ASERS

Theoretical and Practical Research in Economic Fields

Biannually
Volume II
Issue 2(4)
Winter 2011

ISSN 2068 – 7710

Summer 2011
Volume II, Issue 1(3)

ASERS Publishing
http://www.asers.eu/asers-publishing
ISSN 2068 – 7710

Contents:

Call for Papers
Summer_Issue 2012

Theoretical and Practical Research in Economic Fields

Many economists today are concerned by the proliferation of journals and the concomitant labyrinth of research to be conquered in order to reach the specific information they require. To combat this tendency, **Theoretical and Practical Research in Economic Fields** has been conceived and designed outside the realm of the traditional economics journal. It consists of concise communications that provide a means of rapid and efficient dissemination of new results, models and methods in all fields of economic research.

Theoretical and Practical Research in Economic Fields publishes original articles in all branches of economics – theoretical and empirical, abstract and applied, providing wide-ranging coverage across the subject area.

Journal promotes research that aim at the unification of the theoretical-quantitative and the empirical-quantitative approach to economic problems and that are penetrated by constructive and rigorous thinking. It explores a unique range of topics from the frontier of theoretical developments in many new and important areas, to research on current and applied economic problems, to methodologically innovative, theoretical and applied studies in economics. The interaction between empirical work and economic policy is an important feature of the journal.

Theoretical and Practical Research in Economic Fields is indexed in RePEC, IndexCopernicus, EBSCO, ProQuest and very soon in CEEOL databases.

The primary aim of the Journal has been and remains the provision of a forum for the dissemination of a variety of international issues, empirical research and other matters of interest to researchers and practitioners in a diversity of subject areas linked to the broad theme of economic sciences.

All the papers will be first considered by the Editors for general relevance, originality and significance. If accepted for review, papers will then be subject to double blind peer review.

Invited manuscripts will be due till May 25st, 2012, and shall go through the usual, albeit somewhat expedited, refereeing process.

Deadline for submission of proposals:	25st May 2012
Expected Publication Date:	1st June 2012
Web:	www.asers.eu/journals/tpref/
E-mail:	tpref@asers.eu

To prepare your paper for submission, please see full author guidelines in the following file: TPREF_Full_Paper_Template.doc, then send it via email at tpref@asers.eu.

SOME CONVERGENCE RESULTS ON DYNAMIC FACTOR MODELS

Maddalena CAVICCHIOLI
University of Venice, **Italy**
Advanced School of Economics
maddalena.cavicchioli@unive.it

Abstract:

We review some recent papers on a large dynamic factor model (LDFM) and its applications to structural macroeconomic analysis. Then we prove some convergence results concerning with the stochastic variables which define such a model.

Keywords: dynamic factor model, fundamentalness, identification, estimation, consistency, convergence.

JEL Classification: C01, C32, E32.

1. Introduction

Factor models have been used by several authors to address many different economic issue nowadays. Some literature has focused on models specifically designed to handle a large amount of information: the *generalized dynamic factor models* (Forni, Hallin, Lippi and Reichlin, 2000; Forni and Lippi, 2001; Bai and Ng, 2002). Such models have been successfully used for forecasting and, recently, also for structural macroeconomic analysis (Forni, Giannone, Lippi, Reichlin, 2009, FGLR from now on; Forni and Gambetti, 2010).

The main idea underlying factor analysis is that a large set of variables can be explained by a small number of latent variables, the *factors*, which are responsible for all the relevant dynamics. Factor analysis is a technique of dimension reduction that takes the information contained in a large data set and summarizes it by means of few unobservable variables. In this context, it is assumed that macroeconomic observable variables are represented as the sum of two unobservable components, called the *common component* and the *idiosyncratic component*. The common component captures that part of the series which comove with the rest of the economy and the idiosyncratic component is the residual. The idiosyncratic components are not necessarily orthogonal to each other and they are not of direct interest for the analysis since they arise from measurement errors or sectoral sources of variation. The vector of the common components is highly singular, i.e., it is driven by a very small number of shocks (the "common shocks" or "common factors") as compared to the number of variables.

Furthermore, the relation between the common part of the observable series and the factors is assumed to be linear. Structural analysis requires the identification of the macroeconomic shocks and their dynamic effect on macroeconomic variables. The approach is a combination of structural vector autoregression (SVAR) analysis and large-dimensional dynamic factor models. More precisely, the factor model is used to consistenly estimate common and idiosyncratic components of macroeconomic variables. Then the identification of the relationship between common components and macroeconomic shocks can be obtained just in the same way as in SVAR models, and the impulse response functions can be consistently estimated by means of a relatively simple procedure.

In this paper we review some recent papers on a large dynamic factor model and its applications to structural macroeconomic analysis. Then we prove some convergence results concerning with the stochastic variables which define such a model.

2. A Dynamic Factor Model

In this section we illustrate the basic definitions and results concerning with a dynamic factor model, briefly called *the model FGLR*, introduced and studied by Forni, *et al.* (2009).

1) **The model.** Denote by $\mathbf{x}_n^T = (x_{it})_{i=1,\ldots,n;\, t=1,\ldots,T}$ an $n \times T$ rectangular array of observations, and make two preliminary assumptions:

PA1. *The array* $\mathbf{x}_n^T$ *is a finite realization of a real-valued stochastic process*

$$X = \{x_{it}: i \in \mathbb{N}, t \in \mathbb{Z}, x_{it} \in L_2(\Omega, \mathcal{F}, P)\}$$

where the n*-dimensional vector processes* $\{x_{nt} = (x_{1t} \ldots x_{nt})' : t \in Z\}_{n \in N}$ *are stationary, with zero mean and finite second-order moments* $\Gamma_{nk}^x = E(\mathbf{x}_{nt}\mathbf{x}_{n,t-k}')$ *for every* $k \in N$.

PA2. *For all* $n \in N$, *the process* $\{x_{nt} : t \in Z\}$ *admits a Wold representation* $\mathbf{x}_{nt} = \sum_{k=0}^{\infty} C_k^n \mathbf{w}_{n,t-k}$,

where the full-rank innovations $\mathbf{w}_{nt}$ *have finite moments of order four and the matrices* $C_k^n = (c_{ij,k}^n)$ *satisfy* $\sum_{k=0}^{\infty} | c_{ij,k}^n | < \infty$ *for all* $i, j, n \in N$.

The model FGLR is obtained by assuming that each variable x_{it} is the sum of two unobservable components $x_{it} = \chi_{it} + \xi_{it}$, where χ_{it} (resp. ξ_{it}) is called the *common* (resp. *idiosyncratic*) component. The common component χ_{it} is driven by q common shocks $\mathbf{u}_t = (u_{1t} \cdots u_{qt})'$ for q independent of n and $q << n$. More precisely:

FM0. *(Dynamic factor structure of the model FGLR)*

Defining $\boldsymbol{\chi}_{nt} = (\chi_{1t} \cdots \chi_{nt})'$ *and* $\boldsymbol{\xi}_{nt} = (\xi_{1t} \cdots \xi_{nt})'$, *suppose that*

$$\mathbf{x}_{nt} = \boldsymbol{\chi}_{nt} + \boldsymbol{\xi}_{nt} = B_n(L)\mathbf{u}_t + \boldsymbol{\xi}_{nt} \quad (1.1)$$

where $\mathbf{u}_t$ *is a q-dimensional orthonormal white-noise vector, that is,* $E(\mathbf{u}_t\mathbf{u}_t') = \mathbf{I}_q$ *for all* t. *The shocks* $\mathbf{u}_t$ *will be called dynamic factors.*

Moreover, assume that

$$B_n(L) = A_n N(L) \quad (1.2)$$

where:

i) $N(L)$ is an $r \times q$ absolutely summable matrix function of L, that is,

$$N(L) = \sum_{k=0}^{\infty} \mathbf{\Psi}_k L^k \qquad \sum_{k=0}^{\infty} | \Psi_{jh,k} | < \infty \qquad \text{for all } j,h$$

where

$$\mathbf{\Psi}_k = (\Psi_{jh,k})_{j=1,\ldots r;\, h=1,\ldots q}$$

ii) $A_n = (a_{ij})_{i=1,\ldots n;\, j=1,\ldots r}$ is an $n \times r$ matrix, nested in A_m for all $m > n$.

Defining the $r \times 1$ vector $\mathbf{f}_t = (f_{1t} \cdots f_{rt})'$, called the *static factor*, as

$$\mathbf{f}_t = N(L)\mathbf{u}_t \quad (1.3)$$

Equation (1.1) can be rewritten in the static form

$$\mathbf{x}_{nt} = A_n \mathbf{f}_t + \boldsymbol{\xi}_{nt} \quad (1.4)$$

From (1.1) and (1.4) we get

$$\boldsymbol{\chi}_{nt} = A_n \mathbf{f}_t \quad (1.5)$$

hence

$$\chi_{it} = \sum_{j=1}^{r} a_{ij} f_{jt}.$$

This means that all the variables χ_{it}, $i = 1,\ldots,\infty$, belong to the finite dimensional vector space spanned by $\mathbf{f}_t = (f_{1t} \cdots f_{rt})'$.

Following Forni, *et al.* (2009), we are going to illustrate some conditions under which the shocks $\mathbf{u_t}$ can be identified and estimated by means of the observable variables x_{it}. First, we recall the assumptions necessary for the identification and the estimation of the common components χ_{it}.

FM1. (Orthogonality of common and idiosyncratic components)

For all n, the vector $\boldsymbol{\xi}_{nt}$ is stationary, and $E(\mathbf{u}_t \boldsymbol{\xi}_{n\tau}') = 0$ for any $t, \tau \in Z$ and $n \in N$.

Let $\Gamma_{nk}^{x} = E(\mathbf{x}_{nt}\mathbf{x}_{n,t-k}')$, $\Gamma_{nk}^{\chi} = E(\boldsymbol{\chi}_{nt}\boldsymbol{\chi}_{n,t-k}')$ and $\Gamma_{nk}^{\xi} = E(\boldsymbol{\xi}_{nt}\boldsymbol{\xi}_{n,t-k}')$ be the k-lag covariance matrices of $\mathbf{x}_{nt}$, $\boldsymbol{\chi}_{nt}$ and $\boldsymbol{\xi}_{nt}$, respectively. Denote by μ_{nj}^{x}, μ_{nj}^{χ} and μ_{nj}^{ξ} the jth eigenvalues, in decreasing order, of Γ_{n0}^{x}, Γ_{n0}^{χ} and Γ_{n0}^{ξ}, respectively.

FM2. (Pervasiveness of common dynamic and static factors)

a) The complex matrix $N(e^{-i\theta})$ has (maximum) rank q for θ almost everywhere in $[-\pi, \pi]$;

b) There exist positive real constants $\underline{c}_j < \bar{c}_j$, $j = 1,\ldots,r$, such that $\underline{c}_j > \bar{c}_{j+1}$, $j = 1,\ldots,r-1$, and

$$\underline{c}_j \leq \liminf_{n\to\infty} \frac{\mu_{nj}^{\chi}}{n} \leq \limsup_{n\to\infty} \frac{\mu_{nj}^{\chi}}{n} \leq \bar{c}_j$$

Proposition 1.1 *Under assumption FM2, the $r \times r$ matrix $A_n' A_n$ has full rank r for n sufficiently large.*

Assumption FM2 also implies that the common components χ_{it} are identified (see Chamberlain, and Rothschild (1983)), and that the number q is unique, i.e., a representation of type (1.1) - (1.4) with a different number of dynamic factors is not possible (see Forni and Lippi (2001)).

FM3. (Non-pervasiveness of the idiosyncratic components)

There exists a real number d such that $\mu_{n1}^{\xi} \leq d$ for any $n \in N$. This obviously implies that $\mu_{nj}^{\xi} \leq d$ for any $n \in N$ since such eigenvalues are in decreasing order.

Assumption FM3, jointly with the identification of the common components χ_{it}, implies that the vector space spanned by the r static factors $f_{1t},\cdots,f_{rt}$ (in $\mathbf{f}_t$) is identified, or, equivalently, $\mathbf{f}_t$ is identified, up to non-singular linear transformation.

In conclusion, given a model of type (1.1)-(1.4), then under assumption FM0-FM3 the integers q and r, the components χ_{it} and ξ_{it}, and the vector space spanned by $\mathbf{f}_t$ are identified.

2) **Fundamentalness**. First we recall briefly some basic notions on fundamental representations of stationary stochastic vectors. Assume that the n stochastic vector $\boldsymbol{\mu}_t$ admits a moving average (MA) representation

$$\boldsymbol{\mu}_t = K(L)\mathbf{v}_t \qquad (2.1)$$

where $K(L)$ is an $n \times q$ square-summable filter and $\mathbf{v_t}$ is a q-dimensional white noise.

Definition 2.1 *If $\mathbf{v}_t$ belongs to the vector space spanned by present and past values of $\boldsymbol{\mu}_t$, then the MA in (2.1) is said to be fundamental, and $\mathbf{v}_t$ is called fundamental for $\boldsymbol{\mu}_t$.*

Without loss of generality, we can suppose that $q \leq n$ and that $\mathbf{v}_t$ is full rank. Moreover, for our purpose, we can suppose that the entries of $K(L)$ are rational functions of L and that the rank of $K(z)$, $z \in C$, is maximal, i.e., it is q except for a finite number of complex numbers.

Proposition 2.1 *The MA representation in (2.1) is fundamental if and only if the rank of $K(z)$ is q for all complex numbers z such that $|z|<1$.* For the proof, see Rozanov (1967).

Fundamentalness plays an important role for the identification of the structural shocks in SVAR analysis. SVAR analysis starts with the projection of a full rank q-dimensional vector $\boldsymbol{\mu}_t$ on its past, thus producing an q-dimensional full rank fundamental white-noise $\mathbf{w}_t$. Then the structural shocks are obtained as a linear transformation $A\mathbf{w}_t$, where the matrix A arises from economic theory statements. This is equivalent to assume that the structural shocks are fundamental.

Fundamentalness has here the effect that the identification problem is enormously simplified. However, economic theory, in general, does not provide support for fundamentalness, so that all representations that fulfill the same economic statements, but are not fundamental, are ruled out in SVAR analysis with no justification. Such representations, although they imply the same autocovariance structure, cannot be obtained from inversion of estimated VARs. The situation changes if the structural analysis is conducted assuming that n is large with respect to q. The fundamentalness is also required by dynamic factor models but it is a condition less pressing than in VARs. The first reason is that non fundamentalness of structural shocks arises when the econometrician's information set is smaller than the agent's. The second reason comes from a mathematical background. Precisely, a crucial step in our analysis is the dynamic specification of the common components $\boldsymbol{\chi}_{nt}$ as vector autoregression (VAR) driven by only q macroeconomics shocks $\mathbf{u}_t$, i.e., $\boldsymbol{\chi}_{nt} = A_n N(L)\mathbf{u}_t$, where $q < n$. So the model contains only q variables; suppose such variables are χ_{jt}, $j = 1,...,q$, and they cannot ensure fundamentalness of $\mathbf{u}_t$. By Proposition 2.1, the rank of $B_n(z)$ is less than q for some complex number z with $|z|<1$, or equivalently, the polynomials $B_{nj}(z)$, $j = 1,...,q$, have a common root. However, the informational advantage of the agents may disappear if the econometrician observes a large set of additional macroeconomic shocks. The generating process of χ_{jt}, $j = q+1,...,n$, contains parameters that do not belong to the generating process of χ_{jt}, $j = 1,...,q$, and viceversa. Therefore, with all likelihood, their dynamic responses to $\mathbf{u}_t$ are sufficiently *heterogeneous*, with respect to the first q, to prevent the rank reduction of $B_n(z)$. Now Assumption FM2(b) gives that, for n sufficiently large, A_n has full rank r. Then $N(L)$ has full rank q and it is left-invertible. So the concept of fundamentalness can be adapted to our specification of the dynamic factor model, as follows:

FM4. (*Fundamentalness)*

The matrix function $N(L)$ in (1.2) is left invertible, i.e., there exists an $q \times r$ square-summable filter $G(L)$ such that $G(L)N(L) = \mathbf{I}_q$.

The following proposition shows that FM4, jointly with FM2, imply fundamentalness in the sense of Definition 2.1.

Proposition 2.2 *If FM0-FM4 are satisfied,then $\mathbf{u}_t$ is fundamental for $\boldsymbol{\chi}_{nt}$ for n sufficiently large, and therefore fundamental for $\chi_{it}, i = 1,\ldots,\infty$.Moreover, $\mathbf{u}_t$ belongs to the vector space spanned by present and past values of $x_{it}, i = 1,\ldots,\infty$, that is, the shocks u_{ht} can be recovered as limits of linear combinations of the variables x_{it}.* For the proof see Forni, *et al.* (2009).

To introduce the last assumption, a VAR specification for $\mathbf{f}_t$, let us consider the orthogonal projection of $\mathbf{f}_t$ on the space spanned by its past values

$$\mathbf{f}_t = Proj(\mathbf{f}_t \mid \mathbf{f}_{t-1}, \mathbf{f}_{t-2}, ...) + \mathbf{w}_t \tag{2.2}$$

where $\mathbf{w}_t$ is the r-dimensional vector of the residuals. Under our assumptions, $\mathbf{w}_t$ has rank q. Moreover, assumption FM4 implies that $\mathbf{w}_t = R\mathbf{u}_t$, where R is a maximum-rank $r \times q$ matrix. In the sequel we will adopt the VAR(p) specification:

FM4'. *(Fundamentalness: VAR(p) specification)*

The r - dimensional static factors $\mathbf{f}_t$ admit a VAR(p) representation

$$\mathbf{f}_t = D_1\mathbf{f}_{t-1} + \cdots + D_p\mathbf{f}_{t-p} + R\mathbf{u}_t \tag{2.3}$$

where D_i is $r \times r$ and R is a maximum-rank matrix of dimension $r \times q$.

By (2.3) we have

$$\mathbf{f}_t = (I - D_1L - \cdots - D_pL^p)^{-1}R\mathbf{u}_t$$

Hence

$$\chi_{nt} = A_n(I - D_1L - \cdots - D_pL^p)^{-1}R\mathbf{u}_t$$

by (1.5). So Equation (1.1) yields

$$B_n(L) = A_n(I - D_1L - \cdots - D_pL^p)^{-1}R \tag{2.4}$$

called the *impulse-response function* (IRF) of the lags L.

3) Estimation. The following procedure of estimation can be found in Forni and Gambetti (2010) (see also Forni, *et al.* (2009) Sect.4.2).

Step 1) First, we need to set value for the number r of the static factors $\mathbf{f}_t = (f_{1t} \cdots f_{rt})'$. Bai and Ng (2002) proposed some consistent criteria to determine an estimation of r. Let $\hat{r}$ denote an estimation of r obtained by such criteria.

Step 2) We estimate the static factors $\mathbf{f}_t$, up to a non-singular linear transformation, by means of the first $\hat{r}$ ordinary principal components of the variables $\mathbf{x}_{nt}$ in the data set. Setting

$$\hat{\Gamma}_k^x = \frac{1}{T}\sum_{h=k+1}^{T} \mathbf{x}_{nt}\mathbf{x}_{n,t-k}'$$

and $\hat{\mu}_j^x$ the jth greatest eigenvalue of the sample variance matrix $\hat{\Gamma}_0^x$, the ordinary principal components method gives

$$\mathbf{f}_t = (\hat{f}_{1t} \cdots \hat{f}_{\hat{r}t})' = \hat{A}_n'\mathbf{x}_{nt} = \begin{pmatrix} \hat{a}_{11} & \cdots & \hat{a}_{n1} \\ \hat{a}_{12} & \cdots & \hat{a}_{n2} \\ \vdots & & \vdots \\ \hat{a}_{1\hat{r}} & \cdots & \hat{a}_{n\hat{r}} \end{pmatrix} \begin{pmatrix} x_{1t} \\ x_{2t} \\ \vdots \\ x_{nt} \end{pmatrix}$$

where $\hat{A}_n$ is the $n \times \hat{r}$ matrix having on the jth column the normalized eigenvector $\hat{a}_j = (\hat{a}_{1j} \hat{a}_{2j} \cdots \hat{a}_{nj})'$ corresponding to $\hat{\mu}_j^x$ for $j = 1,..., \hat{r} \leq n$, hence

$$\hat{A}_n = \begin{pmatrix} \hat{a}_{11} & \cdots & \hat{a}_{1\hat{r}} \\ \hat{a}_{21} & \cdots & \hat{a}_{2\hat{r}} \\ \vdots & & \vdots \\ \hat{a}_{n1} & \cdots & \hat{a}_{n\hat{r}} \end{pmatrix} = (\hat{a}_1' \hat{a}_2' \cdots \hat{a}_{\hat{r}}')'.$$

Step 3) We set a number of lags $\hat{p}$ and run a VAR($\hat{p}$) as in (2.3) with the estimated static factors $\hat{\mathbf{f}}_t$ to get estimates $\hat{D}(L)$ and $\hat{\boldsymbol{\varepsilon}}_t$ of the matrix function $D(L)$ and the residuals $\boldsymbol{\varepsilon}_t$, respectively. Recall that $\varepsilon_t = R\mathbf{u}_t$ in (2.3).

