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# Contents:

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# GAME COMPLETE ANALYSIS OF BERTRAND DUOPOLY

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**Abstract:**

*In this paper we apply the Complete Analysis of Differentiable Games [introduced by D. Carfì in (Carfi 2010), (Carfi 2009), (Carfi 2009), and (Carfi 2009)] and already employed by himself and others in (Carfi 2011), (Carfi 2010), (Carfi 2009)] to the classic Bertrand Duopoly (1883), classic oligopolistic market in which there are two enterprises producing the same commodity and selling it in the same market. In this classic model, in a competitive background, the two enterprises employ as possible strategies the unit prices of their product, contrary to the Cournot duopoly, in which the enterprises decide to use the quantities of the commodity produced as strategies. The main solutions proposed in literature for this kind of duopoly (as in the case of Cournot duopoly) are the Nash equilibrium and the Collusive Optimum, without any subsequent critical exam about these two kinds of solutions. The absence of any critical quantitative analysis is due to the relevant lack of knowledge regarding the set of all possible outcomes of this strategic interaction. On the contrary, by considering the Bertrand Duopoly as a differentiable game (games with differentiable payoff functions) and studying it by the new topological methodologies introduced by D. Carfì, we obtain an exhaustive and complete vision of the entire payoff space of the Bertrand game (this also in asymmetric cases with the help of computers) and this total view allows us to analyze critically the classic solutions and to find other ways of action to select Pareto strategies. In order to illustrate the application of this topological methodology to the considered infinite game, several compromise pricing-decisions are considered, and we show how the complete study gives a real extremely extended comprehension of the classic model.*

**Keywords:** duopoly, normal form games, microeconomic policy, complete study, bargaining solutions.

**JEL Classification:** D7, C71, C72, C78

## 1. Introduction

We consider a duopoly (1, 2) with *production fixed cost* f and *production variable cost* a function v of the produced quantity, for both the producers; we shall assume the function v equal 0.

*The demand for enterprise* 1 is the affine *reaction function* $Q_1$, from the Euclidean plane of price bi-strategies $R^2$ into the real line of quantities to be produced R – the demand $Q_1$ (p) is the aggregate reaction of consumers in the market to the pair p of prices imposed by the two enterprises (see for a general theory of reactions (Carfi, and Ricciardello 2009), (Carfi 2009), (Carfi 2009), (Carfi, and Ricciardello 2011) - defined by:

$$Q_1(p) = b + a_1p_1 + a_2p_2 \quad (1.1)$$

that is by the equality

$$Q_1(p) = b + (a|p), \quad (1.2)$$

for every pair of prices p, where a is a pair of real numbers whose first component is negative and whose second component is positive, where b is a non-negative real and where (a|p) is the standard Euclidean scalar product of the two vectors *a* and *p*.

The components of the pair *p* are determined by the two enterprises 1 and 2, respectively.

The *demand for the enterprise* ***2*** is the function $Q_2$ defined, in a perfectly analogous way as the first one, by

$$Q_2(p) = b + a_1p_2 + a_2p_1, \text{ that is} \quad (1.3)$$

$$Q_2(p) = b + (a^-|p), \quad (1.4)$$

for every pair of prices p, where $a^-$ is the symmetric pair of a.

The *classic way to solve the duopoly* (see for instance: Davide Vannoni, and Massimiliano Piacenza, University of Torino, Faculty of Economics, *Appunti di Microeconomia - Corso C - Lezione 10*, A. A. 2009/2010) is to determine the curves of best price reaction, for example, for enterprise 1, we consider the profit function $P_1$ defined by

$$P_1(p_1,q_1) = p_1q_1 - f \tag{1.5}$$

that, on the reaction demand function $Q_1$, assumes the form

$$g_1(p) = P_1(p_1, Q_1(p)) = p_1(b + a_1p_1 + a_2p_2) - f, \tag{1.6}$$

for every price $p_2$, fixed by the enterprise 2, the price of maximum profit for enterprise 1 must satisfy the following stationary condition

$$D_1(g_1)(p) = b + 2a_1p_1 + a_2p_2 = 0. \tag{1.7}$$

We note that the second derivative of the function $g_1( . ,p_2)$ is negative ( $2a_1 < 0$), hence the above stationary condition is not only necessary but also sufficient in order to obtain maxima, we so determine the classic reaction curve of enterprise 1, the line of equation

$$p_1 = b/(2a_1) + p_2a_2/(2a_1). \tag{1.8}$$

Symmetrically, the reaction curve of enterprise 2 is the line

$$p_2 = b/(2a_1) + p_1a_2/(2a_1). \tag{1.9}$$

Now, by the intersection of the two reaction curves, we obtain the fixed-point equation

$$p_1 = b/(2a_1) + (b/(2a_1) + p_1a_2/(2a_1))a_2/(2a_1), \tag{1.10}$$

and so finally we obtain the equilibrium price of the enterprise 1, and the same of enterprise 2:

$$p_2 = p_1 = - b/(2a_1 + a_2). \tag{1.11}$$

Another classic solution is the symmetric collusive point C = (c,c) determined by maximization of the function H defined by

$$H(c) = P_1(c, Q_1(c,c)) + P_2(c, Q_2(c,c)) = 2c(b + (a_1 + a_2)c) - 2f, \tag{1.12}$$

for every c.

*But also in this case, an accurate analysis of this point is impossible since we do not know the geometry of the payoff space.*

## 2. Formal Description of Bertrand's Normal Form Game

It will be a non-linear two - players gain game ($f$, >) (see also (6, Carfì 2009), (8, Carfì 2009), and (9, Carfì 2009). The two players/enterprises shall be called *Emil* and *Frances* (following Aubin's books (Aubin 1982) and (Aubin 1998).

*Assumption 1 (strategy sets).* The two players produce and offer the same commodity at the following prices: $x \in \mathbb{R}_\geq$ for Emil and $y \in \mathbb{R}_\geq$ for Frances. In more precise terms: the payoff function $f$ of the game is defined on a subset of the positive cone of the Cartesian plane $\mathbb{R}^2$, interpreted as a space of bi-prices. We assume (by simplicity) that the set of all strategies (of each player) is the interval E = [0, +∞].

*Assumption 2 (symmetry of the game).* The game will be assumed symmetric with respect to the players. In other terms, the payoff pair f(x,y) is the symmetric of the pair f(y,x).

*Assumption 3 (form of demand functions).* Let the demand function $Q_1$ (defined on $E^2$) of the first player be given by:

$$Q_1(x,y) = u - 2x + y, \tag{2.1}$$

for every positive price pair $(x, y)$ and let analogously the demand function of the second enterprise be given by

$$Q_2(x,y) = u - 2y + x, \tag{2.2}$$

for every positive bi-strategy $(x, y)$, where u is a positive constant (representing, obviously, the quantity $Q_i(0,0)$ demanded of good i, by the market, when both prices are fixed to be 0).

*Remark (about elasticity).* The demand's elasticity of the two functions with respect to the corresponding price is:

$$e_1(Q_1)(x,y) = \partial_1 Q_1(x,y)(x/Q_1(x,y)) = -2x/(u - 2x + y), \text{ and} \tag{2.3}$$

$$e_2(Q_2)(x,y) = \partial_2 Q_2(x,y)(y/Q_2(x,y)) = -2y/(u - 2y + x), \tag{2.4}$$

for every positive bi-strategy $(x, y)$.

Their values are negative, according to the economic law: *produced quantities are decreasing with respect to their prices*. So, if Emil (or Frances) increases his price, the consumers' demand will diminish.

*Assumption 4 (payoff functions).* First player's *profit function* is defined, classically, by the revenue

$$f_1(x,y) = x\, Q_1(x,y) - c = x(u - 2x + y) - c, \tag{2.5}$$

for every positive bi-strategy $(x, y)$. Symmetrically, for Frances, the profit function is defined by

$$f_2(x,y) = y\, Q_2(x,y) - c = y(u - 2y + x) - c, \tag{2.6}$$

for every positive bi-strategy $(x, y)$, where the positive constant c is the fixed cost. So we assume the variable cost to be 0 (this is not a great limitation for our example, since our interest is the interaction between the two players and the presence of the variable cost does not change our approach).

The bi-gain function is so defined by

$$f(x,y) = (x(u - 2x + y), y(u - 2y + x)) - (c, c), \tag{2.7}$$

for every bi-strategy $(x, y)$ of the game in the unbounded square $E^2$.

## 3. Study of the Bertrand's Normal Form Game

When the fixed cost is zero, we can assume that Emil and Frances have the compact strategy sets

$$E = F = [0, u], \tag{3.1}$$

indeed we have the following property.

*Property. A necessary condition in order to obtain both the quantities $Q_i(x,y)$ positive is that the pair of prices $(x,y)$ lies in the square $E^2$.*

*Proof.* The reader can easily prove the above interesting property, by imposing the positivity conditions for the affine functions $Q_i$. ♦

*The improper Bertrand game.* Besides, we will consider an extension of the Bertrand game with strategy spaces $E = F = [-u, u]$, in order to obtain a wider vision of the game itself by enlarging the bi-strategy space.

*Payoff function to examine.* When the fixed cost c is zero (this assumption determines only a "reversible" translation of the gain space), the bi-gain function $f$ from the compact square $[0, u]^2$ into the bi-gain plane $\mathbb{R}^2$ (respectively the function $f$ from the square $[-u, u]^2$ into the same plane $\mathbb{R}^2$) is defined by:

$$f(x,y) = (x(u - 2x + y), y(u - 2y + x)), \tag{3.2}$$

for every bi-strategy $(x, y)$ in the square S = $[0, u]^2$ (respectively, in the square S = $[-u, u]^2$ ) which is the convex envelope of its vertices, denoted by A, B, C, D starting from the origin (or from $(- u, u)$) and going anticlockwise.

When the characteristic price $u$ is 1, we will obtain the payoff vector function defined by

$$f(x,y) = (x(1 - 2x + y), y(1 - 2y + x)), \tag{3.3}$$

on the strategy square S = $[0, 1]^2$ (or S = $[-1, 1]^2$).

Now, we must find *the critical space of the game* and its image by the function $f$, before representing $f(S)$. For, we determine (as explained in (3, Carfi 2010), (6, Carfi 2009), (8, Carfi 2009), and (9, Carfi 2009) firstly the *Jacobian matrix* of the function $f$ at any point $(x, y)$ - denoted by $J_f(x,y)$. We will have, in vector form, the pair of gradients:

$$J_f(x,y) = ((y - 4x + 1, x), (y, - 4y + x + 1)), \tag{3.4}$$

and concerning the determinant of the above pair of vectors

$$\det J_f(x,y) = (-4y + x + 1)(y - 4x + 1) – xy \ = -4y^2 + 16xy - 3y - 4x^2 -3x + 1. \tag{3.5}$$

The *Jacobian determinant* is zero at those points $(x_1, y_1)$ and $(x_2, y_2)$ of the strategy square such that

$$y_1 = -\mathrm{sqrt}(192\, x_1^2 - 144\, x_1 + 25)/8 + 2\, x_1 - 3/8, \tag{3.6}$$

and

$$y_2 = \mathrm{sqrt}(192\, x_2^2 - 144\, x_2 + 25)/8 + 2\, x_2 - 3/8. \tag{3.7}$$

From a geometrical point of view, we will obtain two curves (Figure 1 with S = $[0, 1]^2$ and Figure 2 with S = $[-1, 1]^2$); they represent *the critical zone of Bertrand Game.*

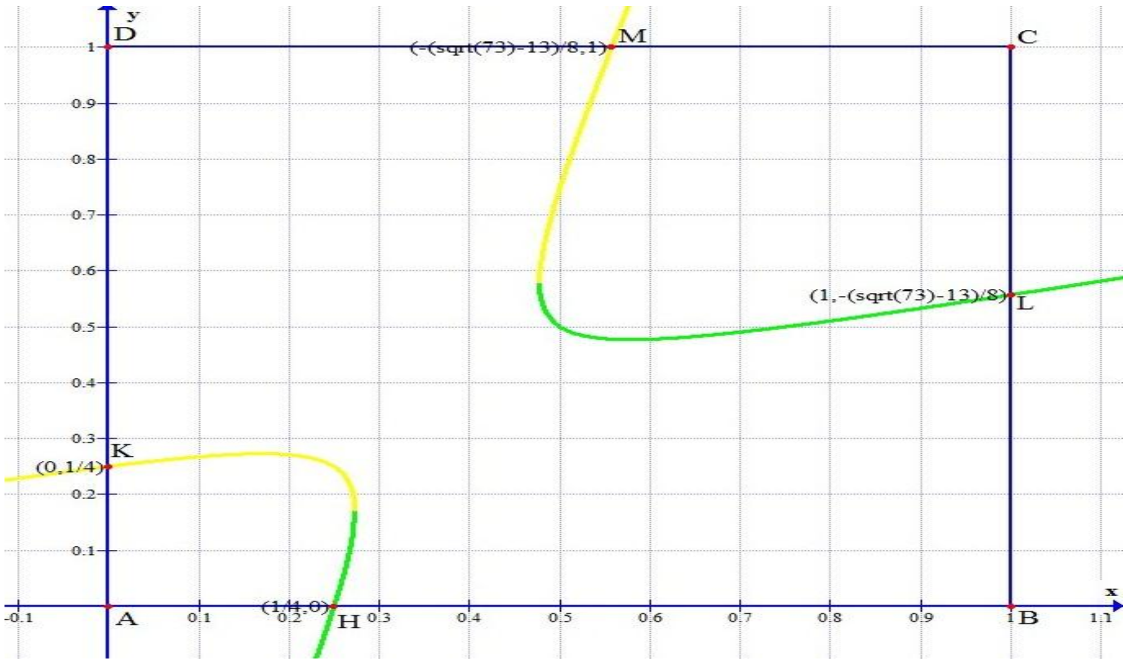


**Figure 1.** Critical zone of Bertrand game

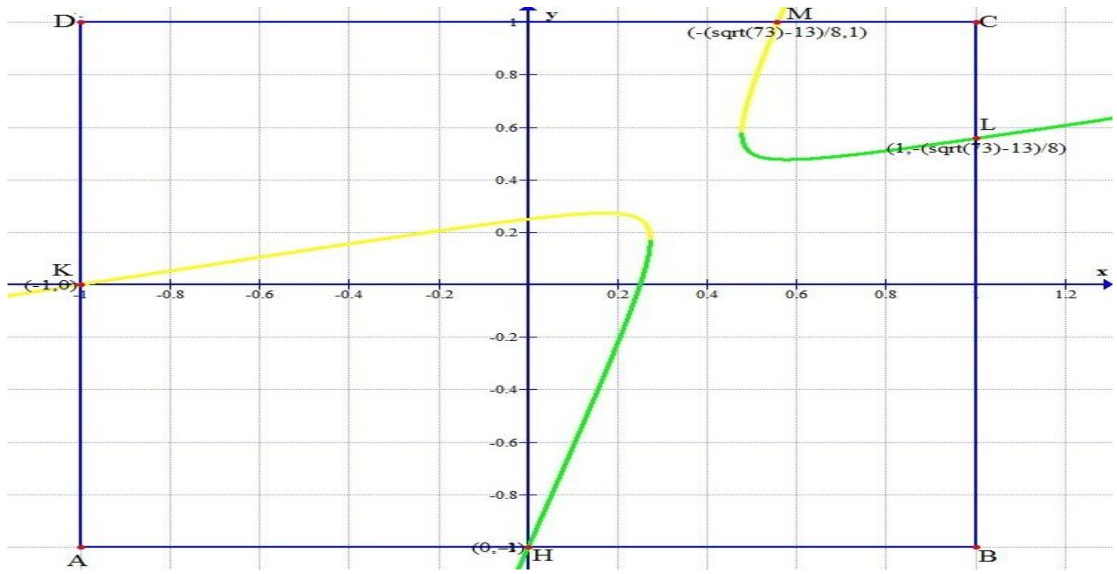


**Figure 2.** Critical zone of improper Bertrand game

## 4. Transformation of the Strategy Space

It is readily seen that the intersection points of the yellow curve with the boundary of the strategic square are the two points

M = (-(sqrt(73) - 13)/8, 1) , K = - (1,0)

*Remark.* The point M is the intersection point of the yellow curve with the segment [C, D], its abscissa μ verifies the non-negative condition and the following equation

$$8 = \text{sqrt}(192\mu^2 - 144\mu + 25) + 16\mu - 3, \quad (4.1)$$

this abscissa is so

μ = -(sqrt(73) - 13)/8

(approximately equal to 0,557).

We start from Figure 1, with $S = [0, 1]^2$. The transformations of the bi-strategy square vertices and of the points H, K, M are the following:

- A' = ƒ(A) = ƒ(0, 0) = (0, 0);
- B' = ƒ(B) = ƒ(1, 0) = (-1, 0);
- C' = ƒ(C) = ƒ(1, 1) = (0, 0);
- D' = ƒ(D) = ƒ(0, 1) = (0, -1);
- H' = ƒ(H) = ƒ(1/4, 0) = (1/8, 0);
- K' = ƒ(K) = ƒ(0, 1/4) = (0, 1/8);
- M' = ƒ(M) = ƒ(μ, 1) = $(2\mu - \mu^2/4, \mu - 1)$ approximately equal to (0,494, -0,443);
- L' = ƒ(L) = ƒ(1, μ) = $(\mu - 1, 2\mu - \mu^2/4)$ approximately equal to (-0,443, 0,4936).

Starting from Figure 1, with $S = [0, 1]^2$, we can do the transformation of its sides.

**Side [A, B].** Its parameterization is the function sending any point $x \in [0, 1]$ into the point $(x, 0)$; the transformation of this side can be obtained by transformation of its generic point $(x, 0)$, we have

$$f(x, 0) = (x - 2x^2, 0). \quad (4.2)$$

We obtain the segment with end points H' and D', with parametric equations

$X = x - 2x^2$ and $Y = 0$, (4.3)

with $x$ in the unit interval.

**Side [B, C].** It is parametrized by:

$(x = 1, y \in [0, 1])$;

the figure of the generic point is

$f(1, y) = (y - 1, -2y^2 + 2y)$. (4.4)

We can obtain the parabola passing through the points C', L', D' with parametric equations

$X = y - 1$ and $Y = -2y^2 + 2y$. (4.5)

**Side [C, D].** Its parameterization is:

$(x \in [0, 1], y = 1)$;

the transformation of its generic point is

$f(x, 1) = (-2x^2 + 2x, x - 1)$. (4.6)

We can obtain the parabola passing through the points B', M', C' with parametric equations

$X = -2x^2 + 2x$ and $Y = x - 1$. (4.7)

**Side [D, A].** Its parameterization is

$(x = 0, y \in [0, 1])$;

the transformation of its generic point is

$f(0, y) = (0, -2y^2)$. (4.8)

We obtain the segment [K', B'] with parametric equations

$X = 0$ and $Y = -2y^2$, (4.9)

with $y$ in the unit interval.

Now, we find *the transformation of the critical zone*. The parameterization of the critical zone is defined by the equations

$y_1 = -\,\mathrm{sqrt}(192x_1^2 - 144x_1 + 25)/8 + 2x_1 - 3/8$ (3.6)

and

$y_2 = \mathrm{sqrt}(192x_2^2 - 144x_2 + 25)/8 + 2x_2 - 3/8$. (3.7)

The parametrization of the GREEN ZONE is

$(x \in [0, 1], y = y_1)$;

the transformation of its generic point is

$f(x, y_1) = (x - 2x^2 + xy_1, y_1 - 2y^2_1 + xy_1)$, (4.10) a parametrization of the YELLOW ZONE is

$(x \in [0, 1], y = y_2)$;

the transformation of its generic point is

$$f(x, y_2) = (x - 2x^2 + xy_2, y_2 - 2y^2_2 + xy_2). \quad (4.11)$$

The transformation of the Green Zone has parametric equations

$$X = x - 2x^2 + xy_1 \text{ and } Y = y_1 - 2y^2_1 + xy_1, \quad (4.12)$$

and the transformation of the Yellow Zone has parametric equations

$$X = x - 2x^2 + xy_2 \text{ and } Y = y_2 - 2y^2_2 + xy_2. \quad (4.13)$$

We have two colored curves in *green* and *black* (Figure 4.1), breaking by two points of discontinuity T and U obtained by resolving the following equation

$$192x^2 - 144x + 25 = 0; \quad (4.14)$$

the solutions of the above equation are

$$x_1 = - (\mathrm{sqrt}(6) - 9)/24, x_2 = (\mathrm{sqrt}(6) + 9)/24, \quad (4.15)$$

and then, replacing them in the parametrical equations of the critical zone, and putting

$t = 9 + \mathrm{sqrt}(6)$ with $s = - \mathrm{sqrt}(t^2/3 - 6t + 25)/8$ and $u = 9 - \mathrm{sqrt}(6)$ with $v = - \mathrm{sqrt}(-6u + u^2/3 + 25)/8$

we obtain:

$T_1 = (t(s + t/12 - 3/8))/24 - t^2/288 + t/24$;

$T_2 = -2(s + t/12 - 3/8)^2 + s +(t(s + t/12 - 3/8))/24 + t/12 - 3/8$,

and,

$U_1 = ((u/12 + v - 3/8)u)/24 + u/24 - u^2/288$;

$U_2 = ((u/12 + v - 3/8)u)/24 + u/12 - 2(u/12 + v - 3/8)^2 + v - 3/8$.

So, we obtain - approximately - the point

$T = (0{,}298,\ 0{,}185)$,

and the point

$U = (0{,}171,\ 0{,}159)$.

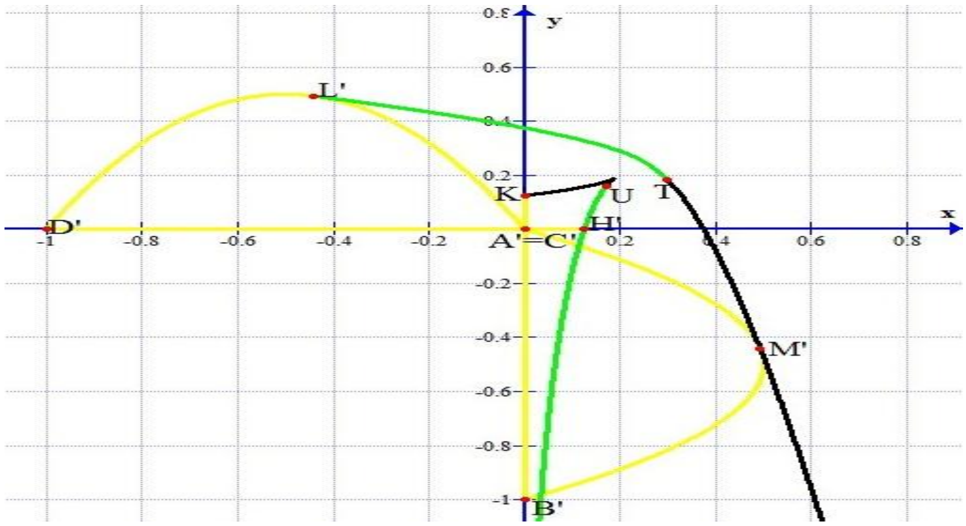


**Figure 3.** Payoff space of Bertrand game

*Payoff space of the improper Bertrand's game*

Starting from the Figure 2 with S = $[-1, 1]^2$; the projections of bi-strategy square's points are the following:

- A' = $f$(A) = $f$(-1, -1) = (-2, -2);
- B' = $f$(B) = $f$(1, -1) = (-2, -4);
- C' = $f$(C) = $f$(1, 1) = (0, 0);
- D' = $f$(D) = $f$(-1, 1) = (-4, -2);
- H' = $f$(H) = $f$(0, -1) = (0, -3);
- K' = $f$(K) = $f$(-1, 0) = (-3, 0);
- M' = $f$(M) = $f$(μ, 1) = ($2\mu - \mu^2/4$, μ - 1) approximately equal to (0,494, -0,443);
- L' = $f$(L) = $f$(1, μ) = (μ - 1, $2\mu - \mu^2/4$) approximately equal to (-0,443, 0,4936).

Starting from Figure 2 with S = $[-1, 1]^2$, we can do the transformation of its sides.

**Side [A, B].** Its parametric form is

$(x \in [-1, 1], y = -1)$;

the transformation of its generic point is

$$f(x, -1) = (-2x^2, -x - 3). \qquad (4.16)$$

We can obtain the parabola passing through the points A', H', B' with

$$X = -2x^2 \text{ and } Y = -x - 3. \qquad (4.17)$$

**Side [B, C].** Its parameterization is

$(x = 1, y \in [-1, 1])$;

the transformation of its generic point is

$$f(1, y) = (y - 1, -2y^2 + 2y). \qquad (4.18)$$

We can obtain the parabola passing through the points B', L', C' with

$$X = y - 1 \text{ and } Y = -2y^2 + 2y. \qquad (4.19)$$

**Side [C, D].** Its parameterization is

($x \in [-1, 1]$, $y = 1$);

the transformation of its generic point is

$$f(x, 1) = (-2x^2 + 2x, x - 1). \tag{4.20}$$

We can obtain the parabola passing through the points C', M', D' with parametric equations

$$X = -2x^2 + 2x \text{ and } Y = x - 1. \tag{4.21}$$

**Side [D, A].** Its parameterization is

($x = -1$, $y \in [-1, 1]$);

the transformation of its generic point is

$$f(-1, y) = (-y - 3, -2y^2). \tag{4.22}$$

We can obtain the parabola passing through the points D', K', A' with parametric equations

$$X = -y - 3 \text{ and } Y = -2y^2. \tag{4.23}$$

For the transformation of the critical zone and the coordinates of the points of discontinuity please refer to the case $S = [0, 1]^2$, we must remember only to widen the interval considered from $x, y \in [0, 1]$ to $x, y \in [-1, 1]$.

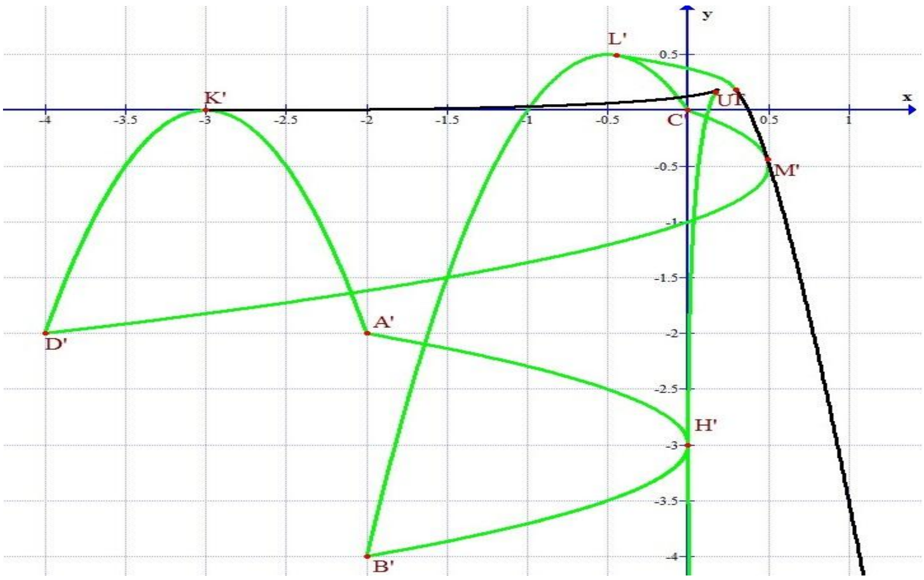


**Figure 4.** Payoff space of the improper Bertrand game

## 5. Non-cooperative Friendly Phase

Examining the Figure 3, in which $S = [0, 1]^2$, we will note that the game has two shadow extremes, that is the *shadow minimum* $\alpha = (-1, -1)$ and the *shadow maximum* $\beta = (1/2, 1/2)$.

The *Pareto minimal boundary of the payoff space* $f(S)$ is showed in the Figure 5 by the union of two segments

[A', B'] ∪ [D', A'], and it is colored in *orange*.

The *Pareto maximal boundary of the payoff space* $f(S)$ is the union of the two curve segments, on the transformations of the critical zone, with extreme points the pair of points (L', T) and (T, M'). They are colored in *green* and in *black*. Both Emil, and Frances do not control the Pareto maximal boundary; they could reach the point L' and M' of the boundary, but the solution is not many satisfactory for them. In fact, a player will suffer a loss and the other one will have a small win. Examining the Figure 4, in which $S = [-1, 1]^2$, we will note that the game has two shadow extremes, that is the *shadow minimum*

$\alpha = (-4, -4)$ and the *shadow maximum*

$\beta = (1/2, 1/2)$.

The *Pareto minimal boundary* $f(S)$ is showed in the Figure 6, it has only two points, the points D' and B'; observe that the weak minimal Pareto boundary is formed by the points D', A', B' and that the curve colored in yellow is Pareto minimal in the *ultra-weak sense*, this means only that if we fix one of the coordinate, in the canonical Cartesian projections of this curve, the other coordinate reaches its minimum exactly on the yellow curve. For *Pareto maximal boundary* $f(S)$, in the case $S = [0, 1]^2$, is the union of the boundary curves with end-points the pairs (L', T) and (T, M'), respectively.

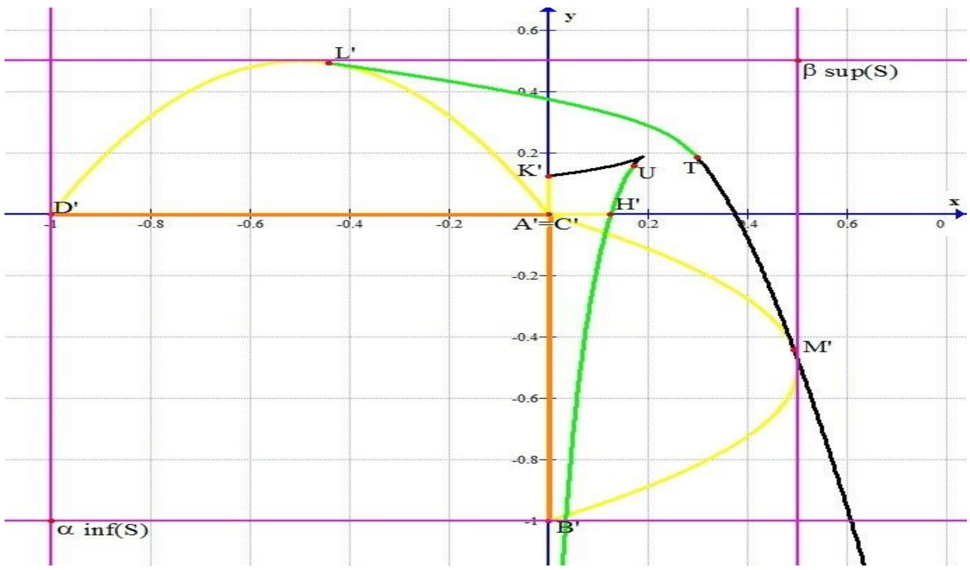


**Figure 5** Extrema of the Bertrand game

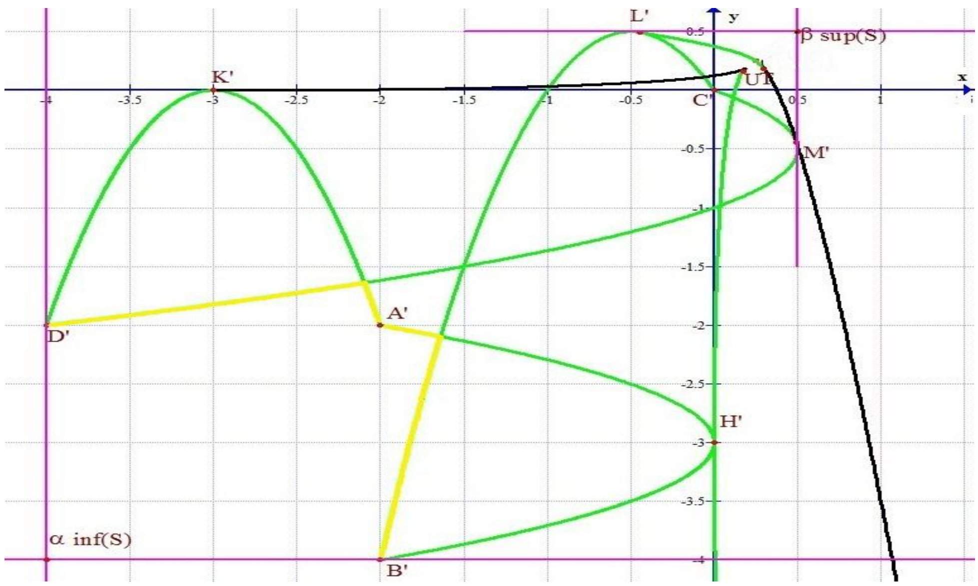


**Figure 6.** Extrema of the improper Bertrand game

## 6. Properly non-cooperative (egoistic) Phase

Now, we will consider *the best reply correspondences* between the two players Emil and Frances, in the cases $S = [0, 1]^2$ and $S = [-1, 1]^2$.

If Frances sells the commodity at the price $y$, Emil, in order to reply rationally, should maximize his *partial profit function*

$$f_1(\cdot, y) : x \mapsto x(1 - 2x + y), \tag{6.1}$$

on the compact interval [0, 1] or [-1, 1].

According to the Weierstrass theorem, there is at least one Emil's strategy maximizing that partial profit function and, by Fermàt theorem, the *Emil's best reply strategy to Frances' strategy* $y$ is the only price

$$B_1(y) = x^* := (1/4)(y + 1). \tag{6.2}$$

Indeed, the partial derivative

$$f_1(\cdot, y)'(x) = -4x + 1 + y, \tag{6.3}$$

is positive for $x < x^*$ and negative for $x > x^*$.

So, the Emil's best reply correspondence is the function $B_1$ from the interval [0, 1] into the interval [0,1], defined by

$$y \mapsto (1/4)(y + 1), \tag{6.4}$$

in the proper case, and $B_1$ from [-1, 1] into [-1,1], defined by

$$y \mapsto (1/4)(y + 1), \tag{6.5}$$

in the improper one.

As we already observe, our Bertrand game is a symmetric game, therefore the Frances' best reply correspondence is the function $B_2$ from [0, 1] into [0,1], defined by

$$x \mapsto (1/4)(x + 1), \tag{6.6}$$

In the proper case, or the function $B_2$ from [-1, 1] into [-1,1] defined by

$$x \mapsto (1/4)(x + 1), \tag{6.7}$$

in the improper one.

The *Nash equilibrium* is the fixed point of the symmetric Cartesian product function B of the pair of two reaction functions $(B_2,B_1)$ defined (canonically) from the Cartesian product of the domains into the Cartesian product of the co-domains (in inverse order), by

$$B : (x, y) \mapsto (B_1(y), B_2(x)), \tag{6.8}$$

that is the only bi-strategy (x,y) satisfying the below system of linear equations

$$x = (1/4)(y + 1),\ y = (1/4)(x + 1), \tag{6.9}$$

that is the point N = (1/3, 1/3) - as we can see also from the two Figures 7 and 8 - which determines a bi-gain

N' = (2/9, 2/9),

as Figures 8 and 10 will show.

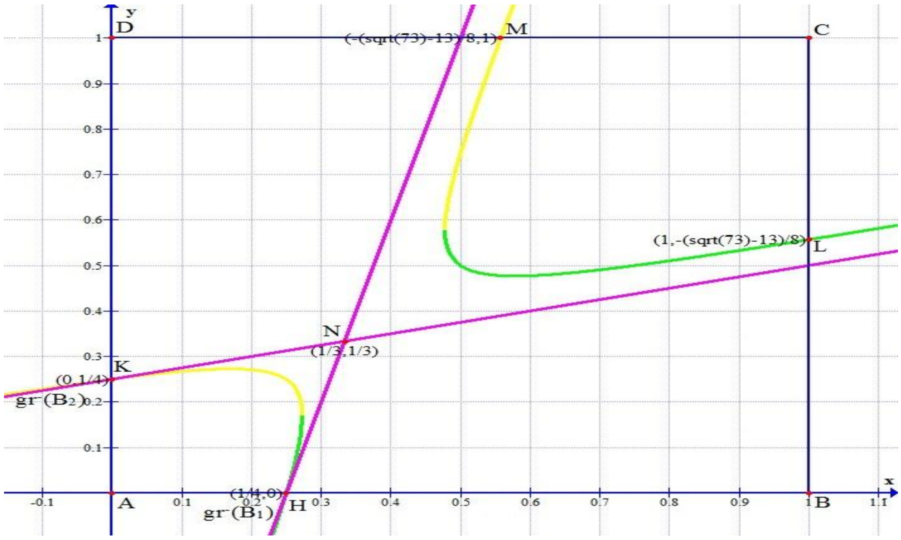


**Figure 7.** Nash Equilibrium of the proper game

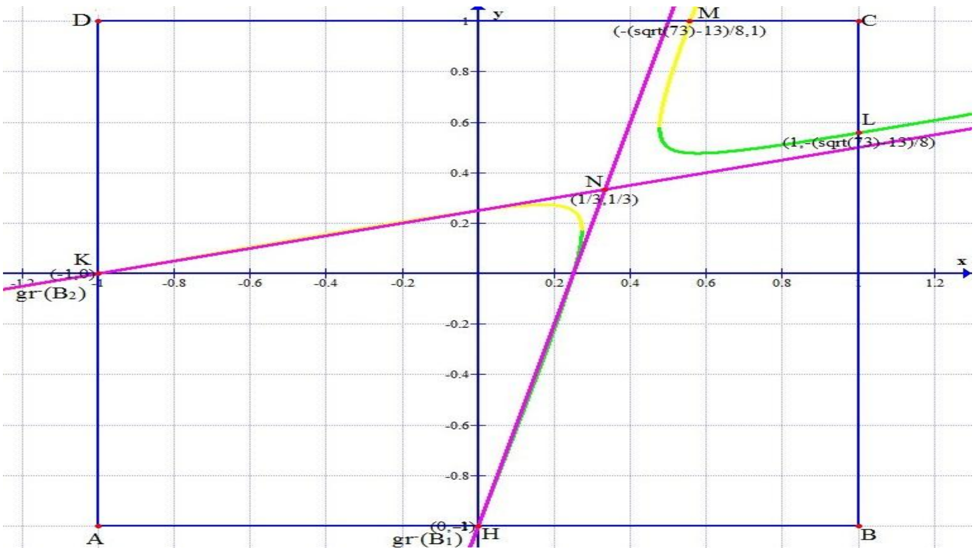


**Figure 8.** Nash equilibrium of the improper game

The Nash equilibrium is not completely satisfactory, because it is not a Pareto optimal bi-strategy, but it represents the only properly non-cooperative game solution.

*Remark (demand elasticity at Nash equilibrium).* Concerning the Nash equilibrium, we can also calculate the demand's elasticity with respect to the corresponding price. At first, we must remember the two demand functions, and then we obtain

$$e_1(Q_1)(x,y) = \partial_1 Q_1(x,y)(x/Q_1(x,y)) = (-2x/(u - 2x + y)), \quad (6.10)$$

and

$$e_2(Q_2)(x,y) = \partial_2 Q_2(x,y)(y/Q_2(x,y)) = (-2y/(u - 2y + x)). \quad (6.11)$$

Then, we have

$$e_1(Q_1)(N) = \partial_1 Q_1(N)((1/3)/Q_1(x,y)) = (-(2/3)/(1 - (2/3) + (1/3))) = -1, \quad (6.12)$$

and

$$e_2(Q_2)(N) = \partial_2 Q_2(N)((1/3)/Q_2(x,y)) = (-(2/3)/(1 - (2/3) + (1/3))) = -1. \quad (6.13)$$

So we can deduce that:
*at the non-cooperative equilibrium N, since*

$$|e_1(Q_1)(N)| = 1 \text{ and } |e_2(Q_2)(N)| = 1, \quad (6.14)$$

*the demands will be elastic with respect to the prices; therefore if the price increases of one unit, demand will reduce of one unit too.*

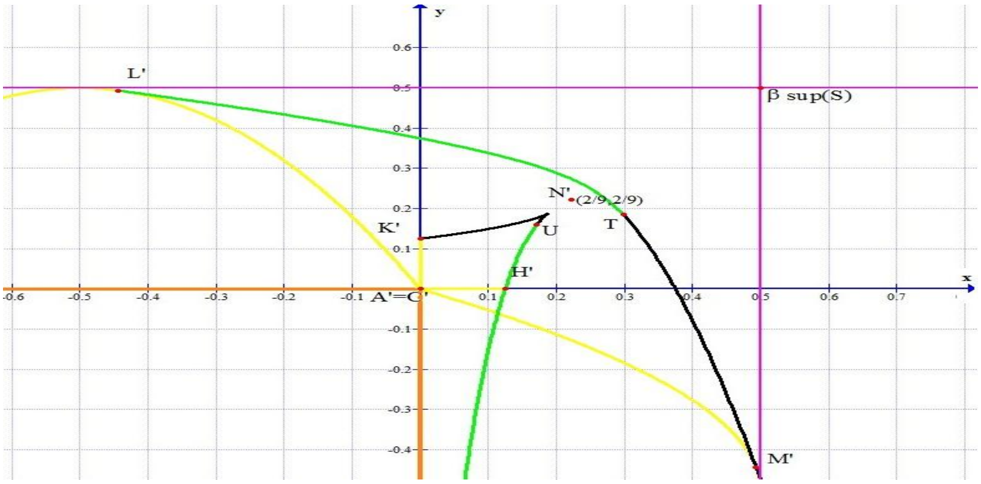


**Figure 9.** Payoff at Nash equilibrium of the proper Bertrand game

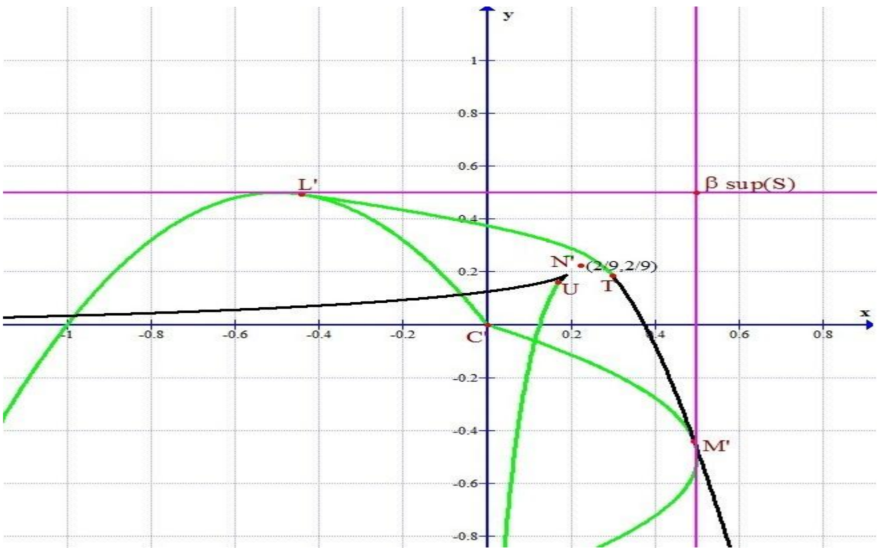


**Figure 10.** Payoff at Nash equilibrium of the improper game

## 7. Defensive and Offensive Phase

Players' conservative values are obtained through their *worst gain functions*.

