			
Explanatory variable	**Coefficient of regression**	**Robust standard error**	**Significance level as given by t Student test**
std(Gross public debt, % GDP)	0,08	(0,036)	p = 0,028
std(Gross public expenditures, % of GDP)	- 0,46	(0,072)	p < 0,001
std(Gross public revenues as % of the GDP)	0,473	(0,072)	p < 0,001
std(Structural fiscal balance (% of GDP)	0,035	(0,028)	p = 0,205
std(Financial assets held by the government, % of GDP)	0,2	(0,024)	p < 0,001
Constant	- 0,152	(0,022)	p < 0,001
Explained variable: std(Gross national savings, as defined by IMF) SAMPLE: CLUSTER #1N = 71 R^2 = 0,408			
Explanatory variable	**Coefficient of regression**	**Robust standard error**	**Significance level as given by t Student test**
std(Gross public debt, % GDP)	0,363	(0,248)	p = 0,147
std(Gross public expenditures, % of GDP)	- 1,73	(0,388)	p < 0,001
std(Gross public revenues as % of the GDP)	1,623	(0,413)	p < 0,001
std(Structural fiscal balance (% of GDP)	- 0,482	(0,138)	p < 0,001
std(Financial assets held by the government, % of GDP)	- 1,114	(0,324)	p = 0,001
Constant	- 0,31	(0,14)	p = 0,031
Explained variable: std(Gross national savings, as defined by IMF) SAMPLE: CLUSTER #2N = 125R^2 = 0,551			
Explanatory variable	**Coefficient of regression**	**Robust standard error**	**Significance level as given by t Student test**
std(Gross public debt, % GDP)	- 0,134	(0,099)	p = 0,177
std(Gross public expenditures, % of GDP)	- 0,22	(0,108)	p = 0,044
std(Gross public revenues as % of the GDP)	0,206	(0,104)	p = 0,051
std(Structural fiscal balance (% of GDP)	- 0,027	(0,049)	p = 0,582
std(Financial assets held by the government, % of GDP)	0,393	(0,087)	p < 0,001
Constant	- 0,16	(0,036)	p < 0,001
Explained variable: std(Gross national savings, as defined by IMF) SAMPLE: CLUSTER #3N = 187R^2 = 0,574			
Explanatory variable	**Coefficient of regression**	**Robust standard error**	**Significance level as given by t Student test**
std(Gross public debt, % GDP)	0,26	(0,063)	p < 0,001
std(Gross public expenditures, % of GDP)	- 0,672	(0,118)	p < 0,001
std(Gross public revenues as % of the GDP)	0,712	(0,13)	p < 0,001
std(Structural fiscal balance (% of GDP)	0,032	(0,04)	p = 0,418
std(Financial assets held by the government, % of GDP)	0,111	(0,043)	p = 0,012
Constant	0,001	(0,083)	p = 0,988

Source: author's

Table 6. Results of regression tests as for the explanation of private investment

Explained variable: std(Private investment) SAMPLE: GENERAL N = 751 R^2 = 0,166			
Explanatory variable	**Coefficient of regression**	**Robust standard error**	**Significance level as given by t Student test**
std(Gross public debt, % GDP)	- 0,062	(0,044)	p = 0,157
std(Gross public expenditures, % of GDP)	- 0,766	(0,086)	p < 0,001
std(Gross public revenues as % of the GDP)	0,589	(0,089)	p < 0,001
std(Structural fiscal balance (% of GDP)	- 0,138	(0,03)	p < 0,001
std(Financial assets held by the government, % of GDP)	- 0,087	(0,028)	p = 0,002
Constant	- 0,01	(0,027)	p = 0,692
Explained variable: std(Private investment) SAMPLE: CLUSTER #1 N = 71 R^2 = 0,472			
Explanatory variable	**Coefficient of regression**	**Robust standard error**	**Significance level as given by t Student test**
std(Gross public debt, % GDP)	- 0,661	(0,527)	p = 0,214
std(Gross public expenditures, % of GDP)	- 0,522	(0,661)	p = 0,443
std(Gross public revenues as % of the GDP)	1,397	(0,508)	p = 0,008
std(Structural fiscal balance (% of GDP)	- 0,452	(0,151)	p = 0,004
std(Financial assets held by the government, % of GDP)	- 2,83	(0,524)	p < 0,001
Constant	- 0,133	(0,234)	p = 0,573
Explained variable: std(Private investment) SAMPLE: CLUSTER #2 N = 125 R^2 = 0,239			
Explanatory variable	**Coefficient of regression**	**Robust standard error**	**Significance level as given by t Student test**
std(Gross public debt, % GDP)	- 0,313	(0,178)	p = 0,080
std(Gross public expenditures, % of GDP)	- 0,334	(0,209)	p = 0,113
std(Gross public revenues as % of the GDP)	0,154	(0,202)	p = 0,450
std(Structural fiscal balance (% of GDP)	- 0,014	(0,102)	p = 0,893
std(Financial assets held by the government, % of GDP)	0,385	(0,152)	p = 0,012
Constant	- 0,069	(0,07)	p = 0,330
Explained variable: std(Private investment) SAMPLE: CLUSTER #3 N = 187 R^2 = 0,252			
Explanatory variable	**Coefficient of regression**	**Robust standard error**	**Significance level as given by t Student test**
std(Gross public debt, % GDP)	- 0,082	(0,08)	p = 0,305
std(Gross public expenditures, % of GDP)	- 0,926	(0,164)	p < 0,001
std(Gross public revenues as % of the GDP)	0,757	(0,195)	p < 0,001
std(Structural fiscal balance (% of GDP)	- 0,224	(0,056)	p < 0,001
std(Financial assets held by the government, % of GDP)	- 0,254	(0,061)	p < 0,001
Constant	0,078	(0,108)	p = 0,468

Source: author's

Table 7. Results of regression tests as for the explanation of the primary completion rate