Step 4) We estimate the number q of the dynamic factors $\mathbf{u}_t = (u_{1t} \cdots u_{qt})'$ obtained by using three criteria which were described in Bai and Ng (2007), Stock and Watson (2005), and Onastki (2009), respectively. Denote this estimate by $\hat{q}$.

Step 5) Now let $\hat{\Gamma}^{\varepsilon}$ denote the sample variance-covariance matrix of the estimated residuals $\hat{\boldsymbol{\varepsilon}}_t$. Having an estimate $\hat{q}$ of the number of dynamic factors $\mathbf{u}_t = (u_{1t} \cdots u_{\hat{q}t})'$, we obtain an estimate of a non-structural representation of the common components by using the spectral decomposition $\hat{\Gamma}^{\varepsilon}$. More precisely, let $\hat{\mu}_j^{\varepsilon}$, $j = 1, \ldots, \hat{q}$ be the jth eigenvalue of $\hat{\Gamma}^{\varepsilon}$, taken in decreasing order, $|\hat{\mu}_1^{\varepsilon}| > \ldots > |\hat{\mu}_{\hat{q}}^{\varepsilon}|$. Let $\hat{M} = Diag(\sqrt{\hat{\mu}_j^{\varepsilon}})$ be the $\hat{q} \times \hat{q}$ diagonal matrix with $\sqrt{\hat{\mu}_j^{\varepsilon}}$ as its (j, j)-entry, and $\hat{K}$ the $\hat{r} \times \hat{q}$ matrix having on the columns the normalized eigenvectors corresponding to $\hat{\mu}_1^{\varepsilon},..., \hat{\mu}_{\hat{q}}^{\varepsilon}$. Then the spectral decomposition states $\hat{\Gamma}^{\varepsilon} = \hat{K}\hat{M}\hat{M}'\hat{K}' = \hat{S}\hat{S}'$, where $\hat{S} = \hat{K}\hat{M}$. Thus our estimated matrix of non-structural impulse-response functions in (2.4) is $\hat{C}_n(L) = \hat{A}_n(\hat{D}(L))^{-1}\hat{S}$. Recall that by definition we have $C_n(L) = B_n(L)H$ and $B_n(L) = A_n D(L)^{-1} R$, where $R = SH'$.

Step 6) Finally, we obtain $\hat{H}$ by imposing our identification restrictions on $\hat{B}_m(L) = \hat{C}_m(L)\hat{H}'$. Thus we get estimates

$$\hat{R} = \hat{S}\hat{H}' \qquad \hat{B}_n(L) = \hat{A}_n \hat{D}(L)^{-1} \hat{R}. \tag{3.1}$$

3. Consistency

Consistency of $\hat{B}_n(L) = \hat{A}_n \hat{D}(L)^{-1} \hat{R}$ as estimator of the impulse-response function $B_n(L)$ for large cross-sections and large sample size, that is, $n, T \to \infty$, was proved in Forni et al.(2009), Proposition 3. For this, it is necessary to state a last assumption.

FM5. *Denote by* $\gamma_{ij,k}^x$ *and* $\hat{\gamma}_{ij,k}^x$ *the* (i, j)*-entries of* Γ_k^x *and* $\hat{\Gamma}_k^x$*, respectively. There exists a positive real number* ρ *such that*

$$T E[(\hat{\gamma}_{ij,k}^x - \gamma_{ij,k}^x)^2] < \rho$$

for $k = 0,1$ *and for all positive integers* i, j *and* T.

Proposition 3.1 *Let $\hat{b}_{ni}(L)$ and $b_{ni}(L)$ denote the i- th rows of the matrix functions $\hat{B}_n(L)$ and $B_n(L)$, respectively. Under assumptions PA1-2 and FM1-5, $\hat{b}_{ni}(L)$, for a fixed i, is a consistent estimator of $b_{ni}(L)$, that is,*

$$\underset{\delta_{nt}\to\infty}{plim}\,\hat{b}_{ni}(L) = b_{ni}(L)$$

where $\delta_{nt} = min(n,T)$, n is the number of variables, and T is the number of observations over time.

For the proof see Forni, *et al.*(2009).

Proposition 3.1 states that consistency is achieved along any path for (n,T) with n and T both tending to infinity. The consistency rate is given by $\sqrt{\delta_{nt}} = min(\sqrt{n},\sqrt{T})$. This implies that if the cross-section dimension n is large relative to the sample size T, that is, $T/n \to 0$, the rate of consistency is $\sqrt{T}$, the same we would obtain if the common components were observed, that is, if the variables were not contamined by idiosyncratic components. On the other hand, if $n/T \to 0$, then the consistency rate is $\sqrt{n}$ reflecting the fact that the common components are not observed but have to be estimated.

Here we give a simplified proof of Proposition 3.1. For an $m\times n$ real matrix $\mathbf{A} = (a_{ij})$, the matrix norm $\|\mathbf{A}\|$ of $\mathbf{A}$ is defined as $\|\mathbf{A}\|= \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^2} = \sqrt{tr(\mathbf{A}'\mathbf{A})}.$ Let $\mathbf{E}$ and $\mathbf{F}$ be two $n\times n$ symmetric matrices and denote by $\sigma_j(\cdot)$, $j=1,\ldots,n$, the eigenvalues in decreasing order of magnitude. We shall use the well--known inequalities due to Weyl:

$$|\sigma_j(\mathbf{E}+\mathbf{F})-\sigma_j(\mathbf{E})|\le\sqrt{\sigma_1(\mathbf{F}^2)}\le\sqrt{tr(\mathbf{F}^2)} =\|\mathbf{F}\|.$$

Denote by Λ_n and $\hat{\Lambda}_n$, the $r\times r$ diagonal matrices having on the diagonal elements the first r largest eigenvalues of $\Gamma_{n0}^{\chi} =E(\boldsymbol{\chi}_{nt}\boldsymbol{\chi}_{nt}')$ and $\hat{\Gamma}_{n0}^{x} =E(\hat{\mathbf{x}}_{nt}\hat{\mathbf{x}}_{nt}')$, respectively. Let us recall here our notation for the eigenvalues of the relevant matrices:

$$\mu_{nj}^{x} := \sigma_j(\Gamma_{n0}^{x}), \hat{\mu}_{nj}^{x} := \sigma_j(\hat{\Gamma}_{n0}^{x}), \mu_{nj}^{\chi} := \sigma_j(\Gamma_{n0}^{\chi}) \quad \text{and} \quad \mu_{nj}^{\xi} := \sigma_j(\Gamma_{n0}^{\xi}).$$

Hence we have $\Lambda_n = Diag(\mu_{n1}^{\chi},\cdots,\mu_{nr}^{\chi})$ and $\hat{\Lambda}_n = Diag(\hat{\mu}_{n1}^{x},\cdots,\hat{\mu}_{nr}^{x}).$

From (1.5) and Step 2, Section 2, A_n and $\hat{A}_n$ are the $n\times r$ matrices having on the columns the normalized eigenvectors corresponding to μ_{nj}^{χ} and $\hat{\mu}_{nj}^{x}$ for $j=1,\cdots,r\le n$, hence we have $\Gamma_{n0}^{\chi} = A_n\Lambda_n A_n'$ and $\hat{\Gamma}_{n0}^{x} = \hat{A}_n\hat{\Lambda}_n\hat{A}_n'$.

Lemma 3.2 *Under assumptions PA1-2 and FM0-5, as $n,T\to\infty$, we have*

(i) $\|\hat{\Gamma}_{n0}^{x} - \Gamma_{n0}^{x}\|^2 = tr[(\hat{\Gamma}_{n0}^{x} - \Gamma_{n0}^{x})^2] = O_p(\frac{n^2}{T})$

(ii) $\frac{1}{n}\hat{\mu}_{nj}^{x} = \frac{1}{n}\mu_{nj}^{\chi} + O_p(\frac{1}{n}) + O_p(\frac{1}{\sqrt{T}})$ for $j=1,\cdots,r$

Proof. By assumption FM5 there exists a positive constant ρ such that for all $T \in N$ and $i.j \in N$, $T\,E[(\hat{\gamma}^x_{ij,0} - \gamma^x_{ij,0})^2] < \rho$ as $T \to \infty$, where $\hat{\gamma}^x_{ij,0}$ and $\gamma^x_{ij,0}$ denote the (i,j)-entries of $\hat{\Gamma}^x_{n0}$ and Γ^x_{n0}, respectively. We have

$$tr[(\hat{\Gamma}^x_{n0} - \Gamma^x_{n0})^2] = \sum_{i=1}^{n}\sum_{j=1}^{n}(\hat{\gamma}^x_{ij,0} - \gamma^x_{ij,0})^2.$$

Taking expectations yields

$$0 \le E\left[\sum_{i=1}^{n}\sum_{j=1}^{n}(\hat{\gamma}^x_{ij,0} - \gamma^x_{ij,0})^2\right] = \sum_{i=1}^{n}\sum_{j=1}^{n}E[(\hat{\gamma}^x_{ij,0} - \gamma^x_{ij,0})^2] < \frac{\rho n^2}{T}$$

hence

$$\| \hat{\Gamma}^x_{n0} - \Gamma^x_{n0} \|^2 = O_p\left(\frac{n^2}{T}\right) \quad \text{which proves (i).}$$

Turning to (ii), from the Weyl inequality, we have (use also(i)):

$$(\hat{\mu}^x_{nj} - \mu^x_{nj})^2 \le tr[(\hat{\Gamma}^x_{n0} - \Gamma^x_{n0})^2] = O_p\left(\frac{n^2}{T}\right)$$

hence

$$0 \le | \hat{\mu}^x_{nj} - \mu^x_{nj} | \le \sqrt{tr[(\hat{\Gamma}^x_{n0} - \Gamma^x_{n0})^2]} = O_p\left(\frac{n}{\sqrt{T}}\right).$$

Moreover, from assumptions FM0-3

$$\frac{1}{n}\mu^x_{nj} = \frac{1}{n}\mu^\chi_{nj} + \frac{1}{n}\mu^\xi_{nj} \le \frac{1}{n}\mu^\chi_{nj} + \frac{1}{n}\mu^\xi_{n1} \le \frac{1}{n}\mu^\chi_{nj} + \frac{d}{n} = \frac{1}{n}\mu^\chi_{nj} + O_p\left(\frac{1}{n}\right)$$

since $\mu^\xi_{nj} \le \mu^\xi_{n1} \le d$ by FM3. Then we have:

$$\frac{1}{n}(\hat{\mu}^x_{nj} - \mu^\chi_{nj}) = \frac{1}{n}(\hat{\mu}^x_{nj} - \mu^x_{nj}) + \frac{1}{n}(\mu^x_{nj} - \mu^\chi_{nj}) = O_p\left(\frac{n}{\sqrt{T}}\right) + O_p\left(\frac{1}{n}\right) \quad \text{which proves (ii).}$$

From Section 2, Step 6, we have:

$$\hat{B}_n(L) = \hat{A}_n\hat{D}(L)^{-1}\hat{R}_n = \hat{A}_n(I + \hat{D}_nL + (\hat{D}_n)^2L^2 + \cdots)\hat{R}_n$$

and

$$B_n(L) = A_n(I + D_nL + D_n{}^2L^2 + \cdots)R_n.$$

Here we assume for simplicity the VAR specification with one lag, the extension to a finite number of lags being immediate.

Lemma 3.3 *Under assumptions PA1-2 and FM0-5, as $n, T \to \infty$, we have:*

(i) $\frac{1}{n}\| \hat{\Lambda}_n - \Lambda_n \| = O_p(\frac{1}{n}) + O_p(\frac{1}{\sqrt{T}})$

(ii) $\| A_n^{'}\hat{A}_n - I_r \| = O_p(\frac{1}{\sqrt{n}}) + O_p(\frac{1}{\sqrt{T}})$

(iii) $\| \hat{D}_n - D_n \| = O_p(\frac{1}{\sqrt{n}}) + O_p(\frac{1}{\sqrt{T}})$

(iv) $\| \hat{R}_n - R_n \| = O_p(\frac{1}{\sqrt{n}}) + O_p(\frac{1}{\sqrt{T}})$

Proof. By Lemma 3.2 (ii) we have:

$$\frac{1}{n}\| \hat{\Lambda}_n - \Lambda_n \| = \frac{1}{n}\sqrt{\sum_{j=1}^{r}(\hat{\mu}_{nj}^{x} - \mu_{nj}^{\chi})^2} = O_p\left(\frac{1}{n}\right) + O_p\left(\frac{1}{\sqrt{T}}\right).$$

Turning to (ii) we have the following decomposition:

$$\begin{aligned}\frac{1}{n}\hat{\Lambda}_n &= \frac{1}{n}\hat{A}_n^{'}\hat{\Gamma}_{n0}^{x}\hat{A}_n = \frac{1}{n}\hat{A}_n^{'}(\Gamma_{n0}^{x} + \hat{\Gamma}_{n0}^{x} - \Gamma_{n0}^{x})\hat{A}_n \\ &= \frac{1}{n}\hat{A}_n^{'}\Gamma_{n0}^{x}\hat{A}_n + \frac{1}{n}\hat{A}_n^{'}(\hat{\Gamma}_{n0}^{x} - \Gamma_{n0}^{x})\hat{A}_n \\ &= \frac{1}{n}\hat{A}_n^{'}(\Gamma_{n0}^{\chi} + \Gamma_{n0}^{\xi})\hat{A}_n + \frac{1}{n}\hat{A}_n^{'}(\hat{\Gamma}_{n0}^{x} - \Gamma_{n0}^{x})\hat{A}_n \\ &= \frac{1}{n}\hat{A}_n^{'}A_n\Lambda_nA_n^{'}\hat{A}_n + \frac{1}{n}\hat{A}_n^{'}\Gamma_{n0}^{\xi}\hat{A}_n + \frac{1}{n}\hat{A}_n^{'}(\hat{\Gamma}_{n0}^{x} - \Gamma_{n0}^{x})\hat{A}_n.\end{aligned}$$

From Lemma 3.2 (i) we get:

$$\frac{1}{n}\| \hat{A}_n^{'}(\hat{\Gamma}_{n0}^{x} - \Gamma_{n0}^{x})\hat{A}_n \| \le \frac{1}{n}\sqrt{tr[(\hat{\Gamma}_{n0}^{x} - \Gamma_{n0}^{x})^2]} = O_p\left(\frac{1}{\sqrt{T}}\right).$$

Moreover, we have:

$$\begin{aligned}\frac{1}{n}\| \hat{A}_n^{'}\Gamma_{n0}^{\xi}\hat{A}_n \| &\le \frac{1}{n}\sqrt{tr(\Gamma_{n0}^{\xi})^2} = \frac{1}{n}\sqrt{\sum_{j=1}^{n}(\mu_{nj}^{\xi})^2} \\ &\le \frac{1}{n}\sqrt{n}|\mu_{n1}^{\xi}| \le \frac{d}{\sqrt{n}} = O\left(\frac{1}{\sqrt{n}}\right)\end{aligned}$$

by assumption FM3. The statement (ii) follows. The statements (iii) and (iv) can be proved by similar arguments.

Proof of Proposition 3.1. By Lemma 3.3 (ii) $A_n^{'}\hat{A}_n$ converges to I_r as $n,T \to \infty$ hence $\hat{A}_n$ converges to A_n as $n,T \to \infty$. By Lemma 3.3 (iv) $\hat{R}_n$ converges to R_n as $n,T \to \infty$. Continuity of the matrix product (notice that D_n has fixed dimension r), implies:

$$(\hat{D}_n)^h = (D_n)^h + O_p\left(\frac{1}{\sqrt{n}}\right) + O_p\left(\frac{1}{\sqrt{T}}\right)$$

hence $(\hat{D}_n)^h$ converges to $(D_n)^h$, as $n,T \to \infty$, for any $h \in N$.

Thus the matrix function $\hat{B}_n(L)$ converges entry-by-entry to $B_n(L)$ as $n,T \to \infty$. This completes the proof of the consistency.

4. Further Convergence Results

The following result states that $\hat{\chi}_{nt}$ converges to χ_{nt} in mean square. See Forni et al.(2000), Proposition (2), for a different proof.

Proposition 4.1

$$\underset{\delta_{nt}\to\infty}{plim}\ var(\chi_{nt}-\hat{\chi}_{nt})=0$$

Proof. Recall that $\chi_{nt}=B_n(L)\mathbf{u}_t$, where $B_n(L)=A_nN(L)$ and $N(L)$ (and hence $B_n(L)$) is an $r\times q$ (resp. $n\times q$) absolutely summable matrix function of L by FM0. So we can set $\chi_{nt}=\sum_{k=0}^{\infty}b_{nk}\mathbf{u}_{t-k}$, where $b_{nk}(L)$ is an $n\times q$ absolutely summable matrix function. Further, $\mathbf{u}_t=(u_{1t}\cdots u_{qt})'$ is an orthonormal vector white noise, hence $E(\mathbf{u}_t\mathbf{u}_t')=I_q$ and $E(\mathbf{u}_t\mathbf{u}_\tau')=0$ for $t\neq\tau$. Then we have:

$$\begin{aligned}var(\chi_{nt}-\hat{\chi}_{nt}) &= E(\chi_{nt}-\hat{\chi}_{nt})(\chi_{nt}-\hat{\chi}_{nt})' \\ &= E(\sum_{h=0}^{\infty}\sum_{k=0}^{\infty}(b_{nk}-\hat{b}_{nk})\mathbf{u}_{t-h}\mathbf{u}_{t-k}'(b_{nk}-\hat{b}_{nk})') \\ &= \sum_{h=0}^{\infty}(b_{nk}-\hat{b}_{nk})(b_{nk}-\hat{b}_{nk})'.\end{aligned}$$

The expectation operator can be moved inside summation because the considered matrix series are absolutely summable. Now the result follows from Proposition 3.1.

This implies that $\hat{\chi}_{nt}$ is a consistent estimator of χ_{nt}, that is, $\underset{\delta_{nt}\to\infty}{plim}\hat{\chi}_{nt}=\chi_{nt}$. The fact $\hat{\chi}_{nt}-\chi_{nt}\xrightarrow{p}0$ gives also the following result on the convergence in distribution: if $\hat{\chi}_{nt}\xrightarrow{L}N(\mu,\sigma^2\mathbf{I})$, then $\chi_{nt}\xrightarrow{L}N(\mu,\sigma^2\mathbf{I})$.

Corollary 4.2

$$\underset{\delta_{nt}\to\infty}{plim}\ var(\mathbf{x}_{nt}-\hat{\chi}_{nt})=E(\xi_{nt}\xi_{nt}')=\Gamma_{n0}^{\xi}$$

Proof. We have:

$$\begin{aligned}&var(x_{nt}-\hat{\chi}_{nt})=E(\chi_{nt}+\xi_{nt}-\hat{\chi}_{nt})(\chi_{nt}'+\xi_{nt}'-\hat{\chi}_{nt}') \\ &=var(\chi_{nt}-\hat{\chi}_{nt})+E((\chi_{nt}-\hat{\chi}_{nt})\xi_{nt}')+E(\xi_{nt}(\chi_{nt}'-\hat{\chi}_{nt}'))+E(\xi_{nt}\xi_{nt}').\end{aligned}$$

Now the idiosyncratic components ξ_{it} are orthogonal to the $\mathbf{u}$'s at any lead and lag. Thus the second and the third summands vanish as n, T go to infinity. By Proposition 4.1, we get the result.