*Worst gain functions*. On the square $S = [0, 1]^2$, *Emil's worst gain function* is defined by

$$f^{\#}_1(x) = \inf_{y \in F} x(1 - 2x + y) = x - 2x^2, \tag{7.1}$$

its maximum will be

$$v^{\#}_1 = \sup_{x \in E} (f^{\#}_1(x)) = \sup_{x \in E} (x - 2x^2) = 1/8, \tag{7.2}$$

attained at the conservative strategy $x^{\#} = 1/4$.

Symmetrically, *Frances' worst gain function* is defined by

$$f^{\#}_2(y) = y - 2y^2, \tag{7.3}$$

its maximum will be $v^{\#}_2 = 1/8$ attained at the unique conservative strategy $y^{\#} = 1/4$.

*Conservative bi-value***.** The *conservative bi-value* is

$$v^{\#} = (v^{\#}_1, v^{\#}_2) = (1/8, 1/8).$$

*The worst offensive multifunctions* are determined by the study of the *worst* gain functions.

The Frances' worst offensive reaction multifunction $O_2$ is defined by $O_2(x) = 0$, for every Emil's strategy $x$; indeed, fixed an Emil's strategy $x$ the Frances' strategy 0 minimizes the partial profit function $f_1(x, .)$. Symmetrically, the Emil's worst offensive correspondence versus Frances is defined by $O_1(y) = 0$, for every Frances' strategy $y$.

*The dominant offensive strategy* is 0 for both players. Indeed the offensive correspondences are constant.

*The offensive equilibrium* A = (0,0) bring to the payoff *A*' = (0, 0), in which the profit is zero for both players.

*The core of the payoff space* (in the sense introduced by J.P. Aubin) is the part of the Pareto maximal boundary contained in the cone of upper bounds of the conservative bi-value $v^{\#}$; the conservative bi-value gives us a bound for the choice of cooperative bi-strategies.

*Conservative phase of the Extended Bertrand game*

If the strategy space is the extended square $S = [-1, 1]^2$, *Emil's worst gain function* is defined by

$$f^{\#}_1(x) = \inf_{y \in F} x(1 - 2x + y) = -2x^2, \tag{7.4}$$

its maximum will be

$$v^{\#}_1 = \sup_{x \in E} (f^{\#}_1(x)) = \sup_{x \in E} (-2x^2) = 0, \tag{7.5}$$

attained at the *conservative strategy* $x^{\#} = 0$.

Symmetrically, *Frances' worst gain function* is defined by

$$f^{\#}_2(y) = -2y^2, \tag{7.6}$$

its maximum will be $v^{\#}_2 = 0$, attained at the *conservative point* $y^{\#} = 0$.

*The conservative bivalue* in the improper case is

$$v^{\#} = (v^{\#}_1, v^{\#}_2) = (0, 0).$$

*The worst offensive multifunctions* can be determined by the study of the *worst* gain functions.

For every strategy Emil could choose, he has the minimum gain when Frances sells his commodity at the price -1. This result is unusual from an economic point of view, but it can make sense in a *short period deep competition*.

Then *Frances' worst offensive multifunction* is defined by $O_2(x)$ = -1, for every Emil's strategy $x$; symmetrically, we obtain $O_1(y)$ = -1, for every Frances' strategy $y$.
The *dominant offensive strategy* is -1 for both players, and the offensive (dominant) equilibrium *A* = (-1, -1) brings to the point *A*' = (-2, -2), in which a severe loss is registered for both players.
The *conservative knot* of the game is the point (0, 0), whose image is the point (0, 0), which coincides with the point *C*'.

*The core of the payoff space* is the part of Pareto maximal boundary contained into the cone of upper bounds of the conservative bi-value $v^{\#}$; this bounds the choice of cooperative bi-strategies.

## 8. Cooperative Phase

When there is an agreement between the two players, *the best compromise solution (in the sense introduced by J.P. Aubin)* is the pair of strategies (1/2, 1/2), showed graphically in the Figures 11 and 12. This compromise bi-strategy determines the bi-gain (1/4, 1/4).

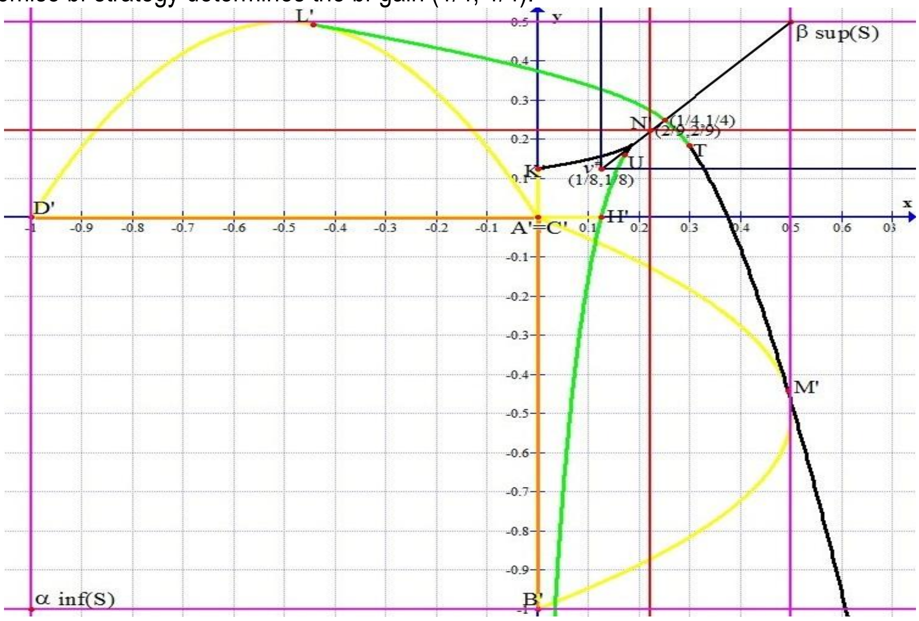


**Figure 11.** Conservative study

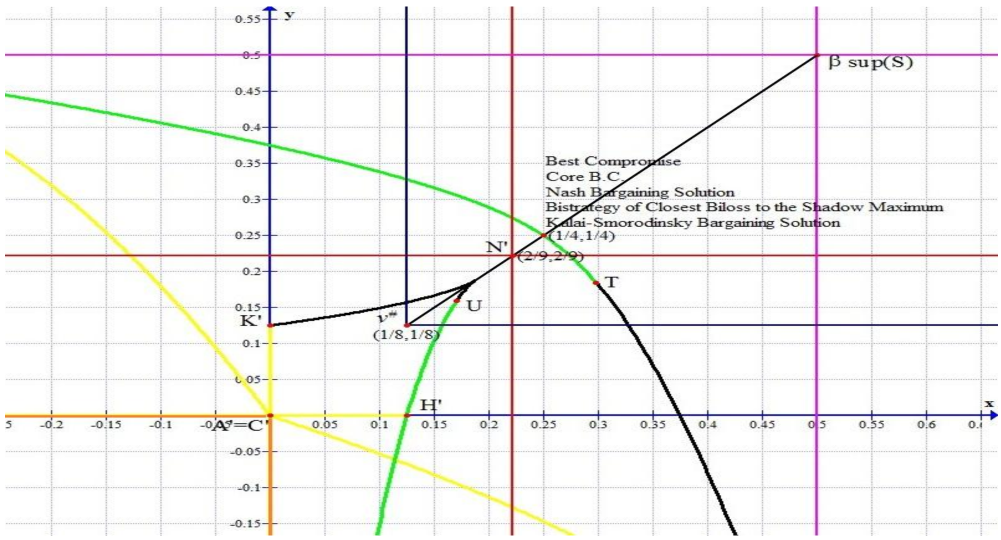


**Figure 11.** The core and the Kalai-Smorodinsky payoff of the proper game

Besides, the best compromise solution coincides with the *core best compromise*, with the *Nash bargaining solution*, with the *bi-strategy with closest bi-gain to the shadow maximum* and with the *Kalai-Smorodinsky bargaining solution*. It coincides also with *the transferable utility solution* which is the unique Pareto strategy that maximizes the aggregate utility function $f_1 + f_2$, this can be easily viewed by geometric evidences considering on the payoff universe the levels of that aggregate function, which are affine lines parallel to the vector (1,-1).

*Selection of Pareto solutions.* The Nash equilibrium can help in the selection of Pareto solutions (Figures 13 and 14). Indeed, if Emil and Frances decide to cooperate their possible choices will lead to those points of Pareto boundary which are also upper bounds of the Nash Equilibrium, in order to obtain a compromise solution strictly better than the non-cooperative one.

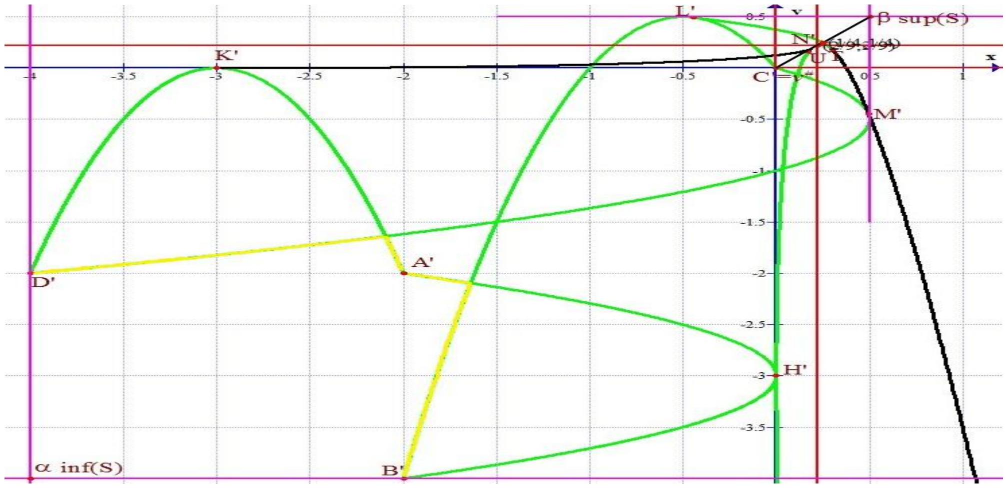


**Figure 12.** Conservative exam of the improper Bertrand game

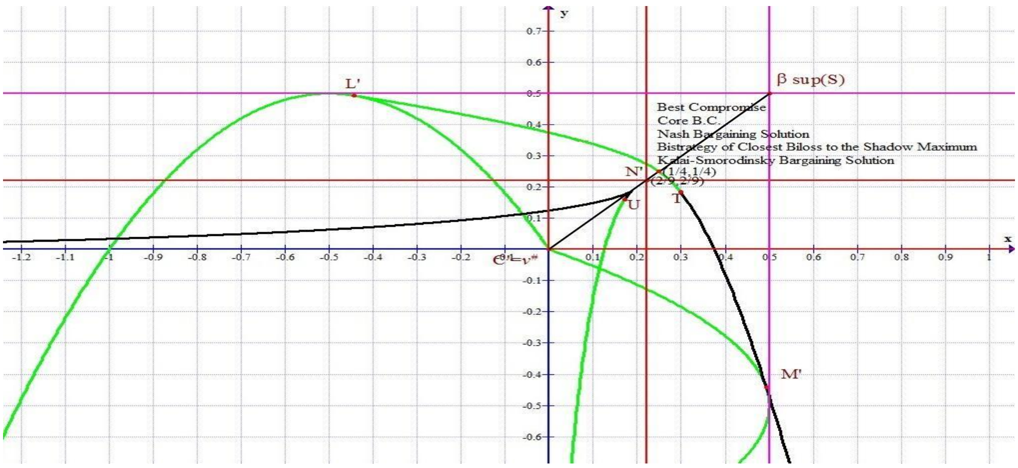


**Figure 13.** The Core of the improper Bertrand game

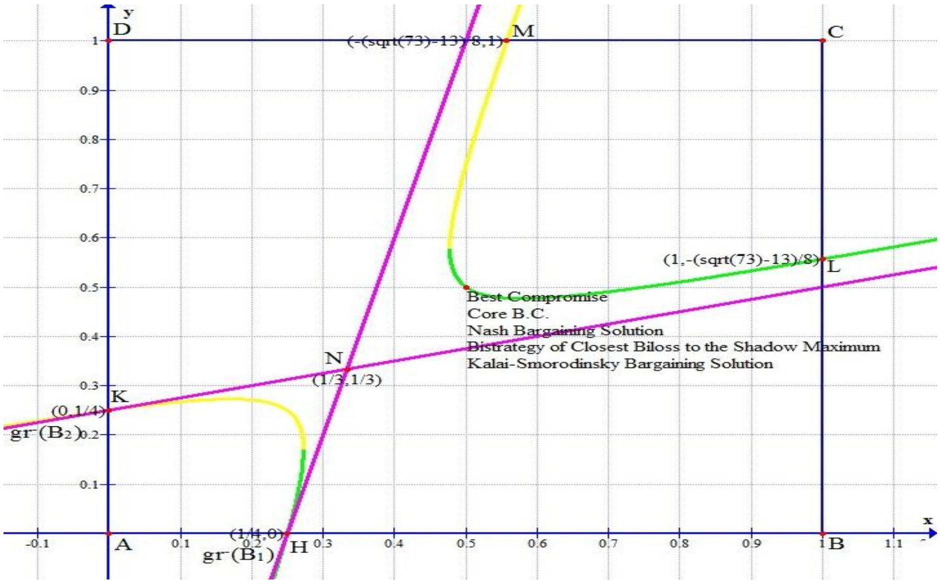


**Figure 14.** Compromise solutions of Bertrand game

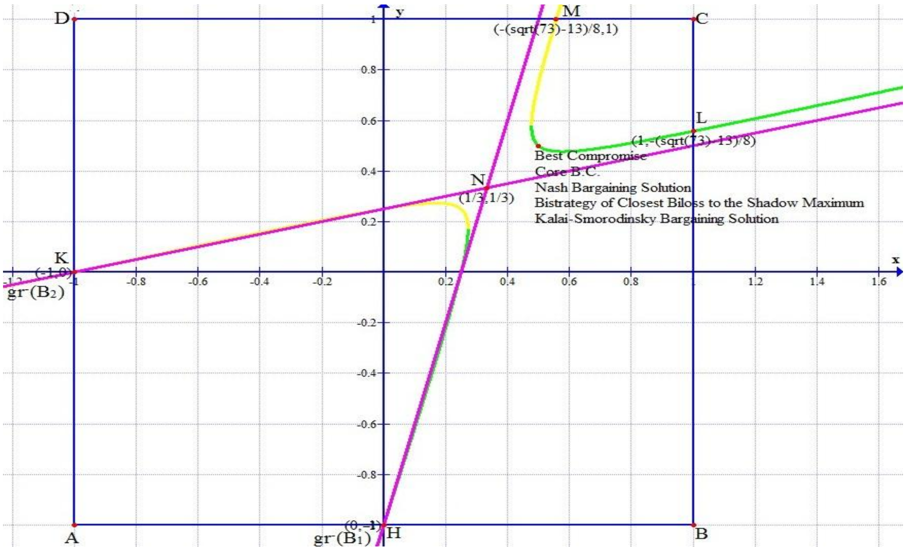


**Figure 16.** Compromise solutions of improper Bertrand game

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# CRISIS IN THE EURO AREA. COOPETITIVE GAME SOLUTIONS AS NEW POLICY TOOLS

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**Abstract**

*The crisis within the euro area has become frequent during 2010. First was the Greek economy to face a default problem of its sovreign debt, in November it was Ireland who has been in a serious financial situation at the verge of collapse causing difficulties to the euro. In this contribution we focus on the Greek crisis and we suggest, through a model of competition based on game theory and conceived at a macro level, feasible solutions in a cooperative perspective for the divergent interests which drive the economic policies in Germany and Greece, with the aim of improving the position of Greece, Germany and the whole euro area, also making a contribution to expand the set of macroeconomic policy tools.*

*By means of our general analytical framework of competition, we show the strategies that could bring to feasible solutions in a cooperative perspective for Germany and Greece, where these feasible solutions aim at offering a win-win outcome for both countries, letting them to share the pie fairly within a growth path represented by a non-zero sum game. A remarkable analytical result of our work consists in the determination of the win-win solution by a new selection method on the transferable utility Pareto boundary of the competitive game.*

**Keywords:** European Monetary Union, competitive games, macroeconomic policy, bargaining solutions

**JEL Classification**: F40, E6, D7, C71, C72, C78

## 1. Introduction[1]

In the present work we analyze the crisis in the euro area and we suggest, through a model of coopetition based on game theory and conceived at a macro level, feasible solutions in a cooperative perspective for the divergent interests which drive the economic policies in Germany and Greece, with the object of improving the position of Greece, Germany and the whole euro area. Our model is also aimed at making a contribution to expand the set of macroeconomic policy tools.

The work is organized as follows: the first section examines the general topic of the crisis in the euro area focusing on the Greek crisis and concentrating on the real aspects of the crisis. The second section suggests a possible way out to reduce the intra-euro zone imbalances through a new macroeconomic tool based on coopetitive solutions within a growth path. The third section introduces a game theory framework of coopetition. The fourth section provides two specific and original models of coopetitive games applied to the euro zone context and shows their solutions. Conclusions end up the paper.

## 2. The Euro and the Crisis

The crisis within the euro area has become frequent during 2010. First was the Greek economy to face a default problem of its sovereign debt last spring, in November it was Ireland who has been in a serious financial situation at the verge of collapse, due mainly to poor quality bank regulation, causing difficulties to the euro. So the euro zone Governments and international institutions are continually trying to solve these problems that create instability and jeopardize the very existence of European Monetary Union. Many of the countries of the European Monetary Union have accumulated large budget deficit to GDP ratios in 2009, which are in turn

[1] Sections 1 and 2 of this paper are written by D. Schilirò, sections 3 and 4 are written by D. Carfì, the Introduction and Conclusions have been drawn by the two authors. We wish to thank Giambattista Dagnino, Davide Provenzano, and Albert E. Steenge for their helpful comments and suggestions. We wish also to thank Samantha Pellegrino for the practical realization of figures.

caused by the global crisis that since 2008 developed itself in the United States for then spreading around the world.

The global crisis has inevitably burdened the public debt of countries such as Greece, Ireland, Portugal, Spain and even Italy, which have found difficult to finance their debt in the financial markets, creating a problem of instability and cohesion of the European Monetary Union. But this is only one aspect of the crisis, the other one, related to the former, is the recession hitting the EMU economies, which are suffering of high unemployment, slowing down of production, difficulties in exporting, further crisis of the welfare state.

In this contribution we focus on the Greek crisis, we know that EMU Governments and IMF agreed to provide Greece with enough financing to cover its refinancing needs for three years, while the Greek government commits to an additional tough austerity program. We also know that Germany is the country of the euro area which has a large trade surplus with Greece and other euro partners, hence strong trade imbalances occur within the euro zone economy.

Germany is the country who has profited most from the euro since the start of the European Monetary Union, according to Adam Posen (2010). Because the benefits received from the German economy have been possible thanks to a cooperative economic system, the main purpose of our paper is to explore win-win solutions for Greece and Germany, involving improvements in domestic demand in Germany.

We do not analyze the causes of the financial crisis in Greece and its relevant political and institutional effects on the European Monetary Union. Rather we focus on some crucial aspects of the Greek economy, with their implications on the euro area. Specifically we concentrate on stability and growth, which should drive the economic policy of Greece, Germany and the other euro countries.

The deep financial crisis of Greece, which was almost causing the default of its sovereign debt, determining also financial instability in the European markets and the devaluation of the euro, has revealed the weaknesses of Greek economy. This crisis has also showed the contradictions that have been characterized the EMU and the euro since their start[2].

Furthermore, EMU is featured by the presence of two countries, Germany and France, which have a major and increasing political and economic role.

Greece is a country with a total population of 11 million and it represents 2,6% of the euro zone's GDP. This country adopted the euro in 2001 and then interest rates fell to near German levels, the lowest in the euro area, fueling consumer spending and house prices. Since joining EMU, Greece has lost competitiveness and, because of that, Greek's unit labor cost rose 34 percent from 2000 to 2009. Thus, Greece relied on state spending to drive growth. With the outbreak of the crisis, debt in Greece has surged as in the other countries, but in 2009 Greece recorded a deficit/GDP ratio of 13.6%, one of the highest of the euro zone economies. This has created deep concerns about its fiscal sustainability.

Greece has also accumulated a huge debt of about 310 billion Euros, thus its financial exposition prevents the Greek government to find capital in the financial markets. The country, therefore, has become at risk of sovereign default. In the meantime the other EMU countries, after a period of uncertainty which raises the cost of the bailout, have decided to help Greece financially also with the support of IMF[3]. This financial contribution is likely to be given until 2012 and it will be very substantial[4]. But tough austerity conditions are requested in return for the emergency loans, which are to be paid with interest rates below the market rates, the Greek Government is required to take courageous and specific actions that will lastingly and credibly consolidate the public budget[5]. The EMU-IMF package also includes measures to enhance competition in many sectors which are still protected; thus the country is expected to reduce its budget deficit from 13.6 per cent of gross domestic product to below 3 per cent by 2014.

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[2] One feature of the institutional setting of the European Monetary Union is the Stability and Growth Pact that guards against the emergence of public deficits and debt, but actually there isn't a true and effective mechanism of enforcement in the Pact. Therefore the budget policy in each country of the euro zone is not under control. Yet the European Commission has, just after the Greek crisis, proposed tougher rules to enforce fiscal discipline in the euro zone and to set up a permanent crisis management mechanism to prevent sovereign debt disasters.

[3] An agreement has been reached on May, 2nd, between the Euro group, the IMF and the Greek Government.

[4] The total sum given to Greece in three years should be of 110 billion Euros.

[5] First, to recover from the budget disequilibrium, Greece is expected to improve the primary balance of 10 percent of GDP over the next three years (This is a heavy task, but other economies like Latvia and Hungary have succeeded in the recent past with the help and the assistance of the IMF and EU). Moreover, the package includes measures to reduce the size of Greece's public sector, cuts in public sector salaries and pensions, a rise in value added tax and other tax increases.

However, although a restrictive fiscal policy and budget austerity are necessarily implemented by the Greek Government, they could be insufficient for Greece to overcome its crisis. The austerity measures are likely to hit hard the Greek economy, since its growth is expected to be negative this year and the next year, making the financial recovery even more problematic[6]. Furthermore, exports are much less than imports, so the trade balance shows a deficit around 10%. Therefore, the focus of economic policy of Greece should be on its productive system and growth must become the major goal for the Greek economy. This surely would help its re equilibrium process.

On the other hand, Germany is considered the soundest European economy. First of all, it accounts for about one-third of the euro zone economy. Secondly, it is the world's second biggest exporter, but its wide commercial surplus is originated mainly by the exports in the euro area, that accounts for about two thirds. Furthermore, since 2000 its export share has gradually increased *vis-à-vis* industrial countries. Thirdly, its government has not allowed itself the extraordinary budget deficits that are threatening economies like Ireland, Italy, Greece, Portugal and Spain. Despite these positive records, the contribution of domestic demand to real GDP from 1999 onwards in Germany has been weak. It is clear, from such a context, that the Germany's growth path has been driven by exports. We do not discuss in this work the factors explaining Germany's increase in export share, but we observe that its international competitiveness has been improving, with the unit labor cost which has been kept fairly constant, since wages have essentially kept pace with productivity. Therefore the prices of the German products have been relatively cheap, favoring the export of German goods towards the euro countries and towards the markets around the world, especially those of the emerging economies (China, India, Brazil, and Russia). Finally, just during 2010 Germany has recovered very well from the 2008-2009 global crisis and is growing at a higher rate than the others euro partners.

Thus, we share the view that Germany (and the other surplus countries of the euro area, i.e. Netherlands) should contribute to overcome the crisis of the EMU economies and of Greece in particular stimulating its domestic demand and relying less on exports towards the euro area.

Germany, as Adam Posen (2010) underlined (Abadi 2010), has benefited from being the anchor economy for the euro zone over the last 11 years. In fact, it enjoyed a wider and deep range of trade in the euro currency than it had under the Dutch Mark. For instance, in 2009, during a time of global contraction, Germany has been a beneficiary, being able to run a sustained trade surplus with its European neighbors. Germany exported, in particular, 6.7 billion Euros worth of goods to Greece, but imported only 1.8 billion Euros worth in return.

Clearly a policy which aims at growth in Greece, Germany and the whole euro area is very important, especially if we take a medium-long term perspective and if we consider that the rate of unemployment in the euro area has reached 10.1%[7], the highest rate in almost 12 years[8].

We believe that a policy that aims at adjusting budget and trade imbalances and looks at improving the growth path of the real economy in the medium and long term in Greece is the only possible one to assure a stable re-balancing of the Greek economy and to contribute to the stability of the euro area. As we have already argued, German modest wage increases and weak domestic demand favored the export of German goods towards the euro countries. This is why Posen, as reported by *Business Week* on March 31, 2010, said that Germany should boost domestic demand and increase wages to ease the lopsided euro-region trade flows that restrict growth in economies like Greece and Portugal. Therefore he suggests a "*win-win solution*"[9] for the EMU countries, which entails that Germany, which still represents the leading economy, should stimulate domestic demand, increase wages in its own country, so that to make its own people better off, and thereby ease some of the pressure on the southern countries of the euro area. In Posen's proposal there is a clear suggestion to

[6] This view, of course, is not shared by the economists who believe that fiscal adjustments not always cause recessions (Giavazzi, Pagano 1990, Von Hagen, Strauch 2001, Alesina, Adagna 2009).

[7] Source: Euro stat. The figure refers to April 2010.

[8] Another aspect to highlight is that despite the new huge rescue plan of 750 billion Euros supported by the EU and IMF to avoid the contagion of the Greek crisis to the other EMU countries, the recent turmoil in the financial markets and the consequent weakening of the euro seem to confirm the poorly optimistic expectations of the financial markets on the future of the Greek economy. Investors are looking for a credible plan that indicated public finances in Greece but also in whole euro area could be kept at a sustainable level. In this context, the view is that a partial debt restructuring by the Greek government might become a sensible and realistic solution.

[9] A win-win solution is the outcome of a game which is designed in a way that all participants can profit from it in one way or the other. In conflict resolution a win-win strategy is a process that aims to accommodate all disputants.

Germany to re-balance its trade surplus. Of course, we are aware that this is a mere hypothesis[10]. Although Germany has been pursuing a strategy of competitiveness based on investments in technology and R&D on the one hand, on industrial relations which are featured by cooperative behaviors between labor and capital on the other, since 2003. We believe that this cooperative attitude, which is a hallmark of German capitalism, can be also taken with respect to its euro partners and the Greece in particular. Thus we pursue our hypothesis and suggest a game theory coopetitive model as an innovative instrument to analyze possible solutions to obtain a win-win outcome for Greece and Germany, which would also help the whole EMU economy.

Giving that Greece must fulfill the conditions of the agreement signed with the euro zone Governments and the IMF for their financial help and, for this reason, it must implement a fiscal policy of government budget consolidations, with current spending cuts and tax increases, to reduce its public and private debt, these changes in current variables (taxes, incentives, provision of public services) would probably also change the expectations about future fiscal policy[11]. In our view, Greece must keep its wages and salaries under control and, at the same time, focus on investments and exports as the two main strategic variables to improve the structure of production and to shift the aggregate demand towards a higher growth path. However, aiming at exports for a country like Greece that has a low “extra euro area” export share on GDP (about 4%) does not mean to rely on the external demand, for instance through the devaluation of the euro, rather to follow an appropriate medium term strategy. In this medium term strategy, Greece should focus on innovative investments, especially investments in knowledge (Schilirò 2010b), to change and improve its production structure and to increase its production capacity and its productivity, which is made possible by the structural change process. As a result of that its competitiveness will raise. An economic policy that focuses on investments and exports, instead of consumptions, will address Greece towards a sustainable growth and, consequently, its financial reputation and stability will get improved.

## 2. A New Tool for Macroeconomic Policy: Coopetitive Solutions for the Greek Crisis

The idea which is driving our model to face the Greek crisis is based on a notion of coopetition where the cooperative aspect will prevail. Thus we are not talking about a situation in which Germany and Greece are competing in the same European market for the same products, rather we are assuming a situation in which Germany stimulates its domestic demand and, in doing so, will create a larger market for products from abroad, but also we are envisaging the case in which Germany purchases a greater quantity of Greek products, in this case Greece increases its exports, selling more products in Germany. The final results will be that Greece will find in a better position, but also Germany will get an economic advantage determined by the higher growth in the two countries and, finally, because it will prevail a greater stability within the EMU system. Therefore we provide, in the present work, a new set of tools based on the notion of coopetition that could be fruitful for the setting of the Greek policy issues.

The concept of coopetition has been devised following different theoretical approaches. Essentially the literature on coopetitive games has a microeconomic origin and has an important point of reference in the seminal paper of Brandenburger, and Nalebuff (1995) who studied the strategic behavior of firms applying some basic notion of game theory and elaborated their theoretical original concept of coopetition within a competitive environment. Brandenburger, and Nalebuff suggest the term *coopetition* (a situation in which the firm must cooperate and compete at the same time) to indicate a situation in which the firm thinks about both cooperative and competitive ways to change the game (1995, p.59).13 Another approach to coopetition represents the synthesis between the competitive paradigm (Porter 1985) and the cooperative paradigm (Gulati, Nohria, Zaheer 2000), a sort of integrative framework between the two, like that offered by Padula, and Dagnino (2007), who define coopetition as the intrusion of competitive elements into a cooperative environment, because of the partially divergent interests among the partners. Thus coopetition is a complex construct and it is the result of the interplay between competition and cooperation.

[10] After the Greek crisis, because of the turmoil in the financial markets, the German government has decided to take austerity fiscal measures, which consists of a seven years plan of government budget consolidations of 70 billion Euros (10 billion Euros for each year), based mainly on structural spending cuts to welfare payments and reduction in the public sector. This plan, however, will also favor investment in education and research to improve Germany’s capacity to compete at a global level.

[11] Regarding the indirect positive effect on aggregate demand see authors Hellwig, and Neumann (1987) that merge the Keynesian view and the expectations view or “German view” on budget cutting. See also Giavazzi, Pagano (1990).

Our model of coopetition is closer to the approach that regards coopetition as a complex construct rooted in a cooperative environment. Thus we suggest a model of coopetitive games, applied at a macroeconomic level, which intends to offer possible solutions to the partially divergent interests of Germany and Greece in a perspective of a cooperative attitude that should drive their policies. Another important goal of the model is to provide a new tool of macroeconomic policy for the crisis in the euro area, thus enriching the toolbox of economic policy.

### 3. The General Definition of Coopetitive Game

In this section we provide an original recent definition of coopetitive game, which we shall use, in the next section, to build up two economic models feasible to represent the interaction of the two EMU countries, Germany and Greece. The two above coopetitive models will show possible new solutions reasonable in a particular coopetitive context, defined by the set of strategy profiles at disposal of the two countries and by a set of possible convenient *ex ante* agreements. This suggested analytical framework enables us to widen the set of possible solutions from a purely competitive into a coopetitive context and moreover it allows "to share the pie fairly" in a win-win scenario. At the same time, it permits to examine the range of possible economic outcomes along a coopetitive dynamic path. Finally, we propose a rational way to limit the space within which the coopetitive solutions can be determined.

*Remark:.* The basic original definition we propose and apply of coopetitive game is that introduced in 2010 by D. Carfì in (Carfì 2010), and [Baglieri, Carfì, and Dagnino 2010.]; the method that we shall use to study the payoff space of a normal form game can be found in (8. Carfì 2010), and (12, Carfì, Ricciardello 2010.); the complete study of a normal form game is presented and applied in (7, Carfì 2009), (9, Carfì 2009), and (10, Carfì 2009); for a general definition and basic properties of Pareto boundaries see (6, Carfì 2008).

*Definition (of coopetitive game):* Let $E$, $F$ and $C$ be three nonempty sets. We define *two person coopetitive gain game carried by the strategic triple* ($E, F, C$) any pair of the form $G = (f, >)$, where $f$ is a function from the Cartesian product $E \times F \times C$ into the real Cartesian plane $\mathbf{R}^2$ and $>$ is the usual strict upper order of the Cartesian plane, defined, for every couple of points $p$, $q$, by $p > q$ if and only if $p_i > q_i$, for each index $i$.

*Remark:* The difference among a two person normal-form gain game and a two person coopetitive gain game is simply the presence of the third strategy Cartesian factor $C$.

*Terminology and notation:* Let $G = (f, >)$ be a two person coopetitive gain game carried by the strategic triple ($E, F, C$). We will use the following terminologies:

- the function $f$ is called the *payoff function of the game G*;
- the first component $f_1$ of the payoff function $f$ is called the *payoff function of the first player* and analogously the second component $f_2$ is called the *payoff function of the second player*;
- the set $E$ is said the *strategy set of the first player*, the set $F$ the *strategy set of the second player*;
- the set $C$ is said *the cooperative strategy set of the two players*;
- the Cartesian product $E \times F \times C$ is called the *coopetitive strategy space of the game G*.

*Memento.* The first component $f_1$ of the payoff function $f$ of a coopetitive game $G$ is the function of the strategy space of the game $G$ into the real line defined by $f_1(x,y,z) = \mathrm{pr}_1(f(x,y,z))$, where $\mathrm{pr}_1$ is the usual first projection of the Cartesian plane; analogously we proceed for the second component $f_2$.

*Strategic interpretation.* We have two players, each of them has a strategy set in which to choose his own strategy; moreover, the two players can *cooperatively choose a strategy* $z$ in a third set $C$. The two players will choose their cooperative strategy $z$ to maximize (in some sense) the gain function $f$.

*Bargaining solutions of a coopetitive game.* The payoff function of a two player coopetitive game is (as in the case of normal-form game) a vector valued function with values belonging to the Cartesian plane $R^2$; so that we should consider the maximal Pareto boundary of the payoff space im($f$) as an appropriate zone for the bargaining solutions (by im($f$) we denote the image of the function $f$).

*The family of normal form games associated with a coopetitive game.* For any cooperative strategy $z$, selected in the cooperative strategy space $C$, there is a corresponding normal form game

$$G_z = (f_z, >) \tag{1}$$

upon the strategy pair ($E, F$) and with payoff function the section

$$1.f(.\,,z) : E \times F \to \mathbf{R}^2, \tag{2}$$

of the payoff function $f$ of the coopetitive game, where the section of $f$ is defined, as usual, on the competitive strategy space $E \times F$ by

$$f(., z)(x) = f(x, z), \quad (3)$$

for every bi-strategy $x$ in the bi-strategy space $E \times F$.

*General solution:* The two players should choose the cooperative strategy $z$ in order that, for instance:

- the Nash equilibrium of Gz are "better" than the Nash equilibrium in each other game Gz';
- the supremum of Gz is greater than the supremum of any other game Gz';
- the Pareto maximal boundary of Gz is "higher" than that of any other game Gz';
- the Nash bargaining solution is better in Gz than that in Gz';

and so on, fixed a common standard kind of solution for any game Gz, say S(z) the set of these kind of solutions, we can consider the problem to find the optimal solutions in set valued path S, defined on the cooperative strategy set C.

We note the fundamental circumstance that in general the above criteria are multi-criteria and so they generate multi-criteria optimization problems.

Let us formalize the concept of game-family associated with a coopetitive game.

*Definition (the family of normal form games associated with a coopetitive game).* Let $G = (f, >)$ be a two players coopetitive gain game carried by the strategic set triple $(E, F, C)$. We call *family of normal-form games associated with the coopetitive game* $G$ the family of normal form games

$$G = (G_z)_{z\in C}, \quad (4)$$

which we will denote by the symbol $G$, having, for any cooperative strategy $z$ selected in the cooperative strategy space $C$, as z-member the normal form game

$$G_z = (f_z, >), \quad (5)$$

upon the strategy pair $(E, F)$, with payoff function the section

$$f(. , z) : E \times F \to \mathbf{R}^2, \quad (6)$$

of the payoff function $f$ of the coopetitive game G.

*Applicative remark.* It is clear that with any family of normal form games

$$G = (G_z)_{z\in C} \quad (7)$$

we can associate

- a family of payoff spaces

$$(\mathrm{im}(f_z))_{z\in C}, \quad (8)$$

- a family of Pareto maximal boundary

$$(\mathrm{bd}^*G_z)_{z\in C}; \quad (9)$$

- a family of suprema

$$(\sup G_z)_{z\in C}; \quad (10)$$

and so on.

And we can interpret any of the above families as set-valued paths in the strategy space $E \times F$.

It is just the study of these induced set-valued paths of solutions which becomes of great interest in the study of a coopetitive game *G*.

## 4. Two Models of Coopetitive Games

In our analysis, Germany is the first exporting country among the EMU countries, which has also experienced a weak domestic demand due to a modest wage increases. Thus our hypothesis is to stimulate Germany's domestic demand and to re-balance its trade surplus in favor of Greece.

On the other hand, Greece is the country that showed a high and rising public debt, which determined its sovereign debt at risk of default. Given that Greece must pursue a budget austerity program externally imposed by the euro area Governments and by IMF in exchange of their financial help, this country has anyway experienced a declining competitiveness of its products. Therefore our hypothesis is that Greece aims at growth by undertaking innovative investments and by increasing its exports primarily towards Germany and also towards the other euro countries[12].

The coopetitive models that we propose hereunder must be interpreted as normative models, in the sense that they will show the more appropriate solutions of a win-win strategy chosen within a cooperative perspective.

The main variables of the two models are:

- strategies x of Germany (*the consumptions of Germany)*, which directly influence only Germany pay-off;
- strategies y of Greece (*the investments of Greece)* which increase only Greece pay-off function;
- a shared strategy z which is determined *ex ante* together by the two countries, Germany and Greece (z is a given amount of Greek exports imported by Germany).

Therefore, in the two models we assume that Germany and Greece define the set of coopetitive strategies.

### 4.1 First Coopetitive Model

Main Strategic Assumptions.

We assume that a real number x, in the unit interval U = [0,1], is the consumption of Germany and a real number y, in the same unit interval U, is the investment of Greece, moreover a real number z, again in the unit interval U, is the amount of Greek exports which is imported by Germany.

We also consider as payoff function of Germany its domestic demand, that we represent in our model as the algebraic sum of the two strategies x and z, and also of the exports of Germany as a reaction function with respect to its domestic consumption.

#### 4.1.1 Payoff Function of Germany

*We assume that the payoff function of Germany is the function g of the unit square U × U into the real line* **R**, *defined by*

$$g(x, z) = x + (x + 1)^{-1} - z, \tag{12}$$

for every pair (x,z) in the square U × U; where the reaction function E, of U into the real line, defined by

$$E(x) = 1/(x+1), \tag{13}$$

*for every consumption x of Germany in U,* is the export of Germany corresponding to the level *x* of consumption. The reaction function E is a decreasing function, randomly chosen, and within certain limits, this choice does not diminish the generality of the model.

#### 4.1.2 Payoff Function of Greece

*We consider as payoff function of Greece the algebraic sum of the economic strategies y and z and of two linear reaction functions M and N.*

*We assume that the payoff function of Greece is the function e of the square U × U into the real line, defined by*

[12] The potential benefit coming from a better trade balance can also contribute to ease the government budget constraint and improve its public debt.

$$e(y, z) = y + z + my + nz = (1+m)\, y + (1+n)\, z\,, \qquad (14)$$

for every pair $(y, z)$ in the Cartesian square $U \times U$, where $m$ and $n$ are two real numbers strictly greater than 1. We note that the function $e$ does not depend upon the strategy $x$ chosen by Germany and that $e$ is a linear function.

*Reaction function M*. The term $my$ represents the quantitative effect of investments on the exports. In fact, the investments, especially innovative investments, contribute at improving the competitiveness of Greek goods, favoring the exports.

*Reaction function* N. The term $nz$ is the cross-effect of the coopetitive variable $z$ that represents the additive level of investment required to support the production of $z$.

#### 4.1.3 Payoff Function of the Game

We so have built up a gain game with payoff function given by:

$$p(x,y,z) = (x + 1/(x+1) - z, (1+m)\, y + z) =$$
$$= (x + 1/(x+1), (1+m)\, y) + z\,(-1,1+n), \qquad (15)$$

with $x,y,z$ in the unit interval [0,1].

#### 4.1.4 Study of the game G = (p, >)

Note that, fixed a cooperative strategy $z$ in the unit interval $U$, the normal form game $G(z) = (p(z), >)$ with payoff function $p(z)$, defined on the square $U^2$ by

$$p(z)(x, y) = p(x, y, z), \qquad (16)$$

is the translation of the game $G(0)$ by the vector

$$v(z) = z(-1,1+n), \qquad (17)$$

so that we can study the game $G(0)$ and then we can translate the various informations of the game $G(0)$ by the vector $v(z)$.

#### 4.1.5 Study of the game G(0)

So let us consider the game $G(0)$. Let the strategic square $S = U^2$ be with vertices $A, B, C, D,$ where $A$ is the origin (0,0), $B$ is the first canonical vector (1,0), $C$ = (1,1), the sum of the two canonical vectors, and $D$ be the second canonical vector (0,1).

The transformation of the side $[A, B]$ is the trace of the parametric curve c defined by

$$c(x) = p(x,0,0) = (x + (x+1)^{-1}, 0), \qquad (18)$$

that is the segment

$$[A', B'] = [(1,0), (3/2,0)] \qquad (19)$$

The transformation of the segment $[A, D]$ is the trace of the curve c defined by

$$c(y) = p(0,y,0) = (1, (1+m)\, y), \qquad (20)$$

that is the segment

$$[A', D'] = [(1,0), (1,1+m)] \qquad (21)$$

The transformation of the segment $[B, C]$ is the trace of the curve c defined by

$$c(y) = p(1, y ,0) = (1 + 1/2, (1 + m)\, y), \qquad (22)$$

that is the segment

$[B', C'] = [(3/2,0), (3/2,1 + m)]$ (23)

So that the payoff space of the game *G*(0) is the rectangle with vertices *A', B', C', D'*.

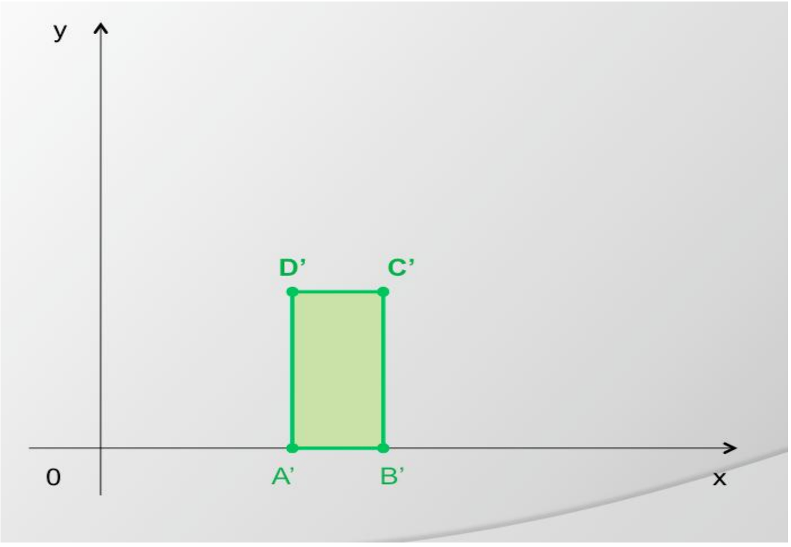


**Figure 1.** The payoff space of the initial game G(0)

*Remark:* it is easily seen that the critical zone of the game is irrelevant (in this case) to determine the payoff space of the game.