Explained variable: std(Primary completion rate) SAMPLE: GENERAL N = 449 R^2 = 0,041			
Explanatory variable	**Coefficient of regression**	**Robust standard error**	**Significance level as given by t Student test**
std(Gross public debt, % GDP)	- 0,068	(0,038)	p = 0,071
std(Gross public expenditures, % of GDP)	0,199	(0,075)	p = 0,008
std(Gross public revenues as % of the GDP)	- 0,158	(0,073)	p = 0,032
std(Structural fiscal balance (% of GDP)	0,053	(0,026)	p = 0,045

std(Financial assets held by the government, % of GDP)	0,101	(0,023)	p < 0,001
Constant	0,219	(0,028)	p < 0,001
Explained variable: std(Primary completion rate) SAMPLE: CLUSTER #1N = 61R² = 0,208			
Explanatory variable	**Coefficient of regression**	**Robust standard error**	**Significance level as given by t Student test**
std(Gross public debt, % GDP)	- 0,366	(0,247)	p = 0,144
std(Gross public expenditures, % of GDP)	0,6	(0,395)	p = 0,134
std(Gross public revenues as % of the GDP)	- 0,28	(0,366)	p = 0,447
std(Structural fiscal balance (% of GDP)	0,292	(0,147)	p = 0,052
std(Financial assets held by the government, % of GDP)	0,727	(0,358)	p = 0,047
Constant	0,577	(0,211)	p = 0,008
Explained variable: std(Primary completion rate) SAMPLE: CLUSTER #2N = 37 R² = 0,430			
Explanatory variable	**Coefficient of regression**	**Robust standard error**	**Significance level as given by t Student test**
std(Gross public debt, % GDP)	0,196	(0,066)	p = 0,006
std(Gross public expenditures, % of GDP)	0,128	(0,085)	p = 0,141
std(Gross public revenues as % of the GDP)	- 0,37	(0,105)	p = 0,001
std(Structural fiscal balance (% of GDP)	0,162	(0,05)	p = 0,003
std(Financial assets held by the government, % of GDP)	- 0,174	(0,066)	p = 0,013
Constant	0,451	(0,038)	p < 0,001
Explained variable: std(Primary completion rate) SAMPLE: CLUSTER #3N = 109 R² = 0,445			
Explanatory variable	**Coefficient of regression**	**Robust standard error**	**Significance level as given by t Student test**
std(Gross public debt, % GDP)	- 0,634	(0,119)	p < 0,001
std(Gross public expenditures, % of GDP)	- 0,075	(0,143)	p = 0,600
std(Gross public revenues as % of the GDP)	- 0,007	(0,191)	p = 0,972
std(Structural fiscal balance (% of GDP)	0,042	(0,059)	p = 0,479
std(Financial assets held by the government, % of GDP)	- 0,023	(0,045)	p = 0,604
Constant	0,384	(0,155)	p = 0,015

Source: author's

Table 8. Results of regression tests as for the vulnerable employment rate

Explained variable: std(Vulnerable employment rate) SAMPLE: GENERALN = 592R² = 0,419			
Explanatory variable	**Coefficient of regression**	**Robust standard error**	**Significance level as given by t Student test**
std(Gross public debt, % GDP)	0,052	(0,056)	p = 0,354
std(Gross public expenditures, % of GDP)	- 0,212	(0,077)	p = 0,006
std(Gross public revenues as % of the GDP)	- 0,313	(0,079)	p < 0,001
std(Structural fiscal balance (% of GDP)	- 0,15	(0,035)	p < 0,001
std(Financial assets held by the government, % of GDP)	- 0,05	(0,029)	p = 0,081
Constant	0,012	(0,033)	p = 0,721
Explained variable: std(Vulnerable employment rate) SAMPLE: CLUSTER #1N = 52R² = 0,244			
Explanatory variable	**Coefficient of regression**	**Robust standard error**	**Significance level as given by t Student test**
std(Gross public debt, % GDP)	- 0,756	(0,451)	p = 0,101
std(Gross public expenditures, % of GDP)	0,327	(0,645)	p = 0,614

std(Gross public revenues as % of the GDP)	- 0,899	(0,618)	p = 0,152
std(Structural fiscal balance (% of GDP)	- 0,163	(0,234)	p = 0,489
std(Financial assets held by the government, % of GDP)	- 1,419	(0,889)	p = 0,117
Constant	- 0,674	(0,459)	p = 0,149
Explained variable: std(Vulnerable employment rate)	SAMPLE: CLUSTER #2N = 99R^2 = 0,386		
Explanatory variable	**Coefficient of regression**	**Robust standard error**	**Significance level as given by t Student test**
std(Gross public debt, % GDP)	0,538	(0,205)	p = 0,010
std(Gross public expenditures, % of GDP)	0,596	(0,307)	p = 0,055
std(Gross public revenues as % of the GDP)	- 1,035	(0,324)	p = 0,002
std(Structural fiscal balance (% of GDP)	- 0,037	(0,129)	p = 0,775
std(Financial assets held by the government, % of GDP)	- 0,507	(0,171)	p = 0,004
Constant	- 0,517	(0,046)	p < 0,001
Explained variable: std(Vulnerable employment rate)	SAMPLE: CLUSTER #3N = 153R^2 = 0,641		
Explanatory variable	**Coefficient of regression**	**Robust standard error**	**Significance level as given by t Student test**
std(Gross public debt, % GDP)	0,129	(0,056)	p = 0,022
std(Gross public expenditures, % of GDP)	0,009	(0,067)	p = 0,895
std(Gross public revenues as % of the GDP)	- 0,356	(0,106)	p = 0,001
std(Structural fiscal balance (% of GDP)	- 0,111	(0,036)	p = 0,002
std(Financial assets held by the government, % of GDP)	- 0,004	(0,025)	p = 0,871
Constant	- 0,14	(0,097)	p = 0,150

Source: author's

4. Case studies

The empirical part of the present research covers a qualitative part too. It consists of case studies connected to the previously signaled identity of three big clusters of political systems. The goal of the present study is contributing to the prediction of changes in fiscal policies that can possibly come out of changes in the partisan structure of the political system. Cases under scrutiny are countries, which migrated to or from any of the three clusters, during the period of observation. The choice of cases was quite intuitive, and the general purpose was to go more in depth of the general patterns observed in quantitative research. The first interesting case is **Bolivia** (see table 9). According to the here-adopted classification of political systems, the country ended up in the cluster #1, yet it was its end of the road, so to say. The span of observation as for this particular country ranged from 2000 through 2012. Constitutionally, Bolivia had been a presidential system with plural elections over the whole period studied. At the beginning of the observation span, the political system of Bolivia was a case of recently implemented democratic reforms, mostly referring to the electoral system (Van Cott 2000, Arnold 2004, Laserna 2009). Still, some authors argue that social inequalities and the resulting underrepresentation of large and poor social groups, made those reforms technical rather than fundamental (O'Donnell *et al.* 2004). Exactly in the middle of the observation span, the country went through deep political change, with the advent in office of the president Evo Morales, in 2006. Since then, some authors call Bolivia "the first post neoliberal democracy in the world", or a "new socialism" (Kohl 2010). In 2009, the constitution of the country was changed, with an important reform of land property, and land management, inclusive of a new policy as for hydrocarbons (Postero 2010). As for the political system, the new constitution claimed to implement a new form of democracy, strongly participatory, and communitarian (Schilling-Vacaflor 2010). Here comes the first interesting contradiction: the Database of Political Institutions indicates that over the years 2000 – 2002 Bolivia displayed high political polarization (POLARIZ = 2), to pass into the zone of moderate polarization (POLARIZ = 1) from 2003 through 2005, and from 2006 on ended up in cluster #1, with no observable polarization. Thus, one body of literature allows assuming that the number of partisan veto players had increased since 2006, whilst other authors suggest just the contrary.