The matrix function $N(L)$, where $B_n(L)=A_nN(L)$, is left invertible (fundamentalness), that is, there exists an $q\times r$ square-summable filter $G(L)$ such that $G(L)N(L)=I_q$. Then we have:

Proposition 4.3 *An estimator $\hat{\chi}_{nt}$ of the common components χ_{nt} can be obtained asymptotically from the sequence $K_n(L)\mathbf{x}_{nt}$, where the square-summable filter $K_n(L)$ is given by $K_n(L)=B_n(L)G(L)\hat{A}_n'$. More precisely, we have:*

$$\underset{\delta_{nt}\to\infty}{plim}\ K_n(L)\mathbf{x}_{nt}=\hat{\chi}_{nt}$$

Proof. Multiplying by $\hat{A}_n^{'}$ on the left the equation:

$$\mathbf{x}_{nt} = \chi_{nt} + \xi_{nt} = B_n(L)\mathbf{u}_t + \xi_{nt} = A_n N(L)\mathbf{u}_t + \xi_{nt}$$

Yields

$$\hat{A}_n^{'}\mathbf{x}_{nt} = \hat{A}_n^{'}A_n N(L)\mathbf{u}_t + \hat{A}_n^{'}\xi_{nt}.$$

By Lemma 3.3(ii) $\hat{A}_n^{'}A_n \xrightarrow{p} I_r$ as n,T go to infinity. Hence for n,T sufficiently large, we can write:

$$\hat{A}_n^{'}\mathbf{x}_{nt} = N(L)\mathbf{u}_t + \hat{A}_n^{'}\xi_{nt}.$$

Thus, for n,T sufficiently large, we get:

$$G(L)\hat{A}_n^{'}\mathbf{x}_{nt} = G(L)N(L)\mathbf{u}_t + G(L)\hat{A}_n^{'}\xi_{nt}$$

hence

$$\mathbf{u}_t = G(L)\hat{A}_n^{'}\mathbf{x}_{nt} - G(L)\hat{A}_n^{'}\xi_{nt}$$

and

$$\chi_{nt} = B_n(L)\mathbf{u}_t = B_n(L)G(L)\hat{A}_n^{'}\mathbf{x}_{nt} - B_n(L)G(L)\hat{A}_n^{'}\xi_{nt}.$$

So, for n,T sufficiently large, we obtain:

$$\hat{\chi}_{nt} = B_n(L)G(L)\hat{A}_n^{'}\mathbf{x}_{nt} = K_n(L)\mathbf{x}_{nt}$$

where $K_n(L) = B_n(L)G(L)\hat{A}_n^{'}$.

From above, we see that the filters $K_{ni}(L)$ can be obtained by their empirical counterparts based on finite realizations of the form $\mathbf{x}_n^T = (x_{n1}\cdots x_{nT})^{'}$. Thus we can write:

$$\hat{K}_n(L) = \hat{B}_n(L)\hat{G}(L)\hat{A}_n^{'}.$$

From the above consistency results, it follows:

$$\underset{\delta_{nt}\to\infty}{plim}\ \hat{K}_n(L) = K_n(L).$$

Finally, the process $\left\{\hat{\chi}_{nt} : t \in Z\right\}$ admits a Wold representation for n and T sufficiently large. Since $\mathbf{x}_{nt}$ has a Wold representation as in (PA2), the process $\left\{\hat{\chi}_{nt} : t \in Z\right\}$ asymptotically can be expressed as

$$K_n(L)\mathbf{x}_{nt} = \sum_{k=0}^{\infty} K_{nk}\mathbf{x}_{n,t-k} = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} K_{nk}C_h^n\mathbf{w}_{n,t-k-h}.$$

5. Conclusion

In this paper we have proved some convergence results concerning with stochastic variables which define our dynamic factor model. Our convergence results show the appropriate statistical properties that qualify such a model. The factor model enables us to handle a large amount of information and then it avoids important limitations of structural VAR models. For this reason, it is an important tool to be used for economic and financia applications.

Acknowledgements

Part of this paper is taken from the author's M.Sc. thesis entitled "*Structural Macroeconomic Analysis for Dynamic Factor Models*" defended at the University of Modena and Reggio Emilia (Italy). A survey of the thesis will appear in Rivista di Politica Economica, Rome, Italy. The author would like to thank the Department of Economics, at University of Venice, Italy, where this work was carried out during the Ph.D. program.

References

[1] Bai, J., and Ng, S., 2002. Determining the number of factors in approximate factor models, *Econometrica* 70(1):191-221.

[2] Bai, J., and Ng, S., 2007. Determining the number of primitive shocks in factor models, *Journal of Business and Economic Statistics*, 25: 52-60.

[3] Chamberlain, G., and Rothschild, M., 1983. Arbitrage, factor structure and mean-variance analysis in large asset markets, *Econometrics*, 51: 1305-1324.

[4] Forni, M., Giannone, D., Lippi, M., and Reichlin, L. 2009. Opening the black box: structural factor models with large cross-sections, *Econometric Theory*, 25: 1319-1347.

[5] Forni, M., and Gambetti, L., 2010. The dynamic effects of monetary policy: a structural factor model approach, *Journal of Monetary Economics*, 57: 203-216.

[6] Forni, M., Hallin, M., Lippi, M., and Reichlin, L. 2000. The generalized dynamic factor model: identification and estimation, *The Review of Economics and Statistics*, 82: 540-554.

[7] Forni, M., and Lippi, M., 2001. The generalized dynamic factor model: representation theory, *Econometric Theory*, 17: 1113-1141.

[8] Onastski, A., 2009. *Testing hypotheses about the number of factors in large factor models, manuscript,* Columbia University, 2009.

[9] Rozanov, Y., *Stationary Random Process*, Holden Day, San Francisco, USA, 1967.

[10] Stock, J., and Watson, M. 2005. *Implications of dynamic factor models for VAR analysis*, NBER Working Papers, No.11467.

ITALY AFTER THE CRISIS: A CASE OF RECOVERYLESS CREDIT GROWTH

Antonio FORTE
Centro Europa Ricerche, **Italy**
forteantonio80@gmail.com

Abstract:

In this study I compare the credit condition with the economic growth in Italy from January 2007 onward. Starting from the literature on the creditless recovery, I highlight the specific features of the Italian situation in which, notwithstanding the prolonged and deep economic crisis, the credit has persistently continued to grow. A comparison with the German case confirms the peculiar characteristics of the Italian condition. An econometric study supports this idea and, in order to depict this Italian economic situation, I propose a new expression: the recovery-less credit growth.

Keywords: Italy, credit, recovery.

JEL Classification: E32, E50.

1. Introduction

Recent studies have analysed a phenomenon that can characterize the post-crisis periods: the so called creditless recovery. When a creditless recovery occurs one can observe economic growth together with a negative, null or very limited credit expansion. In this respect, it is interesting to analyse the Italian situation because Italy is one of the nations that suffered a very deep recession in 2008-2009. The final result of the paper is fascinating because the Italian economic framework after the crisis has been completely different from the so called creditless recovery.

1.2. Literature

Abiad, Dell'Ariccia, and Li (2011), and Coricelli, and Roland (2011) recently focused on this specific issue. In the first cited work the authors stated that the creditless recovery has been not so uncommon in the past. Indeed, the creditless recovery has been frequently observed in their sample, the twenty per cent of the recoveries they analysed occurred without a credit growth. Coricelli, and Roland (2011) focused on the credit flows instead of the credit level. They discovered that, in certain circumstances, a nation can grow even if the credit does not support the economy. According to their work, it is necessary that firms have alternative sources of financing, through a developed financial framework, in order to observe a creditless recovery. Moreover, the authors underlined that, when a creditless recovery occurs, industrial sectors more linked to banking system undergo a slower recovery. The crucial point in this strand of literature is that a recovery can occur even if the credit does not show a brilliant upturn after a recession.

A study by Calvo, Izquierdo, and Talvi (2006) analysed this type of economic framework and found that the creditless recovery has been not so infrequent in the past, especially in the emerging economies. Furthermore, other works, like the one by Claessens, Kose, and Terrones (2008) found that the creditless recoveries occurred in industrialized countries too. In addition, as pointed out by Calvo, Izquierdo, and Talvi (2006), even the Great Depression showed some features that are typical of a creditless recovery. Following this last study, Calvo, and Loo-Kung (2010) proposed a short work in which they focused on the subprime crisis in the US underlining the common traits between the Great Crisis and the so called Phoenix Miracle. According to their opinion, even the US ongoing crisis is showing features that are very similar to the ones of a creditless recovery.

Demirgüç-Kunt, Detragiache, and Gupta (2006) analysed 36 banking crises in 35 countries, included Italy, and, among the other results, they emphasized that the economy returned to pre-crises level in a shorter time if compared to the credit performance. They also stated that, during the banking crises they analysed, recoveries did not seem to be driven by resumption in bank lending. In other words, they found a decoupling of these two indicators in the aftermath of these crises.

This type of recovery is interesting because, as affirmed by Kroszner, Laeven, and Klingebiel (2006), the productive sectors linked in a close way to the banking system and that operates in a country with a well-developed financial system suffer a deeper value added decrease in comparison with sectors that have a lighter linkage with banks. This feature is confirmed by Dell'Ariccia, Detragiache, and Rajan (2007). They found that, in case of a banking crisis, sectors highly dependent on external finance show a very negative performance and this result is even more evident in developing countries or in countries with limited access to foreign finance. This aspect can be useful to understand the different performance of the industrial sectors if the banking system decides to cut loans, creating a credit crunch. Moreover, on this issue, Holmstrom, and Tirole (1997) linked credit tightening to firms condition and established that enterprises with low levels of capital suffered with more intensity the impact of a credit crunch.

Given this strong linkage between credit and real economy, it seemed interesting to analyse the situation in Italy during the last years in order to establish if the inversion of the economic cycle has been characterized by a slow or a rapid credit growth. In fact, in Italy firms are deeply linked to the national banking system and this could act as a brake to recovery if banks decide to limit or reduce the credit flows. Besides, Italy did not experienced bank failures or financial instability but the Great Crisis has had a large impact on real economy.

The results shown in the next pages will highlight a very particular picture for the Italian case. The rest of the paper is organized as follows. A macro scenario and some economic data about Italy are proposed in the second section. In section three I show data on gross domestic product, industrial production and credit in Italy and I compare the Italian case with the German one. The fourth section is focused on the econometric results used to support the main idea of the paper. The last section ends the paper with conclusions.

2. The Macroeconomic Scenario

As pointed out by many studies, creditless recoveries usually take place if the previous economic downturn has been characterized by banking or/and real estate crises. When these two destabilizing economic phenomena led to a crisis, the following recovery has been quite always characterized by a very slow increase in credit. Given the fact that the Great Crisis has been preceded by both global banking and real estate crises the importance of studying the Italian situation after the crisis is increased.

In the previously cited works some explanations of the creditless recovery have been proposed. For example, after a slump it is possible that firms increase the production using the unused capacity and, in so doing, they do not need new or additional funding. As a consequence, one can observe an increase in industrial production and in gross domestic product accompanied by a stable amount of loans.

A second explanation could be linked to the operational choices of the banks. During a post crisis period, banks could focus on high productivity industrial sectors that usually show a better performance in the short to medium run and, on the same time, they can reduce loans to mature sectors that typically have low productivity and a slower upturn. This behaviour can lead to a growth in production and GDP even if the total amount of loans remains stable. So, a different behaviour of the banking system towards the borrowers can explain the decoupling between credit and economic growth.

Table 1. GDP trend in major economies

Yearly percentage change	2007	2008	2009	2010
Australia	4.6	2.6	1.3	2.7
Canada	2.2	0.5	-2.5	3.1
France	2.3	0.1	-2.5	1.5
Germany	2.8	0.7	-4.7	3.5
Italy	1.5	-1.3	-5.2	1.3
Japan	2.3	-1.2	-6.3	3.9
South Korea	5.1	2.3	0.2	6.1
Singapore	8.8	1.5	-0.8	14.5
United Kingdom	2.7	-0.1	-4.9	1.3
United States	1.9	0	-2.6	2.8

Source: International Monetary Fund

A third possible explanation of a creditless recovery is that firms can search for new funding through different channels by-passing the banking system. As a consequence, statistics do not register an increase in banking loans because firms receive new flows of funding from other sources but, at the same time, we observe a recovery.

Finally, another possibility is that firms use their own liquidity and capital to restart the activity in the immediate aftermath of the crisis. This can also lead to a different trend between production and loans.

These possible explanations of the creditless recovery do not reduce the importance of studying what has happened in Italy after the recession. Indeed, Italy has been one of the nations that suffered a very deep decrease in the GDP in 2008 and 2009, see Table 1. It could be interesting to find some linkages between the Italian economic trend and the state of the credit.

It is essential to remember that Italy registered a great reduction in the GDP but the decrease of real estate prices has been very limited and banks did not suffer any problems thanks to a traditional way of making their own business. This framework is compelling because one can suppose that, given a not so dramatic situation for the banking system, the natural consequence should have been a rapid recovery of both the real economy and the credit aggregates. But data depict a different scenario.

3. The Italian Situation

This paragraph shows some data about the Italian economic scenario. After a very deep recession, the industrial production and the GDP showed a recovery. But, the economic growth has been very light during the period analysed in this section. The indicators remain really far from the peaks reached before the crisis. Starting from the first quarter 2007, figure 1 shows the trend of real GDP, industrial production and loans to firms and households, while figure 2 plots the annual change of the loans.

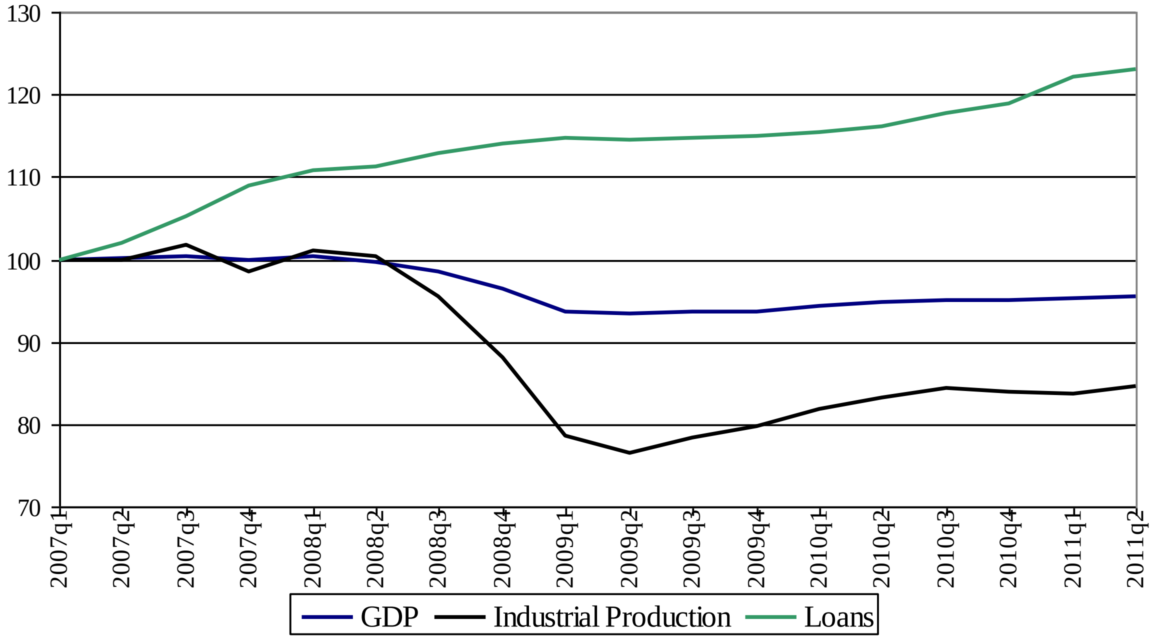


Figure 1. Industrial production, real GDP and loans in Italy

Source: Personal elaboration on Central Bank of Italy and ISTAT data

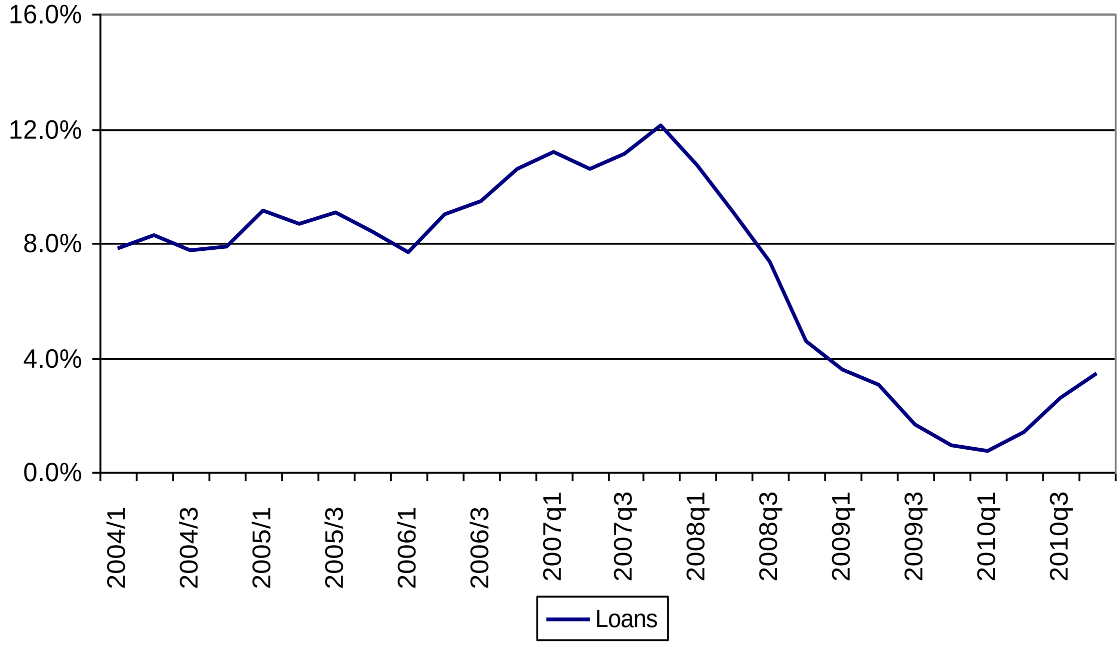


Figure 2. Annual change of loans in Italy

Source: Personal elaboration on Central Bank of Italy data. Loans: firms and households loans by banks

Loans show a positive stable trend during 2007, then we observe a reduction in the growth trend and only during the last quarter it seems that the growth restarts with a more intensive pace, see figure 2 for more details[1].

[1] In June 2010 the Bank of Italy changed the time series calculation for loans, but even if we correct the data for this change the trend of the loans remains positive.

The evolution of real GDP and industrial production is completely different. The real GDP remained stable from the first quarter of 2007 to the mid 2008, then we observe the recession and only in the second part of 2009 the cycle became again positive, even if the growth has been very slow. A similar trend has been traced by the industrial production. But, in this case, the decrease of the index has been stronger. Indeed, the industrial production index declined from a value of 101 in the first quarter of 2008 to 76.5 in the second quarter of 2009.

Summing up these first data, we can observe two main features. GDP and industrial production showed a reduction while loans showed only a slowdown in their trend. Loans annual change has always been positive see figure 2. The second peculiar feature is that the situation showed in the last quarter of the sample is really odd in the light of the previously cited literature: loans are more than 20 points above the value of the first quarter of 2007 while GDP and industrial production are 4,5 and 15,4 points below the starting value.

This situation is completely different from the so called Phoenix Miracle proposed by Calvo, Izquierdo, and Talvi (2006) or by Biggs, Mayer, and Pick (2009). In Italy we did not observe either a reduction of the stock of loans or negative global flows of the credit to firms and households. The consequence is that Italy did not certainly experience a creditless recovery, given the fact that the trend of the loans objectively induces to reject this scenario.

At the same time, we can also reject the hypothesis, made by Kannan (2010), of a linkage between the slow growth of firms that are more linked to banking funding and a reduction of the credit flows supplied by banks after the financial crises. This situation has not been observed in Italy because the total amount of loans has increased.

On the contrary, in Italy the credit condition has been positive during and after the crisis while the economy did not show a rapid inversion of the cycle. Just for this reason it is possible to reverse the structure of the sentence, together with its meaning, by constructing a new expression for the Italian case: the recovery-less credit growth.

This situation has been clearly depicted by Draghi (2011) in his Concluding Remarks during the Ordinary General Meeting of Shareholders of the Bank of Italy in May 2011. He remembered that Italy "recouped only two of the seven points of output lost" during the crisis and, at the same time, "banks have stepped up their lending to firms markedly". He added that in April 2011 the annual rate of growth of loans to firms was "the highest among the main countries of the euro area". Even the Draghi's words confirmed the idea of a *recovery-less credit growth* for Italy.

The Italian economic situation is probably affected by structural problems that reduce the potential growth of the economy, but the really slow recovery is not linked to credit problems.

In order to offer a more complete and robust framework, I compare the Italian situation to the German one. In this case I employ the same sources of the data to make them directly comparable. Data on industrial production and GDP are taken from Eurostat web site while data on loans to non financial corporations and households are taken from the ECB web site.

The situation is the one depicted in figures 3 and 4. It is straightforward to observe that loans to non financial corporations are more than 20 points above the reference period in Italy while in Germany they are 10 points above the first quarter of 2007. Moreover, if we compare the last datum with the pre-crisis pick we observe that loans are above the pre-crisis pick in Italy while they are still below in Germany. The two figures depict the loans to households too. In this case the difference between the German and the Italian trend is remarkable. In Germany this type of credit remained stable for the sample examined in the figure, while it showed a positive trend in Italy for the entire period. It is necessary to stress that between the second and the third quarter of 2010 this type of loans registered a change in the time series in Italy, so the increase registered during that period is abnormal. But, notwithstanding this, the trend remained positive for the whole period[2]. The situation is completely different for the industrial production. The last datum of the industrial production reached the pre crisis level in Germany while in Italy it is more than 15 points below. As regards the real GDP, Germany

[2] In figure 1 and 2 this time series change has been eliminated using data by Central Bank of Italy.

bridged the crisis losses and now the real GDP is at the same level of 2008, while real GDP is still below the 2007-2008 levels in Italy[3].