### 4.1.6 Payoff space of the coopetitive game G

The image of the payoff function *p*, is the union of the family of payoff spaces

$(\text{im } p(z))_z$ , (24)

that is the convex envelope of the of the four points *A', B', C', D'* and of their translations by the vector

$v(1) = (-1,1+n)$ (25)

The *Pareto maximal boundary of the payoff space f*(*S*) is the segment

$[P', Q']$, (26)

where P' = C' and

$Q' = C' + v(1) = (3/2, 1+m) + (-1, 1+n) = (1/2, 2 + m + n)$ (27)

It is important to note that the *absolute slope of the coopetitive Pareto boundary* is the absolute slope of the vector (-1,1+ *n*), that is the real number

$\text{Ab-slope } (-1,1 + n) = 1 + n,$ (28)

this real number is strictly greater than 1 since the factor *n* is strictly positive.

In the following figure we see the payoff space of the coopetitive game *G* as the trace of the path of payoff spaces corresponding to the path of normal form games G. This path of payoff spaces is nothing but a path of translations of a rectangle, namely the payoff space of the "initial" game G(0).

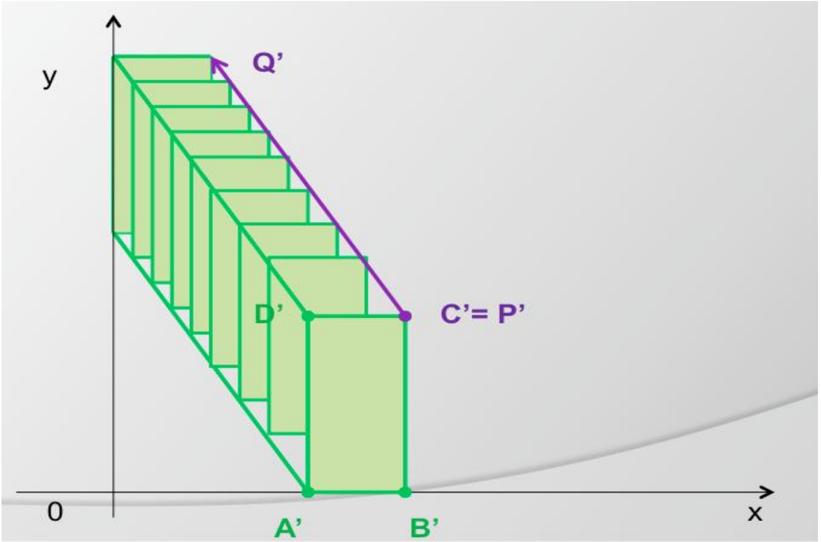


**Figure 2.** The payoff space of the coopetitive game G

Thus the collective payoff function

$g + e$

of the game is not constant on the coopetitive Pareto boundary and, therefore, the game implies possibility of growth, because the minimum value of the aggregate payoff $g + e$ is attained exactly at the supremum of the game $G(0)$.

### 4.1.7 Compromise solutions and the coopetitive compromise

The Nash bargaining solution and the Kalai-Smorodinsky bargaining solution, with respect to the infimum of the Pareto boundary, coincide with the medium point of the segment $[P', Q']$[13].

*Transferable utility solution.* In this coopetitive context it is more convenient to adopt a transferable utility solution: indeed the point of maximum collective gain is the point

$$Q' = (1/2, 2 + m + n), \quad (29)$$

that is the supremum of the game $G(1)$.

Thus we have to propose *a new kind of coopetitive compromise solution* to "share the pie fairly".

We proceed as it follows (in the case $m = 0$):

First, we consider the coopetitive rectangle R having:

a) two sides on the straight lines of equations

$$Y = 1 \text{ and } Y = 2 + n; \quad (30)$$

b) two vertices in $(1/2, 1)$ and $(1/2, 2 + n)$;

c) the diagonal on the straight line S of equation

$$Y + X = 2.5 + n \quad (31)$$

[13] The classic Kalai-Smorodinsky solution that we applied in both models coincides with the solution on the coopetitive Nash path; this result allows us to provide a construct of coopetition which is only "weakly" cooperative, in the sense that it not necessary to cooperate at every stage of the decision process.

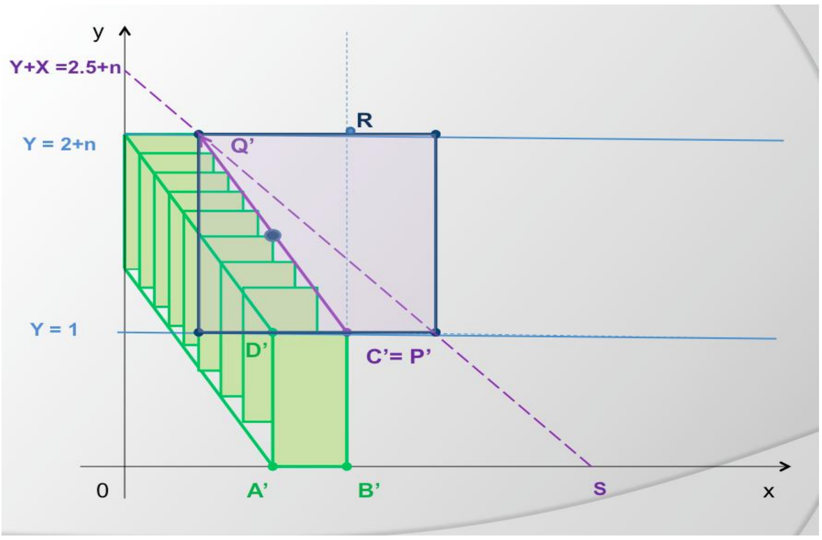


**Figure 3.** The coopetitive bargaining rectangle

*Second*, we consider the segment S' of vertices (3/2,1) (supremum of the game *G*(0)) and the supremum of the rectangle R, the point (3/2+n,2+n), this segment is the set

S' = (3/2,1) + (n,1+n)[0,1] (32)

*Third*, our best payoff coopetitive compromise K is the intersection of the two segments S and S'.

This compromise payoff K represents a win-win solution with respect to the initial supremum (3/2,1), since K is a payoff strongly greater than the initial supremum

C' = sup*G*(0) (33)

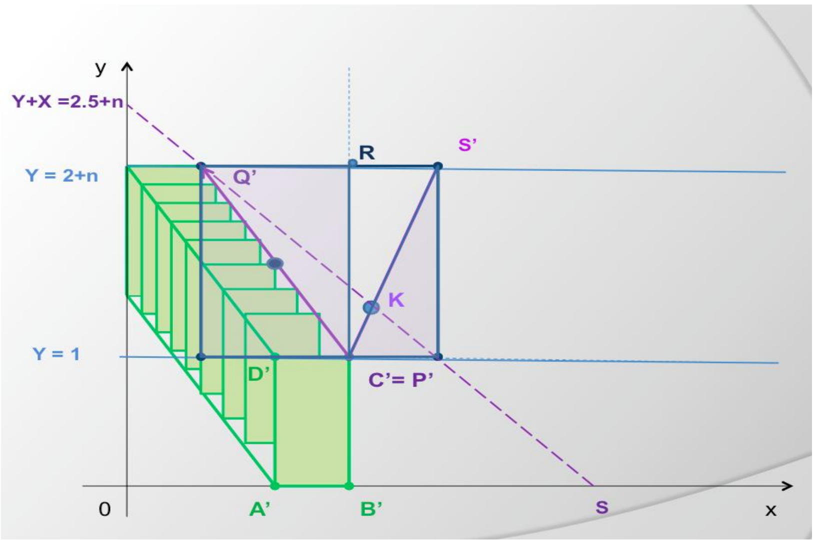


**Figure 4.** Coopetitive compromise K

Remark: in some way, the choices of the coopetitive bargaining interval (rectangle) and of the coopetitive solution are the only reasonable. Indeed:

1) the constraint where we should search for a bargaining solution is the transferable utility Pareto boundary (segment of the line with equation Y + X = 2.5 + n determined by the positive cone of the plane);
2) the possible bargaining Greece's outcomes should belong to the interval [1, 2+n], with end-points the minimum and maximum value of the Greece's payoff function in the coopetitive game;

3) points 2 and 3 determine the coopetitive bargaining rectangle;
4) the solution we propose is nothing but a best compromise solution with utopia point the supremum of this rectangle and threat point the maximum of the initial game G(0).

### 4.2 Second Coopetitive Model

*Let us consider now that a fraction ax of Germany consumption comes from consumption of Greek goods, apart from the given amount of Greek exports that Germany has already determined through an ex ante agreement with Greece (z).*

*Payoff function of Greece*

$$e(x,y,z) = by + z + ax \quad (34)$$

*Payoff function of the game*

$$p(x,y,z) = (x + 1/(x+1) - z, ax+by + cz) = (x + 1/(x+1), ax+ by) + z (-1, c) \quad (35)$$

with a, x,y,z in [0,1] and b,c>1.

Similarly to the previous coopetitive model, but through a more complex procedure, we deduce that the Pareto boundary of the new coopetitive game G = (p, >) – in the payoff space - is the above segment [P', Q'] translated by the vector (0,a).

*The Nash bargaining solution and the Kalai-Smorodinsky bargaining solution,* with respect to the infimum of the Pareto boundary, coincide with the medium point of the segment

$$[P', Q'] + (0,a), \quad (36)$$

which is the optimum of the game $G_{1/2}$.

Analogously the coopetitive compromise solution of this new game is the that of the first model translated by the vector (*0,a*).

## 5. Conclusions

This contribution has tried to provide, through a game theory model of coopetition, feasible solutions in a cooperative perspective to the problems of Greek economy after its crisis. In particular, it has focused on stability and growth as the primary goals, which should drive Greece and Germany economic policy with their positive effects on the whole euro area.

The idea underlying the present work was that of contributing to expand the set of macroeconomic policy tools available to face the economic crisis in Greece, and more generally in the European Monetary Union, where a cooperative attitude should prevail.

In this work we have underlined two aspects which emerged from the crisis. First, the necessity of government budget consolidation of Greece and second the opportunity to re-balance the trade surplus of Germany with respect to Greece (and also with respect to the other euro countries that have a deficit trade balance).

By means of two coopetitive models derived by an original general analytical framework of coopetition, we have showed the strategies that could bring to feasible solutions in a cooperative perspective for Germany and Greece, where these feasible solutions aim at offering a win-win outcome for both countries, letting them to share the pie fairly within a growth path represented by a non-zero sum game. In fact, our analytical results allow us to find a “fair” amount of Greek exports which Germany must import, in order to re-balance the trade surplus of Germany, as well as the investments necessary to improve the Greek economy, thus contributing to growth and to the stability of the Greek economy and, indirectly, of the whole European Monetary Union.

Finally, a remarkable analytical result of our work consists in the determination of the win-win solution by a new selection method on the transferable utility Pareto boundary of the coopetitive game.

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# CREDIT RISK TOOLS. AN OVERVIEW

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**Abstract**

*This document presents several Credit Risk tools which have been developed for the Credit Derivatives Risk Management. The models used in this context are suitable for the pricing, sensitivity/scenario analysis and the derivation of risk measures for plain vanilla credit default swaps (CDS), standardized and bespoke collateralized debt obligations (CDO) and, in general, for any credit risk exposed A/L portfolio.\newline{}. In this brief work we compute the market implied probability of default (PD) from market spreads and the theoretical CDS spreads from historical default frequencies. The loss given default (LGD) probability distribution has been constructed for a large pool portfolio of credit obligations exploiting a single-factor Gaussian copula with a direct convolution algorithm computed at several default correlation parameters. Theoretical CDO tranche prices have been calculated. We finally design stochastic cash-flow stream model simulations to test fair pricing, compute credit value at risk (CV@R) and to evaluate the one year total future potential exposure (FPE) and derive the value at risk (V@R) for a CDO equity tranche exposure.*

**Keywords:** credit default swap, probability of default, direct convolution, loss given default, collateralized, exposure at default, stochastic cash-flow stream model, credit value at risk, future potential exposure, Monte Carlo Simulation.

**JEL Classification:** E44, F37, G15

## 1. Introduction

This brief note presents several applications of credit risk tools developed for the credit derivatives risk management.

The main ingredients are the PD of each single credit-risk bearing exposure, whose hazard rate are estimated with two different methods, and the LGD probability distribution which is constructed via a direct method of calculation. The derivatives considered here synthetically embody the net exposures for different credit operations between two or more counter-parties. The CDS spread represents the fair payment for the stripped risk held by the creditor on a single counter-party's obligation which pays a LIBOR plus cash flow up to maturity. The risk-neutral spread and/or upfront paid on a CDO tranche is the expected present value of the capital loss stood by the tier \QTR{em}{x} capital layer of a financial entity whose asset side is composed by a pool of credits\newline{}. The main risk components can be suitably adapted to the internal rating-based approach (IRB) to credit risk in order to include specific methods of estimation of the PD term structure, the LGD distribution and the EAD computation. Because of their structure and purposes at the cost of light modifications, the credit derivative algorithms developed so far can be exploited for the evaluation of the credit valuation adjustment (CVA) and the estimation of the Economic Capital for any credit risk exposed A/L portfolio.

The structure of each section is very concise and the formulas presented in this work certainly imply broader specifications which have been kept aside to leave the treatment fluent and bring the focus on the results. The organization of this work is as follows. Section 2 presents the main risk components, i.e.\ the market credit spreads and PD; an algorithm for switching from one to another is depicted. Section 3 expands the risk components with the development of the LGD distribution function and presenting the pricing function for a generic collateralized debt obligation. Section 4 incorporates previously developed credit risk tools to provide fairly general credit risk measures, which are the credit adjusted V@R and the FPE distribution.

## 2. CDS Spreads and Probability of Default

In this section an algorithm for estimating the PD from market spreads is presented. On the other hand, given a PD term structure or a hazard rate function, theoretical CDS spreads can be calculated.

Assuming non-stochastic recovery rate $R$ and continuous compounding regime, the fair-valuation CDS spread on maturity $T$ is the $s_T$ such that

$$s_T \int_0^T d\tau B_\tau P_\tau = (1-R)\left\{1 - P_0 - \int_0^T dP_\tau B_\tau\right\} \quad (1)$$

Where $P_\tau$ - is the survival probabilities function $(PD_\tau = 1 - P_\tau)$ and $B_\tau$ is the discounting factor at tenor $\tau$ implied in the term structure of the interest rate swaps.

The pricing formula (1) states that the expected present value of the total cash-flow stream equals the expected present loss for the derivative exposure, i.e., the exposure at default until the maturity of the swap contract.

In order to estimate the market implied default probabilities from the set of market credit default swaps $\left\{s_{T_j}, j = 1, .., n\right\}$ at current time 0, a procedure of forward induction pivoting on (1) has been constructed.

The Figure 1 shows the market CDS spreads available on 22/06/2006 which has been interpolated at ISDA compliant cash-flow dates. The sample is provided by the Mark-it price data for the CDX.NA.HY series 6 index excluding two items, namely the *Charter Comms Holdings* and the *Tembec Industrials*, because of their extreme values in the available sample data.

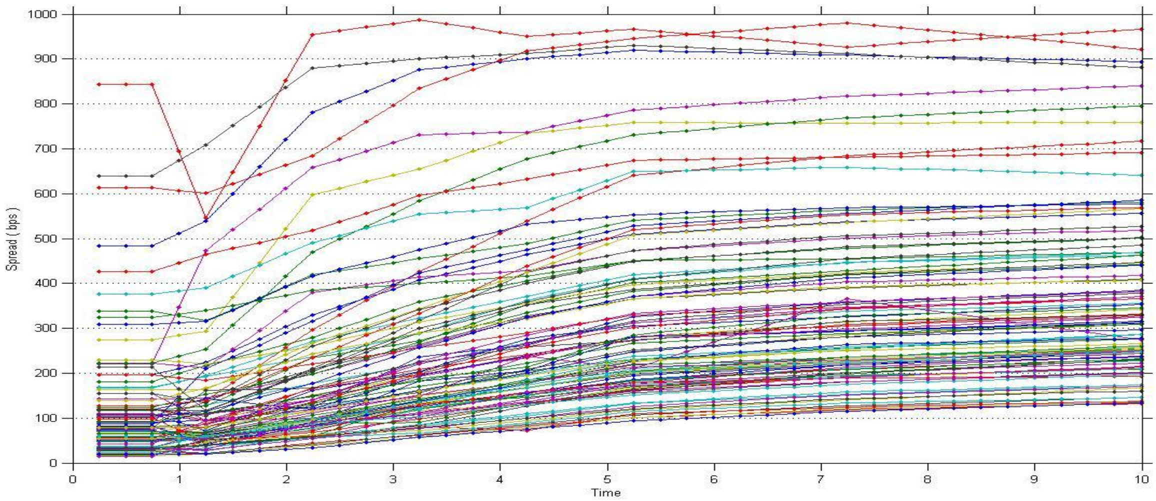


**Figure 1**. Market-it spreads on 22/06/2006 of the CDX.NA.HY series 6 basket components

The companion Figure 2 depicts the (adjusted) implied default probabilities of the CDS market sample.

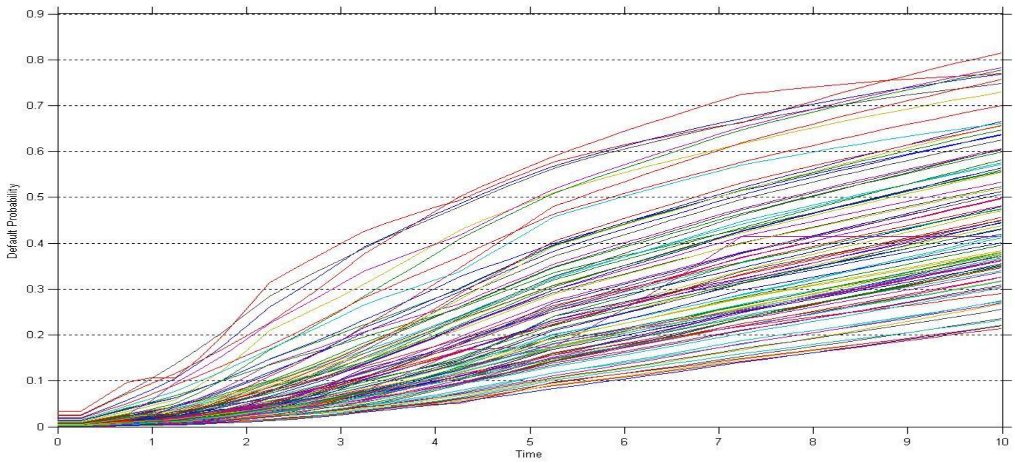


**Figure 2.** Implied probability of defaults of the Market-it spreads on 22/06/2006 of the CDX.NA.HY series 6 basket components

The Table 1 presents the Moody's estimated default frequencies (EDF) within the period 1983-1999. Assuming survival probability functions with constant hazard rate matching the 5 year EDF (Figure 1), the equation (1) has been employed to construct implied theoretical CDS spreads term structures for each rating class on 03/12/2010 (Figure 2).

**Table 1.** Moody's Estimated Default Frequency over different annual horizons

| Rating | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Aaa | 0.00% | 0.00% | 0.00% | 0.06% | 0.20% | 0.28% | 0.37% | 0.48% |
| Aa1 | 0.00% | 0.00% | 0.00% | 0.23% | 0.23% | 0.39% | 0.39% | 0.39% |
| Aa2 | 0.00% | 0.00% | 0.06% | 0.20% | 0.45% | 0.55% | 0.66% | 0.79% |
| Aa3 | 0.07% | 0.10% | 0.19% | 0.29% | 0.41% | 0.55% | 0.55% | 0.55% |
| A1 | 0.00% | 0.03% | 0.33% | 0.52% | 0.66% | 0.82% | 0.89% | 0.97% |
| A2 | 0.00% | 0.03% | 0.14% | 0.39% | 0.60% | 0.79% | 0.91% | 1.24% |
| A3 | 0.00% | 0.13% | 0.25% | 0.34% | 0.40% | 0.53% | 0.78% | 0.88% |
| Baa1 | 0.04% | 0.26% | 0.52% | 0.90% | 1.28% | 1.55% | 2.00% | 2.27% |
| Baa2 | 0.07% | 0.33% | 0.60% | 1.18% | 1.80% | 2.45% | 2.79% | 2.93% |
| Baa3 | 0.31% | 0.81% | 1.34% | 2.15% | 2.84% | 3.82% | 4.73% | 5.66% |
| Ba1 | 0.62% | 2.13% | 3.86% | 6.30% | 8.49% | 10.69% | 12.19% | 13.67% |
| Ba2 | 0.53% | 2.58% | 5.05% | 7.32% | 9.16% | 10.51% | 11.86% | 12.76% |
| Ba3 | 2.52% | 6.96% | 11.89% | 16.47% | 20.98% | 25.05% | 28.71% | 32.61% |
| B1 | 3.46% | 9.29% | 14.81% | 19.63% | 24.48% | 29.79% | 34.85% | 38.35% |
| B2 | 6.88% | 13.95% | 20.28% | 24.84% | 28.45% | 31.16% | 32.57% | 34.39% |
| B3 | 12.23% | 20.71% | 27.27% | 32.53% | 37.54% | 40.66% | 43.95% | 47.84% |
| Caa1-C | 19.09% | 28.37% | 34.23% | 40.07% | 43.37% | 47.73% | 47.73% | 51.33% |
| Investment-Grade | 0.04% | 0.15% | 0.33% | 0.59% | 0.82% | 1.08% | 1.27% | 1.46% |
| Speculative-Grade | 3.68% | 8.26% | 12.66% | 16.56% | 20.17% | 23.38% | 26.17% | 28.73% |
| All Corporates | 1.20% | 2.65% | 4.01% | 5.22% | 6.28% | 7.19% | 7.92% | 8.57% |

## 3. LGD Distribution and CDO Tranche Pricing

In this section we present the algorithm for the estimation of the LGD distribution. Assuming a latent factors default correlation structure, the probability distribution function of the LGD of a credit basket portfolio can be computed as the convolution of the conditional PD of each single exposure weighted with the probability structure of the latent factors. Synthetically:

$$LGD(x) = \int dP(m)\{g_1(x_1|m) * \dots * g_n(x_n|m)\} \tag{2}$$

where $g_i(x_i|m)$ is the (conditional) PD of the $j^{th}$ asset. The $g_j$ have been constructed by mapping a Gaussian copula onto the estimated credit event probabilities.

In the Figure 3 we compute the LGD distribution for a large pool portfolio of credits ranging from Aaa to Caa1-C with equal weights on 03/12/2010 along a five year horizon terminating on 20/12/2015.

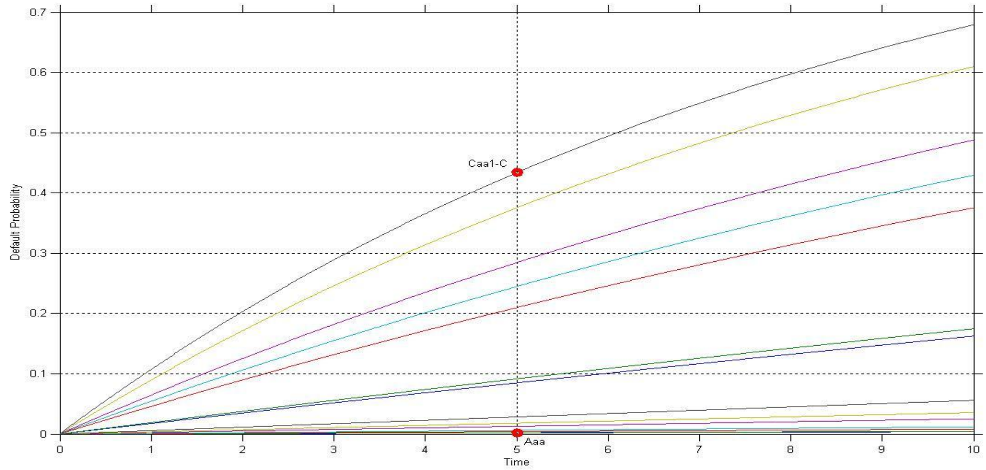


**Figure 3.** PD with constant hazard rate matching the 5 years PD of the Moody's Credit ratings from Aaa to Caa1-C

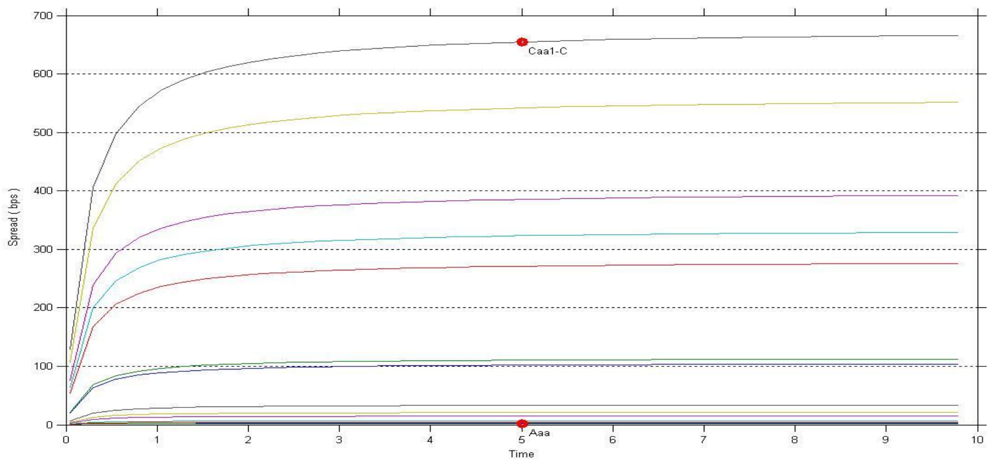


**Figure 4.** PD implied theoretical spreads. The PD are obtained constructing constant hazard rate survival probability functions matching the 5 years PD of the Moody's credit ratings from Aaa to Caa1-C

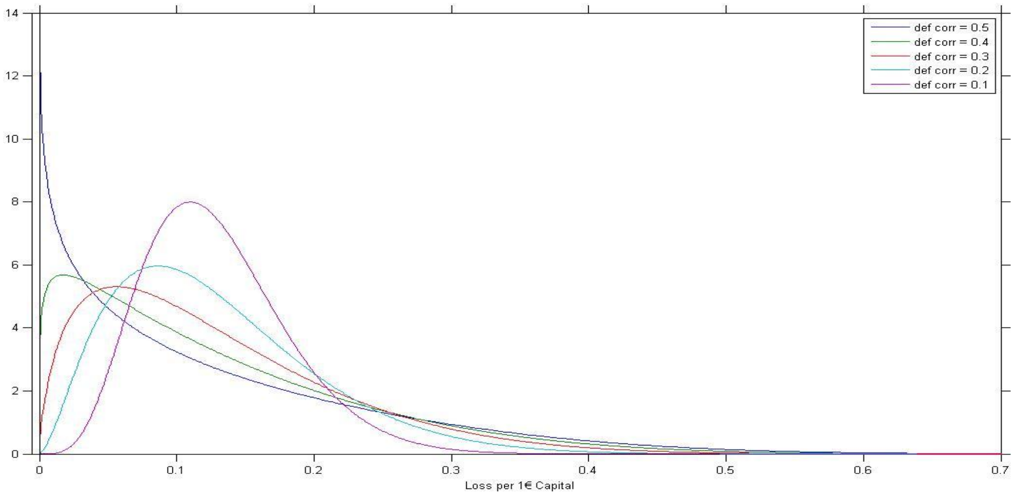

**Figure 5.** Loss given default distribution function for an equally weighted 1400 items basket (Aaa to Caa1-C) at different levels of correlations. The LGD is evaluated over the 5 year period 03/12/2010 to 20/12/2015.

Assuming continuous compounding regime, the fair-valuation CDO tranche *A* upfront payment plus spread are the *u* and *s* such that

$$u + s \int_0^T d\tau B_\tau \left(1 - E\left[\mathcal{L}_\tau^A | \mathcal{F}_0\right]\right) = \mathbb{E}\left[\mathcal{L}_0^A | \mathcal{F}_0\right] + \int_0^T B_\tau d\mathbb{E}\left[\mathcal{L}_\tau^A | \mathcal{F}_0\right] \tag{3}$$

The $\mathbb{E}[.|\mathcal{F}_0]$ is the expectation operator conditional on the information set available at current time. The symbol $\mathcal{L}_\tau^A$ indicates the $\tau$ horizon LGD of the layer *A* notional capital invested in the SPV.

Again, the pricing formula (3) states that the expected present value of the total cash-flow stream equals the expected present loss for the derivative exposure, i.e.\ the exposure at default until the maturity of the swap contract.

In Figure 6 theoretical full upfront prices at different levels of correlation have been computed for the first two tranches, the 0-4% and the 4-8% notional layers, for a theoretical basket portfolio *Q* containing 90 equally weighted names ranging between Aaa and Baa2 Moody's rating buckets and constant recovery rate *R*=40%.

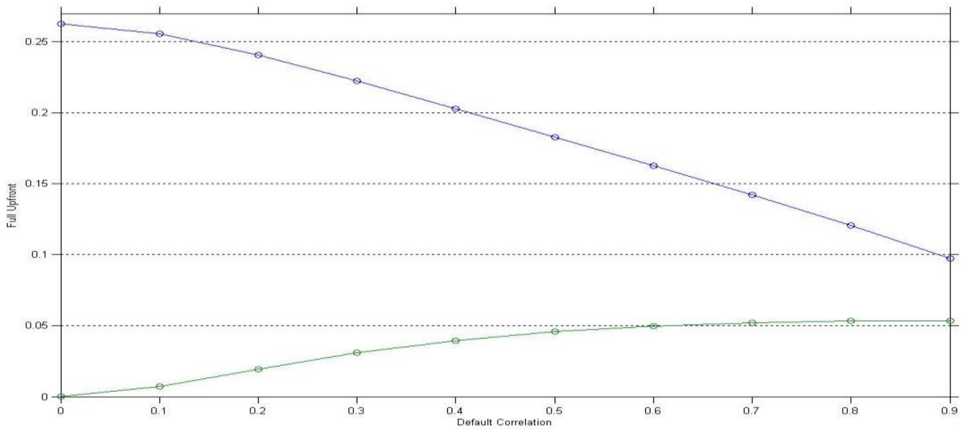


**Figure 6**. Full upfront CDO tranche prices at different default correlations of the tranches 0 - 4% and 4 - 8% of the portfolio Q.9

The valuation has been performed with respect to the reference date 03/12/2010 and maturity 20/12/2013. The prices are expressed in percentage units of the unitary tranche.

## 4. CV@R and Future Potential Exposure

In order to evaluate the correct CDO pricing on a stochastic basis, we have designed and implemented a cash-flow stream model simulation. The basic model components consist in the mapping of the complete portfolio flow of payments and the measurement of the balance sheet consistencies during time evolution. The received/paid cash amounts are accumulated / deducted from a synthetic interest rate generating cash account which can run negative. Stochastic default times $\tau(\omega)$ are generated via single-factor Gaussian copula random numbers mapped onto the prescribed PD. Every random time for each generated cash-flow history sample terminates the corresponding residual cash-flow stream if it happens before maturity.

The Figure 7 shows the theoretical (bars) and simulated (crosses) frequencies of the number of defaults generated for the sample basket *Q* with 0.3 default correlation along the 3 years' time horizon running from the settlement date 03/12/2010 to the maturity date 20/12/2013.

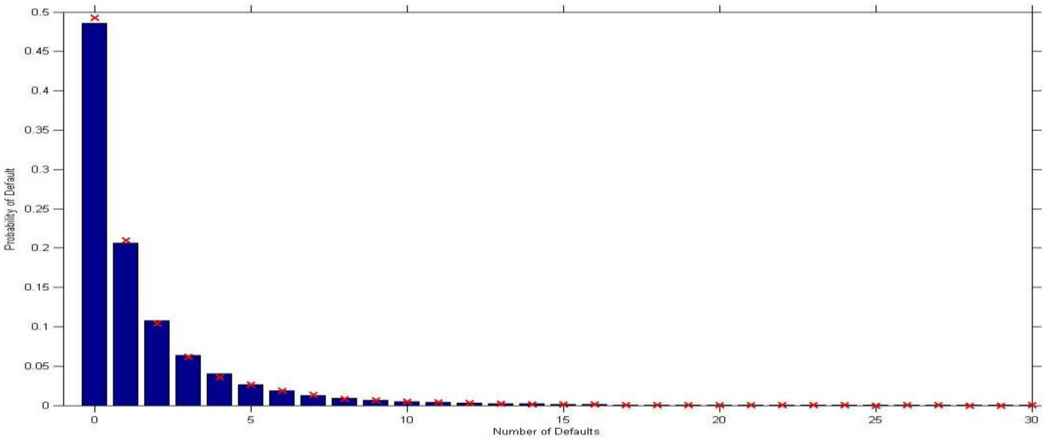


**Figure 7.** Theoretical probabilities (blue bars) and empirical frequencies (red crosses) of the number of defaults of the portfolio Q on maturity

Because of the high flexibility of their structure, Monte Carlo simulations are suitable to allow the user to investigate important characteristics of complex stochastic systems. Indeed, the system which has been just designed can achieve the estimation of several important measures for the credit risk management.

According to (3) the expected present value of a CDO tranche must match the expected present loss, i.e. 22.25%. In economic terms this means that if at time 0 the CDO value is paid into the synthetic cash account, its forward expected value would be zero. The first simulation of this work is run under the described framework plus uniform stochastic recovery centered at $R$=40%. The investor is assumed to have sold protection on the 0-4% equity tranche on the basket of credits $Q$. The full upfront paid by the protection buyer is poured into the cash account on time 0. As it was expected, the average terminal cash value is not significantly different from zero.

The Figure 8 shows the histogram details of the CDO tranche terminal value at maturity, excluding the 0 defaults sub-sample. The tranche consists in the 0-4% (40 €, notional and 8.9 euro, upfront) of the basket $Q$ worth 1,000 euro, notional. The sample trials which incur 0 defaults terminate at +9.2 euro, with an empirical frequency of 49.3%.

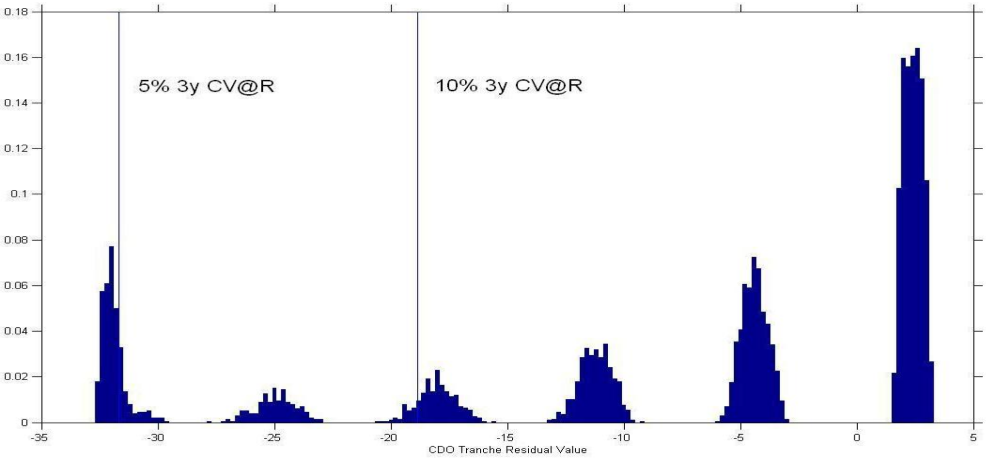


**Figure 8.** Detail of the histogram of the forward value of the 0 - 4% equity tranche on the portfolio Q. The detail excludes the 0 default distribution sub-sample. Time of recording is the maturity 20/12/2013

The Figure 8 presents the remaining 50.7% of the sample forward values which concentrate in the neighborhoods of the portfolios corresponding to each of the sustainable 6 default events. The simulation output consists in the portfolio forward exposure at default.

Finally the Monte Carlo study allows us to compute the Credit Value at Risk (CV@R) of the portfolio at any probability level, which specifically has been calculated at 5% and 10% probability over the time horizon of 3 years. The sought values are CV@R(5%,3y)=-31.66 €, and CV@R(10%,3y)=-18.86 €.

The system which has just been described can be easily modified to study the balance-sheet evolution of a financial entity which invests into LIBOR plus paying obligors and redistributes the revenues between obliges with different seniorities. This program can be regarded as a fully funded cash-flow CDO, with a prescribed cascade redemption scheme. The results of this simulation have not been reported by this work.

The last Monte Carlo study combines together the features of the previously illustrated credit risk tools with a market risk estimation ability in order to provide an instrument capable of deriving measures of the total future potential exposure (FPE) and the V@R of a portfolio of credit derivatives.

With the term *Future Potential Exposure* we mean here the probability distribution of the exposure to a financial obligation evaluated at a future instant in time. By the term *exposure* is intended the market value or replacement cost of the obligation plus the total P/L to date.

Building up on the previous simulation, the total cash-flow stream of the unfunded synthetic portfolio is evaluated at 1 year since settlement, immediately after the ISDA payment date, i.e. on 21/12/2011, when the tranche is assumed to be unwound purchasing an equivalent protection on the residual tranche.

Regarding the simulated dynamics, although the model assumptions are quite restrictive, the understanding of the structure of the stochastic system and the sensitivity/scenario analysis can provide great insight for the sake of timely and effective risk capital allocation and risk management. The market factors which determine the CDO tranche prices are the CDS spreads and the default correlations, which are a direct expression of the credit conditions of the financial markets.

In this study the credit conditions remain unchanged during the 1 year simulation, while at the evaluation time the markets spreads are uniformly shocked by a multiplicative stochastic factor that has a 10\% yearly volatility. The default correlation parameter (0.3) remains unchanged. In order to drastically reduce the computational time, another assumption is taken. The manners in which 6 items can be drawn from a 90 items pool is about 670 millions, therefore only two hypothesis are investigated: the first one, when defaults are assumed to happen from the least likely forward and the second one, when defaults are assumed to happen from the most likely backward, respectively, providing the highest and lowest prices. The numbers of defaults that happen on the evaluation date conform to the expected frequencies. Actually, considering the low spread levels at the shortest maturities and the relatively small time horizon, the simulated market price distribution for the pool components under the two scenarios is not significantly different. Hence, we take into account only the conservative hypothesis.

In Figure 9 the unwinding price frequency distribution of the equity tranche of the *Q* basket on 21/12/2011.

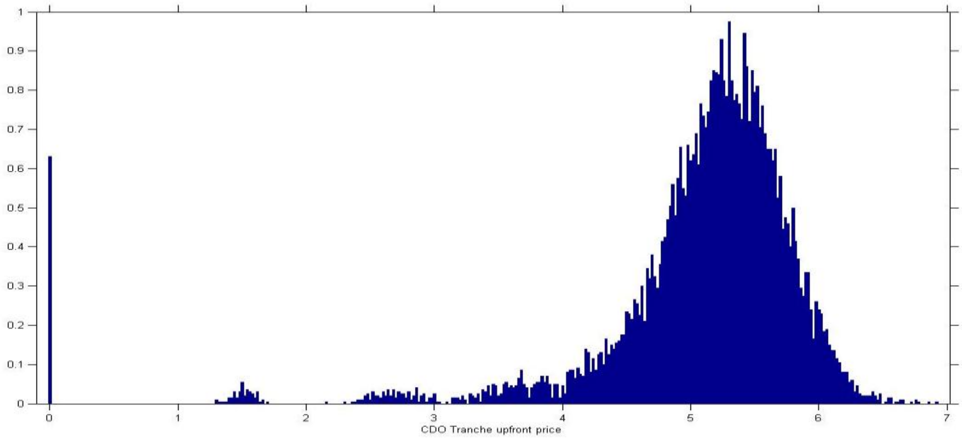


**Figure 9.** Histogram of the simulated market prices (full upfront) of the evolution of the 0 - 4% equity tranche on the portfolio Q generated on 21/12/2011

The final Figure 10 shows the future potential total exposure of the equity tranche under the simulation dynamics. We have estimated the forward V@R measures, comprehensive of the credit and market risk embedded into the derivative exposure.

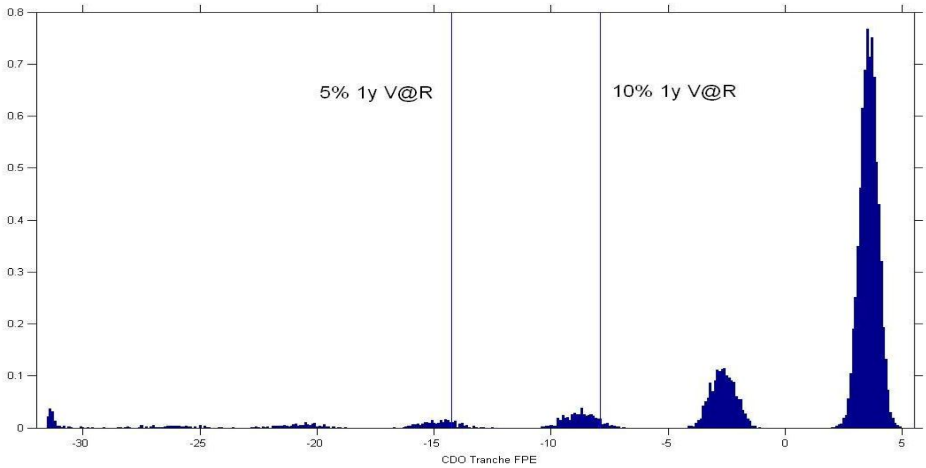


**Figure 10.** Total FPE of the 0 - 4% equity tranche on the portfolio Q generated on 21/12/2011

The sought values are V@R(5%,1y)=-14.26 € thin space and the V@R(10%,1y)=-7.89 €.

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# DEFRAGMENTATION OF ECONOMIC GROWTH WITH A FOCUS ON DIVERSIFICATION: EVIDENCE FROM RUSSIAN ECONOMY[14]

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**Abstract:**

*In this paper, we develop a comprehensive analysis of diversification issues for Russian economy. Assessing diversification for nine different variables, we show that choice of a variable affects the result much, and that, unlike a popular opinion, equiproportional economic diversity measures are still useful in economic analysis. Developing a simple defragmentation of economic growth, we account for labor productivity and labor availability separately, and show that these components depend on different factors. We discover some factors that are rarely studied. We argue that they can become a hard constraint for long-term economic growth.*

**Keywords:** diversification, defragmentation of economic growth, productivity, labor, long-term economic growth

**JEL Classification:** O18, O49, R11, R12, R15.

## 1. Introduction

Many years have passed since *Solow* (1956) introduced his influential model, which has become a starting point in modern theory of economic growth. Since then, theory of economic growth has improved much. Aghion, and Durlauf (2009) describe the evolution of this theory in the latest years, discussing the contributions of Lucas (1988), Romer (1986, 1990), Aghion, and Howitt (1992, 1998, 2006).[15] Aghion (2009) surveys recent attempts at examining the impact of education on economic growth.