As for current fiscal flows, both public expenditures and public revenues increased their share in the GDP over the period studied. Unfortunately, data about the structural balance is not available, yet expenditures and revenues allow calculating the primary fiscal balance, which had passed from a dangerously deep deficit between 2000 and 2005 to a significant, yet hesitating surplus from 2006 on. Both gross and net public debt had been quickly growing between 2000 and 2005, to start falling sharply afterwards. Intriguingly, the amount of financial assets held by the public sector had been growing over the whole period studied, still since 2006 that growth was truly spectacular. As a matter of fact, this particular fiscal variable follows the clearest trend among all the here-adopted descriptors of fiscal stance. Gross national savings followed almost just as spectacular a growth, as that of financial assets held by the public sector. Interestingly, the rate of vulnerable employment displays an overall downward trend to amelioration. As for the investment rate, it dropped between 2000 and 2005, to recover afterwards.

The case of Bolivia presents interesting contradictions. On the one hand, we can observe sudden, and controversial, political changes, which find their expression in some of the fiscal variables. On the other hand, Bolivian economy goes through a process of steady accumulation of capital both in private hands, and the public sector. That process seems to be much steadier than other fiscal and political changes. One can intuitively guess of capital accumulation, under the stirring surface.

Table 9. Bolivia, country profile from the database used in quantitative research

Year	Structural balance, % of potential GDP	Gross public debt, % GDP	Net public debt, % GDP	Financial assets % of GDP (gross minus net debt)	Gross public expenditures, % of GDP	Gross public revenues as % of the GDP	Gross fiscal balance (% of GDP)
2000	n.a.	66,891	58,93	7,961	29,314	25,586	-3,728
2001	n.a.	59,957	51,729	8,228	31,955	25,135	-6,82
2002	n.a.	69,144	62,116	7,028	33,293	24,505	-8,788
2003	n.a.	74,066	66,444	7,622	31,99	24,114	-7,876
2004	n.a.	89,567	81,006	8,561	32,345	26,801	-5,544
2005	n.a.	80,375	71,088	9,287	33,183	30,938	-2,245
2006	n.a.	55,23	41,886	13,344	29,834	34,304	4,47
2007	n.a.	40,506	27,258	13,248	32,653	34,393	1,74
2008	n.a.	37,155	20,607	16,548	35,333	38,902	3,569
2009	n.a.	39,992	23,144	16,848	35,82	35,834	0,014
2010	n.a.	38,52	18,382	20,138	31,5	33,165	1,665
2011	n.a.	34,687	14,438	20,249	35,377	36,209	0,832
2012	n.a.	33,424	11,059	22,365	36,104	37,861	1,757
Year	**Private investment % of GDP**	**Gross national savings % of GDP**	**Current account balance, % of GDP**	**Primary completion rate, total**	**Vulnerable employment, total (% of total employment)**	**POLARIZ, as in DPI**	
2000	18,143	11,019	-5,324	96,041	66,1	2	
2001	14,268	11,252	-3,36	95,85	64,9	2	
2002	16,295	12,319	-4,42	99,17	63,7	2	
2003	13,232	14,59	1,042	97,98	59,2	1	
2004	11,022	17,045	3,694		59,2	1	
2005	14,254	19,877	5,863		60	1	
2006	13,865	26,56	11,225	99,088	61	0	
2007	15,187	28,593	11,396	96,079	57	0	
2008	17,553	28,952	11,859	97,092	57,1	0	
2009	16,971	22,878	4,27	92,345	54,9	0	
2010	17,007	24,969	3,869	92,897		0	
2011	19,562	25,329	0,318	92,293		0	
2012	17,669	25,735	7,812			0	

Source: International Monetary Fund, World Bank

United States (Table 10) are the biggest economy in the world, and for the most part of their observation span, namely since 2001 through 2010 remained in the cluster #1, to move to high polarization for the two remaining years studied. The fiscal stance of the U.S. over that period is a deepening negative, structural balance, quickly accumulating public debt, shrinking public revenues and growing expenditures. Interestingly, the wedge between gross and net public debt (or financial assets held by the government), had been growing steadily between 2001 and 2012, which was accompanied by a substantial decrease in the rate of private savings, and that of private investment. Data about vulnerable employment is not available, and one can notice a significant decrease as for the primary completion rate. Summing up, from 2001 through 2012 the United States were an economy in trouble, with a worsening fiscal stance. A few interesting questions emerge. Firstly, was the passage to higher polarization in the matters of economic policy (POLARIZ shifting from 0 to 2) rather the driver of fiscal changes, or the driven one? Both the growing public indebtedness, and the growing amount of financial assets held by the government allow guessing quick capital accumulation in the public sector. Was it just lax fiscal policy, or did that mean that new partisan veto players were progressively coming into the political game and they needed some financial space for themselves? An immediate comparison with the preceding case of Bolivia reveals an interesting dualism, right in the spirit of John Stuart Mill and his canons of logic: those two countries differ in practically every respect as for the variables studied, excepted two. They both witnessed a change in the structure of partisan veto players, and both accumulated more financial assets in the public sector. The change in political polarization was of opposite direction in each case (Bolivia 2 to 0; the U.S. 0 to 2), still a change there was. As case studies allow heuristic inference, a heuristic hypothesis can be formulated, namely that any change in the structure of partisan veto players in the political system is connected to increased accumulation of capital in the public sector, whatever the vector of change.

Table 10. The U.S. country profile from the database used in quantitative research

Year	Structural balance, % of potential GDP	Gross public debt, % GDP	Net public debt, % GDP	Financial assets % of GDP (gross minus net debt)	Gross public expenditures, % of GDP	Gross public revenues as % of the GDP
2001	-1,682	53,005	33,761	19,244	33,822	32,113
2002	-3,854	55,366	36,342	19,024	34,844	29,774
2003	-4,943	58,507	39,712	18,795	35,395	29,144
2004	-4,971	65,483	46,878	18,605	35,105	29,292
2005	-4,167	64,882	46,259	18,623	35,212	30,645
2006	-3,498	63,635	44,755	18,88	34,972	31,542
2007	-4,327	64,005	44,481	19,524	35,693	31,659
2008	-6,196	72,833	50,435	22,398	37,986	30,174
2009	-7,905	86,054	62,108	23,946	43,121	28,403
2010	-9,745	94,807	69,694	25,113	41,313	28,775
2011	-8,336	99,005	76,223	22,782	40,083	29,049
2012	-6,777	102,355	80,122	22,233	38,718	29,045
Year	Private investment % of GDP	Gross national savings % of GDP	Current account balance	Primary completion rate, total	Vulnerable employment, total (% of total employment)	polariz
2001	22,045	19,426	-3,733	100,83	n.a.	0
2002	21,571	18,112	-4,169	100,281	n.a.	0
2003	21,657	17,301	-4,505	100,218	n.a.	0
2004	22,523	17,512	-5,126	98,777	n.a.	0
2005	23,22	17,849	-5,649	97,456	n.a.	0
2006	23,33	19,141	-5,762	96,307	n.a.	0
2007	22,347	17,311	-4,927	98,284	n.a.	0
2008	20,784	15,502	-4,629	97,903	n.a.	0
2009	17,514	14,369	-2,647	98,235	n.a.	0
2010	18,402	15,075	-3,005	97,826	n.a.	0
2011	18,442	15,845	-2,947	97,538	n.a.	2
2012	19,047	16,336	-2,711	97,866	n.a.	2