This comparison has highlighted the significant diversity between the German case and the Italian one. Given these results, it is possible to classify the German economic situation as a case of creditless recovery while, using the new terminology proposed in this paper it is possible to assert that the Italian economic situation from 2007 to 2011 is a recovery-less credit growth.

In the next section I support this analysis through an econometric study that focuses on Italy.

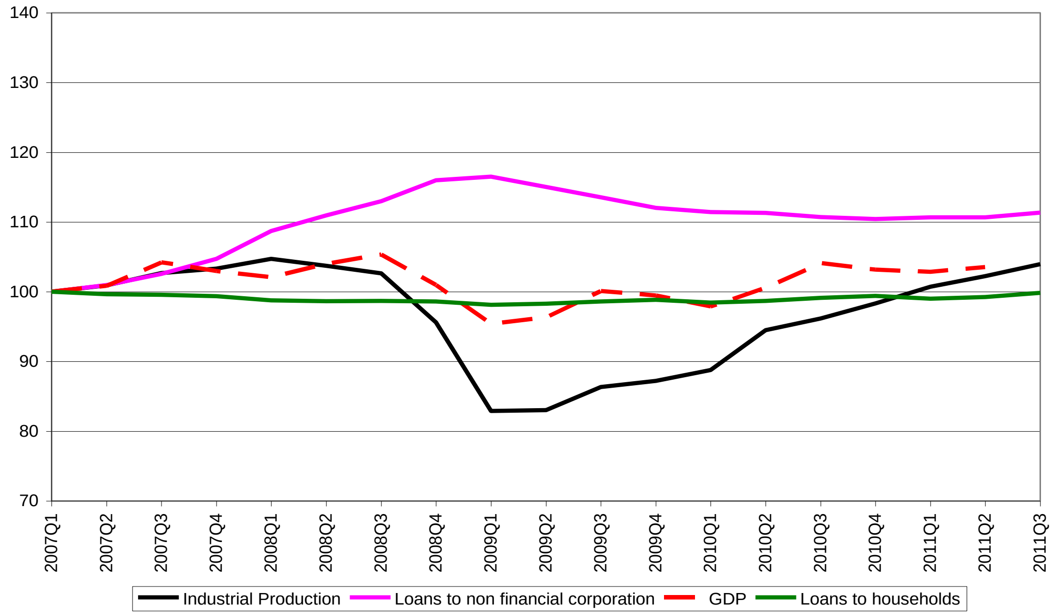


Figure 3. Industrial production, loans and GDP in Germany, reference period: first quarter of 2007

Source: personal elaboration on ECB and Eurostat data.

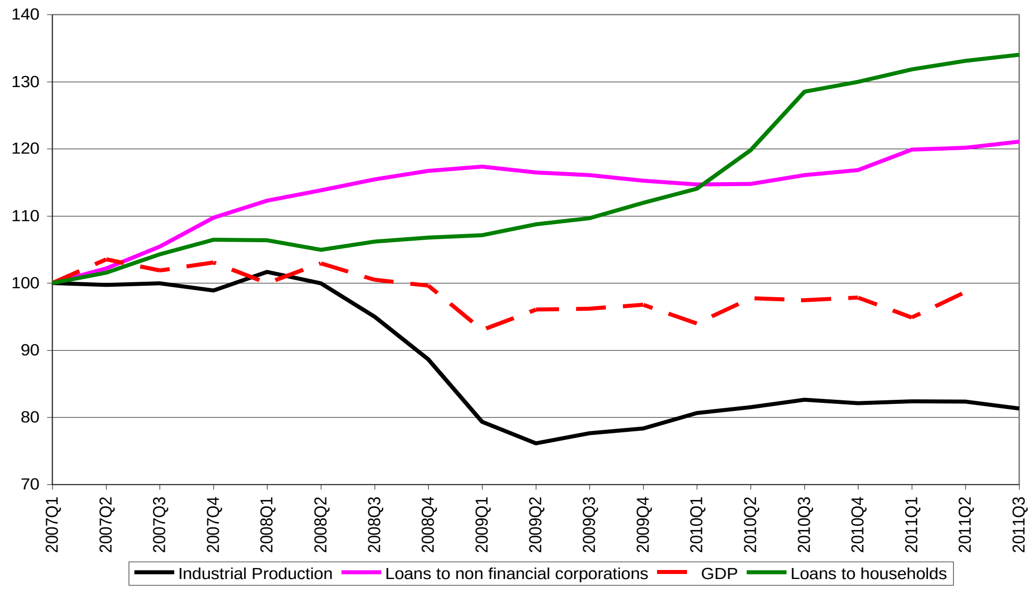


Figure 4. Industrial production, loans and GDP in Italy, reference period: first quarter of 2007

Source: personal elaboration on ECB and Eurostat data

[3] Figure 1 and Figure 4 show slightly different real GDP trends because data in Figure 1, by ISTAT, are corrected for seasonal effects while data in Figure 4, by Eurostat, are not.

4. An Econometric Study on the Italian Case

I decided to support these findings through an econometric study. In this way it is possible to confirm the idea that the Italian economy behaved in a different way during the last years. To this aim, in this paragraph, I show the regressions and the data used for the estimations.

I decided to employ data on GDP, from ISTAT, on 3 months Euribor rate, from Eurostat, on loans to non financial firms and households, from the Central Bank of Italy[4], and data on employees, from ISTAT. The dependent variable is the real GDP. None of the regressors are used in the calculation of the GDP. The choice of the regressors has been made in order to use three indicators of the economic situation: a monetary indicator, the Euribor rate, a credit condition indicator, the loans, and a real economy indicator, the number of employees. All the series have been downloaded in November 2011.

I computed the quarterly percentage change of these series to limit or to eliminate the presence of the unit root. The presence of unit roots has been tested through the Phillips Perron and the KPSS tests. Results are shown in Table 2 and they confirm the absence of stationarity problems.

Using these data, I estimated two equations. The first equation is the following one:

$$\Delta rgdp = c + \beta_1 * \Delta loans(-1) + \beta_2 * \Delta Euribor3m(-2) + \beta_3 * \Delta empl(-1) + \beta_4 * \Delta rgdp(-1) \quad (1)$$

where Δrgdp is the quarter over quarter real GDP growth rate, c is a constant, Δloans is the quarter over quarter percentage change of loans to non financial firms and households, ΔEuribor3m is the quarter over quarter percentage change of the 3 months Euribor rate, Δempl is the quarter over quarter percentage change of the total employees in Italy. There is a lagged dependent variable too. All the regressors are lagged. Loans and employees are one-period lagged while the Euribor rate is two periods lagged. This difference is linked to the slowness of the effects of the monetary policy. The series of the loans has been corrected, using data from the Central Bank of Italy, in order to eliminate the statistical break of June 2010. The results of this regression are shown in Table 3.

Table 2. Unit root analysis 1999:1q-2010:4q

Variable	PP test (lags:4, no trend)	KPSS test (lags:4, no trend)
Δ Euribor3m	-3.087 **	0.057
Δ empl	-4.251 ***	0.443
Δ real gdp	-3.061 ***	0.296
Δ loans	-2.923 *	0.386
* significant at 1 per cent level, ** significant at 5 per cent level, *** significant at 1 per cent level. Δ = quarterly percentage change.		

Table 3. OLS estimation

Dependent variable: real GDP quarter over quarter percentage change; Sample:1999:1-2010:4, robust standard errors			
variable	Coefficient	t statistic	p-value
c	-0,0075	-2,7301	0,00921***
Δ loans (-1)	0,3709	3,1823	0,00275***
Δ Euribor3m (-2)	-0,0246	-3,4990	0,00112***
Δ empl(-1)	0,2908	1,1839	0,24310
Δ real gdp(-1)	0,5277	2,9759	0,00483***
Adj R^2: 0.542	D.W.: 2.24	F test: 6.62 ***	
* significant at 1 per cent level, ** significant at 5 per cent level, *** significant at 1 per cent level. Δ = quarterly percentage change			

[4] As regards data on loans, I employed data published by the Central Banks of Italy instead of the data by ECB because the first series covers a more extended sample.

This equation is useful in order to find a long period linkage between loans and growth. There is a statistically significant linkage between loans and real GDP. The coefficients on Euribor rate and lagged dependent variable are also significant. The coefficient on Euribor rate has the expected sign. β_3 is not statistically significant. This means that from 1999 to the end of 2010 an increase in loans contributed to boost the economic growth.

In order to test the *recovery-less credit growth* hypothesis, I decided to add an interaction dummy variable to observe the magnitude of this linkage in a more limited period. So, the second step is designed to find an econometric pillar to the idea depicted in the previous paragraph.

To this aim, I estimated a second equation with the same structure of the previous one, but with one more regressor.

$$\Delta rgdp = c + \beta_1 * \Delta loans(-1) + \beta_2 * \Delta Euribor3m(-2) + \beta_3 * \Delta empl(-1) + \beta_4 * (dummy * \Delta loans(-1)) + \beta_5 * \Delta rgdp(-1) \qquad (2)$$

The meaning of the symbols is the same of the previous regression but I added an interaction variable, dummy*Δloans. The dummy is equal to 1 from the first quarter 2007 onward and equal to 0 otherwise. I decided to set the dummy in this way because I suppose that the crisis modified the linkage between loans and economic growth in Italy. The results of this second regression are shown in Table 4.

The coefficient on the dummy is statistically significant and it has the expected sign. The interaction between dummy*Δloans and Δloans shows that during the last period of the sample, from the first quarter 2007 to the end of 2010, the linkage between loans and real GDP has been lower than during the previous quarters. This result supports the finding of the previous section: the role of the credit in stimulating the economic growth has changed during the crisis. This linkage has been weak and the consequence is a milder positive relationship between credit and economic growth.

Table 4. OLS estimation

Dependent variable: real GDP quarter over quarter percentage change; Sample:1999:1-2010:4, robust standard error			
variable	coefficient	t statistic	P-value
c	-0,0074	-2,5404	0,01496**
Δ loans (-1)	0,4169	3,0335	0,00418***
Δ Euribor3m (-2)	-0,0230	-3,4371	0,00136***
Δ empl(-1)	0,2712	1,0884	0,28278
dummy*Δloans(-1)	-0,1623	-2,6845	0,01043**
Δ real gdp(-1)	0,4706	2,6309	0,01194**
Adj R2: 0.568	D.W.: 2.21	F test: 7.01***	
* significant at 1 per cent level, ** significant at 5 per cent level, *** significant at 1 per cent level. Δ = quarterly percentage change.			

5. Conclusions

In this paper I studied the relationship between credit and economic growth in Italy from 1999 to 2010. The results have highlighted a specific situation in Italy. The credit condition has been positive during the recession and even during the very light recovery. For this reason the Italian case seems to be in contrast with the finding by Rajan, and Zingales (1998). They stated the importance of the financial sector in supporting the economic growth. Notwithstanding Italy had a well developed financial sector, a banking sector that did not suffer huge problems during the crisis and a flow of loans that has been vigorous during the last years, the economic growth has been very light after the recession. Moreover, this result is not in line with the study by Dell'Ariccia, and Garibaldi (2005) too. In this work, studying the 1991 recession in the US, they found that high credit contraction is a key feature of the cyclical downturn. Even this feature is absent in the Italian case. Finally, a study by Bernanke and Lown (1991), about the same 1991 crisis in the US, underlined that a linkage between credit crunch and economic crisis exists but they also said that demand factors can explain a big portion of the lending slowdown. In Italy even this linkage seemed to be absent during the months of the crisis. In fact, Italy experienced a very deep recession and a slow economic recovery during the sample I examined, while loans continue to grow with a good pace. So, even the demand factors cannot explain this situation.

This is a real new puzzle that economists should investigate. This has led me to mind the expression recovery-less credit growth in order to describe the economic scenario that Italy went through from 2007 to 2010, a period in which a robust increase in loans has been accompanied by a slack or absent economic growth.

References

[1] Abiad, A., Dell'Ariccia, G., and Li, B. 2011. Creditless recoveries, *CEPR Discussion Paper Series No. 8301*.

[2] Bernanke, B.S., and Lown, C.S. 1991. The Credit Crunch, *Brooking Papers on Economic Activity, Economic Studies Program, the Brookings Institution,* 22(2): 205-248.

[3] Biggs, M., Mayer, T., and Pick, A., 2009. Credit and economic recovery, *DNB Working Paper No. 218*.

[4] Calvo, G., Izquierdo, A., and Talvi, E. 2006, Phoenix miracles in emerging markets: recovery without credit from systemic financial crises, *BIS Working Paper No. 221*.

[5] Calvo, G., and Loo-Kung, R. 2010. US Recovery: A New Phoenix Miracle? *Inter-American Development Bank, Discussion Paper No. 106*.

[6] Coricelli F., Roland, I., 2011. How do credit conditions shape economic recoveries?, *CEPR Discussion Paper Series No. 8325.*

[7] Claessens, S., Kose, A., and Terrones, M.E. 2008. What happens during the recessions, crunches and busts? *IMF Working Paper No. 274.*

[8] Dell'Ariccia, G, Detragiache, E and Rajan, R 2007. The real effect of banking crises. *Journal of Financial Intermediation*, 17: 89-112.

[9] Dell'Ariccia, G., and Garibaldi, P. 2005. Gross Credit Flows, *Reviews of Economic Studies,* 72(3): 665-85.

[10] Demirgüç-Kunt, A., Detragiache, E., and Gupta, P. 2006. Inside the crisis: An empirical analysis of banking systems in distress, *Journal of International Money and Finance,* 25(5): 702-718.

[11] Draghi, M. 2011. The Governor's Concluding Remarks, *Banca d'Italia,* 31 May 2011.

[12] Holmstrom, B., and Tirolle, J., 1997. Financial Intermediation, Loanable Funds, and the Real Sector, *The Quarterly Journal of Economics,* 112(3): 663-691.

[13] Kannan, P. 2010. Credit Conditions and Recoveries from Recessions Associated with Financial Crises, *IMF Working Paper No. 83.*

[14] Kroszner, R., Laeven, L., and Klingebiel, D. 2007. Banking Crises, Financial Dependence, and Growth, *Journal of Financial Economics,* 84: 187-228.

[15] Rajan, R.G., and Zingales, L. 1998. Financial Dependence and Growth, *American Economic Review,* 88(3): 559-586.

EXPECTATIONS IMPACT ON THE EFFECTIVENESS OF THE INFLATION-REAL ACTIVITY TRADE-OFF

Gbaguidi S. DAVID
Université des Antilles et de la Guyane (CREDDI-LEAD), **France**
Université de la Méditerranée II (GREQAM), **France**
David.Gbaguidi@univ-ag.fr

Abstract:

The current study takes place in the Phillips curve framework in which first, we look at determining econometrics models leading to characterize the dynamics of the main variables underlying the trade-off in univariate contexts. As a result, it appears that an adequate way to characterize the agents' expectations regarding the dynamics of these variables is to consider a combination of some fixed levels (regimes) in the variables evolutions with an agents' adaptive beliefs notion. This expectation process is empirically captured by a Markov Switching Intercept Heteroskedastic-AutoRegressive (MSIH-AR) model. Finally, based on the implied expectations value of the variables, we show that the Phillips curve seems to disappear when the expected inflation rate's impact on its current value converges to its long-term value.

Keywords: New Keynesian Phillips curve, Markov switching, fractional integration.

JEL Classification: C20; C32; E00; E52.

1. Introduction

In a landmark paper, Phillips (1958) reported a strong inverse and relatively stable relationship over the last century between the unemployment rate and the rate of wage inflation in the United Kingdom. A few years later Solow (1960) highlights a similar correlation between the inflation and the unemployment rates based on United States data. A basic version of this relationship, which took the name of the Phillips curve in the macroeconomic literature, can be written as

$$\pi_t = \xi^{kpc}\psi_t + \varepsilon_t \qquad (KPC) \qquad (1)$$

where π_t, ψ_t and ε_t represent the inflation rate, a macroeconomic variable measuring the real economic activity and an error term with zero mean and constant variance. This Phillips curve, which reflects the basic Keynesian analysis, suggests the existence of a trade-off between changes in the aggregate price level and those in the real economic activity.

However, this curve was challenged in the late 60's as, for the Nobel Milton Friedman nominal variables cannot have permanent effects on real variables such that any Inflation - Real activity arbitration could only be exploited temporarily. Indeed, any macroeconomic policy would eventually lead to agents' behaviors changes. This monetarist perception of the trade-off leads to the following Augmented Phillips Curve

$$\pi_t = b_1^m \pi_t^* + \xi^m (\psi_t - \psi_t^*) + \varepsilon_t \qquad (APC) \qquad (2)$$

in which, ψ_t^* represents a variable measuring the real activity to its natural level and π_t^* the adaptive expected inflation rate. This expected rate can also be regarded as the inflation target of monetary authorities. As reported in the literature, this first integration of agents' expectations in the debate appears very important. This last equation shows that an Inflation-Real activity relationship may only exist in the short term $(\xi^m \neq 0 \text{ and } b_1^m \neq 0)$. In long-term, when agents adjust their decisions, realized and expected rates of inflation should be equal $(b_1^m = 1)$ to ensure a de facto equality between the current and expected (the natural level) values of the variable measuring the real activity. Then, the Phillips curve trade-off disappears $(\xi^m = 0)$.

Basically, any change in the nominal sphere of the economic system should lead to changes in the behaviors of the agents that ultimately inhibit any possible impact on the real activity.

With this basic summary of the trade-off evolution, it appears clear that taking into account the agents' expectations seems essential to fully capture the possible Inflation-Real activity compromise in the short term, while its long term disappearance is almost certain. The speed of the economy's transition to its steady state highly depends on the expectations adjustments. In other words, the effectiveness of the Phillips curve depends on the agents' adaptation, which itself seems closely related to the expected effectiveness of the former trade-off situation. According to many researchers (Samuelson (2008), Sims (2008)), the agents' optimization behaviour and their expectations modes (derived from their rationality) seem relatively clear in the Keynesian framework, but how their rationality is introduced, defined and operated in the analysis should be clarified.

To address this problem, we first extend the results of Boutahar, and Gbaguidi (2009) to characterize all the NKPC-PI variables dynamics. Each of these variables is studied in a univariate context by using a Markov Switching Intercept Heteroscedastic-AutoRegressive (MSIH-AR) model. This approach permits us to consider the unconditional means of these variables as a series obeying the regimes' switching controlled by a Markov chain of order 1. This Markov switching framework allows to characterize the agents' expectations process and to take into account the non-linearities observed in the variables' dynamics. Conceptually, this approach seems to be the most adequate as the trend dynamics of a considered variable come from a random scheme. This first stage estimation represents the background of the empirical analysis of the Phillips curve. It then presents the expected values of the main variables that appear in the Phillips curve. Based on these expected dynamics, we estimate the different versions of the Phillips curve and highlight the contribution of the introduction of agents' expectations in the debate surrounding the Inflation-Real activity trade-off. For that purpose, the evolution of the Phillips curve coefficients are considered as time or state varying parameters. These estimates enable us to show that, as the agents' expectations converge to their rational long term values, the trade-off seems to disappear. A final section summarizes the main results and discusses further research.

2. The Inflation-Real Activity Trade-off Context

In the mid-70's, authors such as Lucas (1972a) extended the monetarist arguments by introducing the «revolutionary» hypothesis of the rational expectations. Taking into account this fundamental assumption of rationality upsets the whole macroeconomic analysis and even deeper vision of the trade-off. From this hypothesis, agents' decisions will reflect their immediate adjustments in response to changing economic environment within which they operate. Therefore, in the absence of nominal rigidities, the Phillips curve relationship disappears even in the short term. The activity fluctuations are real and above all their explanations seem to have no relationship with any interventionist policies.

In the early 80's, research that focused on the Phillips curve was made within a frame of systematic optimization behaviors of economic agents. In this context, the inflation rate dynamic is mainly studied under Time-Dependent models à la Calvo (1983). In this New-Keynesian Phillips Curve (NKPC) framework, the economic system consists of firms in monopolistic competition facing adjustment costs in their prices' set up. Formally, at each moment, each of these firms receives a signal[5] (a probability (1-α)) to adjust its prices. This model is based on an asynchronous, non-global and non-random adjustment of all the firms' prices. For a representative firm, the decision to adjust its price will partially depend on the states of the economic environment in which it solves its profit maximization problem.

[5] By assumption, this probability is exogenous, single, identical for all firms and independent of the firms pricing history.