Recent research studies the interrelationship between institutional quality and economic growth are: Barro (1996) shows that property rights and free markets affect growth much more than democracy. Acemoglu, Johnson, and Robinson (2001, 2004) consider that institutional quality is the fundamental driver of long-term economic growth. Glaeser, La Porta, Lopez de Silanes, and Shleifer (2004) disagree.[16]

Sachs, and Warner (1995, 1999 and 2001) find evidence that economic growth is negatively correlated with resource abundance. According to the commonly shared opinion, institutional quality is the main transmission channel. For details, see Gylfason (2001), Mehlum, Moene, and Torvick (2005), and Papyrakis, and Gerlagh (2004).

At the same time, there is various literature concerning cross-country growth regressions. The pioneers in these are Barro (1991, 1996), and Mankiw, Romer, and Weil (1992). Since then, as shown by Durlauf, and Quah (1999), more than ninety potential growth determinants have been proposed throughout the literature. Choosing the variables to be included in the analysis has become a real challenge.[17] Brock, and Durlauf (2000) therefore propose a methodology to account for model uncertainty in growth empirics.

We are also concerned with the fact that the evolution of economic growth theory brings us to disintegration, isolation of each theory. It is sometimes due to certain difficulties in defining the subject of the analysis. Desired economic outcomes can be defined in different ways, and can include, apart from growth,

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[14] The views expressed in the article are those of the author and do not necessarily reflect the position of Center for Macroeconomic Analysis and Short-Term Forecasting or Institute of Economic Forecasting, RAS. Author's contact e-mail is the following: agni.research@gmail.com.

[15] Romer (1986), and Lucas (1988) propose a model of growth driven by technological knowledge and human capital. Romer (1990) introduces the product-variety paradigm (variety of products matters, not their improvement). Aghion, and Howitt (1992, 1998) argue that quality-improving innovations are at the heart of economic growth.

[16] "Our evidence suggests in contrast that the Lipset-Przeworski-Barro view of the world is more accurate: countries that emerge from poverty accumulate human and physical capital under dictatorships, and then, once they become richer, are increasingly likely to improve their institutions." (p. 27)

[17] Brock, and Durlauf (2000) explain: "This problem occurs because growth theories are open-ended. By open-endness, we refer to the idea that the validity of one causal theory of growth does not imply the falsity of another." (p. 6)

social and ecological parameters. The optimal development strategy in this case often depends on theoretical preferences. For instance, Lin (2010) compares "new" and "old" structural economics and shows that there are more differences than similarities in these two structural approaches. The former recommends changes consistent with comparative advantages of a country (i.e., strictly accounts for economy's factor endowments), and the latter advocates developing advanced capital-intensive industries (i.e., considers advanced economies' structure as a standard).

Economic growth can be export-driven as well. Here, competitive advantages of a country in production of certain goods are crucial to be examined, since specialization historically origins from cross-country comparison. Note that, according to Rodrick (2009), export-driven economic growth is in fact driven by competitive advantages. The ability to produce goods that are useful for other countries stimulates exports, not vice versa.[18]

Gorodnichenko, Mendoza, and Tesar (2009) study the impact of trade on economic growth. They find evidence that the deep economic downturn in Finland in 1991-1993 (Finland's Great Depression) was triggered by the collapse of Finnish trade with the Soviet Union. Besides, they provide an interesting comparison between Finland's downturn and the downturn in transition economies of Eastern Europe. They find that Finland's macroeconomic dynamics during Great Depression mirrors those of the transition economies of Eastern Europe, though Finland did not face large institutional transformations.[19]

Hasan, and Toda (2004) describe the methodology used to measure export diversification. They calculate five export diversity measures for Bangladesh, Nepal and Malaysia. Additionally, they study an interesting empirical distinction between horizontal and vertical diversification.[20]

Wagner (2000), and Raj Sharma (2008) provide an extensive literature review on measuring diversification. Wagner (2000) introduces a classification of diversity measures, dividing them in four broad groups. Raj Sharma (2008) calculates two diversity indices for the US states for 1990, 2000 and 2006, and estimates their impact on economic stability.[21] He describes the shift-share analysis methodology and provides a cluster analysis for Hawaii. Smith, and Gibson (1988) show that indiscriminate diversification does not necessarily foster economic growth or stability.

Wagner (2000) describes a trade-off between specialization and diversification. The former is associated with economic growth, and the latter is associated with economic stability. Wagner (2000) considers that it is quite a difficult task to success in both stimulating economic growth and maintaining stability, since specialization and diversification are almost opposite measures.

In this paper, we revise theoretical and empirical research on economic diversification, and discuss what diversity measures should be applied to analyze modern Russian economy. Regional economic development is at the top of our attention: we find evidence that industrial diversification of a region's economy impacts its economic development.

It's not a common thing to examine an impact of diversification on economic growth, since there is no a diversity measure commonly accepted as best. Two problems are worth considering. The first is the absence of agreement on a standard of perfect diversity. The second is diversity indices' dependence on aggregation level (the number of industries included in diversity indices' calculations). Additionally, Raj Sharma (2008) shows that

[18] It is true while we talk about long-term economic development. Of course, a drop in export taxes would cause an increase in production. However, this effect is substantially lower while considering long periods of time.

[19] "The trade shocks we observe in the data could lead to economic downturns in standard theoretical multi-sector models which are remarkably close to the size of downturns we observe in transition economies. This important finding suggests that alternative explanations such as institutional transformations could have had a smaller effect than thought before." (p. 28)

[20] They find that low-income countries need to develop vertical diversification first (that is, to create new innovative commodities). In the long-run, however, they have to stimulate horizontal diversification as well (that is, to alter the primary export mix). Thus they eliminate the volatility of global commodity prices (for details, see p. 54).

[21] An impact of diversity on economic stability was found to be insignificant. However, Kort (1981), Simon (1988), Izraeli, and Murphy (2003), and Trendle, and Shorney (2003) argue that industrial diversity reduces unemployment. Following the earlier work of Simon (1988), Mizuno, Mizutani, and Nakayama (2006) found evidence that diversity and economic stability are correlated positively (in Japanese economy), but diversity appears to be only one of many factors impacting unemployment instability. However, adding other variables makes the industrial diversity factor insignificant. In general, there is no theoretical consonance on the role of diversification.

the main factor impacting diversity indices seems to be a region's economy size (in terms of GRP *see* Figure 1 and 2).[22]

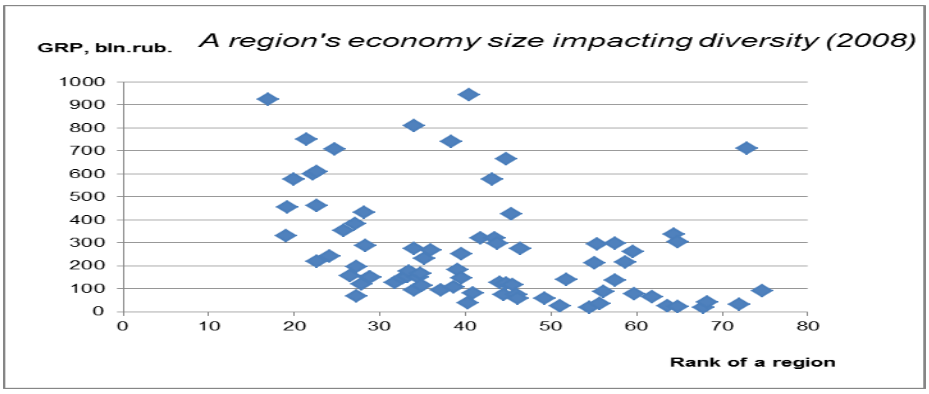


**Figure 1.** GRP and Economic Diversity by Regions

**Source:** Central Statistical Database of Rosstat, author's calculations.

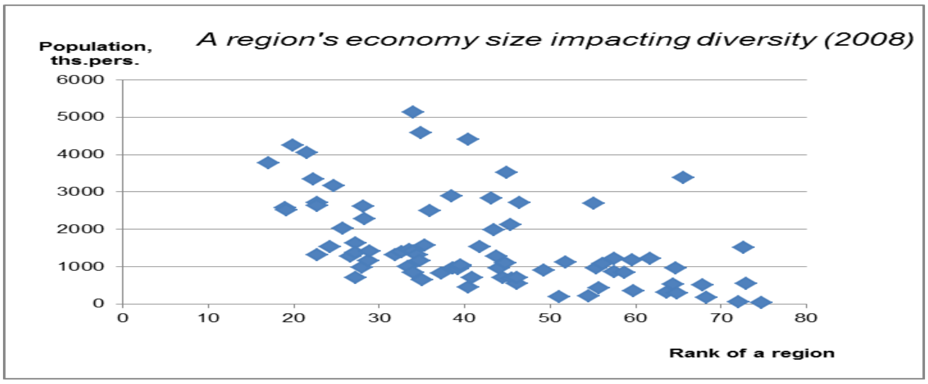


**Figure 2.** Population and Economic Diversity by Regions

**Note.** For each type of diversity indices, we calculate average from different variables. They are presented in Table 7. Final ranking is a simple average from those.

**Source:** Central Statistical Database of Rosstat, author's calculations.

To measure economic diversity, one should choose a standard of perfect diversity. National economy is usually considered as a standard for a region's economy[23] (a standard is also called a reference economy, or a base economy). However, it is a challenge to choose such a standard for national economy. Another problem appears when one tries to reveal competitive advantages of a region in production of certain goods. The knowledge on a region's competitive advantages is incomplete, as it is quite hard to account for a region's trade with other countries and other regions of national economy.[24]

The paper is organized as follows. In Section 2, we provide a brief guide on methodology and describe the data. In Section 3, we discuss the literature on measuring diversification and calculate diversity indices for regions of Russian economy. In Section 4, we develop a simple defragmentation of economic growth. Then we analyze the impact of diversification on GRP per capita through labor productivity, using simple econometric techniques. Section 5 concludes.

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[22] The impact of a region's economy size on diversity is positive. Although Russia is considered to be exposed to the resource curse (Luong, and Weinthal 2001, Ahrend 2005), for Russia this also holds true (see Figure 1, Figure 2).

[23] Of course, if a researcher is not satisfied by equiproportional diversity measures.

[24] As mentioned by Artemyeva *et al.* (2010), a sound statistics on cross-regional trade in Russia is missed.

## 2. Data and Methodology

Analyzing Russian economic development looks like a challenge. Frequent methodological changes in official statistical procedures make it hard to build long time-series.[25] In OKVED, the data[26] on shipment by industry is available only from 2005. The data on employment by industry is available from 1998, and the data on Gross Regional Product (GRP) by industry is available only from 2004.[27] To realize the dynamic incomparability of data, just look at Figure 3.

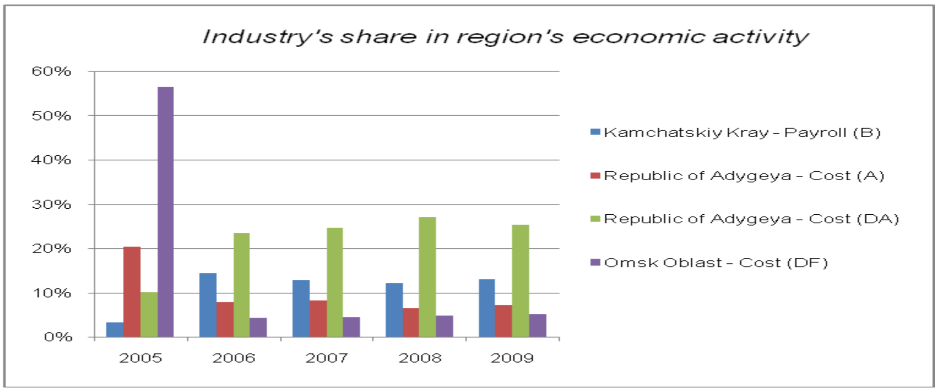


**Figure 3.** Dynamic Incomparability of Data (2005 and 2006)

**Source:** Central Statistical Database of Rosstat, author's calculations.

Due to statistical difficulties outlined above, we do not estimate time series. We build cross section equations, documenting spatial distribution of various characteristics among regions. So, testing the data on unit root would be useless. However, to control for robustness of our results, we estimate the characteristics separately for every year from sample period (2006-2009).

We understand that the sample period outlined is rather geterogenous, and that it has to be divided into three sub-periods at least: 2006-2007 (rapid economic growth in Russia), 2008 (the beginning of economic crisis in Russia – it stroke in August-September 2008) and 2009 (crisis is in full strength). That's why it would be wise to analyze data for each year separately.[28]

There is a critical difference between standard conditions in which one analyzes economic diversification and those conditions that are in Russian economy. Most analysts focus on long-term period while studying diversification process. The data for the latest fifteen, twenty or more years is usually analyzed.[29] In Russia, despite a significant increase in the role of long-term forecasts for official decision-making, it is obviously impossible to forecast long-term economic growth, since there are simply no long-term data sets.

We specially treat a problem of choosing an industry aggregation level. The main difficulty is diversity indices' dependence on the number of industries in the sample. Diversity indices' sensitivity to the level of aggregation is calculated in the next Section.

To calculate diversity indices, we use variables from Table 1 with two-letter aggregation level, except wages and profits. So, we calculate diversity indices for nine different variables.

[25] In 2005, the Federal State Statistics Service (Russian official statistical board, also called Rosstat) introduced All-Russian Classification of Economic Activities (OKVED), instead of All-Union Classification of National Economy Industries (OKONH). OKVED is harmonized with Statistical Classification of Economic Activities in the European Community (NACE Rev. 1).

[26] The majority of data that we use in our analysis goes from the Central Statistical Database of Rosstat. It is worth noticing that Rosstat has significantly improved the availability and transparency of statistical services recently. Henceforth, if no additional reference is provided, assume that we use the following source: *The Central Statistical Database of Rosstat* http://www.gks.ru/dbscripts/Cbsd/DBInet.cgi#1

[27] Moreover, the level of aggregation is quite low (one-letter): manufacturing do not disintegrate into sub-industries.

[28] However, we are not able to estimate econometric equations for 2009 due to the lack of data.

[29] Raj Sharma (2008) calculates diversity indices for 1990-2006, Hasan, and Toda (2004) – for 1975-2000.

**Table 1.** Economic Size Indicators

| Variable | Measure | Period | Description | Aggregation |
|---|---|---|---|---|
| Land.area | Thousand square KM | stable | A region's land area | Aggregate |
| GRP | Million rubles | 1996-2008 | Gross Regional Product | One-letter |
| Pop | Thousand People | 1990-2008 | Permanent population | Aggregate |
| Inv | Million rubles | 2005-2009 | Investment | Aggregate |
| FDI | Million USD | 2003-2008 | Foreign Direct Investment | Aggregate |
| R&D | Million rubles | 2001-2008 | R&D value | Aggregate |
| Exp.R&D | Million rubles | 2001-2008 | Internal expenses on R&D | Aggregate |
| Labor | Million people | 1998-2009 | Permanent labor force | Two-letter |
| Payroll | Million rubles | 2004-2009 | Payroll of permanent labor force | Two-letter |
| Wages | Thousand rubles per month | 2004-2009 | An average monthly wage | Two-letter |
| Shipm | Million rubles | 2005-2009 | Shipment of goods and services | Two-letter |
| Shipm.paid | Million rubles | 2005-2009 | Fraction of shipment paid | Two-letter |
| Rev.s | Million rubles | 2005-2009 | Revenues from sales | Two-letter |
| Cost.s | Million rubles | 2005-2009 | Cost from sales | Two-letter |
| Exp.se | Million rubles | 2005-2009 | Selling and executive expenses | Two-letter |
| Prof.s | Million rubles | 2005-2009 | Profit (loss) from sales | Two-letter |
| Pr.tax.acc | Million rubles | 2003-2009 | Profit tax (accounts) | Two-letter |
| Num.acc | Items | 2003-2009 | Number of companies (accounts) | Two-letter |

***Note***. "Accounts" denote data that is provided to Rosstat by enterprises in their accounts. So, this data is not fully comparable with other data due to possible differences in sample size.

In our econometric analysis, we use three groups of variables: economic size indicators (TABLE I), economic effectiveness indicators (Table 2), and social and institutional indicators (Table 3).[30] Note that regression analysis considers only regional economic development.

**Table 2.** Economic Effectiveness Indicators

| Variable | Measure | Period | Description | Aggregation |
|---|---|---|---|---|
| GRP.pc | Rubles | 1996-2008 | Gross Regional Product per capita | Aggregate |
| LP | Rubles per worker | 1996-2008 | Labor productivity | Aggregate |
| Pop.dens | People per square KM | 1990-2008 | Population density | Aggregate |
| Pop.urb% | % | 1990-2008 | Fraction of urban population | Aggregate |

[30] Classification is explained in details in Section 5, where a simple model for our analysis is presented.

| Variable | Measure | Period | Description | Aggregation |
|---|---|---|---|---|
| FDI.pc | USD | 2003-2009 | Foreign Direct Investment per capita | Aggregate |
| U.lev | % | 1992-2009 | Level of unemployment | Aggregate |
| Cars.pc | Items per thousand people | 1999-2008 | Cars per capita | Aggregate |
| Road.dens | KM per thousand KM of land | 1999-2008 | Road density | Aggregate |
| R&D% | % | 2001-2008 | R&D value as a fraction in GRP | Aggregate |
| R&D.LP | Rubles per worker | 2001-2008 | Labor productivity in R&D | Aggregate |
| APC | % | 2000-2008 | Average propensity to consume | Aggregate |
| Inv.Loan | % | 2005-2009 | Investment financed by loans | Aggregate |

**Table 3.** Social and Institutional Indicators

| Variable | Measure | Period | Description | Aggregation |
|---|---|---|---|---|
| Liv.area | Square meters per person | 2000-2008 | Living area | Aggregate |
| Hous.ac% | Items per million square meters | 2000-2008 | Housing accidents per living area | Aggregate |
| Hous.exp% | % | 1999-2008 | Housing expenses as a fraction of income | Aggregate |
| Pop.pd | People per doctor | 1997-2008 | Population per doctor | Aggregate |
| Stud% | % | 2000-2008 | Fraction of students in population | Aggregate |
| Child.st% | % | 2000-2008 | Fraction of children studying | Aggregate |
| Pre.sch% | % | 2000-2008 | Pre-school centers availability | Aggregate |
| Soc.exp% | Rubles per person | 2006-2009 | Planned social expenses per capita | Aggregate |
| Fines.s.r% | % | 2000-2009 | Fraction of fines paid | Aggregate |
| Crime.pc | Items per thousand people | 1990-2008 | Registered crimes per capita | Aggregate |
| Inc.Prop | % | 2000-2008 | Fraction of income from property | Aggregate |
| Inc.Enter | % | 2000-2008 | Fraction of income from entrepreneurship | Aggregate |

## 3. Measuring Diversification

In this section, we briefly discuss the literature on measuring diversification and calculate diversity indices for regions of Russian economy. Considering the aggregation level problem is of a particular interest for us.

Various ways to assess the level of diversification are described in the literature. Note that diversification is usually measured for a region, not for the national economy. Though, the same formulas could be used to assess the level of diversification in the national economy. Wagner (2000), and Raj Sharma (2008) provide a good review of diversity measures.

We follow the logics of Wagner (2000), who classified diversity measures into four groups: equiproportional, type of industries, portfolio, and input-output.

Equiproportional measures are traditional measures of economic diversity:

$$Entropy_j = \sum_{i=1}^{N} S_{ij} \ln\left(\frac{1}{S_{ij}}\right) = -\sum_{i=1}^{N} S_{ij} \ln(S_{ij}), \qquad (1)$$

$$Herfindahl_j = \sum_{i=1}^{N} S_{ij}^2, \qquad (2)$$

$$NAI_j = \sum_{i=1}^{N} \frac{(S_{ij} - S_j)^2}{S_j}, \qquad (3)$$ [31]

where:
- $j$ – Region;
- $i$ – Industry;
- $N$ – Number of industries;
- $S_{ij}$ – Industry's share of a region's economic activity;[32]
- $S_j$ – Industry's share of economic activity in national economy.

Wagner (2000) criticizes this approach, since a standard of perfect diversification in these measures is equiproportional distribution. He finds several theoretical and empirical concerns on equiproportional diversity measures in the literature (see Table 4, Table 5).

**Table 4.** Theoretical Concerns on Equiproportional Diversity Measures

| References | Extractions from Wagner (2000), p. 6 |
|---|---|
| Conroy (1974 and 1975)<br>Brown and Pheasant (1985) | "selection of an equal distribution of activities across sectors as the reference point for diversity is not based on any a priori rationale, and is indeed, quite arbitrary" |
| Wagner and Deller (1998) | "these measures do not account for any form of interindustry linkages, and the number of industry sectors is usually fixed and not allowed to vary by region" |
| Bahl et al. (1971) and Conroy | "perhaps equality in the distribution of activities is not the key, but rather the specialization in specific industries that tend to be "inherently" stable" |

**Table 5.** Empirical Concerns on Equiproportional Diversity Measures

| References | Extractions from Wagner (2000), p. 6 |
|---|---|
| Wasylenko and Erickson (1978) | "regions defined as highly specialized by the entropy approach, were, in fact, characterized by relative economic stability" |
| Kort (1981) | "policy results were sensitive to the specific entropy measure used" |
| Attaran (1987) | "more specialized regions experienced greater economic growth and there was little relationship between these levels of diversity and unemployment" |
| Kort (1981)<br>Smith and Gibson (1987) | "part of the empirical shortfall might be due to factors, other than diversity, that influence stability and have tended to be ignored in empirical estimation" |
| Malizia and Ke (1993) | "the empirical literature has been lax regarding modeling the relevant economic regions" |

Wagner (2000) names several types of industry measures, but the most interesting for us is location quotient, as it is used to assess specialization and to calculate Hachman index:

[31] NAI stands for "National averages index".
[32] Economic activity is a term to unite different variables of interest, such as employment, production, value added.

$$LQ_{ij} = \frac{S_{ij}}{S_i}. \quad (4)$$

Raj Sharma (2008) describes Hachman index, which is very close to the NAI:

$$Hachman = \frac{1}{\sum_{i=1}^{N}\left[\left(S_{ij}/S_i\right)\times S_{ij}\right]} = \frac{1}{\sum_{i=1}^{N}\left[LQ_{ij}\times S_{ij}\right]}. \quad (5)$$

He also discusses dynamic shift-share analysis:

$$CHANGE = \sum_{i=1}^{N} E_i^{Reg}\cdot g^{US} + \sum_{i-1}^{N} E_i^{Reg}\left(g_i^{US} - g^{US}\right) + \sum_{i-1}^{N} E_i^{Reg}\left(g_i^{Reg} - g_i^{US}\right), \quad (6)$$

where:

- $E_i^{Reg}$ – Labor force in an industry $i$ in a region's economy (base year);
- $g^{US}$ – Average pace of economic growth in national economy;
- $g_i^{US}$ – Average pace of growth in industry $i$ in national economy;
- $g_i^{Reg}$ – Average pace of growth in industry $i$ in a region's economy;
- $\sum_{i=1}^{N} E_i^{Reg}\cdot g^{US}$ – National growth effect;
- $\sum_{i-1}^{N} E_i^{Reg}\left(g_i^{US} - g^{US}\right)$ – Industrial mix effect;
- $\sum_{i-1}^{N} E_i^{Reg}\left(g_i^{Reg} - g_i^{US}\right)$ – Competitive share effect.[33]

We do not calculate portfolio diversity measure and an input-output diversity measure. It is shown in the literature that portfolio diversity measure does not assess diversification separately from stability.[34] So, it isn't accurate to consider it a factor of economic stability. However, unlike the majority of researchers, we are interested in the impact of economic diversity on economic growth, not on stability. Unfortunately, this is hard to estimate too, as we do not have long-term time series to calculate correlation.[35]

Input-output matrices, unfortunately, are not available for Russian economy since 2005.[36] These severe statistical limitations make it impossible to calculate this measure.

Apart from these measures, we also apply variation coefficient which is commonly used to measure variation of a variable, and a simple version of Robin Hood index (or Hoover index), which stands for the value of the variable of interest needed to be redistributed in order to get an equiproportional distribution:

$$Variation_j = \frac{\sigma_j}{S_{ij}^{AVER}}, \quad (7)$$

[33] Combined with location quotient (LQ), competitive share effect (CSE) is used for cluster analysis.

[34] See, for example, Sherwood-Call (1990), and Raj Sharma (2008).

[35] As Wagner, and Lau (1971) show, diversification reduces risk considerably only at the first stage of diversifying a portfolio. If two assets are perfectly correlated, diversification would not bring any gains. So, the more the number of assets is, the less benefits an additional increase in diversification will bring. Consequently, if we could calculate correlation indices between variable X in industry A and variable X in industry B, we would be able to use them as weights to assess diversification in terms of its benefits for stability.

[36] Rosstat will revive the publications only in 2015, according to the message at the official site.

$$Hoover_j = \frac{\sum_{i=1}^{N} E_i - E^{AVER}}{2}, \tag{8}$$

where:

- $\sigma_j$ – Standard deviation of variable of interest in region $j$;
- $S_{ij}^{AVER}$ – Average value of variable of interest in region $j$;
- $E_i$ – Economic activity in industry $i$ and region $j$;
- $E^{AVER}$ – Average level of economic activity in region $j$.

To assess a region's diversity index sensitivity to the level of aggregation, we calculate the listed measures in four different levels of aggregation and nine different variables of interest.

Variables of interest are listed in TABLE I (two-letter aggregation level, except wages and profits). Levels of aggregation are the following:[37]

- One-letter industries; full range;
- Two-letter industries; full range;
- Two-letter industries; agriculture, fishing, mining, manufacturing and energy;[38]
- Two-letter industries; mining, manufacturing and energy.

The procedure is as follows. First, we calculate diversity measures for all four levels of aggregation for nine variables. Then we estimate sensitivity to changes in aggregation levels and sensitivity to changes in indicator type. Usually, employment is used as the variable of interest, since data on employment is published earlier than other data, and since employment is measured in physical volumes, not in dollars. However, it is doubtful that there is an objective need to deflate Gross Regional Product or shipment, as we have a diversity index as a result. If we don't deflate such variables, we assess diversification of income, in fact. If we do deflate them, we assess diversification of production, but we do not account for changes in quality of products (quality is usually assessed through prices).

To get an example of sensitivity analysis, look at Figure 4. For every region, we construct a 9x4 table and use it to calculate an average rank (in the table, nine indicators and four levels of aggregation are listed). We build the table for every indicator type (six indicator types are available).

[37] To be precise, we shouldn't name each of these four variants an aggregation level. In fact, only two first variants are aggregation levels, since in third and fourth variant number of industries is cut. However, it's convenient to name all these with a one word, as we want to vary the list of industries too.

[38] Here, we exclude services, such as construction, wholesale and retail trade, hotels and restaurants, transport and warehousing, finance and insurance, real estate, scientific research, educational services, health care and so on. Thus we try to assess diversification in the real sector of economy. The problem here is correlation between services and manufacturing – for example, between construction and manufacturing of construction materials. Moreover, some advanced statistics is available only for manufacturing (for instance, some surveys concerning expectations). Third, services are mostly non-tradable. However, the role of services in export diversification has been emphasized in some recent research. See, for example, Brenton, Newfarmer, and Walkenhorst (2009) to learn that tourism can be useful in understanding tastes of people from other countries (thus it enhances competitiveness).

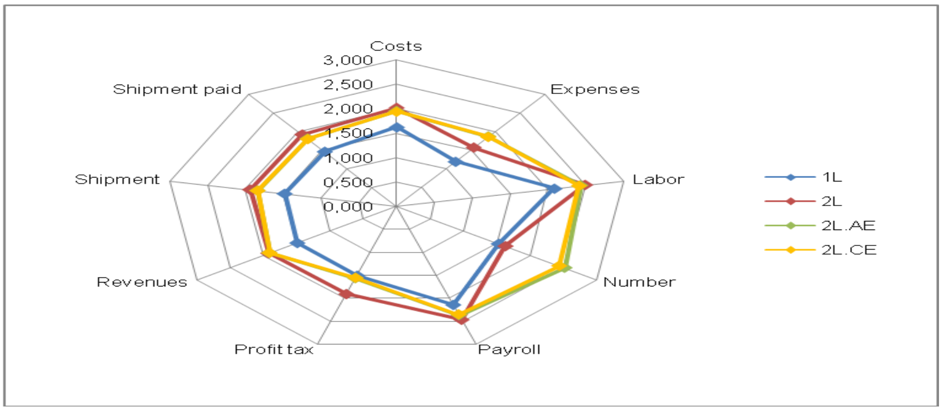


**Figure 4.** Sensitivity Analysis for Moscow City (2009, Entropy Index)

**Source:** Central Statistical Database of Rosstat, author's calculations.

We rank Russian regions by the level of diversification (*see* Table 7) and look at the variation of these ranks by every diversity measure (for results, navigate to Table 6). We find no evidence that equiproportional diversity measures perform worse. Even more, we show that equiproportional diversity measures are still useful in economic analysis. Variation coefficient, Entropy index and Hoover index, which are all equiproportional measures, proved to be the most stable.

**Table 6. Diversity Indices' Sensitivity to the Level of Aggregation**

| Activities | Entropy | Hachman | NAI | HHI | Variation | Hoover | *Sensitivity* |
|---|---|---|---|---|---|---|---|
| Costs | 0.195 | 0.319 | 0.395 | 0.323 | 0.225 | 0.087 | *0.257* |
| Shipment | 0.204 | 0.344 | 0.401 | 0.323 | 0.204 | 0.086 | *0.260* |
| Shipment paid | 0.202 | 0.345 | 0.407 | 0.318 | 0.207 | 0.085 | *0.261* |
| Expenses | 0.220 | 0.348 | 0.418 | 0.286 | 0.264 | 0.080 | *0.270* |
| Revenues | 0.204 | 0.363 | 0.437 | 0.347 | 0.191 | 0.092 | *0.272* |
| Payroll | 0.147 | 0.228 | 0.529 | 0.320 | 0.317 | 0.208 | *0.292* |
| Labor | 0.127 | 0.224 | 0.622 | 0.303 | 0.289 | 0.202 | *0.294* |
| Profit tax | 0.234 | 0.511 | 0.488 | 0.306 | 0.194 | 0.085 | *0.303* |
| Number of firms | 0.137 | 0.197 | 0.545 | 0.399 | 0.383 | 0.251 | *0.319* |
| *Sensitivity* | *0.186* | *0.320* | *0.471* | *0.325* | *0.253* | *0.131* | *0.281* |
| *Sensitivity rank* | *2* | *4* | *6* | *5* | *3* | *1* | |

***Note***. We assess sensitivity calculating variation coefficients for every diversity measure.

**Table 7.** Ranking of Russian Regions by the Level of Diversification

| Size | Region | Entropy | Hachman | NAI | HHI | Variation | Hoover | Ranking |
|---|---|---|---|---|---|---|---|---|
| 7 | Republic of Tatarstan | 18.7 | 12.3 | 12.0 | 21.7 | 18.3 | 18.9 | **17.0** |
| 25 | Saratov Oblast | 22.2 | 8.5 | 8.6 | 24.6 | 29.5 | 20.8 | **19.0** |
| 19 | Irkutsk Oblast | 16.6 | 20.4 | 20.4 | 18.4 | 25.1 | 14.0 | **19.1** |
| 16 | Rostov Oblast | 23.2 | 9.2 | 8.9 | 26.0 | 28.4 | 23.7 | **19.9** |
| 9 | Republic of Bashkortostan | 23.6 | 14.1 | 13.9 | 26.6 | 23.6 | 27.2 | **21.5** |

| Size | Region | Entropy | Hachman | NAI | HHI | Variation | Hoover | Ranking |
|---|---|---|---|---|---|---|---|---|
| 15 | Nizhniy Novgorod Oblast | 31.1 | 7.9 | 7.9 | 27.4 | 26.3 | 32.8 | **22.2** |
| 14 | Perm Kray | 22.1 | 20.7 | 19.9 | 24.8 | 22.5 | 25.9 | **22.7** |
| 18 | Novosibirsk Oblast | 27.6 | 5.6 | 4.6 | 32.9 | 37.4 | 28.1 | **22.7** |
| 40 | Yaroslavl Oblast | 20.8 | 21.1 | 20.8 | 25.4 | 29.1 | 19.1 | **22.7** |
| 38 | Udmurtskaya Republic | 30.6 | 12.8 | 12.1 | 31.6 | 28.9 | 29.3 | **24.2** |
| 12 | Samara Oblast | 25.0 | 25.0 | 24.8 | 26.4 | 25.6 | 21.4 | **24.7** |
| 23 | Omsk Oblast | 32.0 | 17.6 | 17.0 | 28.0 | 26.6 | 33.6 | **25.8** |
| 47 | Chuvash Republic | 24.9 | 25.9 | 25.4 | 25.4 | 29.7 | 28.0 | **26.6** |
| 4 | Moscow Oblast | 30.9 | 19.3 | 19.3 | 34.2 | 31.3 | 25.8 | **26.8** |
| 22 | Leningrad Oblast | 21.8 | 39.0 | 39.5 | 24.1 | 22.4 | 16.2 | **27.2** |
| 71 | Republic of Mariy El | 26.4 | 28.3 | 28.4 | 26.8 | 28.9 | 24.4 | **27.2** |
| 43 | Tver Oblast | 22.4 | 31.8 | 31.5 | 26.8 | 28.1 | 23.1 | **27.3** |
| 59 | Smolensk Oblast | 22.0 | 36.9 | 36.8 | 26.1 | 26.7 | 19.1 | **27.9** |
| 20 | Volgograd Oblast | 35.4 | 15.7 | 16.1 | 36.1 | 33.4 | 32.3 | **28.2** |
| 32 | Voronezh Oblast | 33.5 | 16.4 | 16.0 | 34.0 | 35.9 | 34.1 | **28.3** |
| 51 | Ryazan Oblast | 24.8 | 34.8 | 34.9 | 24.9 | 25.9 | 27.3 | **28.8** |
| 50 | Kirov Oblast | 22.3 | 36.7 | 36.4 | 27.1 | 30.7 | 20.2 | **28.9** |
| 56 | Bryansk Oblast | 34.1 | 25.4 | 25.5 | 32.9 | 36.0 | 37.3 | **31.8** |
| 53 | Penza Oblast | 31.6 | 32.3 | 31.8 | 32.6 | 31.9 | 35.7 | **32.6** |
| 48 | Kaluga Oblast | 30.5 | 41.3 | 41.6 | 31.0 | 27.5 | 28.0 | **33.3** |
| 45 | Vladimir Oblast | 32.2 | 36.6 | 36.4 | 33.6 | 31.9 | 30.2 | **33.5** |
| 64 | Republic of Mordoviya | 31.0 | 45.4 | 46.0 | 26.8 | 26.9 | 27.6 | **33.9** |
| 34 | Khabarovsk Kray | 35.1 | 30.7 | 31.1 | 35.1 | 40.8 | 31.0 | **34.0** |
| 8 | Krasnodarskiy Kray | 37.8 | 23.2 | 23.8 | 40.1 | 41.2 | 37.9 | **34.0** |
| 49 | Ulyanovsk Oblast | 32.8 | 38.3 | 37.9 | 34.7 | 32.9 | 29.7 | **34.4** |
| 46 | Kursk Oblast | 26.7 | 48.4 | 48.3 | 27.6 | 25.7 | 31.8 | **34.8** |
| 5 | Saint Petersburg City | 44.3 | 19.9 | 20.3 | 43.2 | 38.8 | 43.0 | **34.9** |
| 61 | Novgorod Oblast | 28.8 | 49.5 | 49.6 | 28.6 | 26.9 | 26.5 | **35.0** |
| 39 | Tula Oblast | 37.0 | 30.7 | 30.1 | 32.9 | 34.4 | 46.6 | **35.3** |

| Size | Region | Entropy | Hachman | NAI | HHI | Variation | Hoover | Ranking |
|---|---|---|---|---|---|---|---|---|
| 35 | Altayskiy Kray | 38.7 | 26.4 | 26.8 | 42.0 | 41.9 | 39.9 | **36.0** |
| 63 | Oryol Oblast | 41.5 | 33.9 | 33.9 | 38.0 | 37.7 | 38.4 | **37.2** |
| 10 | Krasnoyarsk Kray | 38.5 | 35.0 | 34.1 | 42.9 | 43.1 | 36.9 | **38.4** |
| 62 | Kurgan Oblast | 40.9 | 31.3 | 31.2 | 40.5 | 45.1 | 43.1 | **38.7** |
| 44 | Kaliningrad Oblast | 36.8 | 45.6 | 45.9 | 36.9 | 30.6 | 39.3 | **39.2** |
| 1 | Moscow City | 48.0 | 21.7 | 22.4 | 48.7 | 51.6 | 43.6 | **39.3** |
| 52 | Astrakhan Oblast | 40.3 | 35.6 | 35.6 | 41.6 | 42.8 | 41.4 | **39.5** |
| 37 | Tomsk Oblast | 43.2 | 32.1 | 31.2 | 45.1 | 43.3 | 42.4 | **39.6** |
| 76 | Republic of Adygeya | 36.1 | 45.1 | 44.9 | 39.2 | 40.1 | 36.6 | **40.3** |
| 6 | Sverdlovsk Oblast | 42.7 | 34.9 | 34.7 | 47.8 | 45.3 | 37.3 | **40.5** |
| 67 | Kostroma Oblast | 26.9 | 66.2 | 66.3 | 29.1 | 31.1 | 25.6 | **40.9** |
| 27 | Belgorod Oblast | 39.0 | 53.4 | 53.5 | 33.3 | 28.4 | 42.9 | **41.7** |
| 17 | Kemerovo Oblast | 48.2 | 35.4 | 35.8 | 46.1 | 43.1 | 50.1 | **43.1** |
| 26 | Primorskiy Kray | 33.6 | 55.9 | 56.2 | 37.7 | 42.2 | 34.7 | **43.4** |
| 30 | Arkhangelsk Oblast | 42.7 | 48.5 | 49.2 | 42.6 | 39.3 | 40.0 | **43.7** |
| 57 | Republic of Buryatiya | 41.5 | 49.2 | 49.3 | 41.6 | 41.6 | 40.8 | **44.0** |
| 70 | Pskov Oblast | 44.4 | 44.6 | 44.5 | 43.2 | 40.6 | 49.3 | **44.4** |
| 58 | Tambov Oblast | 48.8 | 38.6 | 38.7 | 45.7 | 47.2 | 49.6 | **44.7** |
| 13 | Chelyabinsk Oblast | 46.7 | 42.7 | 42.6 | 51.0 | 46.9 | 39.1 | **44.8** |
| 21 | Orenburg Oblast | 48.5 | 39.0 | 39.2 | 51.1 | 47.4 | 46.9 | **45.4** |
| 60 | Republic of Kareliya | 36.1 | 64.5 | 64.7 | 34.1 | 34.8 | 38.7 | **45.5** |
| 69 | Republic of Khakasiya | 46.1 | 47.3 | 47.3 | 47.4 | 43.8 | 44.3 | **46.0** |
| 74 | Republic of Northern Osetiya – Alaniya | 51.4 | 37.5 | 38.8 | 47.0 | 48.4 | 53.3 | **46.1** |
| 33 | Stavropol Kray | 48.0 | 40.3 | 40.6 | 47.0 | 53.0 | 49.6 | **46.4** |
| 73 | Kabardino-Balkarskaya Republic | 51.2 | 44.6 | 44.8 | 48.0 | 52.0 | 54.6 | **49.2** |
| 79 | Jewish Autonomous Oblast | 42.3 | 68.9 | 69.0 | 45.1 | 41.6 | 39.3 | **51.0** |
| 54 | Zabaykalskiy Kray | 53.1 | 49.1 | 49.2 | 49.4 | 52.5 | 57.4 | **51.8** |
| 83 | Republic of Altay | 53.5 | 61.4 | 61.6 | 47.9 | 46.9 | 55.4 | **54.5** |

| Size | Region | Entropy | Hachman | NAI | HHI | Variation | Hoover | Ranking |
|---|---|---|---|---|---|---|---|---|
| 42 | Republic of Dagestan | 61.0 | 45.3 | 45.4 | 56.7 | 60.7 | 61.8 | **55.1** |
| 31 | Republic of Komi | 58.2 | 53.3 | 53.6 | 57.0 | 52.5 | 57.8 | **55.4** |
| 77 | Karachaevo-Cerkesskaya Republic | 58.3 | 52.0 | 52.2 | 55.6 | 58.1 | 58.4 | **55.8** |
| 66 | Ivanovo Oblast | 47.5 | 66.9 | 66.8 | 50.5 | 56.6 | 48.8 | **56.2** |
| 29 | Vologda Oblast | 59.4 | 56.6 | 56.0 | 61.1 | 55.5 | 56.3 | **57.5** |
| 55 | Amur Oblast | 54.5 | 64.7 | 65.0 | 51.1 | 55.0 | 54.6 | **57.5** |
| 41 | Murmansk Oblast | 51.6 | 75.5 | 76.4 | 46.9 | 47.8 | 54.0 | **58.7** |
| 36 | Lipetsk Oblast | 63.5 | 57.5 | 57.6 | 62.0 | 55.8 | 61.4 | **59.6** |
| 68 | Kamchatskiy Kray | 52.1 | 76.0 | 79.3 | 48.1 | 49.2 | 53.6 | **59.7** |
| 72 | Chechenskaya Republic | 65.4 | 53.3 | 53.4 | 64.4 | 64.5 | 69.7 | **61.8** |
| 80 | Republic of Tyva | 62.1 | 69.6 | 69.7 | 56.6 | 63.1 | 61.1 | **63.7** |
| 24 | Sakhalin Oblast | 62.4 | 68.4 | 68.5 | 66.8 | 61.5 | 58.6 | **64.4** |
| 28 | Republic of Sakha (Yakutiya) | 60.0 | 75.3 | 75.8 | 59.6 | 57.4 | 60.5 | **64.8** |
| 81 | Republic of Kalmikiya | 65.8 | 62.8 | 62.9 | 63.1 | 65.9 | 68.6 | **64.9** |
| 2 | Tumen Oblast | 66.1 | 65.3 | 65.2 | 68.1 | 64.7 | 64.3 | **65.6** |
| 82 | Republic of Ingushetiya | 73.4 | 61.8 | 63.1 | 67.8 | 67.4 | 73.2 | **67.8** |
| 75 | Magadan Oblast | 63.1 | 79.6 | 80.4 | 61.7 | 61.8 | 63.3 | **68.3** |
| 78 | Chukotskiy Autonomous Okrug | 69.0 | 77.7 | 79.4 | 70.1 | 65.5 | 70.6 | **72.1** |
| 3 | Khanty-Mansiyskiy Autonomous Okrug - Yugra | 73.6 | 70.6 | 70.8 | 74.7 | 71.3 | 75.0 | **72.7** |

| Size | Region | Entropy | Hachman | NAI | HHI | Variation | Hoover | Ranking |
|---|---|---|---|---|---|---|---|---|
| 11 | Yamalo-Neneckiy Autonomous Okrug | 75.5 | 69.6 | 69.8 | 73.4 | 71.7 | 77.8 | **73.0** |
| 65 | Neneckiy Autonomous Okrug | 76.3 | 74.2 | 74.3 | 75.2 | 72.3 | 76.3 | **74.8** |

**Note**. Size is GRP 2008 rank of a region.

Hasan and Toda (2004) provide a good review of export diversity measures. However, this review describes many measures that are used to assess diversification in employment or value added as well. And this is not surprising, as diversification is a solid concept. Of course, there are some special measures in this review, but they are useful considering long periods of time.[39]

## 4. Growth Issues

We start with building a cross-indicator portrait for every region by documenting a set of important characteristics in a radar chart. This proves to be a powerful and simple technique to identify major issues at a glance.