Source: International Monetary Fund, World Bank

Mexico (Table 11), geographically between Bolivia, and the United States, is an interesting case of oscillatory changes with respect to the variables investigated. Within its interval of observation, namely from 1998 through 2012, Mexico started in the cluster #1, left it quite quickly to move towards higher political polarization, but the movement was a wave rather than a trend. Over the period 2001- 2009, political polarization passed from non-existent (POLARIZ = 0), through moderate to high, just to return to moderate from 2010 on. That political oscillation was accompanied, roughly in step, by a wavy change in capital fiscal variables (gross debt, net debt, financial assets), as well as that of the structural balanced. An unequivocally growing share of current fiscal flows in the GDP was to observe, as well as worsening ratios of savings and investment. Interestingly, the social situation seems to have had improved, as seen through the double lens of primary completion rate, and the rate of vulnerable employment. In general, growing political polarization was accompanied by shrinking indebtedness, both in gross and net terms, and by a growing amount of financial assets held by the government. It looks as if growing political polarization in this case went together with the building – up of borrowing capacity from the part of the public sector.

Table 11. Mexico, country profile from the database used in quantitative research

Year	Structural balance, % of potential GDP	Gross public debt, % GDP	Net public debt, % GDP	Financial assets % of GDP (gross minus net debt)	Gross public expenditures, % of GDP	Gross public revenues as % of the GDP
1998	-5,424	44,1	37,718	6,382	22,358	16,727
1999	-5,86	46,327	38,785	7,542	22,412	16,917
2000	-4,005	41,857	35,41	6,447	20,908	17,902
2001	-3,152	41,11	35,59	5,52	21,242	18,174
2002	-3,465	43,468	38,067	5,401	22,119	18,764
2003	-2,587	44,747	35,583	9,164	22,497	20,187
2004	-2,067	40,857	32,847	8,01	20,307	19,058
2005	-2,359	39,041	31,52	7,521	21,66	20,429
2006	-2,5	37,777	29,772	8,005	22,569	21,594
2007	-2,512	37,564	29,089	8,475	22,83	21,675
2008	-2,453	42,85	33,169	9,681	25,637	24,668
2009	-4,823	43,945	36,252	7,693	27,207	22,126
2010	-4,549	42,241	36,248	5,993	26,715	22,445
2011	-4,193	43,272	37,524	5,748	26,287	22,942
2012	-4,457	43,284	37,795	5,489	27,162	23,461
Year	Private investment % of GDP	Gross national savings, % of GDP	Current account balance	Primary completion rate, total	Vulnerable employment, total (% of total employment)	polariz
1998	26,825	23,168	-3,186		34,4	0
1999	25,704	22,355	-2,408	94,628	34,1	0
2000	25,982	22,373	-2,734	94,953	31,8	0
2001	23,382	19,899	-2,443	95,133	32	1
2002	22,608	20,492	-1,906	95,984	32,6	1
2003	21,887	20,718	-1,169	95,373	32,7	1
2004	22,662	21,752	-0,91	96,515	32,4	1
2005	22,284	21,25	-1,034	96,64	31	1
2006	23,457	22,65	-0,806	96,346	29,7	1
2007	23,383	21,977	-1,406	96,517	29,5	2
2008	24,42	22,587	-1,833	95,162	29,2	2
2009	22,887	21,976	-0,911	92,944		2
2010	22,033	21,688	-0,344	92,657		1
2011	22,296	21,244	-1,052	92,722		1
2012	23,242	21,994	-1,248	99,121		1

Source: International Monetary Fund, World Bank

As cases from the cluster #1 are studied, **Poland** (Table 12) is an interesting one: it is a case of truly high, structural instability of the political system with respect to the variables studied. Over its interval of observation, namely 1995 – 2012, Poland jumped between the cluster #1 and other groups of political systems, inclusive of changes in the electoral regime, coming and going between plurality and proportionality. At the very beginning of the span of observation, in the late 1990s, Poland had virtually no net public debt, and an extremely inflated pool of financial assets held by the public sector, in the presence of substantial gross public indebtedness. Since then, gross public indebtedness had slightly grown, which took place against a quickly growing net indebtedness, and an overall decreasing trend in the amount if financial assets in the public sector. One can also observe steady decrease in the share of current fiscal flows in the GDP. The rate of private savings had, on the whole, decreased, whilst private investment climbed slightly. On the social side, we can observe a steady improvement both as for the primary completion rate, and the rate of vulnerable employment. Change in political polarization seem to have been the most reflected in the oscillation of gross public indebtedness, and the amount of financial assets held in the public sector: both tend to be lower in the times of lower polarization, and to increase with higher polarization. Poland is a case of the overall steady economic change, in the presence of slight oscillations of capital fiscal variables, seemingly connected to variations in political fragmentation. The steady, long-term change seems to be attached to an outflow of capital from the public sector. Characteristically, that outflow came along with visible social improvement.

Table 12. Poland, country profile from the database used in quantitative research

Year	Structural balance, % of potential GDP	Gross public debt, % GDP	Net public debt, % GDP	Financial assets % of GDP (gross minus net debt)	Gross public expenditures, % of GDP	Gross public revenues as % of the GDP
1995	-5,824	48,989	-3,383	52,372	47,715	43,3
1996	-5,76	43,39	-1,287	44,677	51,006	46,141
1997	-5,818	42,926	0,068	42,858	46,44	41,809
1998	-5,141	38,889	1,434	37,455	44,344	40,068
1999	-2,944	39,567	6,346	33,221	42,718	40,407
2000	-3,995	36,787	6,918	29,869	41,081	38,053
2001	-4,876	37,562	13,9	23,662	43,803	38,532
2002	-5,269	42,16	13,385	28,775	44,258	39,272
2003	-6,034	47,053	17,626	29,427	44,677	38,485
2004	-5,618	45,686	15,146	30,54	42,623	37,239
2005	-3,421	47,088	15,867	31,221	43,44	39,367
2006	-4,796	47,738	14,991	32,747	43,864	40,235
2007	-2,62	44,986	10,218	34,768	42,187	40,306
2008	-4,162	47,106	9,92	37,186	43,23	39,546
2009	-7,133	50,88	14,921	35,959	44,614	37,206
2010	-7,575	54,838	20,502	34,336	45,424	37,567
2011	-5,51	56,218	26,179	30,039	43,436	38,415
2012	-3,803	55,59	27,556	28,034	42,313	38,381
Year	Private investment % of GDP	Gross national savings % of GDP	Current account balance	Primary completion rate, total	Vulnerable employment, total (% of total employment)	polariz
1995	18,716	19,33	0,614	94,592	25,9	
1996	20,875	18,792	-2,083	97,51	25,4	1
1997	23,429	19,773	-3,657	98,839	25	1
1998	25,061	21,049	-4,012	97,278	23,4	0
1999	25,26	17,817	-7,442	96,5	23	0
2000	24,85	18,811	-6,039	95,451	23,5	0
2001	20,771	17,648	-3,123	97,113	24,3	0
2002	18,624	15,827	-2,797	96,957	24,4	0
2003	18,742	16,218	-2,524	98,284	23,2	0
2004	20,069	14,829	-5,24	98,918	22,5	0