This decision will be primarily influenced by the fact that the firm must wait for some periods before re-optimizing its price[6]. Under the strong assumptions of a zero steady state inflation rate and an instantaneous and costless reallocation of capital, the NKPC will be written

$$\hat{\pi}_t = \tilde{\varrho}^{nkpc}\hat{\pi}_{t-1} + b_1^{nkpc}E_t\hat{\pi}_{t+1} + \xi^{nkpc}\hat{\psi}_t + \varepsilon_t \qquad (NKPC) \quad (3)$$

where, the current inflation rate is defined as a non-negative function $(\xi^{nkpc} \geq 0,\ b_1^{nkpc} \geq 0)$ of the real marginal cost $(\hat{\psi}_t)$ and the one period expected inflation rate $(E_t\hat{\pi}_{t+1})$. The coefficients b_1^{nkpc}, ξ^{nkpc} and $\tilde{\varrho}^{nkpc}$ are calculated as

$$\begin{aligned}
\tilde{\varrho}^{nkpc} &= \frac{\varrho}{1+\varrho\beta} \\
b_1^{nkpc} &= \frac{\beta}{1+\varrho\beta} \\
\xi^{nkpc} &= \frac{(1-\alpha)(1-\beta\alpha)}{\alpha(1+\omega\theta)(1+\varrho\beta)}
\end{aligned} \qquad (4)$$

These coefficients depend on the degree of price rigidity (α), the real discount factor with which firms discount future real marginal costs (β), the elasticity of substitution between goods[7] (θ), the elasticity of the firm marginal cost on its own output[8] (ω) and an indexation parameter of current prices to past inflation. The hat notations indicate that the variables are expressed in their log-deviations form from their steady-state values (e.g. $\hat{\pi}_t = \log(\pi_t/\bar{\pi})$).

In previous research, the inflation rate dynamic was mainly studied in this context of Time Dependent micro-based models à la Calvo (1980). These models are also based on the rational expectation hypothesis and the existence of frictions in the economy. Indeed, under this NKPC label, the inflation dynamics are presented as a forward-looking phenomenon resulting from the optimizing behaviours of economic agents. But, even though studies conducted by Gali, and Gertler (1999), Sbordone (2002) have suggested almost a resurrection of the Phillips curve, one could observe that the NKPC framework does not put to rest the debate surrounding the empirical effectiveness or the theoretical validity of this temporary arbitration.

The econometric weaknesses linked with these last results do not take away the theoretical doubts raised by monetarist or neoclassical approaches since the implied "rejections" of these Keynesian models call for a crucial need to review the way their Time Dependent framework considers the rational expectations process. Also, recalling that in the Calvo (1983) frame, firms are unable to adjust instantaneously their prices (even if they expect changes in their activity's environment), the NKPC pricing approach can be perceived as inappropriate for describing the inflation rate dynamics. An adequate integration of the agents' expectations in these New Keynesian analyses[9] is clearly necessary.

One of the paper's goals is to start from a way of integrating these expectations that will enable desired values of the Phillips curve variables to possibly differ in sample periods, without being a

[6] Like in the Taylor (1980) model, prices are fixed for a predetermined time period and firms are constrained by periodic prices contracts. One of the features of the Calvo (1983) model is that it considers the length of individual contracts to be randomized while the average duration of price contracts is constant.

[7] Which determines the mark-up ((θ/(θ-1))) that a firm can apply over its marginal costs.

[8] This parameter occurs in the equilibrium condition because there is no reallocation of capital between firms.

[9] Ascari (2004), Sahuc (2006), etc. show that the zero steady state inflation rate frame of analysis can only lead to bias in the New Keynesian Phillips Curve estimation so that the NKPC models are actually presented as particularly restrictive when an analysis is done in a changing inflation environment which could affect the firms pricing decisions.

consequence of continuous revisions of the agents' expectations. In fact, continuous updating of these expectations could be perceived as a form of weakening towards agents' rationality since such revisions could only be justified through a frequent need for correction of errors that would be, in sense, systematic. From then on, it would be necessary to look at an empirical approach in which all the variables considered in the New Keynesian Phillips Curve equation could admit trends dynamics and this in spite of the «constraints» imposed by the rational expectations hypothesis.

Fundamentally, it would be about allowing the considered variable to be different from zero at steady state, while having a trend dynamics similar to those one can extract with a Hodrick-Prescott filter (Figure 1a-1d). An explicit consideration of this last point is needed in any empirical evaluation of the trade-off. Moreover, if the evolution of each of the Phillips curve variables is defined by

$$x_t = \mu(x_t) + \varepsilon_t \quad (5)$$

where ε_t is an $NID(0, \sigma_\varepsilon^2)$ shock and $\mu(x_t)$ a term which represents the systematic component (expected value) of a variable (x_t), it appears obvious that every study of the Phillips curve has to adequately characterize the evolution of this component.

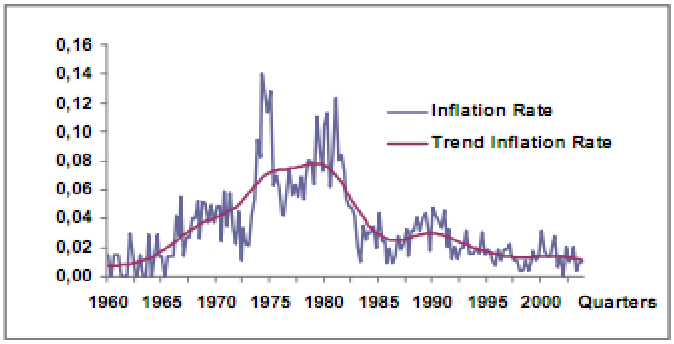


Figure 1a. Inflation rate and trend (Hodrick-Prescott)

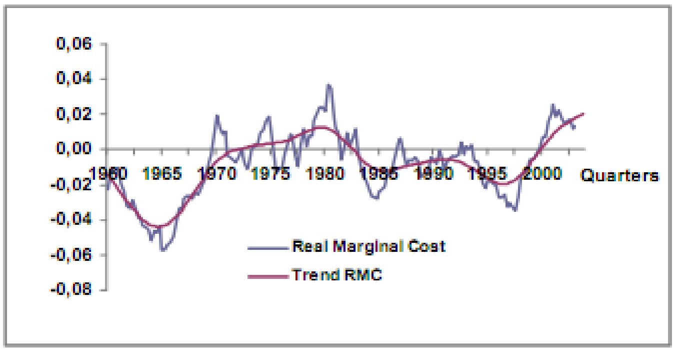


Figure 1b. Real marginal cost and trend (Hodrick-Prescott)

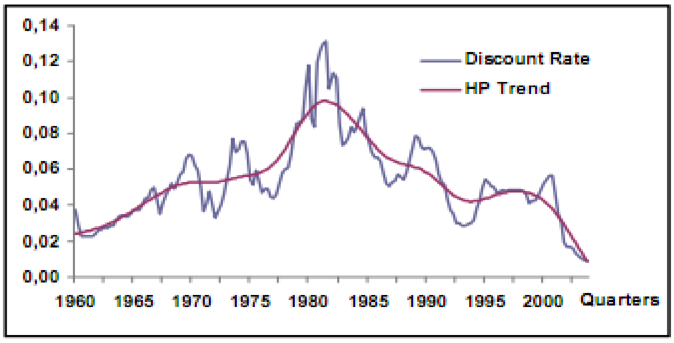


Figure 1c. Discount rate and trend (Hodrick-Prescott)

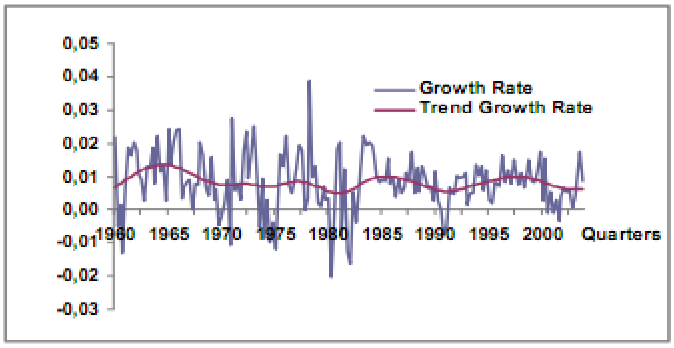


Figure 1d: Real output growth rate and trend (Hodrick-Prescott)

Based on these fundamental limits, Bakshi, *et al*. (2005), Cogley, and Sbordone (2005, 2008) or Groen, and Mumtaz (2008) examine the implications of a positive steady state inflation rate in the New Keynesian framework. Their studies lead to a second generation of New Keynesian models derived under the assumption of a non-zero steady state inflation rate.

Mathematically, the New Keynesian Phillips Curve with Positive Inflation (NKPC-PI) equation can be written as follows:

$$\hat{\pi}_t = \tilde{\varrho}^{nkpc-pi}\hat{\pi}_{t-1} + b_1^{nkpc-pi} E_t\hat{\pi}_{t+1} + \xi^{nkpc-pi}\hat{\psi}_t$$
$$+b_2 \sum_{j=2}^{\infty} \gamma_1^{j-1} E_t\hat{\pi}_{t+j} + \chi(\gamma_2 - \gamma_1) \sum_{j=0}^{\infty} \gamma_1^j \left(E_t\hat{R}_{t+j,t+j+1} + E_t\hat{\gamma}_{y_{t+j+1}}\right) + \varepsilon_t \quad (NKPC-PI) \qquad (6)$$

This equation shows that the inflation rate $(\hat{\pi}_t)$ dynamics are explained by their own expected dynamics $(E_t\hat{\pi}_{t+1},\ E_t\hat{\pi}_{t+j})$, the dynamics of the expected nominal discount rate $(E_t\hat{R}_{t+j,t+j+1})$, the expected output growth rate $(E_t\hat{\gamma}_{y_{t+j+1}})$ and the real marginal cost $(\hat{\psi}_t)$. The NKPC-PI coefficients are functions of the structural parameters of the economy, i.e. $\Psi = [\alpha, \theta, \rho]$ and on the steady state inflation rate (π>1). Formally, we have:

$$\tilde{\varrho}^{nkpc-pi} = \frac{\varrho}{\Delta}$$
$$b_1^{nkpc-pi} = \frac{1}{\Delta}\left(\left(\frac{1-\alpha\varsigma_1}{\alpha\varsigma_1}\right)\phi_1 + \gamma_2\right)$$
$$\xi^{nkpc-pi} = \frac{1}{\Delta}\left(\frac{(1-\alpha\varsigma_1)(1-\gamma_2)}{\alpha\varsigma_1(1+\theta\omega)}\right)$$
$$b_2 = \frac{1}{\Delta}\left(\frac{1-\alpha\varsigma_1}{\alpha\varsigma_1}\left(\frac{\theta(1-\varrho\gamma_1)+\varrho\gamma_1}{1+\theta\omega}\right)(\gamma_2-\gamma_1)\right)$$
$$\chi = \frac{1}{\Delta}\left(\frac{1-\alpha\varsigma_1}{\alpha\varsigma_1(1+\theta\omega)}\right) \qquad (7)$$

and the intermediate terms are given by:

$$\begin{gathered}
\varsigma_1 = \bar{\pi}^{(\theta-1)(1-\varrho)} \\
\varsigma_2 = \bar{\pi}^{\theta(1+w)(1-\varrho)} \\
\tilde{\beta} = \bar{R}\bar{\gamma}_y\bar{\pi} \\
\gamma_1 = \alpha\tilde{\beta}\varsigma_1 \\
\gamma_2 = \alpha\tilde{\beta}\varsigma_2 \\
\Delta = 1 + \varrho\gamma_2 - \left(\frac{1-\alpha\varsigma_1}{\alpha\varsigma_1}\right)\phi_1 \\
\phi_1 = \frac{\varrho\theta(\gamma_1-(1+\omega)\gamma_2)-\varrho\gamma_1}{1+\theta\omega} \\
\phi_2 = \frac{1}{1+\theta\omega}\left(\gamma_2\left(1+\theta\omega\right)+\left(\gamma_2-\gamma_1\right)\left(\theta\left(1-\varrho\gamma_1\right)+\varrho\gamma_2\right)\right)
\end{gathered} \tag{8}$$

The non-zero steady state hypothesis allows a "long term Phillips curve" characterized by an equation tying together the different steady state values of the NKPC-PI variables, i.e.

$$\left(1-\alpha\bar{\pi}^{(\theta-1)(1-\varrho)}\right)^{\frac{1+\theta\omega}{1-\theta}}\left(\frac{1-\alpha\bar{R}\bar{\gamma}_y\bar{\pi}^{1+\theta(1+w)(1-\varrho)}}{1-\alpha\bar{R}\bar{\gamma}_y\bar{\pi}^{\theta-\varrho(\theta-1)}}\right)=\frac{\theta}{\theta-1}\left(1-\alpha\right)^{\frac{1+\theta\omega}{1-\theta}}\bar{\psi} \quad (NKPC_{SS}-PI) \tag{9}$$

in which $\bar{\pi}$, $\bar{\psi}$, $\bar{\gamma}_y$ and $\bar{R}$ are these steady state values. It is to be noticed that that when the steady state inflation rate varies, the structure of the economy may be affected and in this case, the structural parameters may themselves vary[10].

To the extent that the steady state inflation rate can be non-zero, models from this NKPC-PI framework suppose a possibly permanent arbitration resulting from a combination of the short term (NKPC-PI) and the long term (NKPCSS-PI) equations. In short and medium terms, the effectiveness of the arbitration is possible because, like in the NKPC framework, the NKPC-PI approach combines the concepts of the new classical reasoning (rational expectations) and Keynesian basis (nominal rigidities). However, unlike in the basic Keynesian analysis, the Phillips curve is not unique so that we speak rather of arbitration with a prolonged persistence. During the transition of the economy to its steady state (time for a new trade-off), the economic system appears to follow a path characterized by a succession of inconstantly persistent moments of arbitration. As envisaged in the NKPC, the magnitude of the relationship between the inflation rate and the variable measuring the real activity will not necessarily remain the same throughout the time preceding the stationary state[11] where a new link (NKPCSS-PI) is set up.

3. Expected Values of the NKPC-PI Variables

Before considering the non-linear specifications, we present the data upon which our empirical study takes place and conduct a linear analysis as benchmark for the rest of this study.

3.1 Description of the Data

To be able to reconsider previous results of the Phillips curve estimations (Solow (1968), Friedman (1968), Gali, and Gertler (1999), Cogley, and Sbordone (2005), Groen, and Mumtaz (2008)), we focus on a database reflecting the best possible data used in these earlier studies. The main variables appearing in the Phillips curve debate are the inflation rate; a unit labor cost based measure of the real marginal cost, the nominal discount rate and the output growth rate. The sample period covers T=176 quarters from 1960: I to 2003: IV for the U.S. economy.

[10] In a related work, we show that, even if there is structural changes in the economy, these changes remain infrequent and of small magnitude confirming the structural parameters stability.

[11] In short and medium terms, the slope of the NKPC-PI can vary as $\xi^{nkpc-pi} = f\left(\alpha\left(\bar{\pi}\right)\right)$.

The inflation rate is measured from the implicit price deflator as (P_t) recorded in the Bureau of Labor Statistics (BLS) database. From these data, we calculate this series as

$$\pi_t = 4 * [\ln(P_t) - \ln(P_{t-1})] \tag{10}$$

The real activity is measured by the real marginal cost. Assuming a Cobb-Douglas production function, the real marginal cost (ψ_t) is proportional to the labor unit cost (ulc_t) as mathematically[12], we have

$$\psi_t = \ln\left(\frac{W_t N_t}{P_t Y_t}\right) - \ln(1-\kappa) = \ln(ulc_t) - \ln(1-\kappa) \tag{11}$$

where Y_t is the level of output in real terms, N_t is the total amount of labor input and W_t measure wages. Following Cogley, and Sbordone (2005), the output elasticity to hours of work (1-κ) in the production function is set equal to 0.6666 so that the strategic complementarities parameter equal to ω = (κ/(1-κ)) = 0.5001.

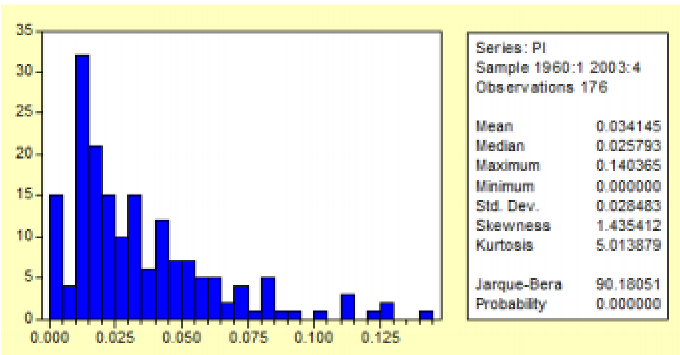
Regarding the discount rate $(R_{t,t+1})$, we use the 3-Month Treasury Bill: Secondary Market Rate (i_t) from the Federal Reserve Bank of St. Louis (FRED) database. To construct $(R_{t,t+1})$, we apply the formula: $R_{t,t+1} = \frac{i_t}{1+i_t}$, where (i_t) was divided by 100.

The output growth rate is calculated on the basis of a weighted sequence of the real Growth Domestic Product (GDP) expressed in 2000 dollars (seasonally adjusted at an annual rate) and recorded in the NIPA.

3.2 The Linear-Benchmark Approach

As shown in Charts 1a-1d, the series appears characterized by periods in which their average levels and their variability differ. Clearly this reveals non-linearity in their dynamics and these series appear to originate from an asymmetric law with a noticeably high flattening coefficient. Their trends or expected values dynamics enable us to foresee possible periods of instability.

Chart 1a. Descriptive statistics of the inflation rate



[12] This real marginal cost series is constructed according to Groen, and Mumtaz (2008).

Chart 1b. Descriptive statistics of the real marginal cost

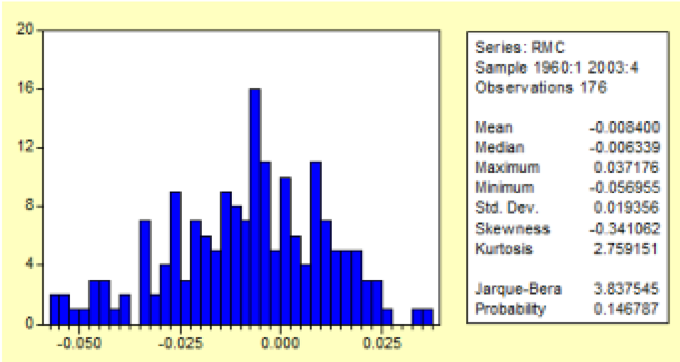


Chart 1c. Descriptive statistics of the discount rate

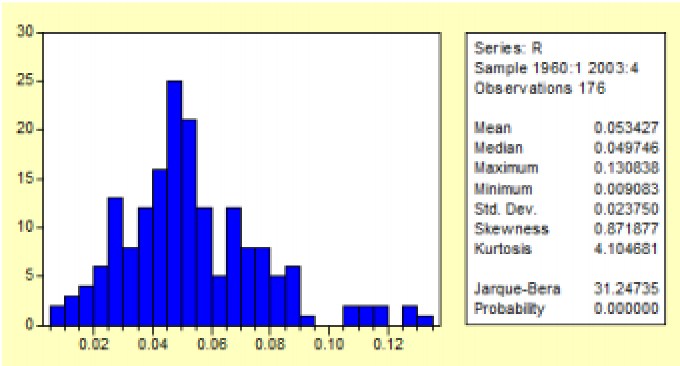
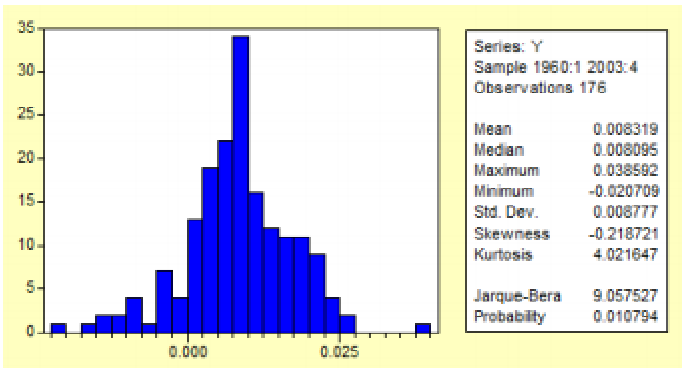


Chart 1d. Descriptive statistics of the real output growth rate



In a linear framework, it appears that these series are generated by the following second order autoregressive processes with a Generalized Autoregressive Conditional Heteroskedasticity[13] GARCH(1,1).

$$x_t = \phi_0 + \phi_1 x_{t-1} + \phi_2 x_{t-2} + \varepsilon_t$$
$$\sigma^2_{\varepsilon,t} = \omega_0 + \omega_1 \varepsilon^2_{t-1} + \omega_2 \sigma^2_{\varepsilon,t-1} \tag{12}$$

Tables 1e-1h give the results of these linear models estimation.