Russia is divided into seven Federal Districts. We present radar charts in a separate figure for each district.[40] For results, see Appendix I. For notation of the variables, *see* Tables 1–3.

Then we provide the analysis of industrial specialization in Russian regions. We slightly modify the methodology applied by Raj Sharma (2008). We also calculate LQ and CSE for each region, but we facilitate constraints on CSE due to crisis effects.[41] Cross-specialization matrices by industry and region are presented in Appendix II.[42]

Then we develop a small and very simple defragmentation of economic growth (in a static version). In mathematics, it is often necessary to reformulate the problem in order to solve it. We do the same in quite a simple way, with our first equation looking obvious and thus even a bit confusing. We even do not account for capital at this stage of our analysis.[43] We start with the following equation:

$$Y = L \times p, \tag{9}$$

where:
- $Y$ – Value added or production;
- $L$ – Employment;
- $p$ – Labor productivity (value added or production divided by employment).

Then we rewrite equation (9) in the following ways:

$$y = l \times p = e \times f \times p, \tag{10}$$

[39] Measuring export diversification is a potential area of interest for us, as we state in Section 5, but this is coupled with a set of difficulties, since classifications for exports and production are not harmonized, and since this requires accounting for many additional variables.

[40] As an example of how useful this technique could be, we also compare Moscow City and Moscow Oblast.

[41] Standard constraints do not consider an industry a growing base industry if an average location quotient (LQ) is less than one or an average growth pace of competitive share effect (CSE) is less than zero. We slightly modify the methodology due to crisis effects and admit that, for a growing base industry, an average LQ and an average growth pace of CSE during 2006-2009 for employment and 2006-2008 for other variables plus their maximum value for the same period should be more than one or zero, respectively. Why is this necessary? If there is a sharp crisis drop in industry A in 2009, but in 2006-2008 this industry followed a good growth pass, an analyst applying standard approach can exclude this industry from the list of perspective ones, though it is may be not so wise.

[42] To explore several example four-quadrant graphs, look at Figures B.1–B.6 in Appendix II.

[43] The reasons to start with equation (9) are the following: 1) We do not have long-term series for Russian industrial structure; 2) We try to separate pure economic effects from social and institutional determinants.

$$\ln\left(\frac{y_t}{y_{t-1}}\right) = \ln\left(\frac{l_t}{l_{t-1}}\right) + \ln\left(\frac{p_t}{p_{t-1}}\right) = \ln\left(\frac{e_t}{e_{t-1}}\right) + \ln\left(\frac{f_t}{f_{t-1}}\right) + \ln\left(\frac{p_t}{p_{t-1}}\right),^{44} \tag{11}$$

*where*:

- $y$ – Value added or production per unit of population (not labor force);
- $l$ – Employment per unit of population (fraction of population working);
- $e$ – Employment per unit of labor force;
- $f$ – Labor force per unit of population.

Equation (10) is a simple defragmentation of GRP per capita, and equation (11) is a simple defragmentation of economic growth. Labor productivity is a component that accounts mainly for pure economic effects.[45] Labor availability (fraction of population working) consists of two indicators: labor force per unit of population (demographic effects), and employment per unit of labor force (household's economic behavior). However, we treat it as a solid indicator, as labor force can be potentially extended by retired people: if market conditions are favorable, many of them are likely to start working hard again. So, demographic factors do not necessarily reflect economic incentives.

We tested the dependence of these components on different variables available, estimating econometric equations for each year separately. The results[46] are clustered in Tables 8–10.

**Table 8.** Factors of Labor Productivity

| *2006* | FDI.pc | div.Rank | Inv.Loan | Inc.Prop | Intercept |
|---|---|---|---|---|---|
| *Coeff.* | 0.132 | 7.978 | -812.541 | 4,316.583 | -74.370 |
| *st.err.* | 0.036 | 1.378 | 347.632 | 768.670 | 57.310 |
| *R-sq.* | 0.55 | | | | |
| | | | | | |
| *prob.* | .000 | .000 | .022 | .000 | .198 |
| *impact* | 3% | 46% | 11% | 31% | 10% |
| *min* | 0% | 19% | 0% | 0% | 4% |
| *max* | 58% | 89% | 40% | 64% | 16% |
| *2007* | FDI.pc | div.Rank | Inv.Loan | Inc.Prop | Intercept |
| *coeff.* | 0.227 | 8.620 | -924.116 | 5,795.075 | -62.719 |
| *st.err.* | 0.034 | 1.391 | 340.905 | 925.085 | 57.732 |
| *R-sq.* | 0.67 | | | | |
| | | | | | |
| *prob.* | .000 | .000 | .008 | .000 | .280 |
| *impact* | 5% | 43% | 12% | 33% | 7% |
| *min* | 0% | 16% | 0% | 0% | 3% |
| *max* | 70% | 90% | 36% | 62% | 12% |
| *2008* | FDI.pc | div.Rank | Inv.Loan | Inc.Prop | Intercept |
| *coeff.* | 0.279 | 8.023 | -833.378 | 9,181.888 | -24.800 |

[44] It is quite obvious that the weights are the following:

$$\chi = \frac{l_{t-1}}{l_{t-1} + p_{t-1}},\ \chi_1 = \frac{e_{t-1}}{e_{t-1} + f_{t-1} + p_{t-1}},\ \chi_2 = \frac{f_{t-1}}{e_{t-1} + f_{t-1} + p_{t-1}},\ \varphi = \frac{p_{t-1}}{l_{t-1} + p_{t-1}}.$$

[45] Of course, it is not exactly so. Investment, no doubt, depends on some institutional characteristics of the economy. In their recent study, Caselli, and Feyrer (2007) argued: "Developing countries are not starved of capital because of credit-market frictions. Rather, the proximate causes of low capital-labor ratios in developing countries are that these countries have low levels of complementary factors, they are inefficient users of such factors." (p. 565-566). So, investment covers some factors that couldn't be measured directly.

[46] We use simple OLS in our econometric analysis and estimate cross sections due to data restrictions.

| 2006 | FDI.pc | div.Rank | Inv.Loan | Inc.Prop | Intercept |
|---|---|---|---|---|---|
| *st.err.* | 0.026 | 1.180 | 249.854 | 936.430 | 47.445 |
| *R-sq.* | 0.81 | | | | |
| | | | | | |
| *prob.* | .000 | .000 | .001 | .000 | .602 |
| *impact* | 9% | 39% | 11% | 38% | 3% |
| *min* | 0% | 16% | 0% | 0% | 1% |
| *max* | 70% | 95% | 31% | 70% | 5% |

Several things are worth noticing here. First, we found an evidence of an educational drain in Russian economy (Table 9). By educational drain, we mean negative effects of education on labor productivity. We interpret this using the work of Jones (2010), who showed that education takes a lot of time and efforts, and thus reduces the amounts of scientific research.[47] Education is also competing with companies for providing occupation for most effective people.

**Table 9.** Factors of Labor Productivity (with Educational Drain)

| 2006 | FDI.pc | div.Rank | Inv.Loan | Inc.Prop | Stud% | Intercept |
|---|---|---|---|---|---|---|
| *Coeff.* | 0.123 | 9.219 | -480.647 | 5,898.131 | -5,300.261 | -22.610 |
| *st.err.* | 0.034 | 1.382 | 350.320 | 899.217 | 1,727.776 | 57.463 |
| *R-sq.* | 0.59 | | | | | |
| | | | | | | |
| *Prob.* | .001 | .000 | .173 | .000 | .003 | .695 |
| *impact* | 2% | 39% | 5% | 30% | 22% | 2% |
| *min* | 0% | 17% | 0% | 0% | 0% | 1% |
| *max* | 50% | 82% | 21% | 63% | 38% | 4% |
| ***2007*** | **FDI.pc** | **div.Rank** | **Inv.Loan** | **Inc.Prop** | **Stud%** | **Intercept** |
| *Coeff.* | 0.222 | 9.756 | -613.141 | 7,857.812 | -5,736.655 | -4.639 |
| *st.err.* | 0.032 | 1.358 | 335.167 | 1,060.319 | 1,661.144 | 57.202 |
| *R-sq.* | 0.70 | | | | | |
| | | | | | | |
| *Prob.* | .000 | .000 | .071 | .000 | .001 | .936 |
| *impact* | 4% | 36% | 6% | 33% | 22% | 0% |
| *min* | 0% | 14% | 0% | 0% | 0% | 0% |
| *max* | 64% | 85% | 21% | 65% | 36% | 1% |
| ***2008*** | **FDI.pc** | **div.Rank** | **Inv.Loan** | **Inc.Prop** | **Stud%** | **Intercept** |
| *Coeff.* | 0.260 | 8.962 | -580.119 | 10,760.346 | -3,742.672 | 12.802 |
| *st.err.* | 0.026 | 1.187 | 257.578 | 1,064.772 | 1,333.046 | 47.732 |
| *R-sq.* | 0.82 | | | | | |
| | | | | | | |
| *Prob.* | .000 | .000 | .027 | .000 | .006 | .789 |
| *impact* | 6% | 35% | 6% | 35% | 15% | 1% |
| *min* | 0% | 13% | 0% | 0% | 0% | 0% |
| *max* | 66% | 84% | 18% | 62% | 28% | 2% |

[47] He states: "As foundational knowledge expands, innovators may naturally extend their training phases, resulting in a delayed start to the active innovative career. Such a delay may be particularly consequential if raw innovative potential is greatest when young." (p. 5)

Second, we calculate an impact of each factor on model values of labor productivity and labor availability.[48] Foreign direct investment is an interesting variable from this perspective, as it has a great dispersion of impact: for one region, it can account for 50-70% of the result, and for the other region it cannot account less than for 10%. A region's diversity rank has a strong and stable impact on labor productivity.[49] The share of households' income from property proved to be a very strong variable. It is a good proxy for institutional characteristics of a region. And two variables – the share of investment in fixed capital financed by loans and the share of students in population – have a negative impact. The latter was discussed above, and the former, we admit, is connected with financial stability of an enterprise.

Third, we failed to build such a strong equation for labor productivity as we managed to for labor availability (Table 10). The only variable that is significant for both dependent variables is the share of households' income from property (but it is a minor variable here). The availability of pre-school centers and the fraction of children studying dominate in the equation. It is easy to interpret this result, as parents who have to sit with their children at home due to the absence of a pre-school center work much less or completely refuse to work. Another variable – the average propensity to consume – has a negative impact on labor availability. This is not striking, since consumption takes time, and since there are fewer incentives to work if you already can afford yourself a good consumption level.

**Table 10.** Factors of Labor Availability

| ***2006*** | **Inc.Prop** | **APC** | **Pre.sch%** | **Child.st%** | **Pop.urb%** | **Intercept** |
|---|---|---|---|---|---|---|
| *Coeff.* | 0.658 | -0.291 | 0.354 | 0.219 | 0.140 | -0.002 |
| *st.err.* | 0.174 | 0.045 | 0.052 | 0.047 | 0.059 | 0.012 |
| *R-sq.* | 0.90 | | | | | |
| | | | | | | |
| *Prob.* | .000 | .000 | .000 | .000 | .019 | .842 |
| *impact* | 5% | 26% | 29% | 26% | 13% | 0% |
| *min* | 0% | 0% | 8% | 21% | 7% | 0% |
| *max* | 17% | 38% | 44% | 72% | 19% | 1% |
| ***2007*** | **Inc.Prop** | **APC** | **Pre.sch%** | **Child.st%** | **Pop.urb%** | **Intercept** |
| *Coeff.* | 0.856 | -0.287 | 0.346 | 0.209 | 0.143 | -0.002 |
| *st.err.* | 0.211 | 0.044 | 0.054 | 0.047 | 0.059 | 0.012 |
| *R-sq.* | 0.90 | | | | | |
| | | | | | | |
| *Prob.* | .000 | .000 | .000 | .000 | .017 | .862 |
| *impact* | 6% | 26% | 29% | 25% | 14% | 0% |
| *min* | 0% | 0% | 9% | 21% | 7% | 0% |
| *max* | 20% | 39% | 44% | 70% | 19% | 1% |
| ***2008*** | **Inc.Prop** | **APC** | **Pre.sch%** | **Child.st%** | **Pop.urb%** | **Intercept** |
| *Coeff.* | 1.311 | -0.244 | 0.365 | 0.179 | 0.116 | -0.003 |
| *st.err.* | 0.260 | 0.041 | 0.055 | 0.045 | 0.059 | 0.012 |
| *R-sq.* | 0.90 | | | | | |
| | | | | | | |
| *Prob.* | .000 | .000 | .000 | .000 | .053 | .799 |
| *impact* | 7% | 24% | 33% | 24% | 12% | 0% |
| *min* | 0% | 0% | 11% | 18% | 6% | 0% |
| *max* | 24% | 39% | 47% | 66% | 18% | 1% |

[48] The methodology is simple. For each data point (i.e., for each region), we sum the absolute values of coefficients multiplied by the absolute values of independent variables, and add the absolute value of an intercept. This sum is the full result. A ratio of the absolute value of each coefficient multiplied by the absolute value of the independent variable to the full result is an impact of each variable. To calculate aggregate impact for all regions, we apply a simple average.

[49] We tried seventy variants of diversity indices: combinations of nine different variables and six types of diversity indices, an average from different variables for each type of diversity indices, and an average from different types of diversity indices for each variable. We found that an aggregate diversity measure (a diversity ranking) performs very well, and few other variants can compete with it. So, we finally use diversity ranking as independent variable.

It's also interesting to look at short-term tendencies. First, the impact of FDI improves fast during latest years, and the share of households' income from property does the same. Second, the impact of diversification is declining, but it is the strongest variable for every year in sample. It is difficult to identify some other tendencies, as the period is very short.[50]

So, our results show that decomposing economic growth into several dependent variables is a useful approach. It can shed some light on consumption, technological and institutional effects (if to treat average propensity to consume as reflecting behavior of a household, diversification as a technological phenomenon, and income from property as a proxy for institutions).[51]

Regretfully, there is the lack of time series on many variables considered here. So, we can't estimate economic growth directly. We can only build cross sections and look at the stability of our results. In fact, we decompose GRP per capita, but it is not tricky to decompose economic growth if the data is present. In years, the research potential of our approach is going to improve.

## 5. A Simple Computational Example

In this section, we build a simple computational model to explore important effects that are lying behind the regression results.[52] Suppose that an individual during one period can only work or look after his child. For simplicity, each individual has one child. Wage of an individual is exogenously determined over the periods: the first individual is the poorest (with the wage of only 10 coins), the second individual obtain 30 coins, the third – 50 coins, and so on. The last has an enormous wage – 190 coins. There are only ten individuals in the economy, with the average wage of 100 coins.

There are only ten kindergartens in the economy in the first period. After a child is placed in a kindergarten, it becomes overloaded, and no any child can be placed in this kindergarten in the same period. In the next period, individuals make the same choice again. Note that individual always want to place his child in a kindergarten. In the first period, all children are placed there, since there are ten kindergartens in the economy. However, in the second period the number of kindergartens decreases to nine, in the third period – to eight, in the fourth period – to seven. This number holds constant then, up to the last (tenth) period. Nevertheless, some other changes occur in the seventh period: the government succeeds in preventing corruption in kindergartens, and this success holds up to the last period.

There are two principally different regimes: a regime with corruption and a regime without corruption. In the first regime, the poorest fail to place their children in a kindergarten, since the richest pay a bribe to have a priority. The poorest therefore have to sit with their children, losing all their wages. In the second regime, each individual has a random rate of luck, and only seven individuals with the best rate of luck place their children in a kindergarten. Others fail to do this and lose their wages. To realize what really happens, look at Figure 5.[53]

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[50] However, our analysis provides a very stable result. Coefficients change slightly from year to year. We don't find evidence that there is a critical difference between years. May be, it is so due to the length of the period. But for us it is desirable to think that it is due to fundamental characteristics of our equations, which cover core incentives.

[51] Note that our analysis covers only short-term tendencies. Of course, education has strong lasting effects on labor productivity, but in a short-term it drains the resources. Average propensity to consume may have positive long-term effects, but in a short-term it reduces incentives to work. So, it is hard to draw serious policy implications from these findings, though an important result is showing that building social infrastructure, such as pre-school centers, is not a net loss. It can be considered as a perspective investment in economic growth.

[52] The model can be easily constructed in MS Excel.

[53] Ten experiments with the same parameters were conducted. The lines after the red dot are ten potential paths of an average wage in the model economy. The principal feature is that they are never higher than the green line.

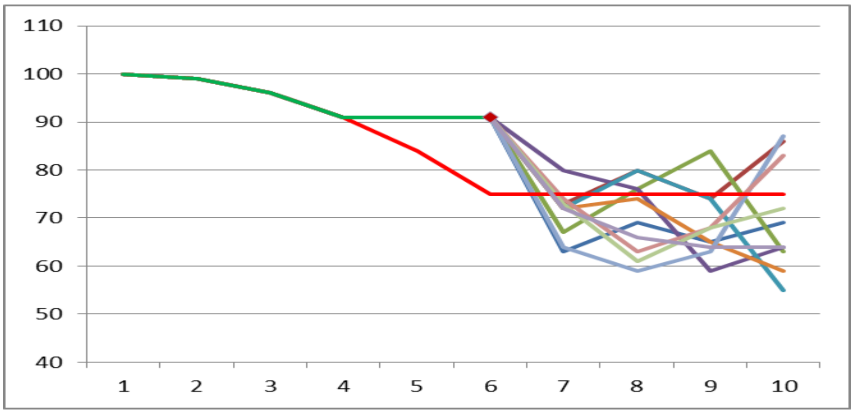


**Figure 5.** Dynamics in a Simple "Kindergarten Economy"

The red line denotes the situation when there are only half of kindergartens left. The green line (before the red dot) denotes the situation when there are seven kindergartens functioning. It is easy to see that it is better for average outcomes to have five kindergartens and an unrestricted corruption, than to have seven kindergartens and no corruption at all. It is better for the economy as well, not only for individuals, as wages correspond much with labor productivity. This result is striking. It clearly shows that every policy should be conducted with caution: in the case with kindergartens, fighting corruption without increasing the number of kindergartens is not growth-enhancing. Moreover, it leads to a sharp drop in labor productivity. Note that such an outcome is a result of agents' heterogeneity. If all individuals in the economy get equal wages, corruption is no more helpful for economic growth.

## 6. Conclusion

As it is stressed in Brock, and Durlauf (2000), modern theory of economic growth tends to be opening ended. Here, we examined only a little piece of the subject. Our attention was focused on empirical analysis of diversification. We calculated diversity indices for Russian regions for nine different variables, accounting for levels of aggregation. We showed that standard measures of economic diversity are still useful in economic analysis, as their sensitivity to aggregation level is relatively low.

Diversification issues have been strangely isolated from economic growth theory. They are usually examined only in regional or land economics.[54] Nevertheless, this technique helps us to understand economic ties among regions that transform a set of separated regions into the united national economy. Second, the right way to construct a diversified economy, in our opinion, is realizing and step-by-step stimulating comparative advantages of every region. Thus, by a set of short-term policy measures, as Wagner (2000) importantly notes, a policy-maker can attain long-term diversification without comparative advantages' bias (i.e., without imposing hard restrains on national leaders, even if they specialize on primary products).

In this research, we developed a very simple defragmentation of economic growth. Labor productivity and labor availability are the two components of economic growth, and they depend on different factors. Regressing economic growth on one or another indicator does not always make much sense. We showed that economic growth is decomposed, and that it is necessary to analyze each of the components separately.

However, there is a huge area for future research. It is interesting to analyze diversification of production in connection with diversification of exports. Doing this, it is good to account for trade openness as a proxy for the level of democracy and distance to technological frontier as a proxy for technological level of an industry, as in Aghion, Alesina, and Trebbi (2007). We expect to extract very useful information from this type of analysis.

This was largely an empirical exercise, without sound theoretical ground. Nevertheless, we showed the importance of some factors that are rarely studied. We built a small computational model showing that bribery in kindergartens can be growth-enhancing. Due to space limitations, we were not able to construct such models for every factor identified in reduced-form equations or to construct a general equilibrium model incorporating these effects. We see this as a potential fruitful area for further research.

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[54] The recent paper of Cuberes, and Jerzmanowski (2009) is one of the pleasant exceptions. In their model, the level of democracy determines diversification, since lower barriers to entry for new firms in democracies induce industrial structure to become more diversified.

The second potential effect that is worth incorporating is educational drain. The problem is that people accumulate human capital not only when they are children, as assumed, for instance, in Galor (2005), but also when they are able to work and to produce goods instead of consuming educational services and thus reducing the amount of resources available at the moment.

This should be a separate model due to the importance of the problem: education, as argued by Lucas (1988) and the followers, fosters long-term economic growth, but there is evidence that it reduces economic outcomes in the short-term. It is easy to recommend that you may stimulate education and technological progress in order to promote growth. But a reasonable question still holds: what are the costs?

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## APPENDIX I

Here, we present a cross-indicator portrait for every Federal District (Figures A.1–A.21). We are able to provide such a portrait for every region, but due to space limitations we present a portrait for two regions – Moscow City and Moscow Oblast (Figures A.22–A.24).

Value of an indicator cannot be lower than zero and greater than ten. We normalized all the variables to get convenient graphs. For each indicator, ten stands for the maximum value of this indicator (where regions are data points). Zero stands for the minimum value of the indicator, not for the absence of value. We use the following formula to calculate the rank:

$$Rank = \frac{x_j - x_{\min}}{x_{\max} - x_{\min}} \times 10, \quad (12)$$

where:

- $x_j$ – Value of a variable for region $j$;
- $x_{\min}$ – Minimum value of a variable;
- $x_{\max}$ – Minimum value of a variable.

Note that the greater rank doesn't necessarily mean the "best" performance of an indicator. We do not normatively rank the variables. We simply take statistical data and work with it. Each indicator may have its own (unknown in our research) "normal values".

In our analysis, we extensively use Microsoft Excel to work with huge volumes of data and construct our tables and graphs. During this research, we managed to effectively standardize the data on regional economic performance. We are going to use this database in our future research, and we are ready to provide some additional information on request (graphs for other regions of Russian economy, raw data by nine variables used to calculate diversification, etc.).