2005	19,266	16,884	-2,382	96,175	21,8	0
2006	21,052	17,205	-3,848	95,508	20,4	0
2007	24,445	18,214	-6,231	95,322	19,4	0
2008	23,9	17,297	-6,603		18,9	2
2009	20,347	16,371	-3,976	94,791	18,6	2
2010	20,998	15,883	-5,115	95,365	18,6	2
2011	22,058	17,206	-4,853	95,451	18,4	1
2012	20,416	16,911	-3,505		18,2	1

Source: International Monetary Fund, World Bank

With the case of **New Zealand** (Table 13), we pass to countries grouped in the cluster #2: parliamentary systems with prevailing plurality in the electoral regime, and no observable political polarization as for the key aspects of economic policy. More specifically, New Zealand remained in the cluster #2 from 1985 through 1994, to leave it for good since then and to pass to high polarization. Over the whole span of observation, New Zealand went through a deep change in public governance, and a substantial part of that change regarded specifically the fiscal policy. Interestingly enough, observation of New Zealand for the purposes of the present research starts at the very moment when important public reforms began. The entry into force of the Public Finance Act 1989 seems to have been a milestone in the process, introducing a novelty at the global scale, namely passing from cash-based budgetary management to the accrual-based one (Goldman and Brashares 1991). In New Zealand, the purpose of the systemic change was to minimize consistently the budgetary slack. As the DPI data shows, as those public reforms had been implemented, from 1985 to 1994, no political polarization as for economic policy was observable (POLARIZ = 0). From 1995 on, polarization jumped to "high" (POLARIZ = 2) and remained such for the rest of the period observed, through 2012.

Quantitative fiscal data about New Zealand, collected for the purposes of the present research, shows clearly that at the beginning of public reforms the public sector was quite greedy, holding over 24% of the GDP in liquid financial assets, and recording a significant gross, and net debt. From 1985 (when our span of observation starts) through 1989 (when the Public Finance Act 1989 was voted), the share of public, financial assets in the GDP shrank significantly, and the structural balance improved. It was probably the most immediate result of passing from cash-based budgetary accounting to the accrual based one. That transition probably terminated a lot of small capital pockets held by public agents through the postponement of cash settlements. The spectacular deflation of public financial assets, and the betterment of the structural balance are the most striking fiscal changes accompanying the reforms. Later on, over the next 15 years, public debt decreased significantly both in gross and net terms. Public financial assets inflated again after 2004, just as public debt, not to the previous levels, though. Interestingly enough, private capital aggregates, namely saving and investment had been changing in close correlation with the public ones. In general, 2003 – 2004 seem to be the moment, when fiscal reforms reached some kind of peak in their quantitative outcomes. Afterwards, the fiscal stance started to revert.

Hence, the following picture emerges. In the 1980s, the political system reached some kind of general agreement about the economic policy to follow, which reflected in the absence of political polarization, and in bold constitutional reforms. The state of partisan unanimity had lasted until 1994, when significant disparities in economic programmes appeared. The striking fact is that the beginning of significant political polarization coincided almost perfectly with the first official publication of the government's balance sheet (Dale and Ball 1996).

The case of New Zealand rouses a few interesting remarks. Was it the process of public reforms that triggered the first visible fiscal change, namely the improvement in structural balance and the deflation of public financial assets, or was it the absence of political polarization? Which factor was decisive: the legal change, or the partisan structure of the political system? On the other hand, why did the private capital aggregates change in such a close correlation with the public ones? What was the connection?

Table 13. New Zealand, country profile from the database used in quantitative research

Year	Structural balance, % of potential GDP	Gross public debt, % GDP	Net public debt, % GDP	Financial assets % of GDP (gross minus net debt)	Gross public expenditures, % of GDP	Gross public revenues as % of the GDP
1985	-4,962	67,076	42,954	24,122	41,853	35,661
1986	-3,991	71,628	46,897	24,731	42,523	37,826
1987	-2,838	65,834	44,866	20,968	41,823	38,853

1988	-1,193	57,232	43,956	13,276	41,834	40,276
1989	-0,807	57,65	49,079	8,571	43,531	41,719
1990	-0,672	58,179	50,136	8,043	45,052	42,723
1991	-2,421	60,776	53,008	7,768	48,144	42,72
1992	-1,984	61,549	53,888	7,661	47,766	42,22
1993	0,707	57,252	48,311	8,941	42,557	41,321
1994	2,011	51,386	44,081	7,305	38,954	40,918
1995	2,699	45,577	38,61	6,967	37,821	41,331
1996	1,822	39,072	32,245	6,827	36,259	38,742
1997	1,422	36,302	29,538	6,764	35,741	36,998
1998	0,78	36,213	26,949	9,264	36,553	36,25
1999	-0,095	33,57	23,543	10,027	35,713	34,827
2000	0,439	31,573	21,742	9,831	34,922	35,205
2001	1,244	29,553	20,129	9,424	33,885	35,183
2002	1,8	27,698	18,871	8,827	33,45	35,901
2003	2,081	25,9	17,255	8,645	33,321	36,802
2004	2,604	23,572	14,42	9,152	32,832	37,047
2005	3,141	21,755	11,301	10,454	33,681	38,478
2006	2,448	19,306	8,781	10,525	34,427	38,77
2007	2,606	17,18	6,495	10,685	33,863	37,283
2008	1,229	20,119	7,369	12,75	35,4	36,886
2009	-1,01	25,731	11,64	14,091	37,075	35,54
2010	-4,017	31,937	16,947	14,99	40,036	34,921
2011	-3,652	36,975	22,08	14,895	39,74	34,873
2012	-0,942	37,487	25,33	12,157	36,395	34,804