[13] On the basis of the Akaike, and Schwartz information criteria, we can select linear specifications of two lags and the Q statistics of the squared residuals indicate the presence of ARCH effects.

Table 1e. Estimates of the linear model for the inflation rate

	$Estimates$ [p−values]
$\hat{\phi}_0$	0.0037 [0.0094]
$\hat{\phi}_1$	0.3749 [0.0000]
$\hat{\phi}_2$	0.4666 [0.0000]
$\hat{\omega}_0$	$6.4 \cdot e^{-6}$ [0.2628]
$\hat{\omega}_1$	0.1969 [0.0042]
$\hat{\omega}_2$	0.7873 [0.0000]
$lnL^{Lin}_{\pi_t} = 502.0563$	

Table 1f. Estimates of the linear model for the real marginal cost

	$Estimates$ [p−values]
$\hat{\phi}_0$	−0.0004 [0.3933]
$\hat{\phi}_1$	1.1250 [0.0000]
$\hat{\phi}_2$	−0.1709 [0.0283]
$\hat{\omega}_0$	$2.2 \cdot e^{-6}$ [0.5852]
$\hat{\omega}_1$	0.0334 [0.5616]
$\hat{\omega}_2$	0.8994 [0.0000]
$lnL^{Lin}_{\psi_t} = 654.9954$	

Table 1g. Estimates of the linear model for the discount rate

	$Estimates$ [p−values]
$\hat{\phi}_0$	0.0022 [0.0088]
$\hat{\phi}_1$	1.3847 [0.0000]
$\hat{\phi}_2$	−0.4429 [0.0000]
$\hat{\omega}_0$	$9.3 \cdot e^{-7}$ [0.0857]
$\hat{\omega}_1$	0.3396 [0.0009]
$\hat{\omega}_2$	0.6840 [0.0000]
$lnL^{Lin}_{R_{t,t+1}} = 691.4020$	

Table 1h. Estimates of the linear model for the output growth rate

	$Estimates$ $[p-values]$
$\hat{\phi}_0$	0.0048 [0.0000]
$\hat{\phi}_1$	0.2061 [0.0211]
$\hat{\phi}_2$	0.2582 [0.0021]
$\hat{\omega}_0$	$2.2 \cdot e^{-6}$ [0.2294]
$\hat{\omega}_1$	0.1735 [0.0145]
$\hat{\omega}_2$	0.8012 [0.0000]
$lnL^{Lin}_{\gamma_{y,t}} = 601.5058$	

One can see that all the variables (except the output growth rate) are characterized by a quite strong global persistence[14].

$$\begin{aligned} \hat{\phi}_1^{\pi_t} + \hat{\phi}_2^{\pi_t} &= 0.8415 \\ \hat{\phi}_1^{\psi_t} + \hat{\phi}_2^{\psi_t} &= 0.9541 \\ \hat{\phi}_1^{R_{t,t+1}} + \hat{\phi}_2^{R_{t,t+1}} &= 0.9418 \\ \hat{\phi}_1^{\gamma_{y_t}} + \hat{\phi}_2^{\gamma_{y_t}} &= 0.4643 \end{aligned} \tag{13}$$

Their unconditional means are given by

$$\begin{aligned} \bar{\pi}^{Lin} &= 0.0233 \\ \bar{\psi}^{Lin} &= -0.0087 \\ \bar{R}^{Lin} &= 5.9978 \\ \bar{\gamma}_y^{Lin} &= 0.0089 \end{aligned} \tag{14}$$

Nevertheless, in order to verify the instability of the parameters ϕ_i; i = 0, 1, 2, in these AR(2) processes, we conduct stability tests (Nyblom (1989)) as described in Hansen (1990, 1992). The results of these tests[15] are described in Table 1i-1l. From these tables and for all the variables, one can globally conclude a weak joint stability of parameters whereas we can't reject the null hypothesis for $\phi_i, \quad i = 0, 1, 2$ taken individually, except for the inflation rate (Boutahar, and Gbaguidi (2009)). Also, one can reject the variance stability only for the inflation and output growth rates[16].

[14] Nevertheless, the Augmented Dickey-Fuller tests show that these series can be considered as stationary.
[15] Noting that these stability tests strictly require the estimates of the models in their linear forms, it is a matter of testing the null hypothesis of the individual or collective stability of the parameters versus the alternative that they follow martingale processes. The L statistic corresponds to the case where only one parameter's stability is tested while the L_c statistic corresponds to the case of joint parameters stability. These statistics follow non-standard laws which essentially depend on the number of tested parameters and the critical values are computed from the theoretical asymptotic distributions. The 5% critical values for these stability tests, taken individually and jointly, are $vc_L = 0.47$ and $vc_{L_c} = 1.24.$.
[16] It seems like the linear model can adequately characterize the discount rate and the real marginal cost dynamics so that their expected values could be consider constant.

Table 1i. Parameters stability tests for the inflation rate

	$Estimates$ $(Std.dev)$	L
$\hat{\phi}_0$	0.0039 (0.0018)	0.2643*
$\hat{\phi}_1$	0.4888 (0.0798)	0.0726*
$\hat{\phi}_2$	0.3984 (0.0693)	0.0735*
$\hat{\sigma}^2_\varepsilon$	0.0002 $(3.5 \cdot e^{-5})$	1.0018

$L_c = 1.2652$

Table 1j. Parameters stability tests for the real marginal cost

	$Estimates$ $(Std.dev)$	L
$\hat{\phi}_0$	−0.0003 (0.0005)	0.1777*
$\hat{\phi}_1$	1.1272 (0.0720)	0.3413*
$\hat{\phi}_2$	−0.1788 (0.0762)	0.2974*
$\hat{\sigma}^2_\varepsilon$	$3.1 \cdot e^{-5}$ $(3.4 \cdot e^{-6})$	0.3906*

$L_c = 0.9303^*$

Table 1k. Parameters stability tests for the discount rate

	$Estimates$ $(Std.dev)$	L
$\hat{\phi}_0$	0.0027 (0.0018)	0.1988*
$\hat{\phi}_1$	1.2008 (0.0956)	0.0544*
$\hat{\phi}_2$	−0.2517 (0.0968)	0.0680*
$\hat{\sigma}^2_\varepsilon$	$4.2 \cdot e^{-5}$ $(1.1 \cdot e^{-5})$	0.4131*

$L_c = 0.8928^*$

Table 1l. Parameters stability tests for the real output growth rate

	$Estimates$ $(Std.dev)$	L
$\hat{\phi}_0$	0.0049 (0.0012)	0.1257*
$\hat{\phi}_1$	0.2427 (0.0779)	0.1086*
$\hat{\phi}_2$	0.1626 (0.0887)	0.1181*
$\hat{\sigma}^2_\varepsilon$	$6.8 \cdot e^{-5}$ $(9.9 \cdot e^{-6})$	0.8715

$L_c = 1.1136^*$

However, in this linear framework, the intercepts are the possible source of non-linearity. To capture this possible non-linearity, the recourse to models with pure or partial parameter instabilities seems necessary. From these AR(2) representations, we then introduce the Markov Switching specification to capture these non-linearity.

3.3 The Markov Switching Approach

The purpose of this specification is that the variables evolve between m regimes (levels) which are controlled by a probability law. We consider a Markov Switching Intercept Heteroskedastic - AutoRegressive (MSIH-AR) type of model in which the intercept parameters characterizing each variable dynamics have the possibility to change at each date according to the Markov chain. We estimate MSIH(m)-AR(2) models defined as

$$x_t = \phi_{0_{S_t=k}} + \phi_1 x_{t-1} + \phi_2 x_{t-2} + \varepsilon_t \qquad (15)$$

where $(\varepsilon_t \mid S_t = k) \sim IIN\left(0, \sigma^2_{\varepsilon_{S_t=k}}\right)$, $k = 1, 2, ..., m$ and S_t is a first order Markov chain with transition matrix defined as

$$P = \begin{bmatrix} p_{11} & \cdots & p_{m1} \\ \cdots & \cdots & \cdots \\ p_{1m} & \cdots & p_{mm} \end{bmatrix} \qquad (16)$$

where $p_{ij} \geq 0$ and $\sum_{j=1}^{m} p_{ij} = 1,\ \forall i, j \in \{1, ..., m\}$. In this specification, we suppose that the intercept $(\phi_{0_{S_t=k}})$ and the variance $(\sigma^2_{\varepsilon_{S_t=k}})$ change with the regimes S_t given the information $(I_{t-1} = (x_{t-1}, ..., x_1))$ available in the beginning of the period t. Those terms vary according to the probability matrix P and the terms $p_{i,j} = P(S_t = i \mid S_{t-1} = j)$ measure the probability that a variable x_t switch from a level j at date t-1 to a level i at date t. In this context, the unconditional mean of a considered variable $(x_t = \{\pi_t, \psi_t, R_{t,t+1}, \gamma_{y_t}\})$ can be measured by

$$\bar{x}_t^{MSIH} = \frac{\phi_{0_{S_t=k}}}{1 - \phi_1 - \phi_2},\ k = 1, 2, ..., m \qquad (17)$$

The estimation of this model is accomplished by the method of maximum likelihood and according to the procedure proposed by Hamilton (1989). The idea is to estimate the probability that an observation x_t has been generated by a regime k and therefore at time t, the intercept and the variance are in a state of $S_t = k,\ k = 1, 2, 3$. This estimation takes the form of a conditional probability $P(S_t = k \mid I_t; \Theta)$ and can be written as

$$\begin{aligned} P(S_t = k \mid I_t; \Theta) &= P(S_t = k \mid x_t,\ I_{t-1}; \Theta) \\ &= \frac{P(S_t = k, x_t \mid I_{t-1}; \Theta)}{f(x_t \mid I_{t-1}; \Theta)} \\ P(S_t = k \mid I_t; \Theta) &= \frac{P(S_t = k \mid I_{t-1}; \Theta) * f(x_t \mid S_t = k, I_{t-1}; \Theta)}{\sum_{i=1}^{3} P(S_t = k \mid I_{t-1}; \Theta) * f(x_t \mid S_t = k, I_{t-1}; \Theta)} \end{aligned} \qquad (18)$$

where $\Theta = \left(\phi_{0_{S_t=k}},\ \phi_1,\ \phi_1,\ \sigma^2_{\varepsilon_{S_t=j}},\ p_{11},\ p_{12},\ p_{21},\ p_{22},\ p_{31},\ p_{33}\right)$ is the set of parameters to estimate and $f\left(x_t \mid S_t = k, I_{t-1}; \Theta\right)$ represents the density of the conditional system of states $S_t = k$. Essentially, it requires a filtering procedure which can be more readily visible when the expression (18) is rewritten under the following compact form

$$\hat{\xi}_{t|t} = \frac{f\left(x_t \mid S_t = k, I_{t-1}; \Theta\right) \odot \hat{\xi}_{t|t-1}}{1_3'\left(f\left(x_t \mid S_t = k, I_{t-1}; \Theta\right) \odot \hat{\xi}_{t|t-1}\right)} \tag{19}$$

where $\hat{\xi}_{t|t}$ is a vector of conditional probabilities containing the predictions of the analyst about the possibility that the observation π_t has been generated by a regime k. The k-th element of this vector represents $P\left(S_t = k \mid I_{t-1}; \Theta\right)$. The term 1' denotes an (3×1) vector all of whose elements are unity. The estimates and the optimal predictions for each date t in the sample are described by the following recursive algorithm

$$\begin{cases} \hat{\xi}_{t|t} = \frac{\eta_t \odot \hat{\xi}_{t|t-1}}{1_3'\left(\eta_t \odot \hat{\xi}_{t|t-1}\right)} \\ \hat{\xi}_{t+1|t} = P\hat{\xi}_{t|t} \end{cases} \tag{20}$$

where η_t represents the vector of conditional densities of which the k-th element is given by

$$f\left(x_t \mid S_t = k, I_{t-1}; \Theta\right) = \frac{1}{\sqrt{2\pi}\sigma_{\varepsilon_{S_t=k}}} \exp\left(-\frac{\varepsilon_t'\varepsilon_t}{2\sigma^2_{\varepsilon_{S_t=k}}}\right) \tag{21}$$

The log-likelihood function is

$$\ln L^{MSI}\left(\Theta\right) = \sum_{t=1}^{T} \ln\left(f\left(x_t \mid I_{t-1}; \Theta\right)\right) \tag{22}$$

and is maximized by using the system (20), given an initial value of the vector of conditional probabilities $\left(\hat{\xi}_{1|0}\right)$ and a vector of initial parameters $\left(\Theta_0\right)$.

First of all, we use some estimations to select the number m of regimes for each of the variables in the MSIH(m)-AR(2) class of models. The results[17] are presented in the Tables 2a-2d.

Table 2a. Selection of the number of regimes for the inflation rate

	LnL	l	AIC
$Linear\ Model$	502.0563	6	−992.1126
$MSI(2)-AR(2)$	507.0065	8	−998.0130
$\mathbf{MSI(3)-AR(2)}$	513.5232	14	**−999.0464**

[17] The statistic AIC is calculate as AIC=-2∗LnL+2∗l, where l is the number of parameters to be estimated in the model.

Table 2b. Selection of the number of regimes for the real marginal cost

	LnL	l	AIC
Linear Model	654.9954	6	**−1297.9908**
$MSI(2)-AR(2)$	655.3994	8	−1294.7988
$MSI(3)-AR(2)$	644.4619	14	−1260.9238

Table 2c. Selection of the number of regimes for the discount rate

	LnL	l	AIC
Linear Model	691.4013	6	**−1370.8026**
$MSI(2)-AR(2)$	683.5747	8	−1351.1494
$MSI(3)-AR(2)$	$n.c$	14	$n.c$

Table 2d. Selection of the number of regimes for the real output growth rate

	LnL	l	AIC
Linear Model	601.5331	6	−1191.0662
$\mathbf{MSI(2)-AR(2)}$	606.2054	8	**−1196.4108**
$MSI(3)-AR(2)$	589.6142	14	−1151.2284

Given the MSIH(m)-AR(2) estimates presented in Tables 3a-3d, we have the following results for the main variables surrounding the Phillips curve debate.

The inflation rate:

Table 3a: Estimates of the MSIH(3)-AR(2) model for the inflation rate

State (j)	n°obs/state	$\hat{\mathbf{P}}(S_t=k)$	$\hat{\phi}_{0_{S_t=k}}$	$\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\sigma}^2_{\varepsilon_{S_t=k}}$
1	26	0.1488 (0.1616)	0.0491 (0.0114)	0.1782 (0.0766)	0.2924 (0.0751)	0.0005 (0.0003)
2	71	0.4106 (0.2546)	0.0189 (0.0042)	0.1782 (0.0766)	0.2924 (0.0751)	0.0001 (0.0000)
3	78	0.4406 (0.3357)	0.0074 (0.0017)	0.1782 (0.0766)	0.2924 (0.0751)	0.0001 (0.0000)

$$\hat{P} = \begin{bmatrix} \underset{(0.0929)}{0.9126} & \underset{(0.0295)}{0.0317} & \underset{(0.0089)}{0.0000} \\ \underset{(0.0954)}{0.0874} & \underset{(0.0402)}{0.9418} & \underset{(0.0375)}{0.0247} \\ \underset{(0.0080)}{0.0000} & \underset{(0.0269)}{0.0265} & \underset{(0.0316)}{0.9753} \end{bmatrix} \qquad lnL^{MSI}_{\pi_t}=513.5232$$

The MSIH(3)-AR(2) model seems to be adequate to characterize the inflation rate dynamics[18]. The unconditional means of the inflation rate calculated from this MSIH(3)-AR(2) estimates are given by $\bar{\pi}_1^{MSIH} = 0.0927$, $\bar{\pi}_2^{MSIH} = 0.0357$ and $\bar{\pi}_3^{MSIH} = 0.0139$. Thereby, the last state ($\hat{\phi}_{0_{S_t=3}} = 0.0074$, the most frequently visited) characterizes about 45% of the observations, whereas the first one, associated with a higher mean $(\bar{\pi}_1^{MSIH})$, appears as an exception because it only covers 26 quarters out of the 174 of the sample. The probability of being in this first state is

[18] According to the likelihood value, the MSIH(3)-AR(2) specification is preferable to the linear model (Garcia (1998)) and the MSIH(2)-AR(3) model. Also, Kang & al. (2009) investigate the existence and timing of changes in U.S. inflation persistence using an unobserved components model of inflation with Markov switching parameters. Their results support using a model with three regimes to capture all of the serial correlation and heteroskedasticity in the inflation rate data.

estimated as $\hat{P}(S_t = 1) = 0.1488$ and naturally, we see that this state is effective over the course of the quarters of «hyper-inflation», namely during the years 74-76 and 80-82. Once in this state, the probability of remaining there is given by $\hat{p}_{11} = 0.9126$ and the probabilities of leaving this regime are given by $\hat{p}_{21} = 0.0317$ and $\hat{p}_{31} = 0.0000$. The probabilities of staying in the other regimes are higher ($\hat{p}_{22} = 0.9418$, $\hat{p}_{33} = 0.9753$) than $\hat{p}_{11}$. In such a case, we can say that the first regime of a high expected inflation rate captures particular dates of this variable dynamics. The probabilities of leaving regimes 2 and 3 to reach the first one are $\hat{p}_{12} = 0.0874$ and $\hat{p}_{13} = 0.0000$. It appears that there is no direct transition between the regimes of low and high expected inflation rate. The second regime $(\bar{\pi}_2^{MSI})$ which covers 71 quarters and can be associated with the average of the inflation rate represents an «intermediate» regime between the two others. We can see that the probabilities to switch between the regimes 2 and 3 are close to each other ($\hat{p}_{23} = 0.0265$ and $\hat{p}_{32} = 0.0247$) and smaller than $\hat{p}_{12}$ and $\hat{p}_{21}$. Consequently, each regime can be perceived as «persistent» because if the inflation starts in its low regime then it will certainly switch to the intermediate level where it will be more attracted by the high inflation rate regime than the low rate regime. Figure 2a illustrates these observations.

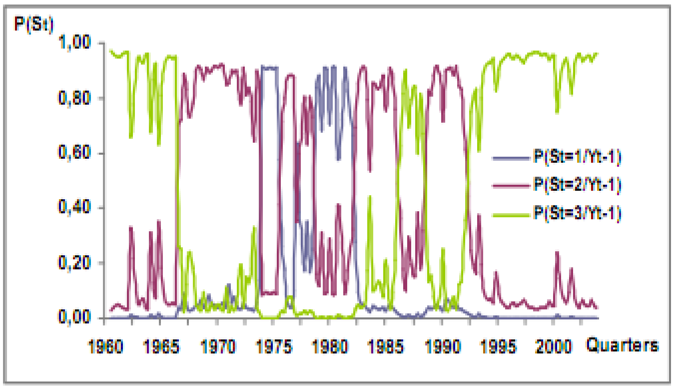


Figure 2a. MSIH(3)-AR(2) regimes probabilities for the inflation rate

To check the adequacy of this specification, a panel of tests[19] based on the score method is executed. The results[20] of these tests are given in Table 4a.

Table 4a. Adequation tests of the MSIH(3)-AR(2) model for the inflation rate

[19] For more extensive details concerning these tests, see Hamilton (1996).
[20] The 5% critical values are given between [.] the p-values are given between {.}.

Tests for parameters stability		
Markov	*mu*	*Res-cov*
0.7361 [1.6800]	0.3344 [1.0100]	1.8652 [1.0100]
Tests for parameters stability in state		
State 1	*State 2*	*State 3*
0.1942 [1.2400]	0.5225 [1.2400]	2.1669 [1.2400]
Test of serially uncorrelated Markov chain		
Wald-test	*F-test*	
1371.2596 {0.0000}	323.1129 {0.0000}	
Misspecification tests based on conditional scores		
Autocorr	*ARCH*	*Markov*
0.6395 {0.7620}	0.6727 {0.7328}	2.0582 {0.0611}

They indicate the instability of the variance of the residuals but not at the level of other parameters of the model in its global specification. Nevertheless, all the parameters of the model appear stable when each state is taken individually except in the last one. The absence of residuals autocorrelation, residuals heteroskedasticity and the hypothesis of a 1st order Markov are not rejected. Consequently, the MSIH(3)-AR(2) specification for the inflation rate appears to be more adequate. Its expected dynamics are illustrated in Figure 3a and its predicted dynamics are given by Figure 4a.