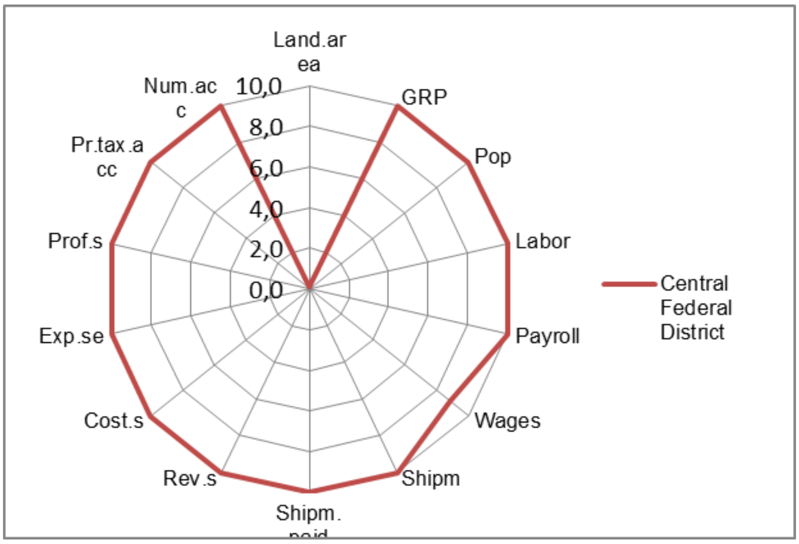


**Figure A.1.** Central Federal District (2008, Economic Size Indicators)

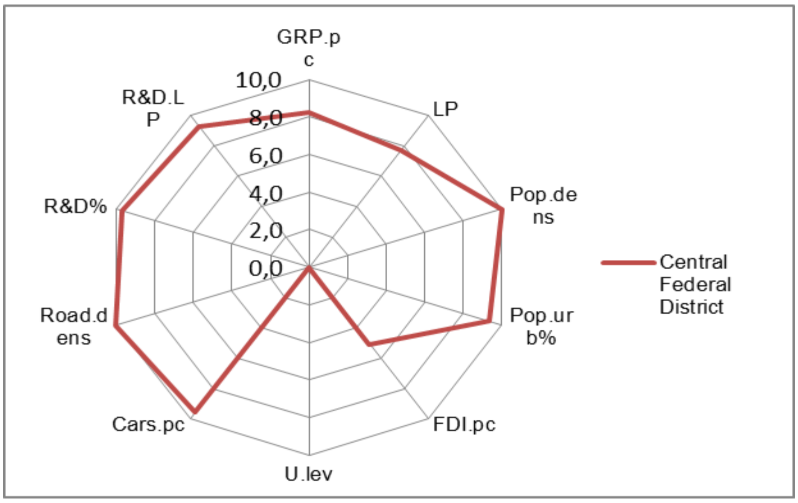


**Figure A.2.** Central Federal District (2008, Economic Effectiveness Indicators)

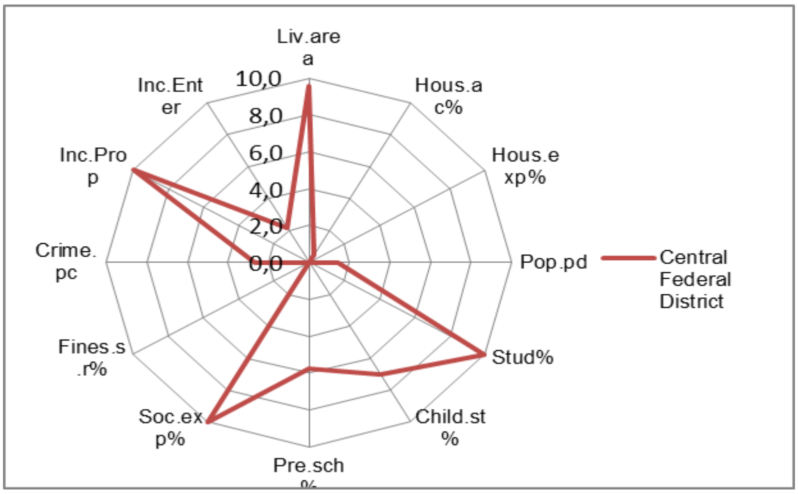


**Figure A.3.** Central Federal District (2008, Social and Institutional Indicators)

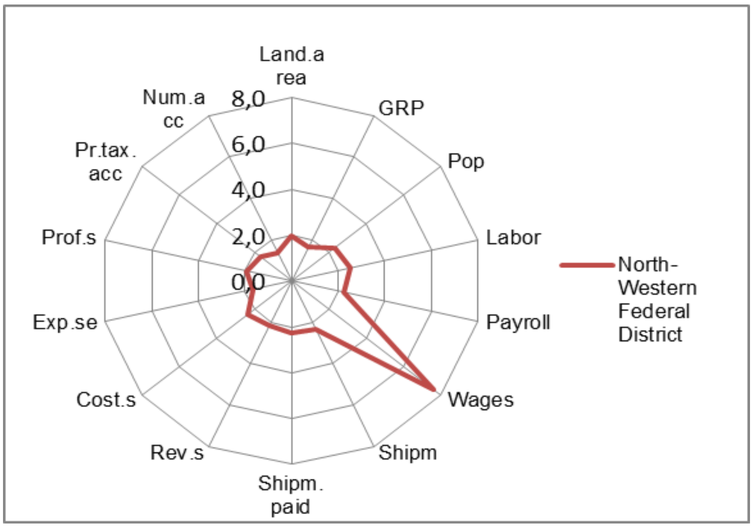


**Figure A.4**. North-Western Federal District (2008, Economic Size Indicators)

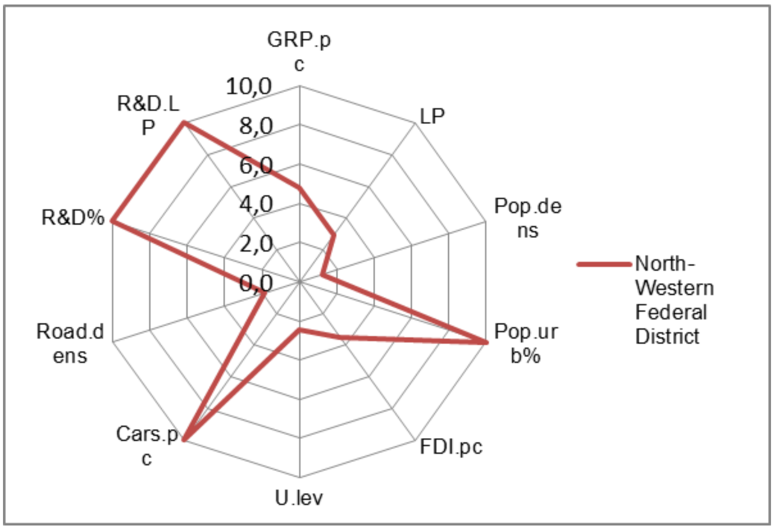


**Figure A.5.** North-Western District (2008, Economic Effectiveness Indicators)

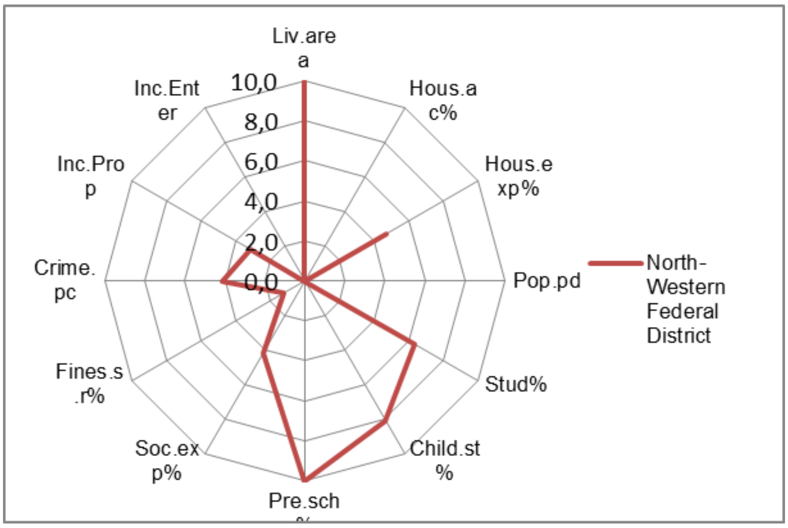


**Figure A.6.** North-Western Federal District (2008, Social and Institutional Indicators)

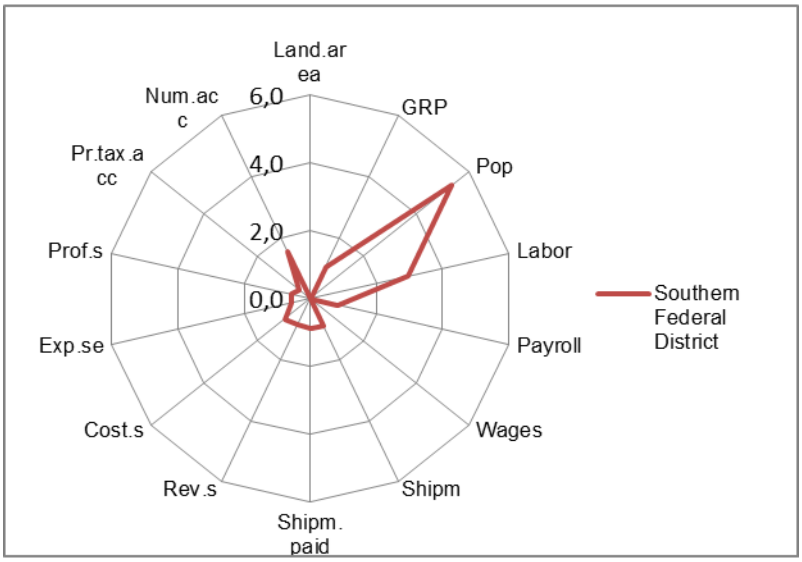


**Figure A.7.** Southern Federal District (2008, Economic Size Indicators)

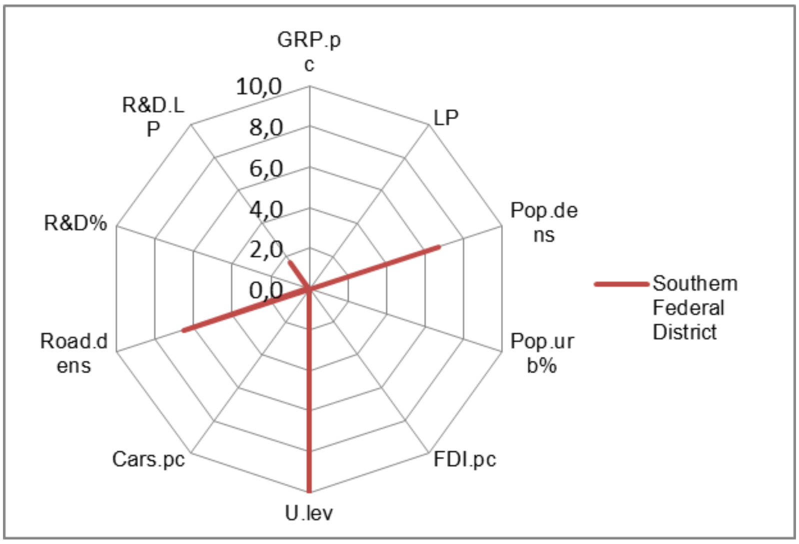


**Figure A.8.** Southern Federal District (2008, Economic Effectiveness Indicators)

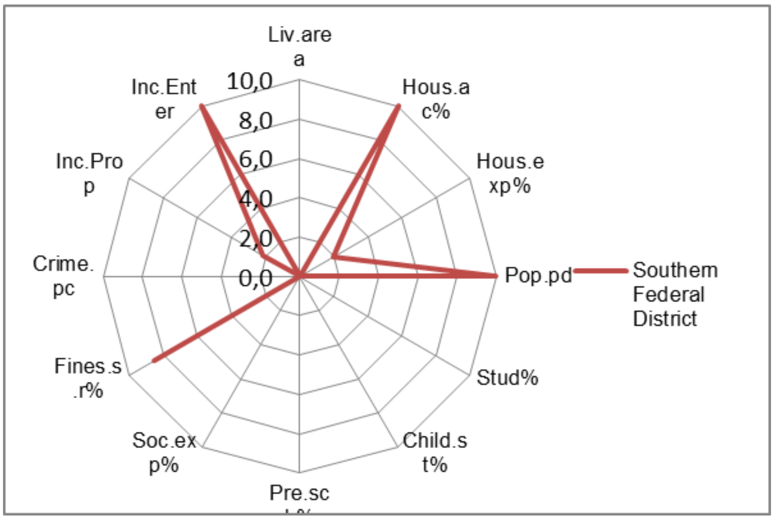


**Figure A.9.** Southern Federal District (2008, Social and Institutional Indicators)

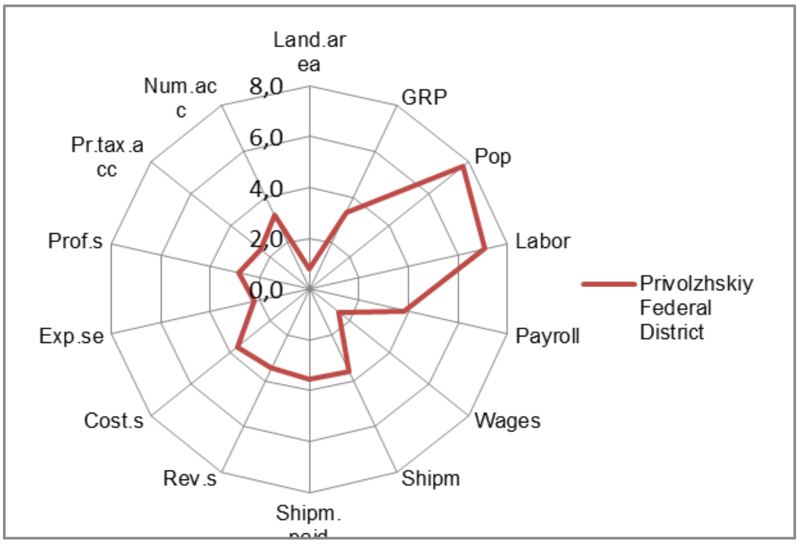


**Figure A.10.** Privolzhskiy Federal District (2008, Economic Size Indicators)

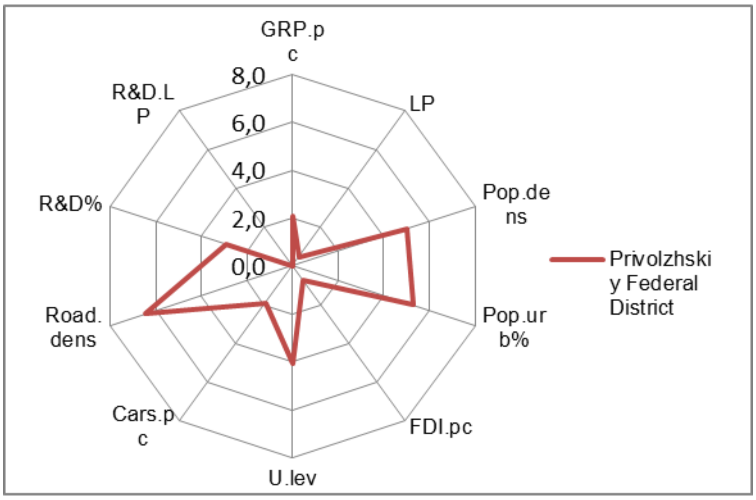


**Figure A.11.** Privolzhskiy Federal District (2008, Economic Effectiveness Indicators)

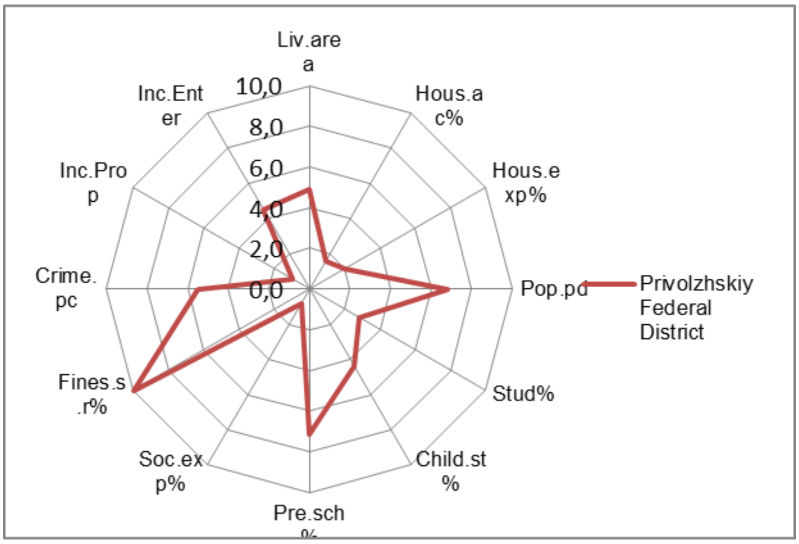


**Figure A.12.** Privolzhskiy Federal District (2008, Social and Institutional Indicators)

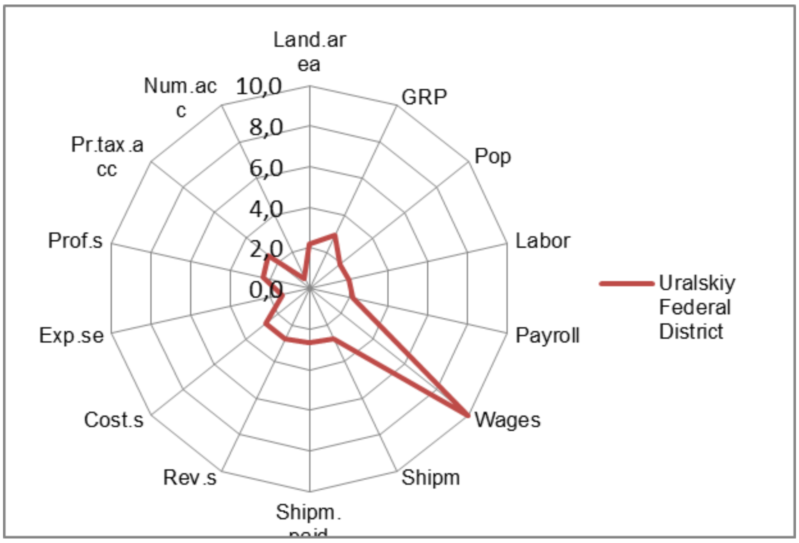


**Figure A.13.** Uralskiy Federal District (2008, Economic Size Indicators)

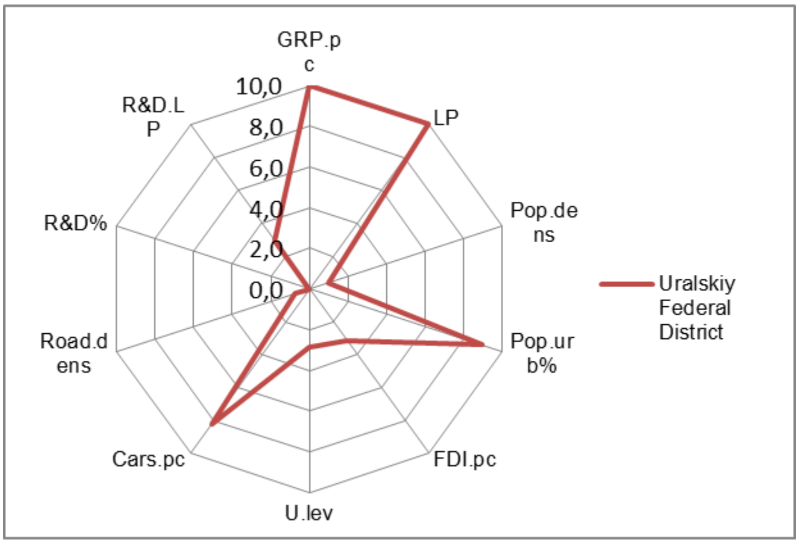


**Figure A.14.** Uralskiy Federal District (2008, Economic Effectiveness Indicators)

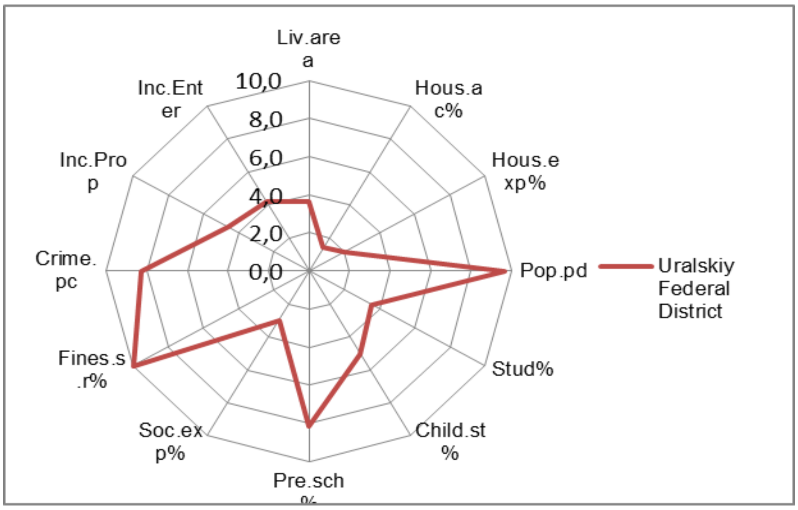


**Figure A.15.** Uralskiy Federal District (2008, Social and Institutional Indicators)

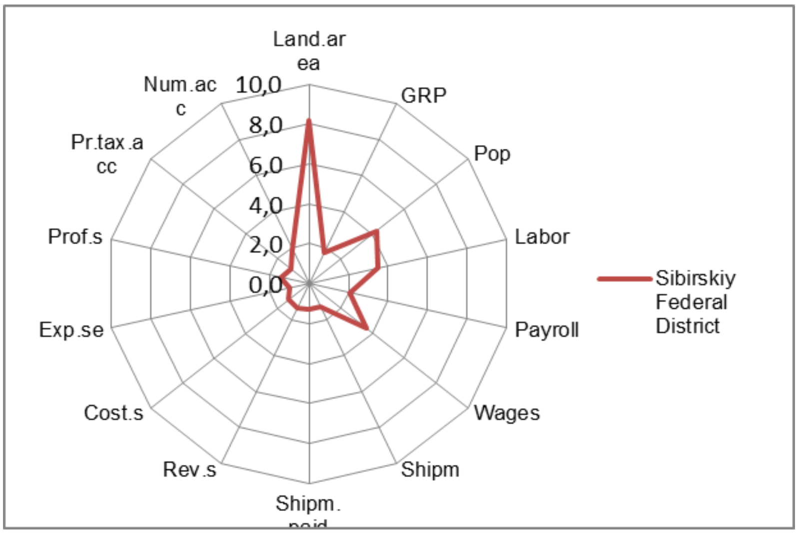


**Figure A.16**. Sibirskiy Federal District (2008, Economic Size Indicators)

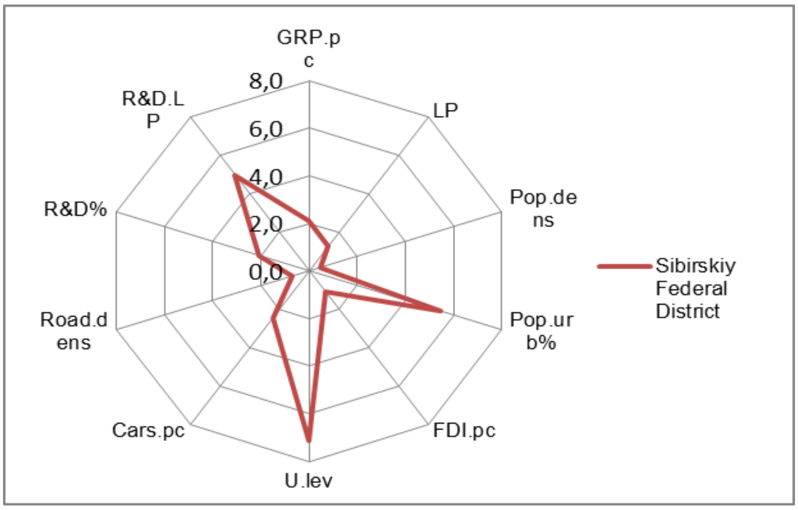


**Figure A.17.** Sibirskiy Federal District (2008, Economic Effectiveness Indicators)

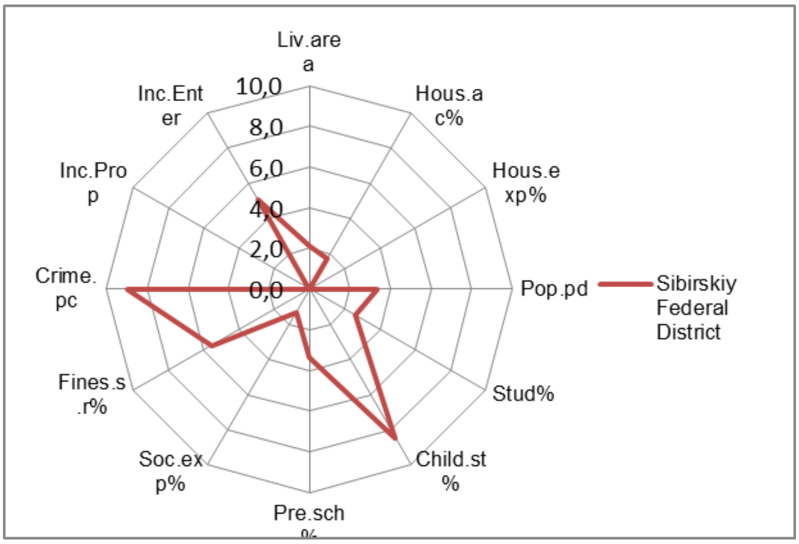


**Figure A.18.** Sibirskiy Federal District (2008, Social and Institutional Indicators)

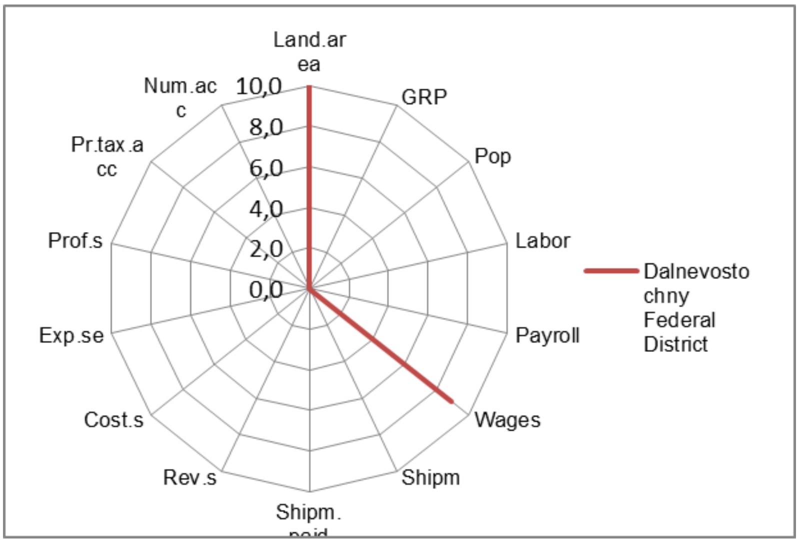


**Figure A.19.** Dalnevostochny Federal District (2008, Economic Size Indicators)

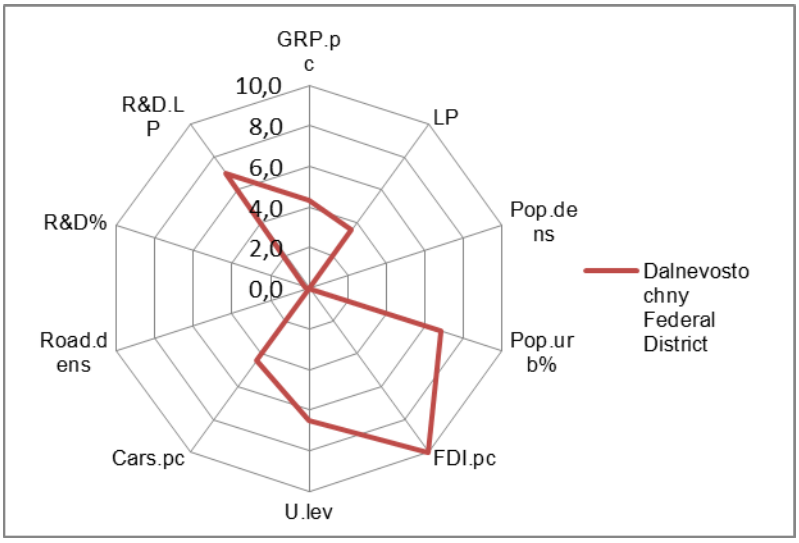


**Figure A.20.** Dalnevostochny Federal District (2008, Economic Effectiveness Indicators)

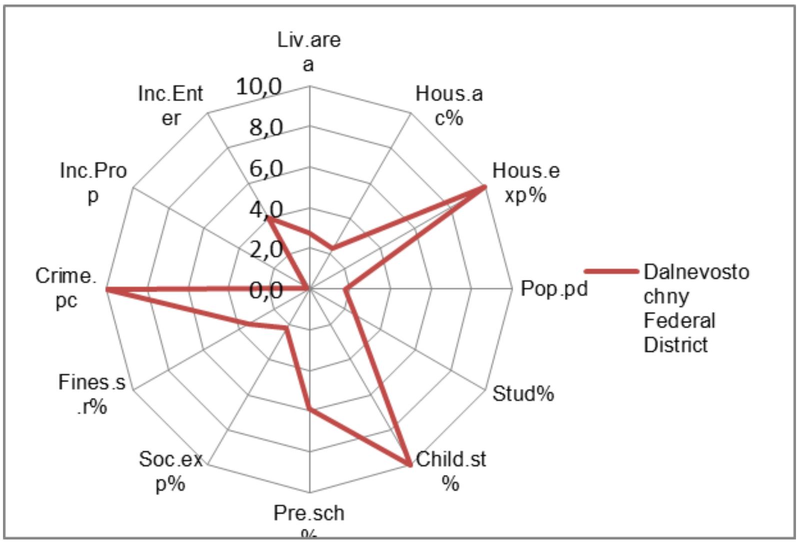


**Figure A.21.** Dalnevostochny Federal District (2008, Social and Institutional Indicators)

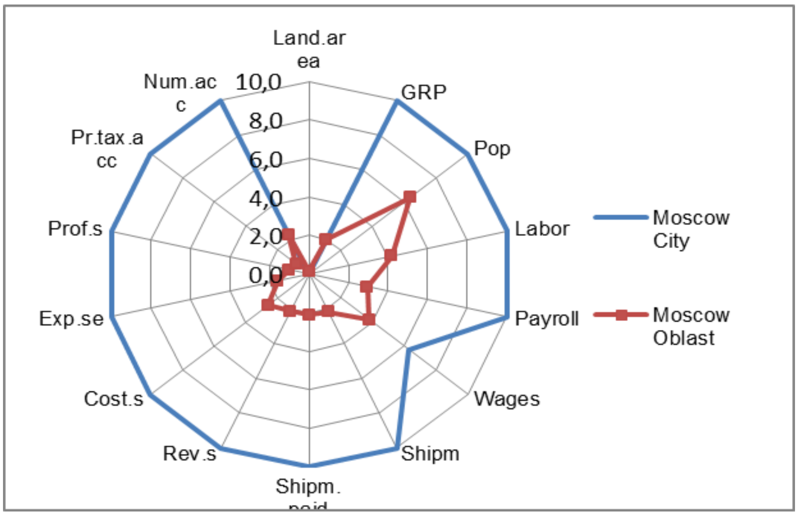


**Figure A.22.** Moscow City and Moscow Oblast (2008, Economic Size Indicators)

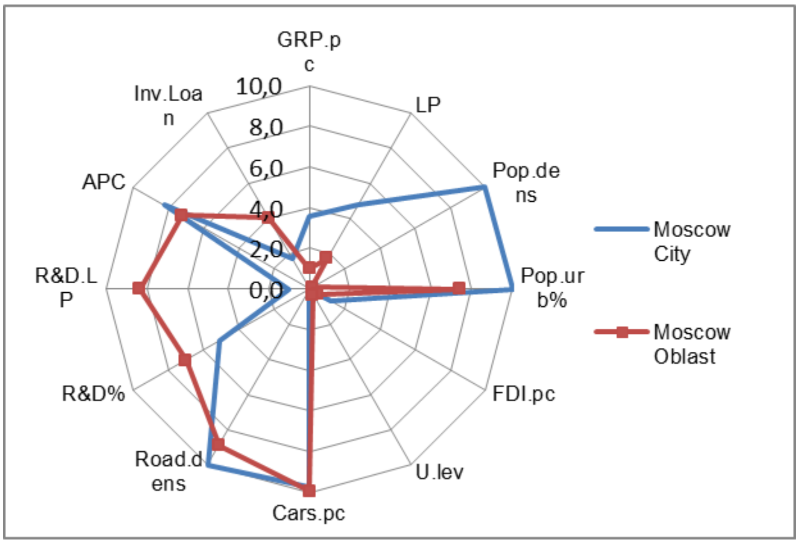


**Figure A.23.** Moscow City and Moscow Oblast (2008, Economic Effectiveness Indicators)

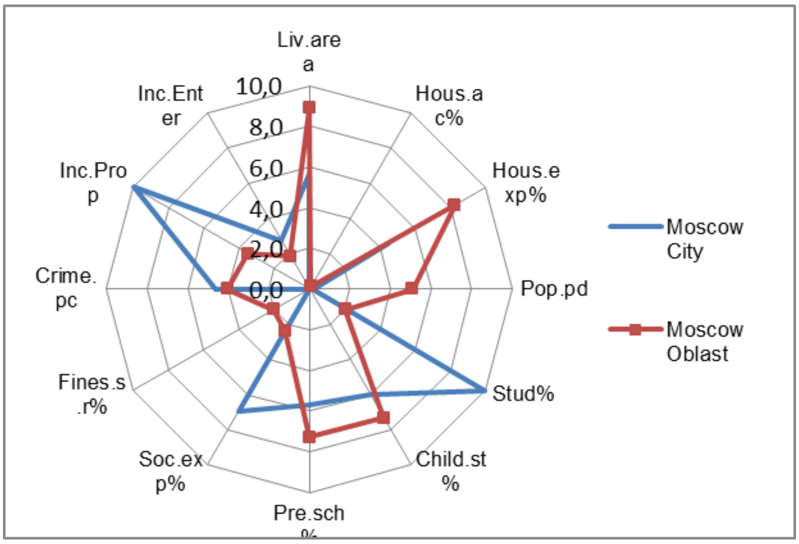


**Figure A.24.** Moscow City and Moscow Oblast (2008, Social and Institutional Indicators)

# APPENDIX II

Here, we provide cross-specialization matrices for three variables: employment, shipment and labor productivity (Tables B.1–B.3). We also describe OKVED in Table B.4.

The methodology is the following. First, we calculate location quotients for every industry and every region by years and indicators (employment and shipment). We use equation (4) to do it. We get a location quotient for labor productivity as a ratio of the one for shipment to the one for employment. Note that we calculate labor productivity for regressions in a different way: we divide value added by employment. However, tables in this Appendix are illustrative and do not influence our core results.

Second, we calculate competitive share effects, using the third part of equation (6). In Raj Sharma (2008), the role of competitive share effect is emphasized: “a positive competitive share effect implies the region’s economic performance is superior to the national average.” (p. 7).

Then we simply combine both indicators in a four-quadrant graph and take those industries that go in the upper-right quadrant. As an example, we present four-quadrant graphs for Republic of Tatarstan for 2008 (FIGURES B.1–B.3). We are able to construct such graphs for every region for 2006, 2007, 2008, and the average. For employment, it is already possible for 2009.

Finally, we combine the result into cross-specialization matrices. These are our technical invention to simultaneously facilitate the analysis of industrial specialization for Russian regions and regional specialization for Russian industries. Since we do not attempt to examine industries separately in this research, we don’t use these tables in our analysis. However, it is right to make them public, since they look like a very powerful instrument for regional research.

The methodology applied here was described by Raj Sharma (2008). Our invention is only applying it to Russian economy and introducing cross-specialization matrices.

**Table B.1.** Cross-Specialization Matrix for Employment

| *Region* | *A* | *B* | *CA* | *CB* | *DA* | *DB* | *DC* | *DD* | *DE* | *DF* | *DG* | *DH* | *DI* | *DJ* | *DK* | *DL* | *DM* | *DN* | *E* | *F* | *G* | *H* | *I* | *J* | *K* | *L* | *M* | *N* | *O* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Republic of Tatarstan | 4 | | | | | | 5 | | | | 1 | 2 | | | 8 | | 3 | | | 6 | | | | 9 | | | 7 | | |
| Saratov Oblast | 4 | | | | | | | | | 6 | | | 5 | | | 1 | | | 2 | | | | | | | | | 3 | |
| Irkutsk Oblast | | | | 3 | | | | 2 | | 1 | 4 | | | | | | | | 6 | | | | | | | 7 | 5 | 8 | |
| Rostov Oblast | 2 | | | | | 1 | | | | | | | 6 | 3 | | | 4 | | 5 | | | | 8 | | | | | 7 | |
| Republic of Bashkortostan | | | 3 | 4 | | | | | | 1 | 2 | | | | 6 | | 5 | | | | 7 | | | | | | | | |
| Nizhniy Novgorod Oblast | | | | | | | 2 | | | | | 3 | | 1 | | 4 | | | | | | | | | | | | | |
| Perm Kray | | | | | 7 | | 1 | 2 | | | | 4 | 5 | | | | 3 | 8 | | | | | | | | 6 | | | |
| Novosibirsk Oblast | | | | | | 5 | | | | | | | 2 | | | 1 | | | | | | | 4 | 3 | | 6 | | | 7 |
| Yaroslavl Oblast | | | | | | 3 | 2 | | 8 | 1 | | | 5 | | 4 | | | | | | | 7 | | | | | | | 6 |
| Udmurtskaya Republic | 2 | | 3 | | | | | 4 | | | | | 7 | | | 1 | | 5 | | | | | | | | | 6 | | |
| Samara Oblast | | | | | | | | | | 2 | 3 | | | | | | 1 | | | | | 7 | 4 | 5 | 6 | | | | |
| Omsk Oblast | 1 | | | | 3 | | | | | | | | | | 4 | 6 | | | | | | | 7 | | | | | 2 | 5 |
| Chuvash Republic | | | | | 6 | 1 | 2 | | | | | | 4 | | | 3 | | | | | | 5 | | | | | | | |
| Moscow Oblast | | | | | 6 | 4 | | 11 | | | 5 | 2 | 3 | | 7 | 8 | | 1 | 13 | | 1 | 9 | 12 | | | | | | |
| Leningrad Oblast | 5 | | | 6 | 9 | 11 | | 4 | 2 | 1 | 7 | | 3 | | | | | 1 | | 8 | | | | | | | | | |
| Republic of Mariy El | | | | | 4 | | | 1 | | 6 | | 1 | | | 5 | 2 | | 3 | | | | 7 | | | | 8 | | | 9 |
| Tver Oblast | | | | | 7 | | 1 | 2 | | | | 4 | 5 | | | | 3 | 8 | | | | | | | | 6 | | | |

| Region | A | B | C A | C B | D A | D B | D C | D D | D E | D F | D G | D H | D I | D J | D K | D L | D M | D N | E | F | G | H | I | J | K | L | M | N | O |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Smolensk Oblast | 6 | | | | 7 | 1 | 8 | 1 | | | 9 | 5 | 2 | | | 3 | | | 4 | | | | 11 | | | | | | |
| Volgograd Oblast | 4 | | | | 8 | | | | | | | | 3 | 1 | 2 | | | | | 5 | 1 | | 6 | | | | 9 | 7 | |
| Voronezh Oblast | 1 | | | | | | | | | | 2 | | | | | | 5 | 4 | | | 3 | | | | | | | 6 | |
| Ryazan Oblast | | | | | | | 1 | | | 2 | | | 3 | | 7 | | 5 | | | | | | | | | 6 | 4 | | |
| Kirov Oblast | | | | | | | 1 | | | | | | | | | | | 2 | 3 | | | | | | | | | | |
| Bryansk Oblast | | | | | | 2 | 1 | | 6 | | | | | | | | 4 | 3 | | | | | | | | | 5 | | |
| Penza Oblast | 2 | | | | 3 | | | 4 | | | | | | | | 1 | | | | | | | | | | | | | |
| Kaluga Oblast | 1 | | | | 9 | 11 | 2 | 7 | 3 | | | 12 | 4 | 6 | 5 | 1 | 8 | | | | | | | | | | | | |
| Vladimir Oblast | | | | | 7 | 1 | 2 | 3 | | | | 5 | | | 4 | | | 6 | 9 | | | 8 | | | | 1 | | | |
| Republic of Mordoviya | 1 | | | | 3 | | | | | | | | 2 | | | | | | | | | | | | | | | | |
| Khabarovsk Kray | | 1 | | | | | | | | 2 | | | | | | | 3 | | | 7 | 6 | | 4 | | | 5 | | | |
| Krasnodarskiy Kray | | | | | 1 | | | | | | | | 4 | | | | | | | 6 | | 2 | 5 | | | | | 3 | 7 |
| Ulyanovsk Oblast | 7 | | | | | 4 | | 3 | | | | | 6 | | | 2 | 1 | 5 | | | | | | | | | | | |
| Kursk Oblast | | | | | 2 | 3 | 1 | | | | | | | | | | | | | | | | | | | | 4 | | |
| Saint Petersburg City | | | | | | | | | | | | | | | | | | | | | 3 | 2 | 4 | 5 | 1 | | | | 6 |
| Novgorod Oblast | | | | | 2 | | | 1 | 3 | | | | | | | | | | 4 | | | 6 | | | | 5 | | | 7 |
| Tula Oblast | | | | | 3 | 7 | 1 | | 8 | | | 4 | 6 | 2 | | 5 | | | | | | | | | | | | | |
| Altayskiy Kray | 1 | | | | 2 | | | | | | | 3 | | | | | | | | | | | | | | | | 4 | |
| Oryol Oblast | | | | | | 1 | | | | | | | 3 | | 2 | | | | | | | | | | | | | | |
| Krasnoyarsk Kray | 5 | | | 1 | | | | 2 | | | | | | 4 | 3 | | | | | | | | 6 | | | | | 8 | 7 |
| Kurgan Oblast | 2 | | | | | | | | | | 4 | | | 5 | 1 | | | | 6 | | | | | | | 3 | | | 7 |
| Kaliningrad Oblast | | 2 | | | 3 | 8 | | 7 | 4 | | | | | | | | 6 | 1 | | | | | | 9 | | 5 | | | |
| Moscow City | | | | | | | 4 | | 2 | | | 6 | | | | 7 | | | | 5 | 3 | | | 1 | | | | | |
| Astrakhan Oblast | | | 6 | | | | | | | 1 | | | | | | | 4 | | 5 | | | | | | | 2 | | 3 | |
| Tomsk Oblast | | | 1 | | 3 | | | 2 | | | | | | | | | | | 6 | 7 | | 5 | 8 | | | | 4 | | |
| Republic of Adygeya | | | | | 4 | | | 1 | 2 | | | | | | | | | | | | | | | | | | 3 | 5 | |
| Sverdlovsk Oblast | | | | | | | | 3 | | | | | | | 1 | 2 | | | 5 | | 6 | 4 | | | | | | | |
| Kostroma Oblast | 5 | | | | | 3 | 4 | 1 | | | | | | | 9 | | | 2 | | | | | | | | 6 | 8 | | 7 |
| Belgorod Oblast | 5 | | | 1 | 2 | | | | | | | | 3 | 4 | 7 | | | | | | | | | | | | | | 6 |
| Kemerovo Oblast | | | | | | | | | | | | | | | 2 | | | | 1 | | | | 3 | | | | | | |
| Primorskiy Kray | | | | 3 | | | | | | | | | | | | | 2 | | | | | | | | | 1 | | | |
| Arkhangelsk Oblast | 5 | 1 | 3 | | | | | | | | | | | | | | 2 | | 4 | | | | | | | | | | |
| Republic of Buryatiya | | 4 | | 1 | | | | | | | | | | | | | 2 | | 5 | | | | | | | 3 | | 6 | 7 |
| Pskov Oblast | 6 | | | | 4 | 2 | 1 | 7 | | | | | 5 | | | 3 | | 9 | 8 | | | | | | | | | | |
| Tambov Oblast | | | | | | 3 | 1 | | | | | | | | | 2 | | | | | | | | | | | | | |
| Chelyabinsk Oblast | | | | 1 | | | | | | | | | 3 | | 2 | | | 4 | | | | | | | | | 5 | | |

| *Region* | *A* | *B* | *CA* | *CB* | *DA* | *DB* | *DC* | *DD* | *DE* | *DF* | *DG* | *DH* | *DI* | *DJ* | *DK* | *DL* | *DM* | *DN* | *E* | *F* | *G* | *H* | *I* | *J* | *K* | *L* | *M* | *N* | *O* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Orenburg Oblast | | | 4 | 1 | | | | | | 2 | | | | 3 | 6 | | | | | 8 | | | 9 | | | | 7 | 5 | |
| Republic of Kareliya | | 1 | | | | | | | | | | | | | | | | | | | | | 2 | | | 3 | | | 4 |
| Republic of Khakasiya | | | | | | | | | | | | | | 2 | | | | | | | | | | | | 1 | 3 | | 4 |
| Republic of Northern Osetiya – Alaniya | | | | | | | | | | | | | | | | | | | 4 | | | | | | | 1 | 2 | 3 | |
| Stavropol Kray | 1 | | | | 2 | | | | | | | 3 | | | | | | | | | | | | | | | | 4 | |
| Kabardino-Balkarskaya Republic | | | | | | 2 | | | | | | | | | | | | | 5 | | | | | | | 4 | 1 | 3 | |
| Jewish Autonomous Oblast | | | | | | 7 | | | | | | | 2 | | | | | 3 | 4 | 8 | | | | | | 1 | | 6 | 5 |
| Zabaykalskiy Kray | | | | | | | | | | | | | | | | | | | 4 | | | | | | | 1 | 3 | 2 | |
| Republic of Altay | 2 | | | 3 | | | | | | | | | | | | | | | | | | | | | | 1 | 4 | 6 | 5 |
| Republic of Dagestan | 2 | | | | | | | | | | | | | | | | | | | | | | | | | 4 | 1 | 3 | |
| Republic of Komi | | | | | | | | 1 | 2 | | | | | | | | | | | | | | 3 | | | | | | |
| Karachaevo-Cerkesskaya Republic | | | | | 2 | | | | | | | | 4 | | | | | | 5 | | | | | | | 1 | 3 | 6 | 7 |
| Ivanovo Oblast | | | | | | 1 | | | | | | | | | 3 | | | 2 | | | | | | | | 4 | 5 | | |
| Vologda Oblast | | | | | 3 | | | | 6 | | 2 | | | | 1 | | | | | | | | | | | 4 | 5 | | |
| Amur Oblast | 6 | | | 1 | | | | | | | | | | | | | | | 2 | 3 | | | | | | 4 | | 5 | |
| Murmansk Oblast | | | | 1 | | | | | | | | | | | | | | | 2 | | | 4 | | | | | | | 3 |
| Lipetsk Oblast | 4 | | | 7 | 2 | | | | | | | | 6 | 1 | 3 | | | | | | | | | | | | | 5 | 8 |
| Kamchatskiy Kray | | 1 | | 3 | 4 | | | | | | | | | | | | | | 2 | | | | 5 | | | | 7 | 6 | |
| Chechenskaya Republic | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Republic of Tyva | 5 | | | 1 | | | | | | | | | | | | | | | | | | | | | | 3 | | 2 | 4 |
| Sakhalin Oblast | | | | | 2 | | | | | | | | | | | | | | 4 | 1 | | 6 | 5 | | 7 | 3 | | 8 | |
| Republic of Sakha (Yakutiya) | | 5 | 2 | 1 | | | | | | | | | | | | | | | | | | | | | | 3 | | | 4 |
| Republic of Kalmikiya | | | | | | | | | | | | | | | | | | | | | | | | | | 1 | | | |
| Tumen Oblast | | | 1 | | | | | | | | | | | | | | | | 4 | 2 | | 5 | 3 | | | | | | |
| Republic of Ingushetiya | | | | | | | | | | | | | | | | | | | | | | | | | | 1 | 2 | 3 | 4 |
| Magadan Oblast | | | | | | | | | | | | | | | | | | | 2 | | | | | | | 1 | | | |
| Chukotskiy Autonomous Okrug | 5 | | | 1 | | | | | | | | | | | | | | | 2 | | | | 4 | | | 3 | | | 6 |
| Khanty-Mansiyskiy Autonomous Okrug – Yugra | | | 1 | | | | | | | 2 | | | | | | | | | 5 | 3 | | 6 | 4 | | | | | | |
| Yamalo-Neneckiy Autonomous Okrug | | 1 | | | | | | | | | | | | | | | | | 3 | | | | 2 | | 4 | | | | |

| Region | A | B | CA | CB | DA | DB | DC | DD | DE | DF | DG | DH | DI | DJ | DK | DL | DM | DN | E | F | G | H | I | J | K | L | M | N | O |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Neneckiy Autonomous Okrug | 6 | 2 | 1 | | | | | | | | | | | | | | | | 8 | 4 | | 3 | 5 | | | | | | 7 |

**Note**. Figures denote ranks of an industry in a region's economic activity (only growing base industries have a rank different from zero).

OKVED codes are disclosed in Table B.4.

**Table B.2.** Cross-Specialization Matrix for Shipment

| Region | A | B | CA | CB | DA | DB | DC | DD | DE | DF | DG | DH | DI | DJ | DK | DL | DM | DN | E | F | G | H | I | J | K | L | M | N | O |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Republic of Tatarstan | 3 | | | | | | | | | | | 1 | | | | | 2 | | | 4 | | | | | | | | | |
| Saratov Oblast | 1 | | | | 5 | | | | | | 3 | | | | | | | 6 | 2 | 8 | | | | | | | 4 | 7 | |
| Irkutsk Oblast | | | | 1 | | | | | | | | | | | | | 3 | | 4 | 7 | | | 2 | | | 6 | | 5 | |
| Rostov Oblast | 2 | | | | 5 | 1 | 3 | | | | | | 1 | | | | 4 | 6 | | 7 | | 11 | | | | 9 | 8 | 12 | |
| Republic of Bashkortostan | | | 3 | 5 | | | | | | | | | 4 | | 1 | | | | 6 | | | | | | | | 2 | | |
| Nizhniy Novgorod Oblast | | | | | | | 3 | | 5 | 1 | | 2 | | 4 | | | | | | 6 | | | | | | | | | |
| Perm Kray | | | | | 8 | | 1 | | 1 | | | 7 | 9 | | | 4 | 2 | | 3 | | | | 6 | | | 5 | | | |
| Novosibirsk Oblast | | | | | 1 | | | | | | | 5 | 6 | | | 3 | | | | | | | | | 2 | | 4 | | |
| Yaroslavl Oblast | | | | | 2 | | | | 7 | | | | | | 4 | 1 | | 6 | 5 | | | | 3 | | | | | | |
| Udmurtskaya Republic | 4 | | 2 | | 6 | | | | | | | | | | | 1 | 3 | 5 | | | | | | | | 7 | | | |
| Samara Oblast | | | | | | | | | | | | | | | | 1 | | | | | | 3 | 2 | | | | | | |
| Omsk Oblast | | | | | 2 | | | | | | | 1 | | | | | | | | | | | | | | 4 | | 3 | |
| Chuvash Republic | | | | | 4 | 1 | | | | | | | 6 | | | 2 | 3 | 7 | | | | | | | | | | 5 | |
| Moscow Oblast | | | | | 5 | | 14 | 6 | 11 | | | 1 | 2 | | 3 | 7 | | 4 | 13 | 9 | 12 | 8 | | | | 15 | 16 | 1 | |
| Leningrad Oblast | | | | | 2 | | | 4 | 1 | | 5 | 6 | | | | | 8 | 9 | 7 | 3 | | | | | | | | | |
| Republic of Mariy El | | | | | 7 | 4 | 1 | 1 | | 6 | | | | | | 2 | | 5 | | 8 | | | | | | 3 | | | 9 |
| Tver Oblast | | | | | 8 | | 1 | | 1 | | | 7 | 9 | | | 4 | 2 | | 3 | | | | 6 | | | 5 | | | |
| Smolensk Oblast | | | | | | | | 4 | | | 5 | 2 | | | | 1 | 7 | | | | | | 9 | | | 3 | 6 | 8 | |
| Volgograd Oblast | 2 | | | | | 6 | | | | 1 | | 4 | 3 | | 5 | | | | | | | | | | 7 | | | | |
| Voronezh Oblast | 1 | | | | 3 | | | | | | 2 | | 5 | | 1 | 7 | | 9 | 6 | | | | 11 | | | | 4 | | 8 |
| Ryazan Oblast | 2 | | | | 6 | 4 | | | | | | | 3 | | 7 | 1 | 8 | 9 | | | | | | | | | 5 | | |
| Kirov Oblast | | | | | | 4 | | | | | 1 | | | | | 5 | | 2 | | | | 6 | | | | | | 3 | |
| Bryansk Oblast | | | | | 7 | 5 | 1 | 2 | 1 | | | 11 | | | 6 | 13 | 3 | | 12 | | | 9 | 8 | | | 4 | | | |
| Penza Oblast | | | | | 5 | | 2 | 4 | | | | | | | 1 | 3 | | 8 | | 1 | | | | | | 7 | 6 | 9 | 11 |
| Kaluga Oblast | | | | | 4 | 7 | | 5 | | | | 9 | 6 | 1 | | 1 | 3 | 2 | 11 | 8 | | | | | | | | | |
| Vladimir Oblast | | | | | 3 | | 4 | 6 | | | 12 | 2 | | | 1 | | | 7 | 8 | | | | | 5 | | 9 | 11 | 1 | |
| Republic of Mordoviya | 2 | | | | 4 | | | | | | | | 1 | | 8 | | | 3 | 7 | 6 | | | | | | 5 | | | |
| Khabarovsk Kray | | | | | | | | | | | | | | | | | 1 | | | 2 | | | | | | | | | |

| Region | A | B | CA | CB | DA | DB | DC | DD | DE | DF | DG | DH | DI | DJ | DK | DL | DM | DN | E | F | G | H | I | J | K | L | M | N | O |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Krasnodarskiy Kray | 1 | | | | 3 | | | | | | | | 7 | | | | | | | 5 | | 2 | 4 | | | 8 | 9 | 6 | |
| Ulyanovsk Oblast | | | | | | 1 | | 3 | | | | | | | | | | 2 | 4 | | | | 5 | | | | | 6 | |
| Kursk Oblast | 2 | | | 1 | 3 | | | | | | | | | | | | | | | | | | | | | | 4 | | |
| Saint Petersburg City | | | | | | | 6 | 1 | 12 | | | | 8 | | 3 | 1 | | | 13 | 9 | 11 | 5 | 7 | | 4 | | | | 2 |
| Novgorod Oblast | | | | | 3 | | | 1 | | | 2 | | | | 4 | | | | 5 | | | | | | | 6 | 7 | | |
| Tula Oblast | | | | | 2 | 4 | 1 | | 5 | | | 11 | | 3 | | 6 | | | 9 | | | | | | 7 | | 1 | 8 | |
| Altayskiy Kray | 1 | | | | 3 | 7 | | 4 | | | | 2 | | | | | 5 | | | | 6 | | | | | | | | |
| Oryol Oblast | 2 | | | | 3 | 1 | | | | | | | | | | | | | | 5 | | | 4 | | | | | | |
| Krasnoyarsk Kray | 8 | | | 4 | | | | 1 | | | | | 5 | | 2 | | | 3 | 7 | | | | | | | 6 | | | |
| Kurgan Oblast | 1 | | | | 1 | | | | | | | | | | 2 | | 3 | 7 | | 9 | | 8 | 5 | | | 4 | | 6 | |
| Kaliningrad Oblast | 11 | | | | 4 | 3 | | | | | | | | | 1 | 1 | 2 | 5 | | 7 | | | | | | | 6 | 8 | 9 |
| Moscow City | | | | | | | | | 2 | 3 | | | | | | | | | | | | | | | 4 | | | | 1 |
| Astrakhan Oblast | 7 | 2 | | | | 9 | 4 | | | | 1 | | 6 | | | | 3 | | | 5 | | 8 | | | | | 1 | | |
| Tomsk Oblast | | | | | 1 | | | | | | | | 2 | | | | | | | 3 | | | | | | | | 4 | |
| Republic of Adygeya | 1 | | | | 3 | | | 4 | 2 | | | | | | | | | | | 5 | | | | | | | | | |
| Sverdlovsk Oblast | | | | 1 | | | | | | | | | | | 2 | 3 | | | 6 | | | | | | | | 5 | 4 | |
| Kostroma Oblast | | | | | | | 3 | 1 | | | | | | | 4 | | 5 | 2 | | | | | 6 | | | | | | |
| Belgorod Oblast | 3 | | | 1 | 2 | | | | | | | | 5 | 4 | | | | 6 | | 7 | | | | | | | | | |
| Kemerovo Oblast | | | 2 | | | | | | | | | | 5 | 1 | | | | | | | | 3 | | | | 4 | | | |
| Primorskiy Kray | | | | | 3 | | | | | | | | | | | | | | 2 | | 4 | | | | | | | 1 | |
| Arkhangelsk Oblast | | 1 | 2 | | | | | | | | | | | | | | 4 | | | 3 | | | 6 | | | 5 | | | |
| Republic of Buryatiya | | | | | | | | | | | | | | | | | 1 | | | | | 3 | | | | 2 | | | |
| Pskov Oblast | | | | | | 3 | 2 | 1 | | | | | | | 5 | 1 | | 7 | 9 | | | 8 | | | | 4 | | 6 | |
| Tambov Oblast | 1 | | | | 4 | 3 | | | | | | | | | 6 | 2 | | | | 5 | 7 | | | | | | | | |
| Chelyabinsk Oblast | 5 | | | | | | | | | | | | 3 | 1 | 4 | | 6 | 2 | | | | | | | | | | | |
| Orenburg Oblast | 3 | | 1 | 2 | | | | | | | | | | 4 | | | | | 5 | | | | | | | | | | |
| Republic of Kareliya | | 1 | | | | | | | | | | | | | 4 | | | | | | | | | | | 2 | | 3 | |
| Republic of Khakasiya | 8 | | | 5 | | | | | | | | | | 2 | 7 | | | | 1 | | | | | | | 3 | 4 | 6 | |
| Republic of Northern Osetiya – Alaniya | 1 | | | | | | | | | | | | | 4 | | | | | 6 | 5 | | | | | | 2 | 3 | | |
| Stavropol Kray | 1 | | | | 3 | 7 | | 4 | | | | 2 | | | | | 5 | | | | 6 | | | | | | | | |
| Kabardino-Balkarskaya Republic | | | | | 3 | | | | | | | | | | | | | | | | | 4 | | | | 1 | | 2 | |
| Jewish Autonomous Oblast | | | | | | | | 5 | | | | | 3 | | | | | 8 | 7 | 6 | | | 1 | | | 2 | | 4 | |
| Zabaykalskiy Kray | | | | 5 | | | | | | | | | | | | | | | | 6 | | | 2 | | | 1 | 4 | 3 | |
| Republic of Altay | 1 | | | 7 | | | | | | | | | | | | | | | 4 | 3 | | 5 | | | | 2 | | | 6 |
| Republic of Dagestan | | | | | | | | | | | | | | | | | | | | 2 | 6 | 1 | | | | 4 | 3 | 5 | |
| Republic of | | | 1 | | | | | | | | | | | | | | | | 3 | | | | | | | 2 | | | |

| Region | A | B | CA | CB | DA | DB | DC | DD | DE | DF | DG | DH | DI | DJ | DK | DL | DM | DN | E | F | G | H | I | J | K | L | M | N | O |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Komi | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Karachaevo-Cerkesskaya Republic | 1 | | | 6 | | 7 | | | | | | 2 | | | | | | | 5 | | | | | | | | 4 | 3 | |
| Ivanovo Oblast | | | | | | 1 | | 2 | | | | | | | 3 | | | 8 | | | | 7 | | | | 4 | 5 | | 6 |
| Vologda Oblast | | 4 | | | | | | | | | 2 | | | 1 | 3 | | | | | | | | | | | | | | |
| Amur Oblast | | | | 1 | | | | | | | | | | | | | | | | 3 | | | | | | 2 | | | |
| Murmansk Oblast | | 1 | | 2 | | | | | | | | | | | | | | | | | | | | | | 3 | | 4 | |
| Lipetsk Oblast | 3 | | | | 2 | | | | | | | | 4 | 1 | | | | | | | | | | | | | | | |
| Kamchatskiy Kray | | 1 | | 3 | | | | | | | | | | | | | | | | | | | | | | 2 | | | |
| Chechenskaya Republic | 4 | | | | | | | | | | | | | | | | | | | 2 | | | | | | 1 | 3 | 5 | |
| Republic of Tyva | 3 | | | | | | | | | | | | | | | | | | | | | | | | | 2 | 1 | | |
| Sakhalin Oblast | | | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Republic of Sakha (Yakutiya) | | | | | | | | | | | | | | | | | | | | 1 | | | | | | 2 | | | |
| Republic of Kalmikiya | 1 | | | | | | | | | | | | | | | | | | | | | | | | | 2 | | | |
| Tumen Oblast | | | | | | | | | | | | | | | | | | | | 1 | | | | | | | | | |
| Republic of Ingushetiya | | | | | | | | | | | | | | | | | | | | 3 | | | 5 | | | 1 | 2 | 4 | |
| Magadan Oblast | | 1 | | | | | | | | | | | | | | | | | | | | | | | | 2 | | | |
| Chukotskiy Autonomous Okrug | | | | 1 | | | | | | | | | | | | | | | 2 | | | | | | | 3 | | | |
| Khanty-Mansiyskiy Autonomous Okrug - Yugra | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Yamalo-Neneckiy Autonomous Okrug | | | | | | | | | | | | | | | | | | | | 1 | | | | | | | | | |
| Neneckiy Autonomous Okrug | | 3 | 1 | | | | | | | | | | | | | | | | | 2 | | | | | | | | | |

**Note**. Figures denote ranks of an industry in a region's economic activity (only growing base industries have a rank different from zero).

OKVED codes are disclosed in Table B.4.