Year	Private investment % of GDP	Gross national savings % of GDP	Current account balance	Primary completion rate, total	Vulnerable employment, total (% of total employment)	polariz
1985	26,594	19,254	-7,288	n.a.		0
1986	24,104	19,255	-6,395	n.a.		0
1987	22,445	18,586	-4,803	n.a.		0
1988	20,005	18,398	-0,924	n.a.		0
1989	22,038	18,157	-3,665	n.a.		0
1990	20,478	16,884	-3,465	n.a.		0
1991	16,193	14,402	-3,047	n.a.	12,7	0
1992	17,576	14,477	-4,397	n.a.	13	0
1993	20,003	17,03	-4,115	n.a.	13,1	0
1994	21,532	18,817	-3,98	n.a.	12,7	0
1995	22,508	19,662	-4,953	n.a.	12,8	2
1996	22,684	18,262	-5,813	n.a.	13,4	2
1997	21,667	16,755	-6,162	n.a.	12,8	2
1998	20,127	15,757	-3,635	n.a.	12,7	2
1999	21,199	15,249	-6,05	n.a.	13,6	2
2000	21,763	16,73	-4,596	n.a.	13,5	2
2001	20,861	19,625	-2,256	n.a.	12,6	2
2002	22,383	19,991	-3,619	n.a.	12,3	2
2003	22,535	20,755	-2,478	n.a.	12,1	2
2004	24,157	19,437	-4,616	n.a.	12,1	2
2005	24,532	17,088	-7,158	n.a.	11,9	2
2006	23,021	15,929	-7,234	n.a.	12,1	2
2007	23,88	16,513	-6,91	n.a.	11,8	2

2008	22,742	15,345	-7,799	n.a.	12,1	2
2009	18,978	16,683	-2,274	n.a.		2
2010	19,239	16,956	-2,256	n.a.		2
2011	18,926	16,009	-2,892	n.a.		2
2012	20,189	16,051	-4,115	n.a.		2

Source: International Monetary Fund, World Bank

Now, we pass to some countries included into **Cluster #3**: parliamentary systems with proportional elections and high political polarization. Two cases seem particularly interesting: Finland, and Israel. **Finland** (Table 14), is some kind of fiscal champion in the sample studied, along with other Nordic countries. Observed, in the sample, since 1980 through 2012, Finland maintains a positive fiscal balance, both at the primary, and at the structural level. On the other hand, it combines all the factors of political fragmentation, both constitutional and partisan. Thus, it is a living proof that strongly differentiated political systems can generate high fiscal discipline. Its reserves of public financial assets, combined with a noticeable gross public debt, and a net claim on the rest of the world (negative net debt) call for the metaphor of a bank-country. As for political polarization, Finland jumps in to and out of cluster #3. Still, political polarization as for economic policy never descends below POLARIZ = 1, which, in turn, means that there is always a significant number of distinct, partisan veto players in the political system. It is to notice that the amount of financial assets held by the public sector, as well as net public debt, both change in close correlation with and proportionally to political polarization. Once more, the more partisan veto players, the more capital held by public agents.

The phenomenon of Finland consists in the fact that the country has developed a whole structure made of the so-called peripheral agencies, *i.e.* relatively small, and prudently endowed agencies of the government, in charge of carrying out many innovative projects in the broadly spoken field of economic development. Those agencies are staffed with people coming from many political parties and fractions, and are supposed to bring together the different economic programmes into concrete, specific projects (Breznitz and Ornston 2013). Hence, the Finish political class has managed to turn a curse into a blessing. The well-known mechanism of “my-friend’s-cousin-should-have-that-job-in-your-ministry”, usually bringing about the worst cases of budgetary slack, in this case is used as a tool for improvement and development.

Table 14. Finland, country profile from the database used in quantitative research

Year	Structural balance, % of potential GDP	Gross public debt, % GDP	Net public debt, % GDP	Financial assets % of GDP (gross minus net debt)	Gross public expenditures, % of GDP	Gross public revenues as % of the GDP
1980		10,823	-177,082	187,905	44,611	46,919
1981		11,477	-175,615	187,092	45,331	49,539
1982		13,76	-161,868	175,628	47,216	49,008
1983		15,308	-153,105	168,413	49,11	48,828
1984		15,124	-152,463	167,587	48,466	50,518
1985		15,803	-159,856	175,659	50,458	52,372
1986		16,416	-164,49	180,906	51,138	53,843
1987		17,622	-164,47	182,092	52,104	52,147
1988		16,514	-172,02	188,534	50,654	54,907
1989		14,275	-196,067	210,342	47,927	54,515
1990		13,839	-208,271	222,11	48,142	54,207
1991		21,9	-200,579	222,479	57,107	56,664
1992		39,361	-146,494	185,855	62,371	56,615
1993		54,226	-94,942	149,168	65,203	56,171
1994		56,532	-96,843	153,375	64,063	56,714
1995		55,518	-23,998	79,516	61,802	54,922
1996		55,723	-39,912	95,635	60,123	56,145
1997		52,854	-44,515	97,369	56,568	54,93
1998		47,619	-86,782	134,401	52,921	54,506
1999	0,835	45,664	-50,277	95,941	51,789	53,445

2000	5,33	43,793	-31,092	74,885	48,416	55,359
2001	3,937	42,46	-31,65	74,11	47,997	53,076
2002	3,65	41,468	-31,318	72,786	49,04	53,147
2003	2,252	44,511	-38,443	82,954	50,336	52,776
2004	1,341	44,387	-46,674	91,061	50,24	52,489
2005	1,685	41,703	-58,591	100,294	50,348	53,038
2006	2,236	39,632	-69,424	109,056	49,195	53,274
2007	2,069	35,158	-72,521	107,679	47,389	52,728
2008	1,599	33,939	-52,292	86,231	49,212	53,557
2009	0,182	43,522	-62,848	106,37	56,122	53,406
2010	-1,22	48,664	-65,561	114,225	55,794	52,984
2011	-0,994	49,193	-54,255	103,448	55,267	54,113
2012	-1,126	53,616	-55,422	109,038	56,61	54,446