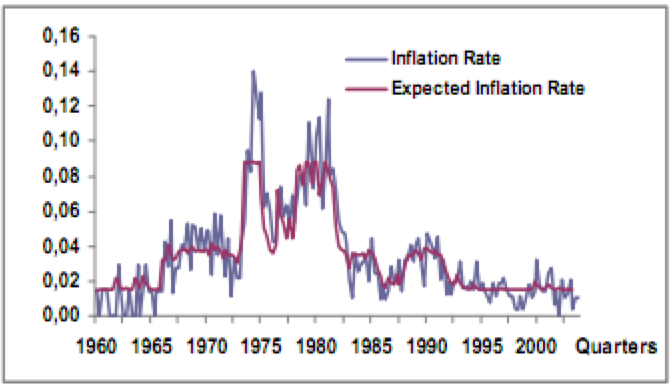


Figure 3a. MSIH(3)-AR(2) expected inflation rate

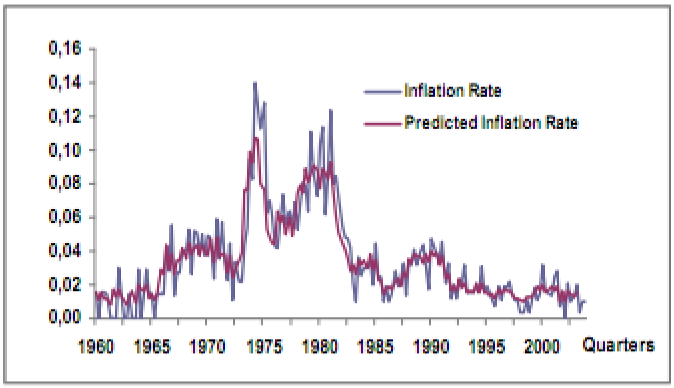


Figure 4a. Inflation rate as described by the MSIH(3)-AR(2) specification

The real marginal cost:

Table 3b. Estimates of the MSIH(2)-AR(2) model for the real marginal cost

State (j)	n°obs/state	$\hat{P}(S_t = k)$	$\hat{\phi}_{0S_t=k}$	$\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\sigma}^2_{\varepsilon S_t=k}$
1	57	0.3264 (0.3332)	0.0027 (0.0016)	1.0455 (0.1278)	−0.0898 (0.1153)	0.0000 (0.0000)
2	117	0.6736 (0.3332)	−0.0017 (0.0020)	1.0455 (0.1278)	−0.0898 (0.1153)	0.0000 (0.0000)

$\hat{P} = \begin{bmatrix} 0.7578 & 0.1173 \\ (0.2185) & (0.1757) \\ 0.2422 & 0.8827 \\ (0.2185) & (0.1757) \end{bmatrix}$	$lnL^{MSI}_{\psi_t}=655.3994$

The results suggest that the linear model can be considered as adequate even if the MSIH(2)-AR(2) specification is preferred to the MSIH(3)-AR(2). The unconditional means calculated from the MSIH(2)-AR(2) specification are $\bar{\psi}_1^{MSIH} = 0.0028$ and $\bar{\psi}_2^{MSIH} = -0.0017$. These regimes reflect opposite values of the real marginal cost series. The probabilities of remaining in each of these two regimes are $\hat{p}_{11} = 0.7578$ and $\hat{p}_{22} = 0.8827$ $\hat{p}_{11} = 0.9126$ while the probabilities of leaving these regimes are given by $\hat{p}_{12} = 0.2422$ and $\hat{p}_{21} = 0.1173$. Figure 2b illustrates the probabilities of being in each regime at t given the information at t-1. These probabilities vary significantly across the sample indicating that the estimated regimes are not stable.

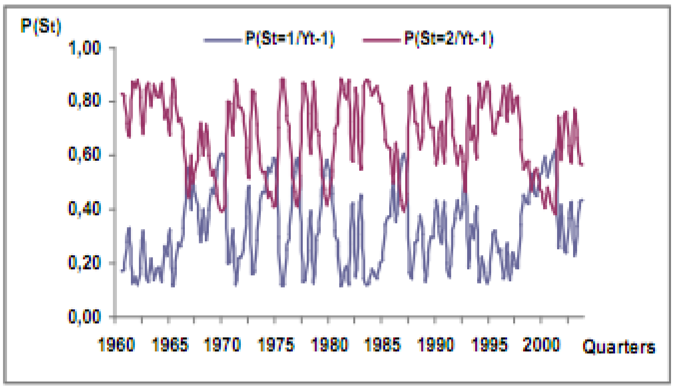


Figure 2b. MSIH(2)-AR(2) regimes probabilities for the real marginal cost

Globally and when each state is picked up individually, the results of the adequacy tests (Table 3b) indicate that all the parameters can be considered stable in this specification.

Table 4b: Adequation tests of the MSIH(2)-AR(2) model for the real marginal cost

Tests for parameters stability		
Markov	*mu*	*Res-cov*
0.1993 [0.7500]	0.3139 [0.7500]	0.5361 [0.7500]
Tests for parameters stability in state		
State 1	*State 2*	
0.6563 [1.2400]	1.0752 [1.2400]	
Test of serially uncorrelated Markov chain		
Wald-test	*F-test*	
4.0937 {0.0430}	3.9290 {0.0491}	
Misspecification tests based on conditional scores		
Autocorr	*ARCH*	*Markov*
0.1210 {0.9748}	0.1362 {0.9687}	6.1520 {0.0001}

The absence of residuals autocorrelation, heteroskedasticity and the hypothesis of a 1st order Markov chain are not rejected. The expected real marginal cost dynamics associated with the MSIH(2)-AR(2) specification is illustrated in Figure 3b and the real marginal cost predicted by this specification is given by Figure 4b.

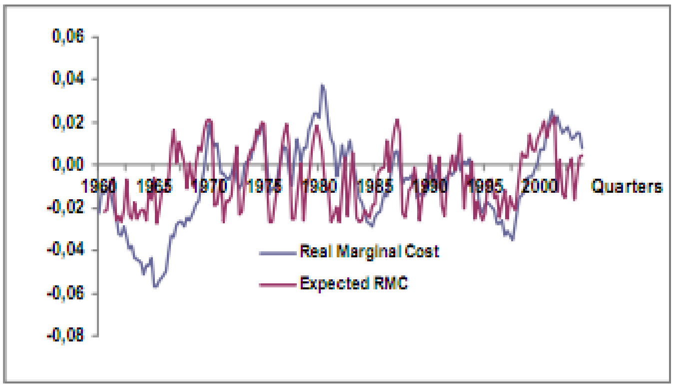


Figure 3b. MSIH(2)-AR(2) expected real marginal cost

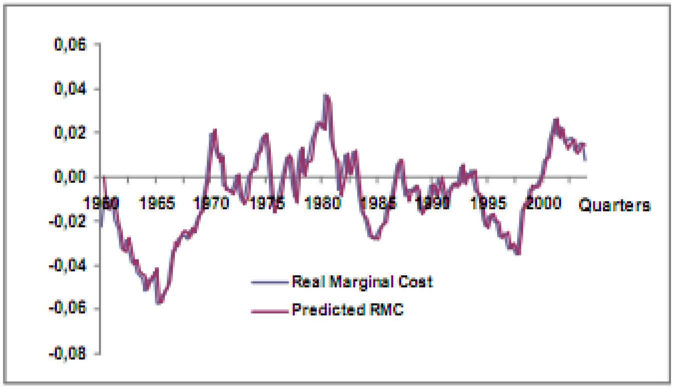


Figure 4b. MSIH(2)-AR(2) predicted real marginal cost

The discount rate:

Table 3c. Estimates of the MSIH(2)-AR(2) model for the discount rate

State (j)	n°obs/state	$\hat{P}(S_t = k)$	$\hat{\phi}_{0_{S_t=k}}$	$\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\sigma}^2_{\varepsilon_{S_t=k}}$
1	57	0.3294 (0.1529)	0.0019 (0.0020)	1.4403 (0.0672)	−0.4750 (0.0677)	0.0001 (0.0000)
2	117	0.6706 (0.1529)	0.0018 (0.0008)	1.4403 (0.0672)	−0.4750 (0.0677)	0.0000 (0.0000)

$$\hat{P} = \begin{bmatrix} \underset{(0.0506)}{0.9173} & \underset{(0.0244)}{0.0406} \\ \underset{(0.0506)}{0.0827} & \underset{(0.0244)}{0.9594} \end{bmatrix} \qquad lnL^{MSI}_{R_{t,t+1}} = 683.5747$$

The linear model appears to be the best one can use to characterize the discount rate dynamics. The MSIH(2)-AR(2) estimates (Table 3c) confirm this result as the calculated unconditional means in each regime, i.e. $\bar{R}_1^{MSIH} = 0.0547$ and $\bar{R}_2^{MSIH} = 0.0518$, are close to each other. One can also notice that once in each of these states, the probabilities of remaining there are close to 1, given by $\hat{p}_{11} = 0.9173$ and $\hat{p}_{22} = 0.9594$ so that, these regimes can be considered as a unique one. However, we note that the infrequent switching between these two regimes are observed during the reported inflation «crisis» episodes from the end of the 60's to the mid 80's and another episode[21] in 2001. Figure 2c illustrates the evolution of these probabilities of being in each regime at t given the information at t-1.

[21] This last episode may reflect the September 11 terrorist attack.

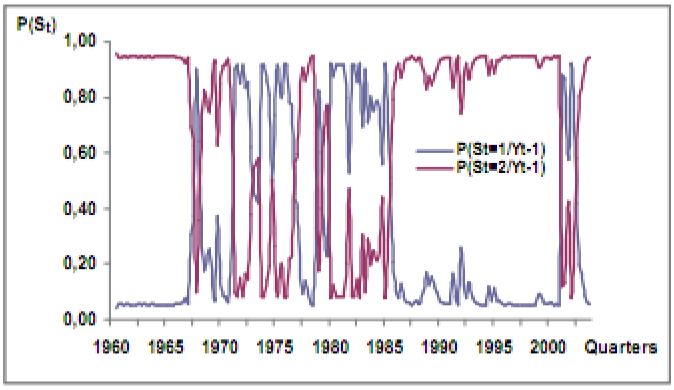


Figure 2c. MSIH(2)-AR(2) regimes probabilities for the discount rate

The results of the specification adequacy (Table 4c) indicate a global stability of the all the parameters even when each state is picked up individually. The presence of residuals autocorrelation cannot be rejected while, residuals heteroskedasticity and the hypothesis of a Markov chain of the 1st order can. Consequently, the MSIH(2)-AR(2) specification for the discount rate does not appear to be the most adequate one.

Table 4c. Adequation tests of the MSIH(2)-AR(2) model for the discount rate

Tests for parameters stability		
Markov	*mu*	*Res-cov*
0.2906 [0.7500]	0.1870 [0.7500]	0.3513 [0.7500]
Tests for parameters stability in state		
State 1	*State 2*	
0.5039 [1.2400]	0.6882 [1.2400]	
Test of serially uncorrelated Markov chain		
Wald-test	*F-test*	
191.7423 {0.0000}	184.0285 {0.0000}	
Misspecification tests based on conditional scores		
Autocorr	*ARCH*	*Markov*
1.3445 {0.2556}	0.3268 {0.8597}	0.4142 {0.7982}

The expected rate dynamics associated with this MSIH(2)-AR(2) specification is illustrated in Figure 3c and the predicted rate is given by Figure 4c.

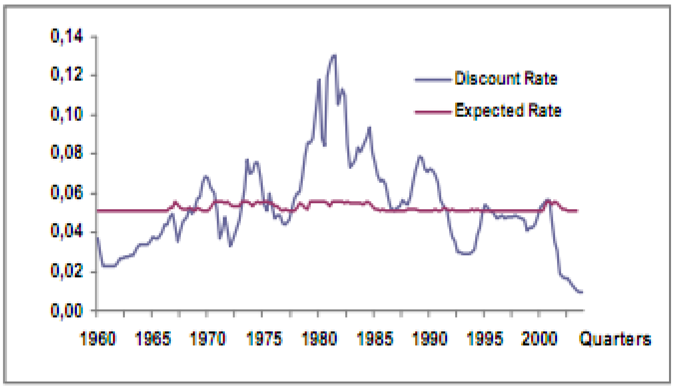


Figure 3c. MSIH(2)-AR(2) expected discount rate

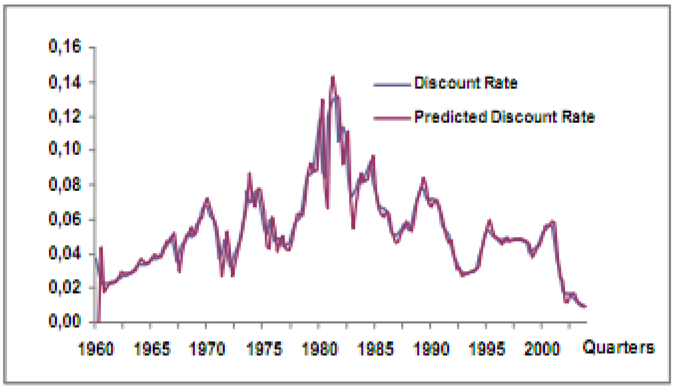


Figure 4c. Discount rate as described by the MSIH(2)-AR(2) specification

The real output growth rate:

Almost clearly, it seems like there is no break and, like in the discount rate case, the two regimes detected by the MSIH(2)-AR(2) specification $(\bar{\gamma}_{y,1}^{MSIH} = 0.0087 \text{ and } \bar{\gamma}_{y,2}^{MSIH} = 0.0080)$ are close.

Table 3d. Estimates of the MSIH(2)-AR(2) model for the real output growth rate

State (j)	n°obs/state	$\hat{P}(S_t = k)$	$\hat{\phi}_{0S_t=k}$	$\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\sigma}^2_{\varepsilon S_t=k}$
1	94	0.0000 (0.5304)	0.0048 (0.0012)	0.2214 (0.0728)	0.2312 (0.0773)	0.0001 (0.0000)
2	80	1.0000 (0.5304)	0.0044 (0.0009)	0.2214 (0.0728)	0.2312 (0.0773)	0.0000 (0.0000)

$\hat{P} =$	0.9894 (0.0109)	0.0000 (0.0056)	$lnL^{MSI}_{\gamma_{y,t}}$=606.2054
	0.0106 (0.0109)	1.0000 (0.0056)	

The probabilities of remaining in each of these regimes can be assimilated to one ($\hat{p}_{11} = 0.9894$ and $\hat{p}_{22} = 1.0000$) while the probabilities of leaving them are close to zero ($\hat{p}_{12} = 0.0106$ and $\hat{p}_{21} = 0.0000$). The «high» growth regime takes place during the years 1973 to 1983 as shown by Figure 2d.

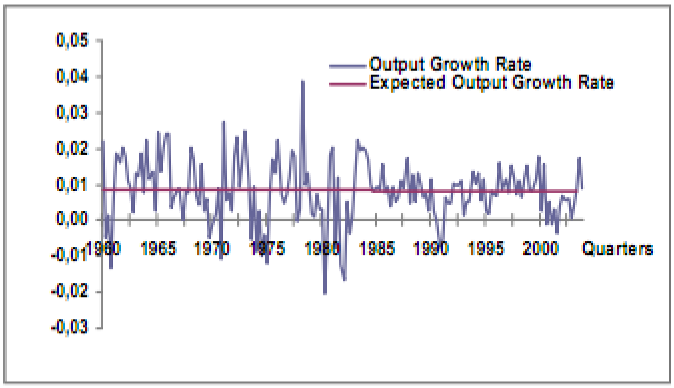


Figure 2d. MSIH(2)-AR(2) expected real output growth rate

The results of the adequacy tests are given in Table 4d.

Table 4d. Adequation tests of the MSIH(2)-AR(2) model for the real output growth rate

Tests for parameters stability		
Markov	*mu*	*Res-cov*
0.2022 [0.7500]	0.0943 [0.7500]	0.3758 [0.7500]
Tests for parameters stability in state		
State 1	*State 2*	
0.5649 [1.2400]	0.3362 [1.2400]	
Test of serially uncorrelated Markov chain		
Wald-test	*F-test*	
5978.1938 {0.0000}	5737.6917 {0.0000}	
Misspecification tests based on conditional scores		
Autocorr	*ARCH*	*Markov*
1.5096 {0.2017}	1.9186 {0.1097}	0.3875 {0.8174}

The dynamics of the expected output growth rate associated with this MSIH(2)-AR(2) specification are illustrated in Figure 3d and the predicted series by this specification are given by Figure 4d.

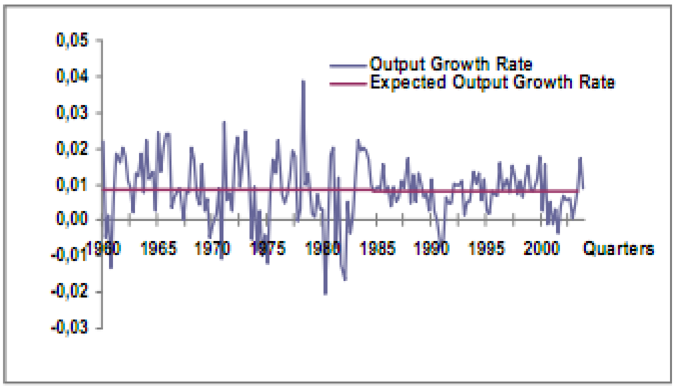


Figure 3d. MSIH(2)-AR(2) expected real output growth rate

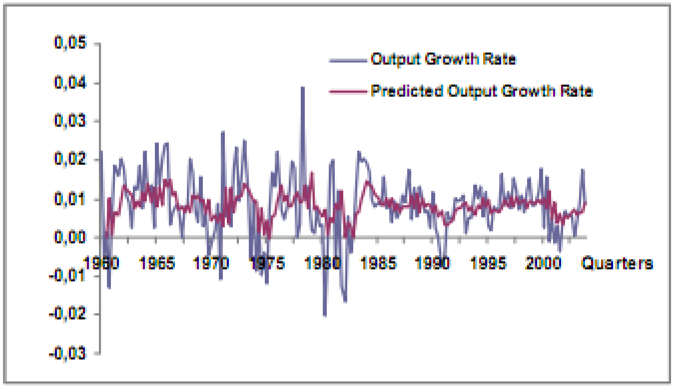


Figure 4d: MSIH(2)-AR(2) predicted real output growth rate

In summary, the results indicate that only the inflation rate switches between three clearly identified regimes. In the real marginal cost case, even if the linear specification seems to be the preferred one, the MSIH-AR specification identifies two distinct regimes and some frequent switches between these regimes. The discount rate and the output growth rate seem to be adequately characterized by the linear specification. Based on this first stage results, we estimate the different versions of the Phillips curve according to different econometric specifications. The choice of these specifications is based on the theoretical background of each version of the Inflation-Real activity trade-off as discussed in the introduction of this paper.

4. Estimation of the Phillips Curves

Insofar, as we try to measure the impact of expectations on the theoretical validity of the trade-off, we focus on empirical aspects of the different versions of the Phillips curve presented in the introduction of this paper. We estimate these major versions of the post-Keynesian views of the trade-off assuming that all the coefficients in these versions could be time or states varying. Building on the expected values of the main variables, one can consider the following econometric approaches.

4.1 Classical Estimation of the Keynesian Trade-off

As a benchmark version of the trade-off, we estimate the Keynesian Phillips Curve. This basic version, described by equation (KPC), is estimated assuming an intercept term and a Generalized Autoregressive Conditional Heteroskedasticity GARCH(1,1) process as

$$\pi_t = c + \xi^{kpc}\psi_t + \varepsilon_t$$
$$\sigma^2_{\varepsilon,t} = \omega_0 + \omega_1 \varepsilon^2_{t-1} + \omega_2 \sigma^2_{\varepsilon,t-1} \quad (23)$$

The results (Table 5) indicate that the KPC trade-off is weakly effective over the sample period as $\xi^{kpc} = 0.0787$ is only significant at 12%.

Table 5: Estimates of the Keynesian Phillips Curve

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.020560	0.001489	13.80881	0.0000
RMC	0.078766	0.049736	1.583683	0.1133
Variance Equation				
C	2.59E-05	1.91E-05	1.351911	0.1764
ARCH(1)	0.440608	0.120539	3.655309	0.0003
GARCH(1)	0.533628	0.124963	4.270273	0.0000
R-squared	-0.204524	Mean dependent var		0.034145
Adjusted R-squared	-0.232700	S.D. dependent var		0.028483
S.E. of regression	0.031624	Akaike info criterion		-5.039496
Sum squared resid	0.171015	Schwarz criterion		-4.949426
Log likelihood	448.4757	Durbin-Watson stat		0.306373

4.2 Time Varying Parameters' Estimation of the Monetarist Trade-off

Recalling that in the monetarist vision of the trade-off, agents are assumed to adaptively make their expectations, we estimate the following Time Varying Parameters - Augmented Phillips Curve

$$\pi_t = b^m_{1,t}\pi^*_t + \xi^m_t\left(\psi_t - \psi^*_t\right) + \varepsilon_t$$
$$b^m_{1,t} = b^m_{1,t-1} + \eta_{b_1,t}$$
$$\xi^m_t = \xi^m_{t-1} + \eta_{\xi,t} \quad (24)$$

In this framework, the expected inflation rate is calculated based on the MSIH(3)-AR(2) estimates $(\pi_t^* = \pi_t^{MSIH})$ and the natural real activity is extracted from the MSIH(2)-AR(2) estimates of the real marginal cost $(\psi_t^* = \psi_t^{MSIH})$.