**Table B.3.** Cross-Specialization Matrix for Labor Productivity

| Region | A | B | CA | CB | DA | DB | DC | DD | DE | DF | DG | DH | DI | DJ | DK | DL | DM | DN | E | F | G | H | I | J | K | L | M | N | O |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Republic of Tatarstan | 5 | | 4 | | | | | | | | | 2 | | | | | 1 | | | 3 | | | | | | | | | |
| Saratov Oblast | 1 | | | | | | | | | | 2 | | | | | | | | 3 | | | | | | | | | | |
| Irkutsk Oblast | 6 | | | | | | | | | | | | | 3 | | 2 | 1 | 7 | | 4 | | | 5 | | | | | 8 | |
| Rostov Oblast | 2 | | | | 6 | 1 | 3 | | | | | | | | | | 4 | 7 | | 5 | | | | | | | | | |
| Republic of Bashkortostan | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Nizhniy Novgorod Oblast | | | | | | | | | | 1 | | | | | | | | | | | | | | | | | | | |
| Perm Kray | | | | | | | 1 | | | | | | | | | | 2 | | 3 | | | | 4 | | | | | | |
| Novosibirsk Oblast | | | 1 | | | | | | | | | 2 | | | | | | | | | | | | | | | 3 | | |
| Yaroslavl | | | | | 2 | | | | | | | | | | | 1 | | | 4 | | | | 3 | | | | | | |

| Region | A | B | CA | CB | DA | DB | DC | DD | DE | DF | DG | DH | DI | DJ | DK | DL | DM | DN | E | F | G | H | I | J | K | L | M | N | O |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Oblast | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Udmurtskaya Republic | | | 2 | | | | | | | | | | | | | | 1 | | | | | | | | | | | | |
| Samara Oblast | 3 | | | | | 4 | | | | | | | | | | 2 | | 1 | | | | | | | | | 5 | | |
| Omsk Oblast | | | | | 1 | | | | | | | 2 | | | | | | | | | | | | | | | | | |
| Chuvash Republic | | | | | | | | | | | | | | | | 1 | 2 | | | | | | | | | | | | |
| Moscow Oblast | | | | | 4 | | 8 | | 7 | 1 | | 5 | | | 3 | 12 | 15 | | 13 | 1 | 14 | 11 | | 2 | | | 9 | 6 | |
| Leningrad Oblast | | | | | | | | 6 | | | 4 | 2 | | | | | | | 3 | 1 | | | | | | | | 5 | |
| Republic of Mariy El | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Tver Oblast | | | | | | | 1 | | | | | | | | | | 2 | | 3 | | | | 4 | | | | | | |
| Smolensk Oblast | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Volgograd Oblast | 2 | | 1 | | | | | | | 3 | | | | | | | | | | | | | | | | | | | |
| Voronezh Oblast | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Ryazan Oblast | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Kirov Oblast | | 2 | | | | | 1 | | | | 3 | | | | | | | | | | | | | | | | | | |
| Bryansk Oblast | | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Penza Oblast | | | | | | | | | | 1 | | | | | | | | | | | | | | | | | | | |
| Kaluga Oblast | | | | | 1 | | | | | | | | | | | | 2 | | | | | | | | | | | | |
| Vladimir Oblast | | | | | | | | | | | | | | | | | | | | | | | | 1 | | | | | |
| Republic of Mordoviya | | | | | | | | | | | | | 1 | | | | | 2 | | | | | | | | | | | |
| Khabarovsk Kray | | | | | | | | | | | | | | | | | | | | | | | | | | | | 1 | |
| Krasnodarskiy Kray | 1 | | | | | | 4 | | | | | | | | | | | | | 5 | | 3 | 2 | | | | | | |
| Ulyanovsk Oblast | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Kursk Oblast | | | | 1 | | | | | 2 | | | | | | | | | | | | | | | | | | | | |
| Saint Petersburg City | | | | | | 7 | 8 | 2 | | | | 11 | 6 | | 5 | 4 | | 9 | 1 | | | | 1 | | | | 3 | | |
| Novgorod Oblast | | | | | 5 | | | 1 | 3 | | 2 | | | | 4 | | | | | | | | | | | | | | |
| Tula Oblast | | | | | 1 | | | | | | | | | | | | | | | | | | | | | | | | |
| Altayskiy Kray | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Oryol Oblast | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Krasnoyarsk Kray | 6 | | | | | | | | | | | | 3 | | 2 | | 5 | 1 | | 4 | | | | | | 7 | | | |
| Kurgan Oblast | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Kaliningrad Oblast | 3 | | 1 | | | 4 | | | | | | | | | 5 | | 2 | | | | | | | | | | 6 | 7 | |
| Moscow City | | 15 | | | | 9 | | 14 | 12 | 1 | 11 | | 3 | | 5 | | 16 | | 2 | | | | 6 | | 13 | 8 | 7 | 1 | 4 |
| Astrakhan Oblast | 3 | | | | | 2 | | | | | 1 | | 5 | | | | 4 | | | | | | | | | | | | |
| Tomsk Oblast | 2 | | | 1 | | | | | | | | | | | | | | | | | | | | | | | | 4 | 3 |
| Republic of Adygeya | 1 | | 2 | | | | | | | | | 3 | | | | | | | | | | | | | | | | | |
| Sverdlovsk Oblast | | | | | | | | | | | | | | 2 | | | | | | | | | | | | | | 1 | |
| Kostroma Oblast | | | | | | | 3 | 1 | | | | | | | 2 | | | 4 | | | | | | | | | | | |
| Belgorod Oblast | 4 | | | 2 | 3 | | | | | | | | | | | | | 1 | | | | | | | | | | | |
| Kemerovo Oblast | 2 | 1 | | | | | | | | | | | 4 | 3 | | | | | | | | | | | | | | | |
| Primorskiy Kray | | 1 | | | | | | | | | | | 2 | | | | | | | | | | | | | | | 3 | |
| Arkhangelsk Oblast | | | 3 | | | | | | 1 | | | | 4 | | 5 | | | 7 | | 2 | | | | | | 6 | | | |

| Region | A | B | CA | CB | DA | DB | DC | DD | DE | DF | DG | DH | DI | DJ | DK | DL | DM | DN | E | F | G | H | I | J | K | L | M | N | O |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Republic of Buryatiya | | | | | | | | | | | | | | | | | | | | | | 2 | 1 | | | | | | |
| Pskov Oblast | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Tambov Oblast | 3 | | | | | | | | | | | | | | | | | | | 1 | 2 | | | | | | | | |
| Chelyabinsk Oblast | 3 | | | | | | 4 | | | 1 | | | | | | | | 2 | | | | | | | | | | | |
| Orenburg Oblast | | | 1 | | | | | | | | 2 | | | | | | | | | | | | | | | | | | |
| Republic of Kareliya | | 4 | | 1 | | | | | | | | | | | 2 | | | | | | | | | | | 3 | | 5 | |
| Republic of Khakasiya | | | | | | | | | | | | | | | | | | | 1 | | | | | | | | | | |
| Republic of Northern Osetiya – Alaniya | 1 | | | | | | | | | | 2 | | | | | | | | | 4 | 3 | | | | | | | | |
| Stavropol Kray | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Kabardino-Balkarskaya Republic | | | | | | | | | | 2 | | | | | | 1 | | | | 4 | | 3 | | | | | | | |
| Jewish Autonomous Oblast | | | | | | | | 3 | | | 1 | | | | | | | | | | | | 2 | | | | | 4 | |
| Zabaykalskiy Kray | | | | | | | | | | | | | | | | 3 | | | | 2 | | 1 | | | | | | | |
| Republic of Altay | | | | | | | | | | | | | | | | | | | | 1 | | | | | | | | | |
| Republic of Dagestan | 4 | | | | | | | | | | | | | | | | | | | 2 | 3 | 1 | 5 | | | | | | |
| Republic of Komi | | | | | | 2 | 7 | | | 3 | | | | 1 | | | | 5 | 8 | 4 | | | | | | 6 | | | |
| Karachaevo-Cerkesskaya Republic | 2 | | | | | 3 | | | | | | | | | | | 1 | | | | | | | | | | | | |
| Ivanovo Oblast | | | | | | | | 1 | | | | | | | | | | | | | | | | | | | | | |
| Vologda Oblast | | 1 | | | | | | | | | 3 | | | 2 | | | | | | | | | | | | | | | |
| Amur Oblast | | | | | | | | | | | | | | | | | | | | | | | 1 | | | | | | |
| Murmansk Oblast | | 2 | | 8 | | 3 | | | | | | | 7 | | 5 | | | 1 | | | | | | | | 9 | 6 | 4 | |
| Lipetsk Oblast | | | | | 3 | | | | | | 5 | 4 | | 2 | | | | 1 | | | | | | | | | | | |
| Kamchatskiy Kray | | 4 | | | 7 | 6 | | | | | | | 5 | | 3 | 1 | | | | | | | | | | 2 | | | |
| Chechenskaya Republic | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Republic of Tyva | 3 | | | | | | | 4 | | | | | 1 | | | 2 | | | | | | | | | | | 5 | | |
| Sakhalin Oblast | | 14 | 5 | 4 | 15 | 17 | | 2 | | 1 | | 8 | 1 | | 3 | 6 | | | | | 13 | 11 | | | 16 | 12 | 9 | 7 | |
| Republic of Sakha (Yakutiya) | | | | 2 | | | | 7 | | 1 | | 6 | | | | | | 3 | | 4 | | | | | | | 5 | | |
| Republic of Kalmikiya | | | | | | | | | | | | | | | | | | | | 1 | | | | | | | | | |
| Tumen Oblast | 5 | | | | | | | | | | | | | | | | | | 6 | 4 | | 3 | | 1 | 2 | | | | |
| Republic of Ingushetiya | | | | | | | | | | | | | | | | | | 4 | | 1 | 3 | 5 | | | | 2 | | | |
| Magadan Oblast | 4 | 1 | | | | 7 | | 6 | | | | | | | | 5 | | 2 | | | | | | | | 3 | | | |
| Chukotskiy Autonomous Okrug | | 3 | | 4 | | | | | | | | | | | | | | | 2 | | | | | | | 1 | | | |
| Khanty-Mansiyskiy Autonomous Okrug - Yugra | 5 | | | | | | | | | | | | | | | | 2 | 6 | | | 4 | 7 | | 1 | 3 | | | | |
| Yamalo-Neneckiy Autonomous | | | 5 | 6 | | | | | | | | | | | | | | | 4 | 1 | | 2 | | | | | 3 | | |

| Region | A | B | CA | CB | DA | DB | DC | DD | DE | DF | DG | DH | DI | DJ | DK | DL | DM | DN | E | F | G | H | I | J | K | L | M | N | O |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Okrug | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Neneckiy Autonomous Okrug | | | 5 | | | | 9 | | | | | | | | 4 | | | | 8 | 1 | | 6 | 7 | | | | 3 | 2 | |

**Note**. Figures denote ranks of an industry in a region's economic activity (only growing base industries have a rank different from zero).

OKVED codes are disclosed in Table B.4.

**Table B.4.** OKVED (Two-Letter Level of Aggregation)

| | |
|---|---|
| A | AGRICULTURE, HUNTING AND FORESTRY |
| B | FISHING; FISH HATCHERIES; FISH FARMS AND RELATED SERVICES |
| CA | MINING AND QUARRYING OF ENERGY PRODUCING MATERIALS |
| CB | MINING AND QUARRYING EXCEPT ENERGY PRODUCING MATERIALS |
| DA | FOOD PRODUCTS, BEVERAGES AND TOBACCO |
| DB | TEXTILES AND TEXTILE PRODUCTS |
| DC | LEATHER, LEATHER PRODUCTS AND FOOTWEAR |
| DD | WOOD AND PRODUCTS OF WOOD AND CORK |
| DE | PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING |
| DF | COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL |
| DG | CHEMICALS AND CHEMICAL PRODUCTS |
| DH | RUBBER AND PLASTICS PRODUCTS |
| DI | OTHER NON-METALLIC MINERAL PRODUCTS |
| DJ | BASIC METALS AND FABRICATED METAL PRODUCTS |
| DK | MACHINERY AND EQUIPMENT, N.E.C. |
| DL | ELECTRICAL AND OPTICAL EQUIPMENT |
| DM | TRANSPORT EQUIPMENT |
| DN | MANUFACTURING NEC; RECYCLING |
| E | ELECTRICITY GAS AND WATER SUPPLY |
| F | CONSTRUCTION |
| G | WHOLESALE AND RETAIL TRADE; RESTAURANTS AND HOTELS |
| H | HOTELS AND RESTAURANTS |
| I | TRANSPORT STORAGE AND COMMUNICATIONS |
| J | FINANCIAL INTERMEDIATION |
| K | REAL ESTATE, RENTING AND BUSINESS ACTIVITIES |
| L | PUBLIC ADMINISTRATION AND DEFENCE COMPULSORY SOCIAL SECURITY |
| M | EDUCATION |
| N | HEALTH AND SOCIAL WORK |
| O | OTHER COMMUNITY SOCIAL AND PERSONAL SERVICE ACTIVITIES |

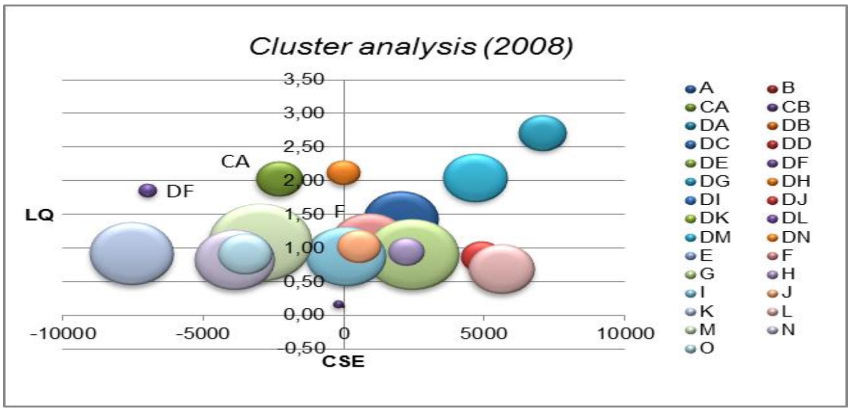


**Figure B.1.** Republic of Tatarstan (Employment)

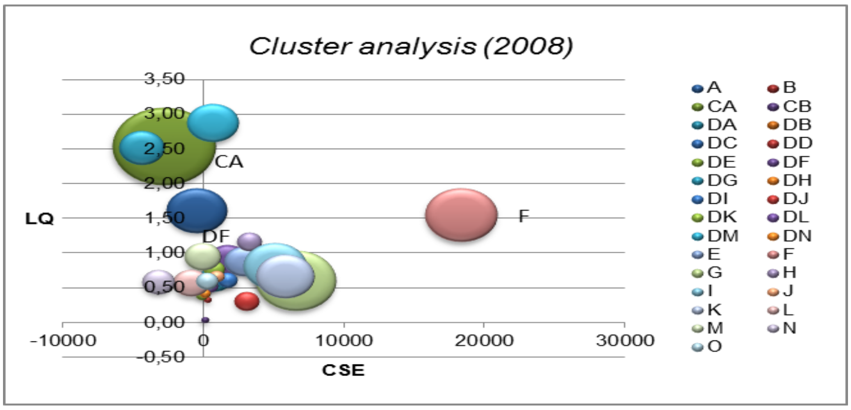


**Figure B.2.** Republic of Tatarstan (Shipment)

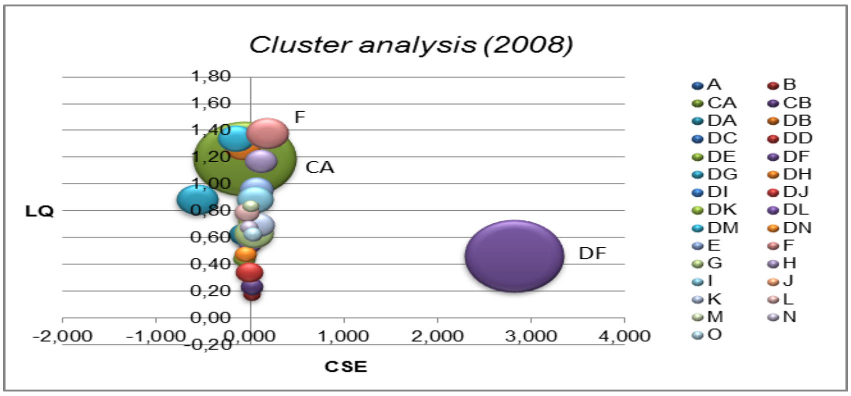


**Figure B.3.** Republic of Tatarstan (Labor Productivity)

# THE EVOLUTION OF FIRM SIZE DISTRIBUTION

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**Abstract**

*Significant differences in the evolution of firm size distribution for various industries in the United States have been revealed and documented. For theoretical considerations, this finding puts major constraints on the modelling of firm growth. For practical purposes, the observed differences create a solid basis for selective investment strategies.*

**Keywords**: firm size distribution, Pareto distribution, the USA, evolution, investment

**JEL Classification**: L11, L17, G1

## 1. Introduction

This is a common observation that firms have different sizes. It would not go far beyond this fact if the distribution of sizes was not characterized by a simple functional form, e.g. the power law, which is common for numerous objects in physics and other natural sciences. The similarity of frequency distribution of sizes can be considered as strong evidence in favor of the existence of universal intrinsic mechanisms giving birth to the distributions. Therefore, investigation into the processes behind the observed distribution of firm sizes is supported by the whole strength of the natural sciences. On the other hand, some specific features of various size distributions in economics and their evolution over time are likely to be helpful for the hard sciences.

There are two principal topics related to the study of firm sizes. Many researches are focused on the properties of the Pareto distribution of firm sizes (Axtell 2001, Coad 2009, de Wit 2005). Modeling the processes of firm growth matching the Pareto law is another area of active research (Kaizoji, Iyetomi, Ikeda 2006, Kitov, I. 2009, Kitov, I., Kitov, O. 2008). In general, results of the previous studies allow building a promising new branch of economics with very tight links to the hard sciences.

The purpose of our study is potentially related to both principal topics. We are trying to describe the dynamics of firm sizes distribution (FSD) in the United States, both aggregate and at industry level. Results of economic censuses in the United States are available in electronic format since 1992. Therefore, one might expect to document some measurable changes. The FSD dynamics could be helpful for the modeling of firm growth, especially if the industries under study evolve in different directions. This could evidence the existence of inhomogeneous internal structure, which could potentially be explained by some firm growth model.

For the effectiveness of financial markets, it would be useful to evaluate the dependence of labor unit productivity or labor unit efficiency on firm size. Moreover, if these characteristics evolve with time, one might be able to design a sound investment strategy.

A recognized source of certified and detailed information on a variety of firms' characteristics in the United States is the Bureau of the Census, which conducts Economic Census every five years (http://www.census.gov/econ/census02/). (According to the Census Bureau documentation, there is an important difference between terms "firm" and "establishment": Establishment - A single physical location where business is conducted, or where services are performed; Firm - A business organization or entity consisting of one or more domestic establishment locations under common ownership or control. In this paper, we examine the size distribution of firms or business organizations). Because the results of the 2007 census will be published during 2009 and 2010 we are restricted to three censuses between 1992 and 2002, as the only available in electronic format from the CB web-site. Comprehensive reports for the censuses conducted before 1992 are available in printed form.

There is a problem with compatibility of data before and after 1997 because of the change from Standard Industrial Classification (SIC) to North American Industry Classification System (NAICS). According to the Census Bureau "Changes between 1997 and 2002 affect only the construction, wholesale trade, retail trade and information sectors." Hence, it is likely that these industries demonstrate some artificial changes in relevant FSDs.

The methodology of economic censuses states that those large and medium-size firms and those firms, which operate more than one establishment, have to fill a questionnaire. The size of the smallest firms is taken from relevant administrative records. Therefore, the data on the frequency distribution of firm sizes are prone to many sampling and non-sampling errors, which might result in somewhat biased conclusion.

## 2. Firm Size Distribution

First, one has to define the frequency distribution of firm size, i.e. define the measure of firm size and corresponding intervals for bin counting. There are two general approaches to the size definition – total sales of a firm as expressed in monetary units and the number of employees. Due to the inhomogeneous character of the economic censuses, one or both measures may not be available for selected industries. This also reduces the comprehensiveness of our analysis.

Figure 1 compares overall FSDs for 2002, as obtained using total sales and the number of employees. Essentially, these are the same firms counted in different bins, but their total number under investigation might not be the same for the two definitions. We exclude from calculations all firms without employees, which may have some sales, however. Also excluded are all firms in the open-end bin "… more than …". The boundary of 10,000 employees may not coincide with the $250,000,000 threshold. In our study, a firm included in the original statistics may be counted either under both definitions (most common case) or less than one of two definitions or is excluded.

Both FSDs in Figure 1 are normalized to the total number of firms (4,927,805 for the number of employees and 5,696,868 for total sales) and the widths of corresponding measurement bins. As a result, both curves in the Figure represent density with the following units of measurements: the portion of the total number of firms per 1K$, and the portion of the total number of firms per 1 employee, respectively. The former definition provides a better resolution for small firms and one can clearly observe the transition from quasi-exponential to power law distribution. The number of employees provides a slightly better coverage at larger sizes. For the purposes of illustration and regression analysis, the estimates of density are assigned to average firm size, which is also reported for all predefined bins. An alternative is to associate the readings with centers of the bins or with the theoretical points, to which the density would belong according to the Pareto law. In all cases, a slight bias in the OLS regression would be observed and for the sake of the balance between simplicity and accuracy we have chosen the average firm size.

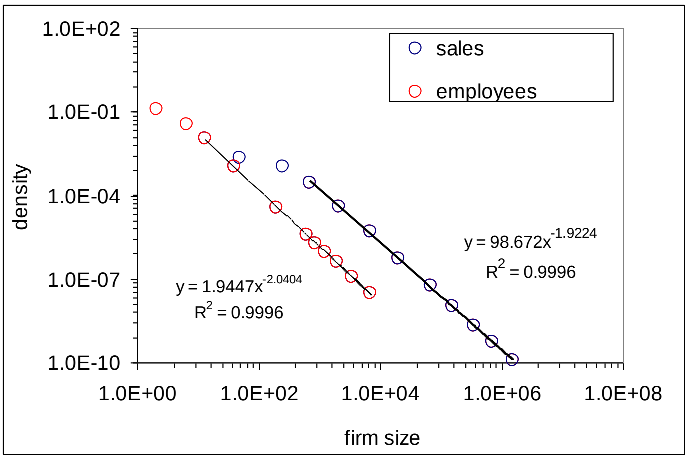


**Figure 1.** Two definitions of firm size: total sales in K$ and the number of employees.

Both frequency distributions of size for 2002 are normalized to the total number of firms and to the widths of corresponding measurement bins. The former definition provides better resolution for small firms. Both distributions give exponents close to -2.0 consistent with k=1 in the Pareto law.

For larger sizes, both distributions in Figure 1 are characterized by the Pareto law (power law). Theoretically, the cumulative distribution function (CDF) of a Pareto distribution is defined by the following relationship:

$$CDF(x) = 1 - (x_m / x)^k$$

for all $x > x_m$, where k is the Pareto index. Then, probability density function (pdf) is defined as

$$pdf(x) = kx_m^k/x^{k+1} \quad (1)$$

We have estimated the Pareto index k for FSDs in Figure 1. Both exponents (slopes in the log-log coordinates) are close to 2.0, and according to (1), k=1. These estimates do not differ from those obtained in other studies. Now we are ready to examine the dynamics of firm size distribution in the US between 1992 and 2002.

### 3. The dynamics of FSD

For 1992, the overall FSD is available only for the number of employees, and for 1997 – only for total sales. In 2002, both representations are available and supported by a smaller economic survey conducted in 2004. For better resolution of the underlying dynamics, we have naturally chosen the longest possible period between 1992 and 2002. Figure 2 displays corresponding density curves illustrating the evolution of the FSD in the US. Density for small firms practically did not change. However, there are visible discrepancies between 1992 and 2002 for mid- (the 1992 curve is above that for 2002) and large-size (the 2002 curve is above that for 1992) firms.

One has to bear in mind the transition from SIC to NAICS, however. It might add some definitional bias in the FSDs: our statistics include ~4,200,000 firms in 1992 and ~4,920,000 in 2002, for firms with more than 0 employees and less than 10,000 employees. Some firms have no employees and the bin above 10,000 employees is an open-ended one. In both cases, density cannot be estimated. It is worth noting that the statistics of firm sizes measured in K$ is richer: ~4,600,000 firms in 1997 and ~5,700,000 in 2002. All in all, there were some changes in the FSD between 1992 and 2002, which might manifest themselves in different measurable features.

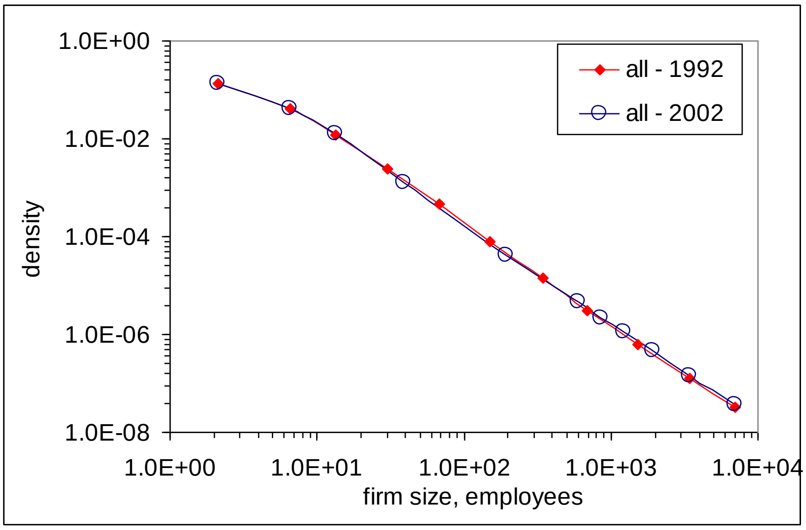


**Figure 2.** Dynamics of the overall FSD - between 1992 and 2002

There might be just slightest discrepancies between 1992 and 2002 for mid- and large-size firms. Otherwise, the curves are indistinguishable

An important characteristic of a firm is the efficiency of labor, as expressed by the ratio of total sales over payroll (a proxy to labor expenditures) with a direct relationship between the ratio and efficiency. In Figure 3 (left panel) we show the dependence of the efficiency on firm size. Surprisingly, the curves for 1992 and 2002 both have minimum at some size near 10 employees. Small and large firms demonstrate an elevated efficiency. The curve for 2002 is below that for 1992 for sizes smaller than 2500 employees. At larger sizes, the 2002 curve is above its counterpart. The right panel in Figure 3 illustrates the evolution of labor efficiency as a function of size – it decreases for small- and middle-size firms and increases for the largest firms. The increase reached 20% during the decade between 1992 and 2002. It would be instructive to compare the change observed in the end of the 20th century to that occurred in the first decade of the 21st century. If the trend is retained in both rate and direction, one might use it as a basis for long-term investments.

There is one obvious consequence of the labor efficiency increasing with firm size that is also observed in Figure 2. During the last decade on the 20th century, a wiser investment was in bigger firms with higher efficiency. As a result, one observed firm size redistribution and an increase in relative number of firms larger than 1000 employees. Not surprisingly, the red line (1992) in Figure 2 lies below the blue line (2002) for sizes above 1000 employees. Slightly lower labor efficiency for the firms between 10 and 1000 employees is expressed in a density decrease between 1992 and 2002. Hence, there is no contradiction between the processes in Figures 2 and 3.

This result is obtained for the overall FSD, however, and it would be of theoretical and practical importance to learn the behavior of smaller parts of the economy. Of special interest are the cases of different or even opposite dependence of labor efficiency on firm size. In other words, is the US economy homogenous or inhomogeneous in terms of the evolution of frequency distribution of firm sizes? Inhomogeneous structure of an economy would need more elaborated models of firm growth.

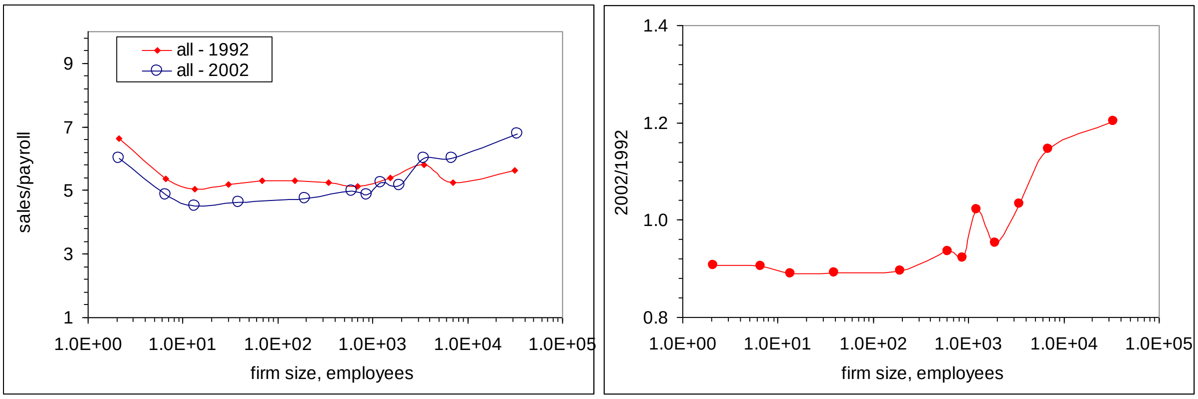


**Figure 3**. Labor unit efficiency (sales / payroll) as a function of firm size (left panel) and its evolution over time (right panel). Overall, larger firms provide an elevated rate of productivity growth

Similar FSD pattern to the one discussed above is observed in the evolution of total sales per employee as a function of firm size. Figure 4 presents the dependence for both economic censuses and the 2002/1992 ratio. Larger firms demonstrate higher output per employee, which also increases with time. Some fluctuations near the size of 1000 employees are likely related to measurement errors.

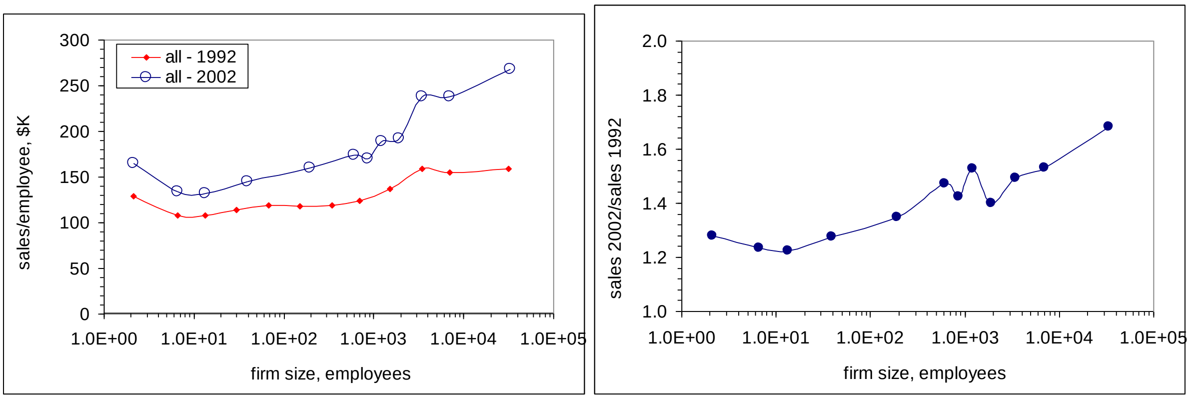


**Figure 4.** The evolution of total sales per employee as a function of firm size

Having studied the overall FSD and its evolution, we now present similar curves for several selected industries. Retail industry is being analyzed first. The number of firms, as defined by employees, in retail decreased from ~887,000 in 1992 to 615,000 in 1997, and finally to 599,000 in 2002. Same figures are observed for the number of firms with size expressed in K$. Figure 5 displays density curves for both definitions. The left panel reveals some problems with the enumeration of firms in K$. The curve for 2002 is far above those for 1992 and 1997. The distributions over the number of employees do not show such dramatic changes. Thus, one could suggest that the discrepancy in the left panel if of artificial character.

In the right panel of Figure 5, the curves for 1992 and 2002 diverge over the whole length of the distribution. At lower sizes, the 1992 curve is below that for 2002, and in the mid-section and at the largest sizes the former curve is above the latter one. This observation is different from the pattern in Figure 2. However, the explanation of the behavior of the retail curves is similar to that given for the overall FSD – the dependence of labor efficiency on firm size and time. Figure 6 evidences that larger distances between the 1992 and 2002 curves correspond to relative decrease in density in Figure 5, and vice versa. For example, the largest distance is observed for sizes between 100 and 1000 employees, where the 1992 FSD curve is clearly above the 2002 curve. This observation supports the mechanism of predominant investment in more labor effective firms, as discussed above.

There is a very specific feature characterizing the curves in Figure 6 – a deep trough for sizes between 100 and 250 employees. The trough also slightly deepened relative to the peak, which jumped from the smallest firms in 1992 to those near 1000 employees in 2002. This implies a relative decrease in labor efficiency for the mid-size firms. To develop a reliable investment strategy in the segment of retail, one should keep track of the observed tendency. In any case, the decreasing density of the retail FSD in the range between 50 and 250 employees implies a higher probability of such firms to shrink or even to fail.

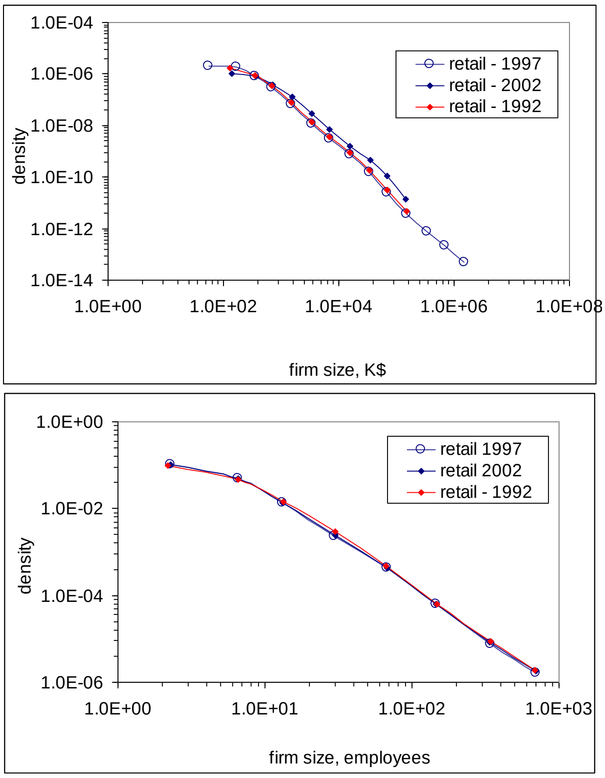


**Figure 5.** Dynamics of the FSD for retail between 1992 and 2002

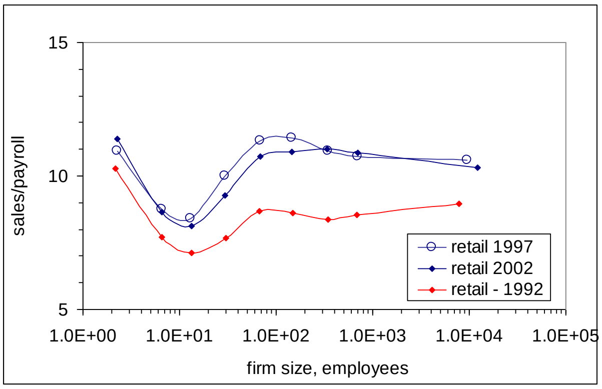


**Figure 6.** Retail: labor efficiency (sales/payroll) as a function of firm size and its evolution over time

Wholesale is the next industry to present. Corresponding FSDs are depicted in Figure 7. In general, the evolution is similar to that observed for the overall FSD – relative decrease in density for mid-size firms and increase – for the larger firms. The effect is more prominent for the wholesale firms. However, with ~436,000 firms in 1992 and ~377,000 in 2002, the influence of the wholesale on the overall FSD is weak.

The sales/payroll ratio in Figure 8 demonstrates a robust increase with firm size. Hence, bigger firms are characterized by higher labor efficiency, which induces the observed increase in density of the FSD at sizes above $2,500,000 (left panel). It is important that the level of the ratio uniformly decreases with time for all sizes. Unfortunately, there are no data on total sales and payroll for 1992, which could help to better resolve the evolution.

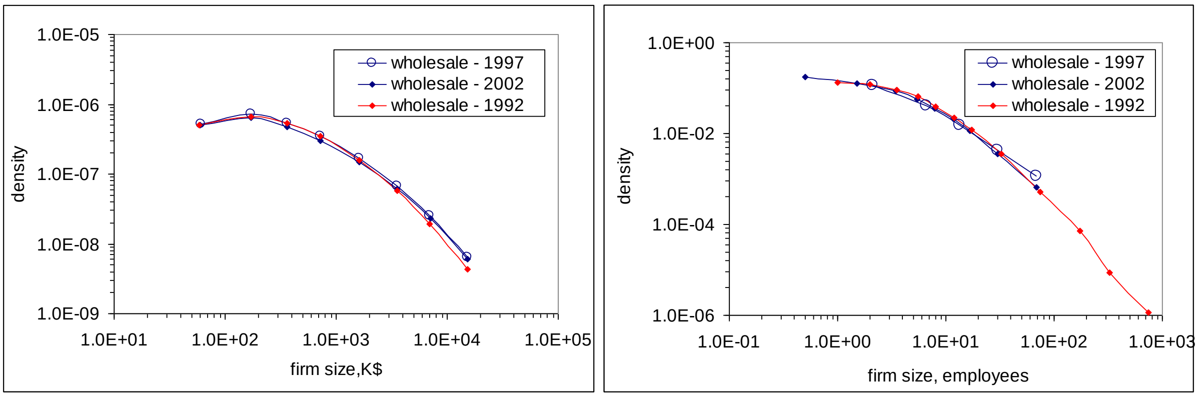


**Figure 7.** Dynamics of the FSD for wholesale between 1992 and 2002

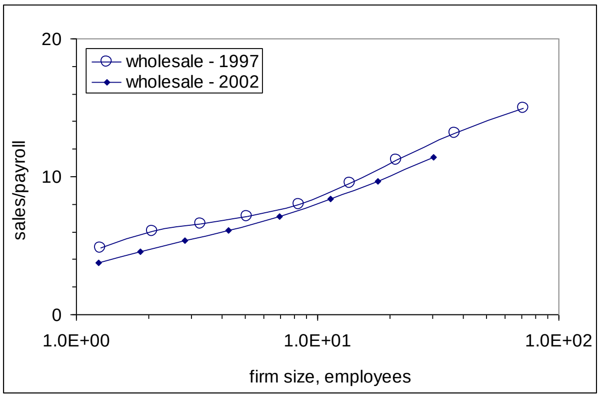


**Figure 8.** Labor unit efficiency (sales/payroll) as a function of firm size and its evolution over time

Manufacturing reveals a more stable FSD – the curves for 1992, 1997, and 2002 in Figure 9 almost coincide over the entire range with only slight deviations for very small and the largest sizes. Labor efficiency demonstrates a plateau at small sizes and then increases with size. The ratio also shows a weak tendency to decrease over time. For very large sizes, statistics is not reliable with only ~1000 firms having from 1,000 to 2,500 employees.

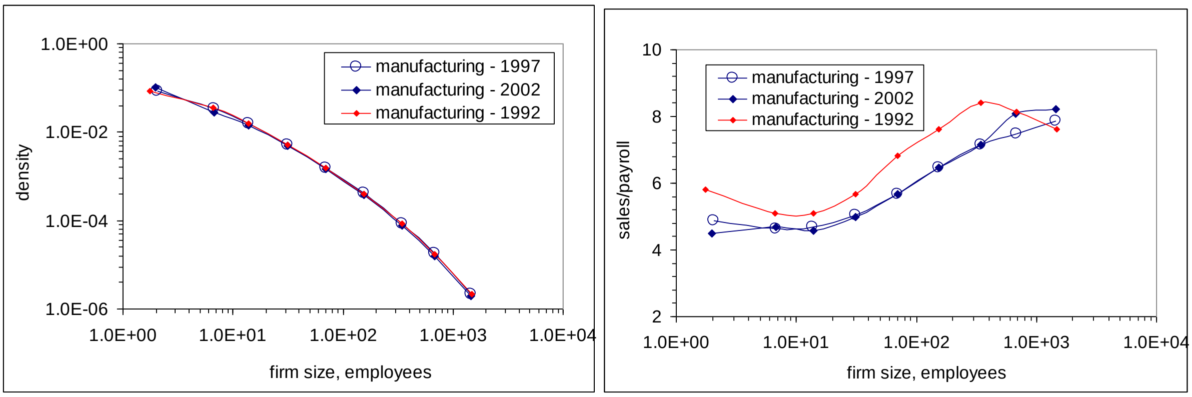


**Figure 9.** Left panel: dynamics of the FSD for manufacturing between 1992 and 2002.
Right panel: labor unit productivity (sales/payroll) as a function of firm size and its evolution over time

The FSDs for construction displayed in Figure 10 reveal a very clear evolutionary picture – density at larger sizes monotonically increases over time. This observation is partly supported by the increase in labor efficiency at larger sizes, as depicted in Figure 11. Another remarkable feature demonstrated by the sales/payroll ratio for the construction industry is the efficiency diminishing with firm size.

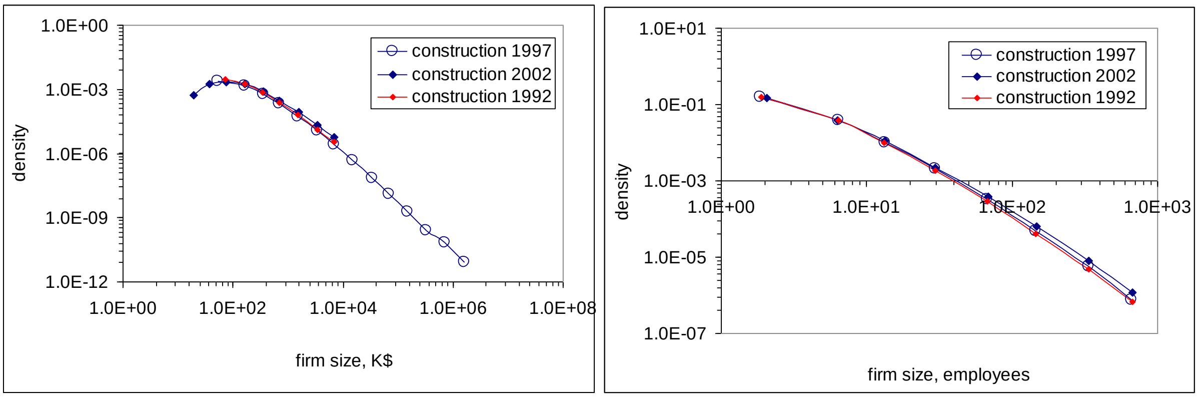


**Figure 10**. Dynamics of the FSD for construction between 1992 and 2002

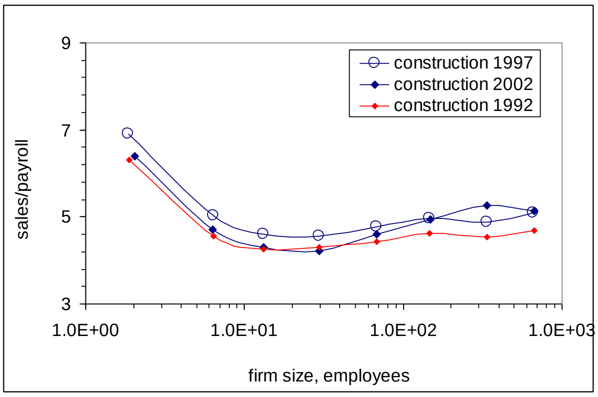


**Figure 11.** Labor unit efficiency (sales/payroll) as a function of firm size and its evolution over time

Mining industry presented in Figure 12 is characterized by a very stable FSD. The sales/payroll ratio is similar to that of the manufacturing – a general decrease in labor efficiency with size, but the effect is less prominent. In this regard, construction and mining are unique cases among all industries considered in the study. There are some other industries to examine, however.

Any tactics or strategy for those who want to invest in mining should take into account the discouraging behavior of labor efficiency. The returns from biggest companies will likely not be growing with time. On the contrary, in relative terms, the biggest companies suffer a higher failure rate than the smallest ones.