Year	Private investment % of GDP	Gross national savings % of GDP	Current account balance	Primary completion rate, total	Vulnerable employment, total (% of total employment)	polariz
1980	30,141	27,364	-2,726			2
1981	27,328	26,238	-0,803			2
1982	27,445	24,876	-1,702			2
1983	27,01	24,444	-2,087			2
1984	25,909	25,308	0,074			2
1985	25,418	24,345	-1,338			2
1986	24,347	24,086	-0,934			2
1987	25,059	23,676	-1,904	104,13		2
1988	27,43	25,931	-2,521	105,881		2
1989	30,44	25,398	-4,943	105,588		2
1990	28,46	23,658	-5,02	101,788		2
1991	22,129	16,33	-5,355	97,388		1
1992	18,765	13,655	-4,618	96,741		1
1993	16,308	14,758	-1,288	96,113		1
1994	17,501	18,064	1,087	97,197		1
1995	18,19	21,663	4,09	100,636	12,1	1
1996	17,775	20,679	4,01	101,391	12,7	2
1997	19,171	23,753	5,566	100,522	10,9	2
1998	20,364	24,786	5,612	98,154	10,8	2
1999	19,506	24,656	5,342	95,983	9,6	2
2000	20,814	28,59	7,776	96,332	9,2	2
2001	20,425	28,779	8,354	102,291	9	2
2002	19,101	27,56	8,459	101,012	8,9	2
2003	19,371	24,199	4,828	102,075	8,7	1
2004	19,949	26,147	6,198	100,35	8,3	1
2005	21,798	25,149	3,351	100,36	8,8	1
2006	21,264	25,421	4,157	96,908	8,8	1
2007	22,852	27,117	4,265	98,161	8,6	1
2008	22,224	24,839	2,615	98,562	9	1
2009	18,518	20,286	1,769	97,428	9,6	1
2010	18,447	19,964	1,517	97,965	9,2	1
2011	20,537	19,036	-1,5	97,424	9,3	1
2012	19,849	17,787	-1,663	99,262	9,6	2

Source: International Monetary Fund, World Bank

Israel (Table 15), another country from the cluster #3, presents a different profile. With the political system just as fragmented into veto players, as that of Finland, but very poor a fiscal stance, Israel presents two peculiarities. Firstly, in the presence of relatively high public indebtedness, and substantial current fiscal flows, the public sector of Israel holds very few financial assets, and over the period of observation those assets plunged close to null. Secondly, Israel is one of the rare countries in the whole sample, where the rate of savings has increased over the period studied, and the social situation has clearly improved in spite of the on-going armed conflict. Israel seems to have developed a network of public, peripheral agencies focused on economic development, similarly to Finland (Breznitz and Ornston 2013, Getz and Goldberg 2015), thus finding positive employment for various partisan veto players. As compared to Finland, in the same cluster, Israel seems to have developed some sort of capital transmission from current fiscal flows directly into privately held assets, without bulking financially the public sector.

Table 15. Israel, country profile from the database used in quantitative research

Year	Structural balance, % of potential GDP	Gross public debt, % GDP	Net public debt, % GDP	Financial assets % of GDP (gross minus net debt)	Gross public expenditures, % of GDP	Gross public revenues as % of the GDP
2000	-6,375	81,402	71,6	9,802	49,189	45,387
2001	-7,001	85,969	76,8	9,169	51,487	45,303
2002	-6,53	93,072	85	8,072	52,928	45,315
2003	-5,558	95,567	87,6	7,967	51,577	43,737
2004	-4,38	94,116	87,6	6,516	48,653	42,701
2005	-3,595	90,626	83,8	6,826	47,369	42,47
2006	-1,815	81,627	74,8	6,827	45,695	43,136
2007	-1,727	74,622	69,2	5,422	43,956	42,434
2008	-3,699	72,925	69,1	3,825	43,215	39,471
2009	-5,736	75,269	70,8	4,469	43,074	36,743
2010	-4,906	71,451	69,1	2,351	42,177	37,623
2011	-4,431	69,706	68	1,706	41,883	37,712
2012	-5,396	68,202	67,4	0,802	41,012	36,378

Year	Private investment % of GDP	Gross national savings % of GDP	Current account balance	Primary completion rate, total	Vulnerable employment, total (% of total employment)	polariz
2000	21,583	19,993	-1,59		7,1	2
2001	20,886	19,291	-1,594		6,9	1
2002	19,423	18,325	-1,098	104,484	7	1
2003	18,541	19,055	0,514	104,555	7,3	1
2004	18,516	20,154	1,637	108,145	7,2	2
2005	19,472	22,432	2,96	104,141	7,5	2
2006	19,683	24,346	4,662	102,726	7,9	2
2007	20,45	23,627	3,177	103,486	7,4	2
2008	20,053	21,494	1,441	100,204	7,2	2
2009	17,624	21,471	3,846	103,292		2
2010	18,126	21,221	3,095	101,597		2
2011	20,184	21,445	1,261	100,975		2
2012	20,677	21,006	0,33			2

Source: International Monetary Fund, World Bank

Conclusion

The present paper was written with the intention to follow up onto the path indicated by the seminal paper by Roubini and Sachs 1989, namely to explore the intuition that the political system in place is a strong determinant of fiscal policy. Quantitative research suggests strong, cross-sectional disparities between political systems as for their typical fiscal stance, and those disparities seem to refer mostly to the amount of capital held by the public sector, rather than to current fiscal flows. The number of veto players in the political system, as possible to estimate on the grounds of constitutional rules, and of political polarization, seems to be strongly, and positively correlated with the amount of liquid capital held in the natural and temporary possession of public agents.

The more veto players in the political system, the greater seems to be the impact of fiscal policy upon some socio-economic outcomes, such as the formation of private savings, private investment or primary completion rate. Clearly, what we use to designate as efficiency of fiscal policy is very specific regarding the political system in place. In a broader perspective, the present paper comes to a somewhat different conclusion than the seminal work by Roubini and Sachs. Whilst these authors claimed that fiscal discipline clearly varies across political systems, the present research seems to prove that fiscal discipline is pretty homogenous, whilst the ways that public agents adopt to govern capital in their possession, and the outcomes of that governance, are clearly system-specific.

On the other hand, qualitative case studies allow noticing that the capital held by public agents, estimated mostly as public debt and financial assets in the public sector, changes over time in close correlation with changes in political polarization. Still, at the level of case studies, no visible pattern emerges as for cross-sectional correlations in that respect. Any increase in political polarization almost inevitably leads to an increase in public indebtedness and/or the endowment of public agents with financial assets, with decreasing political polarity acting in the opposite way. In some cases, though not in all of them, that change in the appropriation of financial assets goes along with a proportional change in public indebtedness. Yet, national political systems seem to be strongly idiosyncratic as for the exact share of GDP held as capital by public agents.

On the whole, empirical research presented in this paper strongly substantiates the general claims of the pork barrel theory, as formulated by Weingast and others. Still, the theory of veto players, as presented by Tsebelis, seems to add much precision to the pork barrel theory.

Case studies inspire an interesting question, namely that of the relative strength of fiscal, and political factors in the shaping of public policies and constitutional orders. Does the amount of capital appropriated by the public sector adapt to the political system, or, conversely, does the political system adapt to the available capital resources? Moreover, is the fiscal stance of the government informative about the actual, partisan structure of the political system? In other words, are substantial shifts in the amount of capital held in the public sector informative about the emergence or disappearance of partisan veto players, not officially disclosed as such? Can we assume, for example, that some partisan veto players start appropriating capital in the public sector even before they have officially emerged as political parties?

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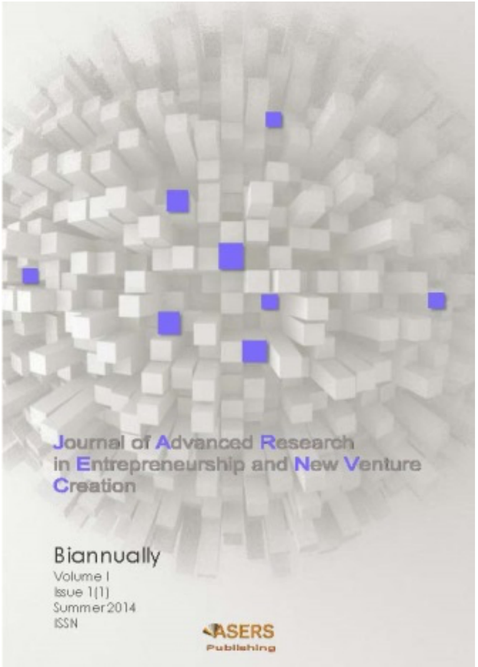

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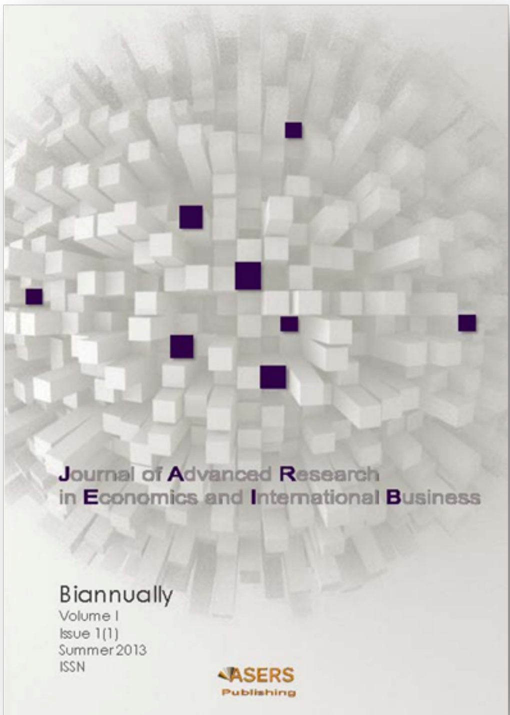

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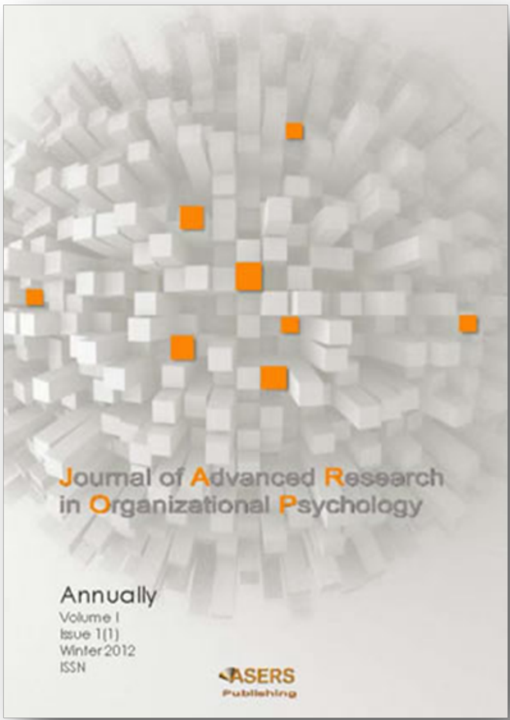

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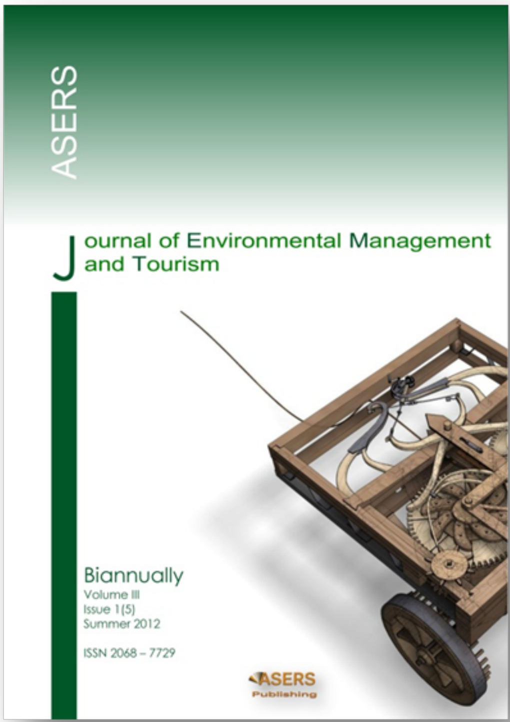

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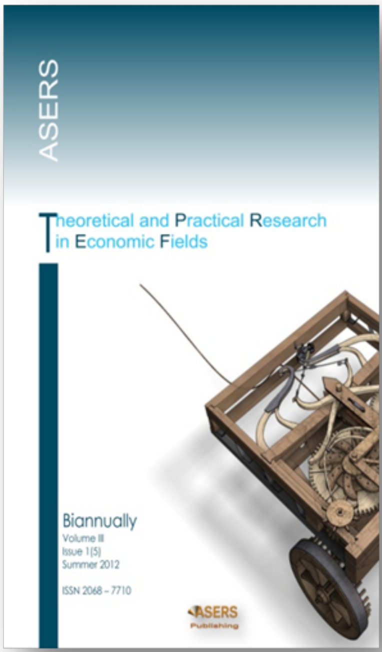

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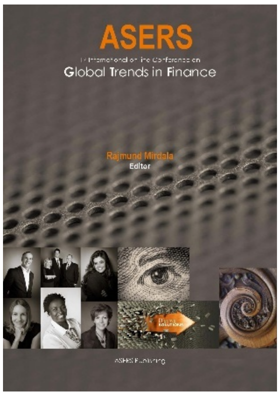

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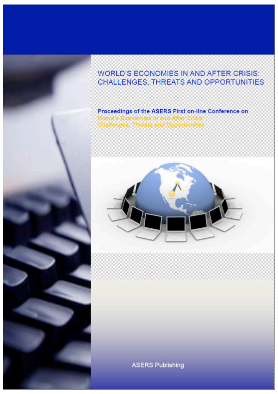

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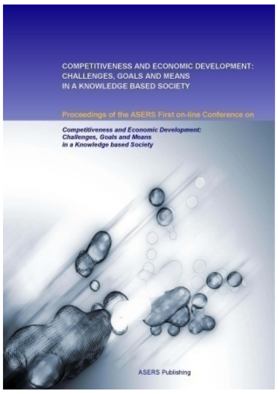

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