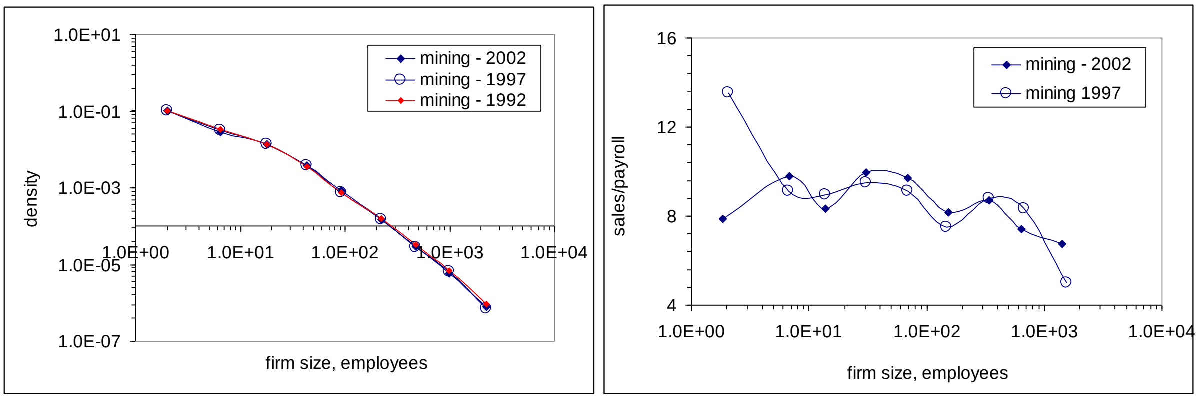


**Figure 12.** Left panel: dynamics of the FSD for mining between 1992 and 2002. Right panel: labor unit efficiency (sales/payroll) as a function of firm size and its evolution over time

## 4. Conclusion

We have documented different types of the evolution of FSD as demonstrated by various industries in the United States. For theoretical purposes, one has to bear in mind the entire diversity when building a model of firm growth. There should be one or several factors which define the observed behavior. The sales/payroll ratio is a good candidate for the explanation of the overall pattern, but its own behavior has to be modelled as well. In any case, the US economy is inhomogeneous in terms of firm size distribution, when it is decomposed in several sectors according to industrial classification system. Despite the flavor of artificiality of such decomposition for theoretical consideration, the inhomogeneous structure waits for a quantitative description.

For practical purposes, these inhomogenieties and the dependence of labor efficiency on firm size provide a reliable and fruitful basis for the development of long-term investment strategies. Those robust trends, which were observed in the size dependencies of the sales/payroll and sales/employee ratios for the studied industries between 1992 and 2002, when and if confirmed by the 2007 census, would be the first profitable candidates. One can easily choose appropriate industry and optimal firm size to the best long-term investment. The strategy might be enhanced by a sound choice of an industry, which provides the highest rate of price growth relative to other sectors (Stanley, Buldyrev, Havlin, *et al.* 1995, Sutton 1997).

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# THE NEXUS BETWEEN PUBLIC EXPENDITURE AND INFLATION IN THE MEDITERRANEAN COUNTRIES

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**Abstract**:

*The aim of this article is to assess the empirical evidence of the nexus between public expenditure and inflation for the Mediterranean countries during the period 1970-2009, using a time-series approach. After a brief introduction, a concise survey of the economic literature on this issue is shown, before discussing the data and introducing some econometric techniques. Stationarity tests reveal, generally, that public expenditure/GDP ratio is a I(1) process, while prices index is a I(2) process. Moreover, a long-run relationship between the share of public expenditure and inflation is found for Cyprus, France, Greece and Portugal. Furthermore, Granger causality tests results show a short-run evidence of a directional flow from inflation to expenditure for Cyprus, France and Spain; and of a bidirectional flow for Italy, Malta and Portugal. Some notes on the policy implications of our empirical results conclude the paper.*

**Keywords:** public expenditure; inflation; Mediterranean countries; time series; unit root; cointegration; causality.

**JEL Classification**: C32; E31; E62; H50.

## 1. Introduction

The optimal size of public sector is one of the most appealing topics in fiscal policy and public finance studies. Several theories have been advanced to explain this problem in different countries. Among them, one can find Wagner's Law (1912) of increasing state activities, Peacock, and Wiseman hypothesis (1961), critical-limit hypothesis (Clark 1945, 1964), Leviathan hypothesis (Brennan, and Buchanan 1980), differential productivity hypothesis (Baumol 1967), and the relative price hypothesis (Balassa 1964, Samuelson 1964). So, economic literature identified several determinants of public expenditure growth: inflation (Clark 1933, 1937, 1945, 1964), total revenue (De Viti De Marco 1893, 1898, 1934, Dalena, and Magazzino 2010), debt service or burden ratio (Ricardo 1817, Barro 1974, 1989, Reinhart, and Rogoff 2010), GDP growth rate (Barro 1989, 1990, Scully 1994, Armey 1995, Forte, and Magazzino 2010, Magazzino 2008, 2009b, 2009c, 2010a, 2010b), strategic transfers from federal government to the state governments, population growth, urbanization effect (Wagner 1912), and taxation. Over the past three decades, some studies – using the concepts of cointegration and Granger causality – focused on several countries and time periods. Yet, empirical findings are mixed and, for some countries, controversial. The results differ even on the direction of causality and the short-term versus long-term effects on economic policies. Depending upon what kind of causal relationship exists, its policy implications may be significant.

The aim of our study is to analyze the nexus between public expenditure and prices for the Mediterranean countries in the period 1970-2009. The data used in this work were taken from the IMF *Government Finance Statistics* database. In addition, Italy has a high public debt to GDP ratio and a high share of public expenditure; so, the reduction in public expenditure could represent a valid way for the consolidation of public finances. However, reducing the size of public sector should focus on the expenditure items that have less impact on GDP.

The outline of this paper is as follows. Section 2 provides a survey of economic literature on this issue. Section 3 provides an overview of the applied empirical methodology and a brief discussion of the data used. Section 4 discusses the empirical results. Finally, Section 5 presents our concluding remarks and some policy implications.

## 2. Literature Survey

Already Ricardo (1824) stressed the importance of separation of the central bank from political institutions, and the prohibition of monetary financing by the excesses of government spending (deficit monetization), only to clearly enunciate the key principles of the theory of today' central bank independence from political power.

Clark (1933, 1937, 1945, 1964) warned the most economically advanced countries of the danger of letting go beyond the relationship between public expenditure and national income as the threshold value of 25%. Clark lies down that when government tax proceeds reach this critical ratio, a progressive tax system generates increasing proportions of additional income from taxpayers, whose productivity falls. In fact, high levels of taxation would have reduced incentives to work and saving. Moreover, people become less resistant to the inflationary methods of government financing. According to the analysis of Clark, the higher taxes would have decreased the profits of enterprises, which, passing it on to prices, have increased the prices of final goods. So, the overall effect is a fall of the aggregate supply (due to the falls of private incentives) and an expansion of the aggregate demand (due to the inflationary financing techniques) and, hence, inflation results.

Through an analysis of time series on prices, taxes and public spending of a large group of countries for the inter-War period, Clark establish precisely the threshold of 25% as a ratio of public expenditure on national income. If it is true that inflation is a “social evil”, it is true that inflation reduces the costs of the public sector, since certain groups in society cannot defend. Moreover, the fiscal drag – the crop that inflation gives policy-makers in countries with progressive tax systems of type – is disappearing in many states, since the awareness of citizens in this respect has increased in recent years. Yet, recent decades have, however, proved that many countries have crossed the 25% limit without much inflationary tendencies (Jain 1989).

Already Bernstein (1936) had investigated the possibility of using the public know which specific anti-recession tool, highlighting the effects of inflation. According to the scholar, in the first three decades of the twentieth century great attention was given to possible use of public expenditure in order to minimize cyclical fluctuations on employment and production (emphasized by the report of the “Royal Commission on the Poor Laws”), while others economists – as Keynes, Martin, Foster, Catchings, and Pigou – had suggested the use of public spending as an instrument of economic policy, whereas periods of depression as a stage characterized by a low cost. Bernstein came to the conclusion that if these conditions were not favorable, considerable increases in public spending during periods of economic depression would lead to increased prices and production.

Bullock (1934), about the crisis of the thirties, put it on the rise-to the effects of economic policy choices of the Administration status, stressing the inadequacy and the lateness of the spending policies enacted in the years 1933-1934, also in view of the level of prices and sharp decline in tax revenue. Basically, if the start conditions of monetary stability are preserved, then the government will have ample room in the policies of deficit.

Pechman, and Mayer (1952) discussed the limits to the inflation taxation outlined by Clark, concluding that in the period between the two world wars, the empirical evidence supports the thesis Clark in only two cases (Britain, and Norway). Similarly, the price indices calculated for the period 1945-1948 grew annually in 53 of the 71 countries considered here: Clark arguments do not prove that prices grew faster where the tax burden exceeded the limit set by him.

Eltis (1983), analyzing the causes of the difficulties of the British economy in the seventies, found a double bond between inflation and public spending on the one hand, inflation was seen as the effect of deficit policies, useful - through increases supply of money - to finance the excess expenditure. Secondly it was originated by the wage increases put forward by workers to protect their purchasing power. Furthermore, Eltis found a strong empirical evidence to support the view that robust budget deficits create inflationary pressures.

Tanzi, Blejer, and Teijeiro (1987) moved from the consideration that the different parts of the public budget respond differently to inflationary pressures. However, scientists spotted in public debt service a strong link between public spending and the price trend.

Buiter (1987) studied the consequences for inflation of public expenditure cuts, emphasizing the important distinction between cuts in public consumption expenditure (which will tend to reduce the deficit) and cuts in public sector capital formation (which may have the perverse effect of increasing the deficit). This will happen if the expenditure effect is swamped by the direct and indirect effects of a reduced public sector capital stock on government revenues. If the public sector deficit increased, the cuts in public sector capital formation will raise the demand for seigniorage revenue.

Özatay (1997) studying the Turkish experience in the period 1997-1995 emphasizes the importance of coordination of fiscal and monetary policies in achieving price stability. Results indicate that, despite the rapidly changing financial environment, there are stationary long-run money-income relationships. Moreover, the growth rates of various monetary aggregates have predictive power for future movements in the Consumer Price Index. However, as the Turkish case clarifies, in an economy with persistent budget deficits these properties are not sufficient to conduct successful monetary policies. By a credible policy, it is possible to substantially reduce the inflation rate from 85% to 10% in a 4-year period. Yet, this necessitates that the Public Sector Borrowing Requirement should not exceed 1.5% of GNP.

Ruge-Murcia (1999) developed a dynamic, rational expectations model of inflation where the money supply is endogenously determined by the government’s use of newly created money to finance its current spending and by the effect of past rates of inflation on the real value of taxes. In an empirical application to Brazil

(1980-1989, monthly data), estimates indicate that there are steady-state inflation and money growth rates associated with each of the two possible government spending regimes. The low regime would be characterized in equilibrium by rates of inflation and money growth of 8.22% and 7.29% per month, respectively, and a share of GDP devoted to government outlays of 22.73%. The high spending regime would be associated with an expenditure level amounting to 33.43% of GDP, a monthly rate of inflation of 19.12%, and a monthly money growth rate of 19.25%.

Aizenman, and Hausmann (2000) investigated budgetary rules for an economy characterized by inflation and volatile relative prices. In the absence of shocks, the design of the budget is that the Treasury allocates funds once in every budgetary cycle. In the presence of volatile shocks, one would observe occasional budgetary revisions, the outcome of which is that the actual expenditure differs from the projected one. They use a panel data for Argentina, Brazil, Chile, Columbia, Costa Rica, Caribbean, Salvador, Guatemala, Honduras, Mexico, Peru, and Venezuela, for 1970-1994. The correlation between the budget error and the inflation variable turned out to be high, and highly significant. Similar results are found for the case where inflation is decomposed into the expected and the unexpected components, confirming that both the expected and the unexpected inflation increase the budget error.

Alavirad (2003) studied the effect of inflation on government revenue and expenditure for Islamic Republic of Iran. His major finding is that the government budget deficit increases in the inflationary condition. In addition, the deficit increases money supply, and this tends to increase inflation in Iran.

Ezirim, and Muoghalu (2006), starting from Clark's hypothesis, found that when the size of the public sector (measured by the share of expenditure on GDP) exceed a certain threshold, incentives to produce are discouraged (because of high tax burden). The reduction in aggregate supply, in addition, is even more pronounced in the case of budget balance (viewed as a fiscal constraint). The net result of such a bad adjustment between demand and supply is an inflationary spiral.

Kia (2006), studying Iranian economy for the period 1970-2002, focused on internal and external factors, which influence the inflation rate in developing countries. According to the estimation results, over the long run, a higher exchange rate leads to a higher price in Iran. So, a policy regime that leads to a stronger currency can help to lower inflation. However, a higher money supply when it is anticipated does not lead to a higher price level, but an unanticipated shock in the money supply results in a permanent rise in the price level. So, an unanticipated reduction in the money supply should be a powerful tool to reduce inflation in Iran. It is also found that the fiscal policy is very effective in Iran to fight inflation as the increase in the real government expenditures as well as deficits cause inflation, but if the changes are unanticipated they cause the opposite effect. While a high debt per GDP is deflationary.

Ezirim, Muoghalu, and Elike (2008) studied the relationship between growth rate of public spending and inflation rate for the United States of America in the period 1970-2002 found that the two variables move in the same direction. According to their analysis, inflation affects spending decisions of the U.S. federal government, but is in turn influenced both the short and long term. The dual causality was confirmed, however, the conclusions were reached and Ezirim, and Ofurum (2003). The conclusion drawn by these scholars is that, in order to bend inflation, governments should appropriately reduce the levels of expenditures; on the other hand, to reduce the growth in the size of the public, policy-makers should diminish price dynamics. A further consequence would be that fiscal policy would be a valuable tool for controlling inflation, by virtue of their ability to act directly on public spending (content).

Pekarski (2010) analyzed budget deficits and inflation in high inflation economies. The main finding is that recurrent outbursts of extreme inflation in these economies can be explicitly explained by the hysteresis effect associated with the action of two mechanisms: the arithmetic of the wrong side of the ITLC and the Patinkin effect. Another finding is that changes in different items of the budget balance sheet may have very different effects on inflation (apart from their different effects on the real economy). Varvarigos (2010) constructed a stochastic, dynamic general equilibrium model of endogenously sustained growth of an economy whose government finances volatile public spending via seigniorage. The resulting volatility in money supply, combined with the effects of money on human capital formation, yielded some interesting and important results concerning macroeconomic performance. The model predicts a negative correlation between long-run output growth and policy volatility. In addition, given that both the mean and the variance of the inflation rate are elevated by volatility in public spending, the model provides a possible account for the strong positive correlation between inflation and its variability, as well as their negative correlation with output growth.

## 3. Data and methodology

For the purpose of this paper, the variables analyzed have been expressed in a logarithmic form. The data that have been used are annual and cover the time period 1970-2009, for Mediterranean countries.

The data used in this work were taken from the IMF *Government and Finance Statistics* database, which provide current and internationally comparable data on the finances and fiscal policies of Fund member governments[55]. Most of time series have unit root as many studies indicated, including Nelson, and Plosser (1982), and as proved by Stock, and Watson (1988), and Campbell, and Perron (1991) among others, that most of the time series are non-stationary. The presence of a unit root in any time series means that the mean and variance are not independent of time. Conventional regression techniques based on non-stationary time series produce spurious regression and statistics may simply indicate only correlated trends rather than a true relationship (Granger, and Newbold 1974). Spurious regression can be detected in regression model by low Durbin-Watson statistics and relatively moderate $R^2$.

One of the most widely used unit root tests is the ADF (Dickey, and Fuller 1979, 1981). Alternatively, Phillips (1987), and Phillips, and Perron (1988) proposed a non-parametric method to correct a wide variety of serial correlation and heteroskedasticity (PP). Perron (1989, 1990) demonstrates that if a time series exhibits stationary fluctuations around a trend or a level containing a structural break, then unit root tests will erroneously conclude that there is a unit root. PP and ADF tests have the same asymptotic distributions.

Elliott, Rothenberg, and Stock (ERS, 1996) proposed a modified Dickey-Fuller *t*-test (known as the DF-GLS test). Essentially, this test is an Augmented Dickey-Fuller test, except that the time series are transformed via a generalized least squares (GLS) regression before performing the test. The Augmented Dickey-Fuller test involves fitting a regression of the form:

$$\Delta y_t = \alpha + \beta y_{t-1} + \delta t + \xi_1 \Delta y_{t-1} + \xi_2 \Delta y_{t-2} + \ldots + \xi_k \Delta y_{t-k} + \varepsilon_t \quad (1)$$

and then testing the null hypothesis $H_0$: $\beta=0$. The DF-GLS test is performed analogously but on GLS-detrended data. The null hypothesis of the test is that $y_t$ is a random walk, possibly with drift.

Finally, the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS 1992) test differs from those unit root tests in common use (such as ADF, PP, and DF-GLS) by having a null hypothesis of stationarity. The test may be conducted under the null of either trend stationarity (the default) or level stationarity. Inference from this test is complementary to that derived from those based on the Dickey-Fuller distribution.

Then we examine the unit root (or stationarity) properties of the variables, accounting for structural breaks. The present paper employs Zivot, and Andrews (ZA, 1992) test to address this issue. The Zivot, and Andrews test is performed by running the following regression:

$$x_t = \mu + \beta t + \alpha x_{t-1} + \sum_{i=1}^{k} \delta_i \, \Delta x_{t-i} + \varepsilon_t \quad (2)$$

for $t=1,\ldots,T$, where $x_t$ is a potentially non-stationary time-series, and the terms $\Delta x_{t-i}$, $i=1,\ldots,k$ are included to purge any serial correlation among the residuals. Furthermore, Clemente, Montañés, and Reyes (CMR 1998) have developed a procedure allowing for a gradual shift in the mean to test more than one break point.

The non-stationary series with the same order of integration may be cointegrated if there exists some linear combination that can be tested for stationarity. The Johansen and Juselius procedure (Johansen 1988, Johansen, and Juselius 1990) is preferable to test for cointegration for more than two series.

Moreover, Johansen, and Juselius procedure is considered better than Engle-Granger even in two time series case and has better small sample properties since it allows feedback effects among the variables under investigation where it is assumed, in the Engle, and Granger procedure, that there are no feedback effects between the variables. The procedure is based on likelihood ratio (LR) test to determine the number of cointegration vectors in the regression. Johansen technique enables to test for the existence of non-unique cointegration relationships. Three tests statistics are suggested to determine the number of cointegration vectors: the first is the Johansen's "trace" statistic method, the second is his "maximum eigenvalue" statistic method, and the third method chooses $r$ to minimize an information criterion. Having established the long-run equilibrium relationship between government expenditure and revenues, the short-run adjustments are estimated using the Error Correction Model (ECM). This model is based on the two following equations:

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[55] See: http://www.esds.ac.uk/international/support/user_guides/imf/gfs.asp.

$$\Delta X_t = \alpha_0 + \alpha_1 e_{t-1} + \sum_{i=1}^{m} \alpha_i \Delta X_{t-i} + \sum_{i=1}^{n} \alpha_j \Delta Y_{t-i} + \varepsilon_t \quad (3)$$

$$\Delta Y_t = \beta_0 + \beta_1 u_{t-1} + \sum_{i=1}^{m} \beta_i \Delta Y_{t-i} + \sum_{i=1}^{n} \beta_j \Delta X_{t-i} + \eta_t \quad (4)$$

where $e_{t-1}$ and $u_{t-1}$ represent the error-correction terms which are the lagged residuals from the cointegration relations.

The error correction terms will capture the speed of the short-run adjustments toward the long-run equilibrium. Furthermore, the error correction model equations (3) and (4) allow testing for short-run as well the long-run causality between government expenditure and aggregate income.

The short-run causality is based on a standard *F*-test statistics to test jointly the significance of the coefficients of the explanatory variable in their first differences. The long-run causality is based on a standard *t*-test. Negative and statistically significant values of the coefficients of the error correction terms indicate the existence of long-run causality.

## 4. Econometric results

We present and discuss an empirical analysis of the nexus between public expenditure and inflation, applied to the Mediterranean countries. In Table 1 variables of the model are summed up. All series contain yearly data in real terms.

**Table 1.** List of the variables.

| *Variable* | *Explanation* |
|---|---|
| *TEGG* | Total Expenditure of General Government, % of GDP |
| *NCPI* | National Consumer Price Index, 2000=100 |

**Source**: IMF.

In Figure 1 the first differences of NCPI (*ΔNCPI*) for the Mediterranean countries from 1970 to 2009 are shown:

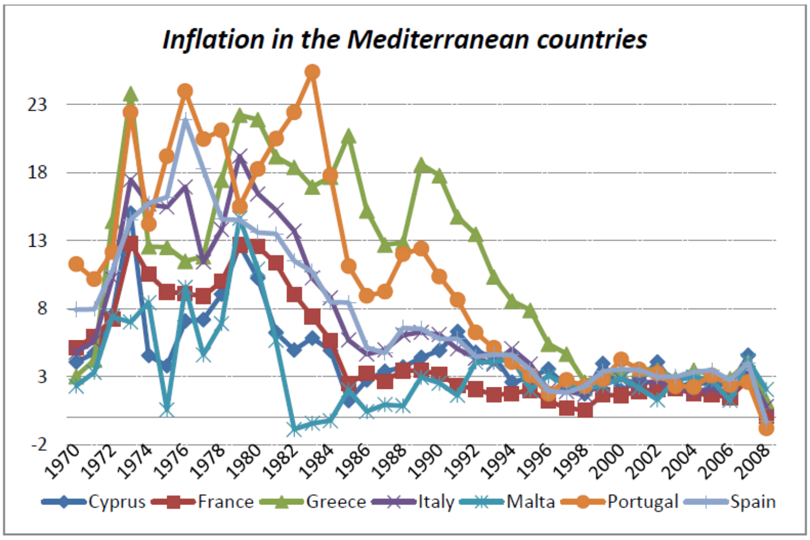


**Figure 1.** Inflation in the Mediterranean countries (1970-2009).

**Source**: our elaborations on IMF data.

As a preliminary analysis, some descriptive statistics are shown in the following Table 2.

**Table 2**. Exploratory data analysis (mld EUR, 1970-2009).

| Variable | Mean | Median | Standard Deviation | Skewness | Kurtosis | Range |
|---|---|---|---|---|---|---|
| ***TEGG*** | 45.2512 | 44.8566 | 5.5844 | -0.2273 | 2.5783 | 26.6596 |
| ***NCPI*** | 66.1517 | 71.2841 | 38.5341 | -0.1515 | 1.7357 | 130.3617 |

**Source**: our calculations on IMF data.

Correlation coefficients summarized in Table 3 indicate a low positive correlation between real total public expenditure and price index. These findings indicate that higher values of real public expenditure are associated with higher values of NCPI (except in Italy and Spain).

**Table 3.** Correlation analysis between public expenditure and price index.

| Country | Correlation coefficient between TEGG and NCPI | Correlation coefficient between ΔTEGG and ΔNCPI |
|---|---|---|
| ***Cyprus*** | 0.9095 | -0.0617 |
| ***France*** | 0.8344 | 0.3088 |
| ***Greece*** | 0.6284 | -0.1408 |
| ***Italy*** | 0.1241 | 0.3503 |
| ***Malta*** | 0.5106 | -0.0642 |
| ***Portugal*** | 0.9422 | -0.1384 |
| ***Spain*** | -0.1412 | -0.3202 |

**Notes**: Bonferroni adjustment applied.
**Source**: our calculations on IMF data.

First of all, we obtained log-transformations of the time-series. The Inter-Quartile Range analysis shows the absence of outliers in our samples. Then, we applied time-series techniques on stationarity and unit root processes, in order to check some stationarity properties. Table 4 contains the results of common unit root tests for our variables.

**Table 4**. Results for stationarity tests.

| ountry | Variable | | Stationarity tests | | | |
|---|---|---|---|---|---|---|
| | | Deterministic component | ADF | ERS | PP | KPSS |
| ***Cyprus*** | ***TEGG*** | intercept, trend | NS: -2.357 | NS: -2.336 | NS: -2.357 | TS: 0.113 |
| | ***NCPI*** | intercept, trend | NS: -2.887 | NS: -1.217 | NS: -2.871 | NS: 0.357 |
| | ***ΔTEGG*** | intercept | DS: -3.418 | DS: -2.298 | DS: -3.418 | DS: 0.067 |
| | ***ΔNCPI*** | intercept | DS: -4.694 | NS: -1.929 | DS: -4.647 | NS: 0.534 |
| ***France*** | ***TEGG*** | intercept, trend | NS: -3.369 | NS: -2.465 | NS: -2.421 | NS: 0.188 |
| | ***NCPI*** | intercept, trend | NS: -2.128 | NS: -1.083 | NS: -1.388 | NS: 2.030 |
| | ***ΔTEGG*** | intercept | DS: -3.159 | DS: -2.631 | DS: -3.159 | DS: 0.129 |
| | ***ΔNCPI*** | intercept | NS: -1.942 | NS: -1.412 | DS: -1.923 | NS: 0.433 |
| ***Greece*** | ***TEGG*** | intercept | NS: -2.183 | NS: -0.247 | NS: -2.183 | NS: 0.555 |
| | ***NCPI*** | intercept, trend | NS: -2.619 | NS: -2.268 | NS: -2.676 | NS: 0.451 |
| | ***ΔTEGG*** | intercept | DS: -5.183 | NS: -2.059 | DS: -5.183 | DS: 0.138 |
| | ***ΔNCPI*** | intercept | NS: -1.610 | NS: -1.092 | NS: -1.579 | NS: 1.140 |
| ***Italy*** | ***TEGG*** | intercept | LS: -2.855 | NS: -0.733 | LS: -2.855 | LS: 0.269 |
| | ***NCPI*** | intercept, trend | NS: -2.153 | NS: -1.818 | NS: -2.937 | NS: 0.208 |
| | ***ΔTEGG*** | intercept | DS: -3.708 | NS: -1.481 | DS: -3.708 | DS: 0.352 |
| | ***ΔNCPI*** | intercept | NS: -2.622 | NS: -1.373 | NS: -2.567 | NS: 0.676 |
| ***Malta*** | ***TEGG*** | intercept | LS: -2.917 | NS: -1.360 | LS: -2.917 | LS: 0.399 |
| | ***NCPI*** | intercept, trend | NS: -2.399 | NS: -2.445 | NS: -1.549 | NS: 0.168 |
| | ***ΔTEGG*** | intercept | DS: -4.732 | NS: -1.937 | DS: -4.732 | DS: 0.131 |
| | ***ΔNCPI*** | intercept | DS: -3.715 | DS: -2.703 | DS: -3.725 | DS: 0.143 |
| ***Portugal*** | ***TEGG*** | intercept, trend | NS: -3.277 | TS: -3.434 | NS: -3.086 | NS: 0.148 |
| | ***NCPI*** | intercept, trend | NS: -2.120 | NS: -2.125 | NS: -2.972 | NS: 0.309 |
| | ***ΔTEGG*** | intercept | DS: -4.125 | DS: -2.783 | DS: -4.098 | DS: 0.064 |
| | ***ΔNCPI*** | intercept | NS: -1.527 | NS: -1.454 | NS: -1.640 | NS: 0.610 |
| ***Spain*** | ***TEGG*** | intercept | NS: 0.720 | NS: -1.389 | NS: -1.218 | LS: 0.232 |
| | ***NCPI*** | intercept, trend | TS: -4.910 | NS: -1.573 | TS: -4.711 | NS: 0.270 |
| | ***ΔTEGG*** | intercept | DS: -3.222 | DS: -2.321 | DS: -3.299 | DS: 0.458 |
| | ***ΔNCPI*** | intercept | NS: -2.575 | NS: -1.209 | NS: -2.697 | NS: 0.673 |

**Notes**: LS: Level Stationary; NS: Non Stationary; TS: Trend Stationary; DS: Difference Stationary.
**Source**: our calculations on IMF data.

The second column presents results for Augmented Dickey, and Fuller (1979) test; the third one for Elliott, Rothenberg, and Stock (1992) test; the fourth column contains results for Phillips, and Perron (1988) test; at last, in the fifth column there are results for Kwiatkowski, Phillips, Schmidt, and Shin (1992) test. Here, results indicate that public expenditure is clearly a *I(1)* process in five countries (Cyprus, France, Greece, Portugal and Spain); a *I(0)* process for Italy and Malta (where it seems to be level-stationary). While, prices index is a *I(2)* process everywhere, except Malta (*I(1)*).

**Table 5**. Results for unit root tests with structural breaks.

| Country | Variable | TB | k | t-stat | 1% Critical Value | 5% Critical Value |
|---|---|---|---|---|---|---|
| ***Cyprus*** | ***TEGG*** | 2003 | 0 | -4.573 | -5.57 | -5.08 |
| | ***ΔTEGG*** | | 0 | -5.504 | -5.57 | -5.08 |
| | ***ΔNCPI*** | | 0 | -5.408 | -5.57 | -5.08 |
| | ***Δ²NCPI*** | | 1 | -6.428 | -4.93 | -4.42 |
| ***France*** | ***TEGG*** | 1992 | 1 | -3.424 | -5.57 | -5.08 |
| | ***ΔTEGG*** | | 0 | -5.752 | -5.57 | -5.08 |
| | ***ΔNCPI*** | | 2 | -3.918 | -5.57 | -5.08 |
| | ***Δ²NCPI*** | | 0 | -5.573 | -4.93 | -4.42 |
| ***Greece*** | ***TEGG*** | 2006 | 0 | -3.663 | -5.57 | -5.08 |
| | ***ΔTEGG*** | | 0 | -7.309 | -5.57 | -5.08 |
| | ***ΔNCPI*** | | 0 | -4.395 | -5.57 | -5.08 |
| | ***Δ²NCPI*** | | 0 | -5.601 | -5.57 | -5.08 |
| ***Italy*** | ***TEGG*** | 2005 | 0 | -2.508 | -5.57 | -5.08 |
| | ***ΔTEGG*** | | 0 | -6.282 | -5.57 | -5.08 |
| | ***ΔNCPI*** | | 0 | -4.679 | -5.57 | -5.08 |
| | ***Δ²NCPI*** | | 0 | -5.983 | -4.93 | -4.42 |
| ***Malta*** | ***TEGG*** | 2003 | 0 | -5.230 | -5.57 | -5.08 |
| | ***ΔTEGG*** | | 0 | -5.324 | -5.57 | -5.08 |
| | ***ΔNCPI*** | | 0 | -3.880 | -4.93 | -4.42 |
| | ***Δ²NCPI*** | | 2 | -4.892 | -4.93 | -4.42 |
| ***Portugal*** | ***TEGG*** | 1997 | 1 | -4.512 | -5.57 | -5.08 |
| | ***ΔTEGG*** | | 1 | -4.439 | -4.93 | -4.42 |
| | ***ΔNCPI*** | | 0 | -3.055 | -5.57 | -5.08 |
| | ***Δ²NCPI*** | | 2 | -4.835 | -4.93 | -4.42 |
| ***Spain*** | ***TEGG*** | 2007 | 0 | -1.008 | -5.57 | -5.08 |
| | ***ΔTEGG*** | | 0 | -7.911 | -5.57 | -5.08 |
| | ***ΔNCPI*** | | 1 | -2.892 | -5.57 | -5.08 |
| | ***Δ²NCPI*** | | 0 | -6.047 | -4.93 | -4.42 |

**Source**: our calculations on IMF data.

The results of the Zivot, and Andrews's unit root test are summarized in Table 5. An examination of these results for public expenditure series indicate that the null hypothesis of a unit root cannot be rejected in levels (the only exception is Malta, at a 5% significance level). If we take the first differences, we can reject the null hypothesis for all countries. So, we can conclude that public expenditure is clearly a *I(1)* process in six countries (Cyprus, France, Greece, Italy, Portugal and Spain); a *I(0)* process for Malta. Inflation is a *I(1)* process everywhere.

**Table 6**. Results for additive outlier unit root tests.

| Country | Variable | SB | k | t-stat | 5% Critical Value |
|---|---|---|---|---|---|
| **Cyprus** | ***TEGG*** | 2000 | 0 | -3.366 | -3.560 |
| | ***ΔTEGG*** | | 0 | -6.378 | -3.560 |
| | ***ΔNCPI*** | | 4 | -3.637 | -3.560 |
| | ***$Δ^2$NCPI*** | | 1 | -8.361 | -3.560 |
| **France** | ***TEGG*** | 1994 | 1 | -3.954 | -3.560 |
| | ***ΔTEGG*** | | 0 | -3.964 | -3.560 |
| | ***ΔNCPI*** | | 5 | -3.177 | -5.490 |
| | ***$Δ^2$NCPI*** | | 1 | -4.237 | -3.560 |
| **Greece** | ***TEGG*** | 2008 | 0 | -4.184 | -3.560 |
| | ***ΔTEGG*** | | 0 | -6.796 | -3.560 |
| | ***ΔNCPI*** | | 5 | -2.300 | -5.490 |
| | ***$Δ^2$NCPI*** | | 0 | -6.516 | -5.490 |
| **Italy** | ***TEGG*** | 1989, 1996 | 4 | -1.310 | -5.490 |
| | ***ΔTEGG*** | | 0 | -5.559 | -5.490 |
| | ***ΔNCPI*** | | 2 | -5.962 | -5.490 |
| | ***$Δ^2$NCPI*** | | 2 | -3.591 | -3.560 |
| **Malta** | ***TEGG*** | 2000 | 2 | -3.472 | -3.560 |
| | ***ΔTEGG*** | | 1 | -3.891 | -3.560 |
| | ***ΔNCPI*** | | 0 | -4.627 | -5.490 |
| | ***$Δ^2$NCPI*** | | 1 | -6.566 | -3.560 |
| **Portugal** | ***TEGG*** | 1987, 1992 | 0 | -2.330 | -5.490 |
| | ***ΔTEGG*** | | 1 | -5.001 | -3.560 |
| | ***ΔNCPI*** | | 5 | -3.127 | -5.490 |
| | ***$Δ^2$NCPI*** | | 2 | -3.892 | -3.560 |
| **Spain** | ***TEGG*** | 1998, 2007 | 3 | -3.646 | -5.490 |
| | ***ΔTEGG*** | | 1 | -3.754 | -3.560 |
| | ***ΔNCPI*** | | 0 | -4.205 | -3.560 |
| | ***$Δ^2$NCPI*** | | 2 | -3.658 | -3.560 |

**Source**: our calculations on IMF data.

From the Table 6 above, we note that the Clemente *et al.* test results are quite different to those found with the Zivot, and Andrews test. For *TEGG*, despite the structural break, we are unable to reject the null hypothesis of a unit root in five countries (Cyprus, Italy, Malta, Portugal and Spain); as a conclusion, public expenditure seems to be a *I(1)* process in these countries, but a *I(0)* process in France and Greece. Inflation is *I(0)* for Cyprus, Italy and Spain, and *I(1)* otherwise.

The lag-order selection has been chosen according to the Final Prediction Error (FPE), Akaike's Information Criterion (AIC), Schwarz's Bayesian Information Criterion (SBIC), and the Hannan, and Quinn Information Criterion (HQIC).

Cointegration tests have been subsequently applied, in order to find the long-run relationship between the share of public expenditure on GDP (*TEGG*) and inflation (*ΔNCPI*), since these two economic variables are integrated at the same order (1). As is shown in Table 7, the Johansen, and Juselius cointegration method suggests that there is a cointegrating relationship in four cases (Cyprus, France, Greece and Portugal). In these cases, the trace statistic and the maximum-eigenvalue statistic reject $r$=0 in favour of $r$=1 at the 5% critical value. Yet, for Spain we have a contradictory result: in fact, the trace statistic suggests $r$=0, while the maximum-eigenvalue statistic suggests $r$=1. As in the lag-length selection problem, choosing the number of cointegrating equations that minimizes either the SBIC or the HQIC provides a consistent estimator of the number of cointegrating equations. As a conclusion, we find rank=0 for Italy, Malta and Spain. While, for the other four countries we find the presence of cointegration (rank=1).

**Table 7**. Results for cointegration tests between public expenditure and inflation (*TEGG* and *ΔNCPI*)

| Country | Trace statistic | Maximum-eigenvalue statistic | SBIC<br>HQIC<br>AIC | Rank |
|---|---|---|---|---|
| *Johansen and Juselius procedure* | | | | |
| ***Cyprus*** | 3.9243<br>(9.42) | 3.9243<br>(9.24) | 16.1207<br>15.7016<br>15.7730 | r=1 |
| ***France*** | 5.0004<br>(9.42) | 5.0004<br>(9.24) | 13.8819<br>13.6278<br>13.5083 | r=1 |
| ***Greece*** | 2.1861<br>(9.42) | 2.1861<br>(9.24) | 15.6044<br>15.2839<br>15.2061 | r=1 |
| ***Italy*** | 16.3700<br>(25.32) | 6.7739<br>(12.52) | 14.5055<br>14.3073<br>14.2201 | r=0 |
| ***Malta*** | 11.0718<br>(25.32) | 7.2492<br>(18.96) | 15.7444<br>15.4300<br>15.4836 | r=0 |
| ***Portugal*** | 6.2125<br>(12.25) | 6.2125<br>(12.52) | 14.7726<br>14.4608<br>14.3100 | r=1 |
| ***Spain*** | 24.8214<br>(25.32) | 18.3598<br>(18.96) | 14.9829<br>14.6685<br>14.7221 | r=0 |

**Notes**: 5% Critical Values in parenthesis.
**Source**: our calculations on IMF data.

Granger causality tests suggest a bi-directional flow, at 1% significance level, between public expenditure and inflation for Italy, Malta and Portugal in the short-run, and for Greece in the long-run; a unidirectional flow, running from inflation to public expenditure for Portugal (in the long-run, at a 1% level), as well as for Cyprus (1%), France (1%) and Spain (10%) in the short-run; a unidirectional flow, but in the opposite direction (from public expenditure to inflation), for Cyprus (at 1% level) and France (1%) in the long-run (see Table 8).

**Table 8**. Results for short and long-run causality tests

| *Country* | *Lags* | *Log-likelihood* | *SBIC* | *Causality in the long-run* | *Causality in the short-run* |
|---|---|---|---|---|---|
| ***Cyprus*** | 3 | -67.9675 | 15.8456 | G → P | P → G |
| ***France*** | 1 | -203.0899 | 14.2196 | G → P | P → G |
| ***Greece*** | 1 | -137.3424 | 16.0068 | G ↔ P | - |
| ***Italy*** | 4 | -152.0508 | 14.4817 | - | G ↔ P |
| ***Malta*** | 4 | 142.6019 | 24.3757 | - | G ↔ P |
| ***Portugal*** | 4 | -181.8044 | 15.3662 | P → G | G ↔ P |
| ***Spain*** | 4 | -181.8044 | 15.3662 | - | P → G |

**Source**: our calculations on IMF data.

For all our equations, a Lagrange-multiplier (LM) test for autocorrelation in the residuals of Vector Error-Correction Model (VECM) clarifies as at the 5% significance level we cannot reject the null hypothesis that there is no serial correlation in the residuals for the orders 1,…,5 tested. Checking the eigenvalue stability condition in a VECM, the eigenvalues of the companion matrix lie inside the unit circle, and the real roots are far from 1. As regard the Wald lag-exclusion statistics, we strongly reject the hypothesis that the coefficients either on the first lag or on the second lag of the endogenous variables are zero in all two equations jointly. The Jarque, and Bera normality test results present statistics for each equation and for all equations jointly against the null hypothesis of normality. For our models, results suggest normality. Finally, the analysis of ARCH effects shows the absence of this problem for the estimated models.

## 5. Conclusions and policy implications

The purpose of this paper is to contribute to the literature on the nexus between public expenditure and inflation, using recent econometric techniques. So, we studied the relationship between public expenditure and inflation for Mediterranean countries, using annual data covering the period 1970-2009. The time-series properties of the data were assessed using several unit root tests (ADF, DF-GLS, PP, and KPSS). Furthermore, in order to evaluate the presence of eventual structural breaks, some tests (ZA and CMR) have been conducted. Empirical findings indicate that public expenditure is clearly a *I(1)* process in five countries (Cyprus, France, Greece, Portugal and Spain); and a *I(0)* process for Italy and Malta. While, prices index is a *I(2)* process everywhere, except Malta.

Cointegration analysis reveals that there is a long-run relationship between public expenditure/GDP ratio and inflation in four cases (Cyprus, France, Greece and Portugal). Granger causality tests suggest a bi-directional flow between public expenditure and inflation for Italy, Malta and Portugal in the short-run, and for Greece in the long-run; a unidirectional flow, running from inflation to public expenditure for Portugal (in the long-run), as well as for Cyprus, France and Spain in the short-run; a unidirectional flow, but in the opposite direction, for Cyprus and France in the long-run.

Yet, we find some evidence of government spending causing prices dynamics. In other words, the original Clark's proposition of an excessive government spending as a cause of pressure on prices in the economy is well supported by the data for the Mediterranean countries. Certainly, this result is subject to the time period examined and statistical methods used; nevertheless, our empirical findings show some evidence in favour of the opposite direction of causality flow. In fact the inflation Granger causes public expenditure growth in three cases.

As a main policy implication, the countries where a bi-directional causality flow has been found can comfortably regulate the levels of inflation in the economy controlling the share of its public expenditure. Furthermore, restrictive monetary policies can contain the size of Government.

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- Environment and Trade;
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- Environmental Accounts and Accounting;
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- Ecological Economics: Ecosystem Services; Biodiversity Conservation; Bioeconomics; Industrial Ecology;
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**Important Dates:**
25 October, 2011 - Abstract submission deadline;
5 November, 2011 - Notification of acceptance/rejection;
10 November, 2011 - Deadline for payments (100€ for attendance at the Conference);
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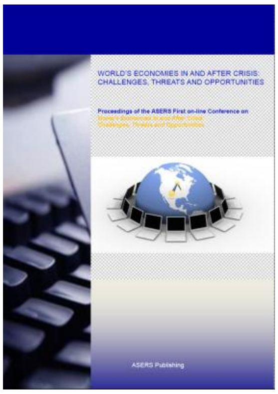


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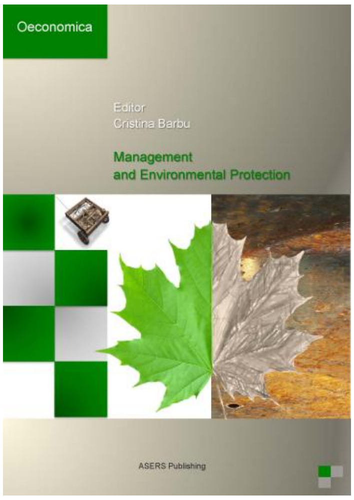


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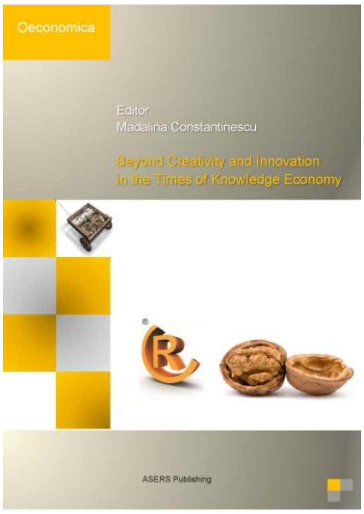


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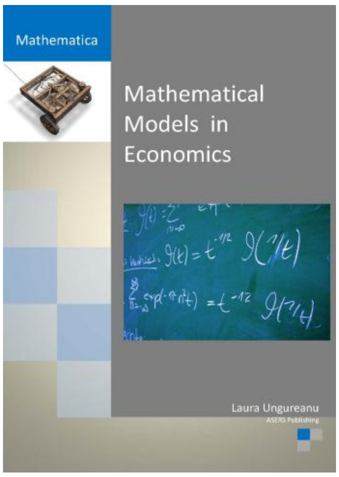


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