The results, obtained using a linear Kalman filter procedure, are given by Table 6.

Table 6. Estimates of the Augmented Phillips Curve

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	-8.431297	0.089729	-93.96400	0.0000
	Final State	Root MSE	z-Statistic	Prob.
B1	1.053917	0.028252	37.30439	0.0000
KSI	0.051449	0.061573	0.835571	0.4034
Log likelihood	467.4473	Akaike info criterion		-5.361463
Parameters	1	Schwarz criterion		-5.343308
Diffuse priors	2	Hannan-Quinn criter.		-5.354098

The extracted filtered series of the APC coefficients $b_{1,t}^m$ and ξ_t^m are illustrated in Figures 5a and 5b.

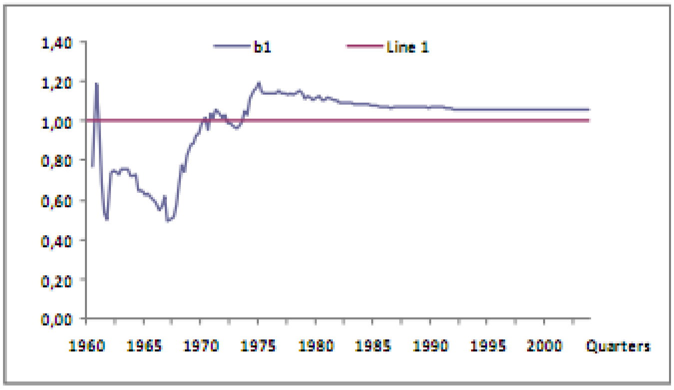


Figure 5a. Dynamic of the expected inflation rate impact when $\pi_t^* = \pi_t^{MSI}$

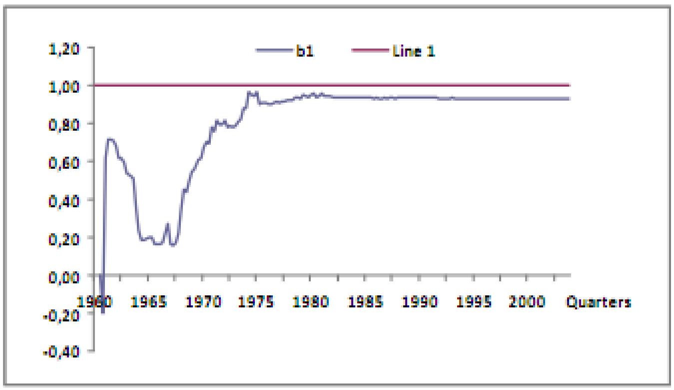


Figure 5b. Dynamic of the expected inflation rate impact when $\pi_t^* = \pi_{t-1}$

One can show that before 1969, the expected inflation rate impact on the current rate evolves under the frontier line $(b^{m}_{1,t} < 1)$. After this year, this series converge to this long term value. The coefficient measuring the real marginal cost impact on the inflation rate (ξ^{m}_{t}) is positive except during the years 1965-1970 and 1972-1974 which can be considered as a period during which the Phillips curve was temporally ineffective. After 1975, the series tend to the estimated KPC value (ξ^{kpc}_{t}) even if the coefficient $b^{m}_{1,t}$ stays close to its long term value. The effectiveness of this monetarist trade-off is weaker than the Keynesian one.

When we estimate the APC equation measuring the expected rate of inflation with the one period lag series $(\pi^{*}_{t} = \pi_{t-1})$, the expected rate impact never reaches its frontier line even if it is close to it after 1973 (Figure 5c). The coefficient $\xi^{m}_{t} \mid \pi_{t-1}$ is positive before 1973 but stays negative after this year (Figure 5d).

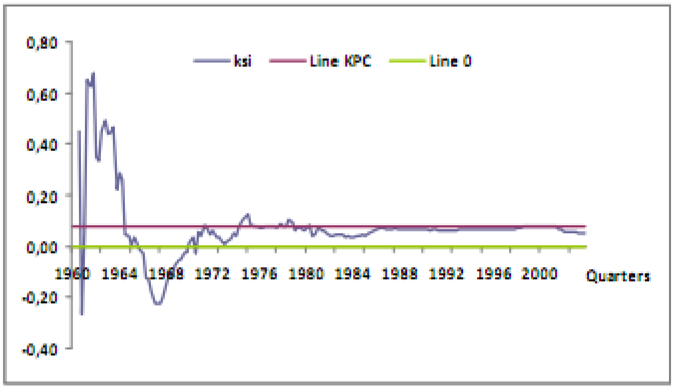


Figure 5c. Dynamic of the real marginal cost impact when $\pi^{*}_{t} = \pi^{MSI}_{t}$

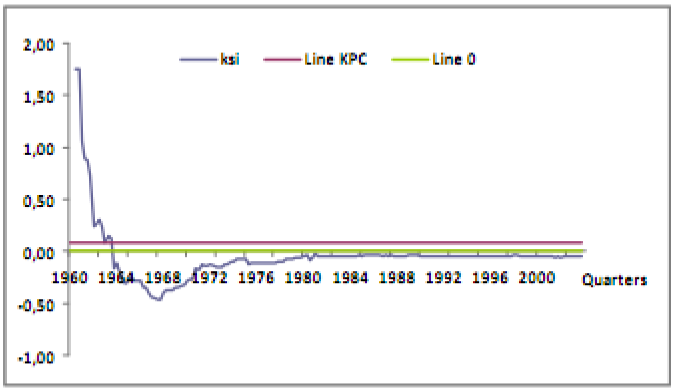


Figure 5d. Dynamic of the real marginal cost impact when $\pi^{*}_{t} = \pi_{t-1}$

Put together, these results seem to confirm the intuition behind the monetarist view of the trade-off as they globally highlight an advent of the long-term conditions impulse by the agents' myopic correction.

4.3 Two-Steps Estimation of the New Keynesian Phillips Curves

The main problem which is raised in estimating equations (NKPC-PI and NKPCSS-PI) is related to the presence of expectations terms $(E_{t}\,[.])$. In order to respond to this problem, we follow a two steps strategy to estimate the New Keynesian Phillips Curves coefficients. In the New Keynesian framework, it is almost assumed that the inflation rate is a stationary process. But, one can assume a possible long memory process in the inflation rate or in all the main NKPC variables' dynamics.

4.3.1 Fractional Integrated - Vectorial AutoRegressive reduced Forms' Estimation

In a first step of these estimations of the New Keynesian Phillips Curves coefficients, we study the dynamics of the variables considering a Fractional Integrated - Vectorial AutoRegressive (FI-VAR) reduced form. This reduced form permits us to investigate the inflation persistence hypothesis. Also, building on the evidence of asymmetries in the evolution of the inflation rate, we combine the techniques based on fractional integration with the results of the non-linear model estimations outline in the first stage of this study.

In this Fractional Integration framework, we first fit ARFIMA(1,d,0) univariate models to the series based on demeaned data and using maximum likelihood procedure. The demeaned data are calculated using the sample mean of the series for the real marginal cost $(\bar{\psi}^{Lin})$, the discount rate $(\bar{R}^{Lin})$ and the real output growth rate $(\bar{\gamma}_y^{Lin})$. In the inflation rate case, we used the MSIH(3)-AR(2) means calculated for each of the three regimes, (i.e. $\bar{\pi}_1^{MSIH} = 0.0927$ from 1974:I to 1982:I, $\bar{\pi}_2^{MSIH} = 0.0357$ in the periods 1966:II - 1973:IV and 1982:II - 1992:I and finally $\bar{\pi}_3^{MSIH} = 0.0139$ in the periods 1960:I - 1966:III and 1992:II - 2003:IV).

Considering these inflation rate regimes subdivisions of the sample, we estimate a FI-VAR model with one lag to reduce the number of parameters to be estimated. The model can be written as

$$A(L)\,D(L)\,Z_t = \varepsilon_t \tag{25}$$

$$D(L) = \begin{bmatrix} (1-L)^{d_{\hat{\pi}_t}} & 0 & 0 & 0 \\ 0 & (1-L)^{d_{\hat{\psi}_t}} & 0 & 0 \\ 0 & 0 & (1-L)^{d_{\hat{R}_{t,t+1}}} & 0 \\ 0 & 0 & 0 & (1-L)^{d_{\gamma_{y_{t,t+1}}}} \end{bmatrix} \tag{26}$$

where $Z_t = \left(\hat{\pi}_t, \hat{\psi}_t, \hat{R}_{t,t+1}, \hat{\gamma}_{y_{t,t+1}}\right)'$, $A(L) = I_4 - A_1 L$, $\varepsilon_t = \left(\varepsilon_t^{\pi}, \varepsilon_t^{\psi}, \varepsilon_t^{R}, \varepsilon_t^{\gamma_y}\right)'$ with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = \Sigma$.

This fractional departures from the linear VAR specification have very different long-run implications as in equation (25), each variable in Z_t can be non-stationary but non-explosive depending on the values of the differencing parameters $d_{\cdot}$. When these parameters are equal to 0.5 the variables are non-stationary and the non-stationary increasing towards $d_{\cdot} = 1$, Z_t can be viewed as becoming "more non-stationary", but it does so gradually. Non-linearity and the order of integration of inflation rates can, therefore, be considered as a key point to understand the dynamics of the inflation rate and to measure the expectations impact on the Inflation - Real activity trade-off. Noting that fractional integration and non-linearity are issues which are intimately related (Diebold, and Inoue (2001), Davidson, and Terasvirta (2002), Caporale, and Gil-Alana (2008), etc.), we take into account the analysis of the order of integration of the variables in the first stage Markov Switching Intercept Heteroskedastic - AutoRegression framework. To estimate this model, we follow the procedure described by Sela, and Hurvich (2009).

4.3.2 The Structural Parameters' Estimation

In a second step, we run the estimation of the structural parameters using the cross-equation restrictions that the model requires for the considered reduced form. Specifically, the estimation performed in the first step offers a set of FI-VAR coefficients describing the data through these reduced forms which, combined with the restrictions imposed by the theoretical model, lead to a moment conditions F that capture the gap between the data and the model.

Starting from the FI-VAR(1) estimates and considering equation (NKPC-PI), one can express the conditional expectations of the deviations of the variables relative to their steady states as[22]

$$\begin{aligned} E\left(\hat{\pi}_t \mid \tilde{Z}_{t-1}\right) &= e'_{\pi} A_1 \tilde{Z}_{t-1} \\ E\left(\hat{\psi}_t \mid \tilde{Z}_{t-1}\right) &= e'_{\psi} A_1 \tilde{Z}_{t-1} \\ E\left(\hat{R}_{t,t+1} \mid \tilde{Z}_{t-1}\right) &= e'_{R} A_1 \tilde{Z}_{t-1} \\ E\left(\hat{\gamma}_{y_{t,t+1}} \mid \tilde{Z}_{t-1}\right) &= e'_{\gamma_y} A_1 \tilde{Z}_{t-1} \end{aligned} \tag{27}$$

Under the assumption that $E\left(\varepsilon_t \mid \tilde{Z}_{t-1}\right) = 0$, we are able to obtain the conditional expectations of each variable by projecting the left and right terms of equation (NKPC-PI) on $\tilde{Z}_{t-1} = D(L) Z_{t-1}$, i.e.

$$E\left(\hat{\pi}_t \mid \tilde{Z}_{t-1}\right) = e'_{\pi} A_1 \tilde{Z}_{t-1} = f\left(A_1, \Psi\right) \tilde{Z}_{t-1} \tag{28}$$

$$\begin{aligned} E\left(\hat{\pi}_t \mid \tilde{Z}_{t-1}\right) = \bar{\rho}^{nkpc-pi} e'_{\pi} \tilde{Z}_{t-1} + \xi^{nkpc-pi} e'_{\psi} A_1 \tilde{Z}_{t-1} + b_1^{nkpc-pi} e'_{\pi} A_1^2 \tilde{Z}_{t-1} \\ + b_2 e'_{\pi} \gamma_1 \left(I - \gamma_1 A_1\right)^{-1} A_1^3 \tilde{Z}_{t-1} + \chi\left(\gamma_2 - \gamma_1\right) e'_R \left(I - \gamma_1 A_1\right)^{-1} A_1 \tilde{Z}_{t-1} \\ + \chi\left(\gamma_2 - \gamma_1\right) e'_{\gamma_y} \left(I - \gamma_1 A_1\right)^{-1} A_1^2 \tilde{Z}_{t-1} \end{aligned} \tag{29}$$

where e_k terms are column vectors of value 1 at the position corresponding to the variable k and 0 elsewhere and are used to select separately each of the four variables in the vector Z_t.

We then obtain a first set of moment conditions that capture the difference between data and model as

$$F_{1,S_t}\left(A_1, \Psi\right) = e'_{\pi} A_1 - f\left(A_1, \Psi\right) \tag{30}$$

Similarly, one can use the NKPCSS-PI equation to form the second set of moment conditions linking the steady state values of all the model variables

$$\begin{aligned} & F_{2,S_t}\left(A_1, \Psi\right) \\ = & \left(1 - \alpha \bar{\pi}^{(\theta-1)(1-\rho)}\right)^{\frac{1+\theta\omega}{1-\theta}} \left(\frac{1 - \alpha \bar{R} \bar{\gamma}_y \bar{\pi}^{1+\theta(1+w)(1-\rho)}}{1 - \alpha \bar{R} \bar{\gamma}_y \bar{\pi}^{\theta-\rho(\theta-1)}}\right) - \frac{\theta(1-\alpha)^{\frac{1+\theta\omega}{1-\theta}}}{\theta - 1} \bar{\psi} \end{aligned} \tag{31}$$

[22] Their empirical steady state equivalents are given by

$$\begin{aligned} \bar{\pi} &= \exp\left(\bar{\pi}^{MSI}\right) \\ \bar{\psi} &= \exp\left(\bar{\psi}^{Lin}\right) \\ \bar{R} &= \bar{R}^{Lin} \\ \bar{\gamma}_y &= \exp\left(\bar{\gamma}_y^{Lin}\right) \end{aligned}$$

These two sets of moment conditions define an overall distance measure that enables us to judge the adequacy of the model to the data

$$F_{S_t}(\Psi) = \left(F'_{1,S_t}, F'_{2,S_t}\right)' \tag{32}$$

so, the model fits the data, if and only if, there is a vector of structural parameters (Ψ) that solves the following constrained minimization problem

$$\underset{\Psi}{Min} F_{S_t}(\Psi)' F_{S_t}(\Psi) \tag{33}$$

subject to $\alpha \in]0,1[$, $\rho \in [0,1]$ and $\theta \in [0,+\infty[$.

4.3.3 FI-VAR Estimation of the NKPC Model

Based on the first stage and the previous results but also on previous works (Cogley, and Sbordonne (2005), Groen, and Mumtaz (2008)), we estimate the first generation of the New Keynesian Phillips Curve given by equation (NKPC) following the two-steps strategy. In the first step, the FI-VAR estimation[23] procedure starts with the univariate estimated differencing parameters (Tables 7a-7c) for the all variables and setting all the initial off-diagonal elements of A_1 and Σ to zero.

Table 7a. FI-AR(1) estimates based on the first inflation rate regime

	$Inflation$	$Real\ Marg.\ Cost$
ϕ_1	0.1489 $(1.04e^{-1})$	0.6118 $(7.61e^{-2})$
σ_ε^2	0.0005 $(1.89e^{-8})$	$1.0 \cdot e^{-4}$ $(2.43e^{-8})$
d	0.2819 $(4.47e^{-2})$	0.3496 $(4.77e^{-2})$
$\ln L$	110.5020	145.8362

Table 7b. FI-AR(1) estimates based on the second inflation rate regime

	$Inflation$	$Real\ Marg.\ Cost$
ϕ_1	0.7977 $(2.92e^{-2})$	0.8050 $(1.95e^{-2})$
σ_ε^2	0.0002 $(1.43e^{-9})$	$1.0 \cdot e^{-4}$ $(6.30e^{-10})$
d	0.4199 $(4.26e^{-2})$	0.2001 $(4.49e^{-2})$
$\ln L$	261.7809	312.5373

Table 7c. FI-AR(1) estimates based on the third inflation rate regime

---

[23] Note that in this NKPC context, the FI-VAR model will be written as

$$A(L)\,D(L)\,Z_t = \varepsilon_t$$

$$D(L) = \begin{bmatrix} (1-L)^{d_{\hat{\pi}_t}} & 0 \\ 0 & (1-L)^{d_{\hat{\psi}_t}} \end{bmatrix}$$

where $Z_t = \left(\hat{\pi}_t, \hat{\psi}_t\right)'$, $A(L) = I_2 - A_1 L$, $\varepsilon_t = \left(\varepsilon_t^{\pi}, \varepsilon_t^{\psi}\right)'$ with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = \Sigma$.

	Inflation	*Real Marg. Cost*
ϕ_1	0.0017 (5.33e−2)	0.9000 (8.85e−4)
σ^2_ε	$1.0 \cdot e^{-4}$ (7.06e−10)	$1.0 \cdot e^{-4}$ (6.06e−9)
d	0.1075 (3.42e−2)	0.1157 (1.34e−3)
$\ln L$	302.5667	311.8143

From these univariate estimates, the bivariate FI-VAR(1) results are reported in Tables 8a-8c.

Table 8a. FI-VAR(1) estimates in the first inflation rate regime

	Maximum Likelihood with regression approximation	
A_1	0.0983 (1.73e−1)	0.7739 (6.34e−2)
	0.0645 (5.97e−1)	0.3210 (8.67e−1)
Σ	$4.2 \cdot e^{-4}$ (6.77e−6)	$5.8 \cdot e^{-6}$ (3.20e−6)
	$5.8 \cdot e^{-6}$ (3.20e−6)	$1.0 \cdot e^{-4}$ (4.15e−6)
d	0.2456 (2.85e−2)	0.4671 (1.80e−3)
$\ln L$	259.6892	

Table 8b. FI-VAR(1) estimates in the second inflation rate regime

	Maximum Likelihood with regression approximation	
A_1	0.8062 (2.31e−1)	0.0412 (3.41e−1)
	−0.0239 (6.59e−2)	0.8041 (9.19e−1)
Σ	$2.0 \cdot e^{-4}$ (1.48e−6)	$-6.2 \cdot e^{-7}$ (1.44e−6)
	$-6.2 \cdot e^{-7}$ (1.44e−6)	$1.0 \cdot e^{-4}$ (6.18e−6)
d	0.4488 (5.91e−3)	0.1976 (6.38e−2)
$\ln L$	574.5333	

Table 8c. FI-VAR(1) estimates in the third inflation rate regime

	Maximum Likelihood with regression approximation	
A_1	−0.0390 (4.72e−2)	0.1005 (1.41e−1)
	−0.0634 (5.46e−3)	0.8930 (2.1616)
Σ	$1.0 \cdot e^{-4}$ (9.48e−7)	$-2.0 \cdot e^{-5}$ (1.84e−6)
	$-2.0 \cdot e^{-5}$ (1.84e−6)	$1.0 \cdot e^{-4}$ (2.70e−6)
d	0.1285 (2.57e−2)	0.1221 (2.71e−2)
$\ln L$	616.7422	

One can show that in the second regime, the inflation rate has a larger differencing parameter than in the other two. In this skepticism regime, the inflation rate differencing parameter is quite close to 0.5 implying a long memory in the series. In the real marginal cost case, the highest differencing parameter is associated to the third regime. These results imply that, when price start to increase from the third regime to the second one (decrease from the first regime to the second one), reflecting the departure from the optimism (pessimism) regime to reach the skepticism one, the agents become more concerned by the level of the inflation rate. Similarly, when the economy moves to the pessimism regime, firms seem to pay more attention to the level of the real marginal cost.

The corresponding results of the distance minimization[24], obtained from the bivariate FI-VAR reduced form estimates, are presented in Table 9.

Table 9. Structural parameters based on the FI-VAR(1) estimates

	α	ρ	θ
$S_t = 1$	0.2500	0.0010	9.8580
$S_t = 2$	0.8500	0.7500	24.6060
$S_t = 3$	0.9500	0.0010	36.4040

The results indicate that the parameter measuring the degree of price rigidity is estimated as $\hat{\alpha}_{S_t=1} = 0.2500,\ \hat{\alpha}_{S_t=2} = 0.8500 \text{ and } \hat{\alpha}_{S_t=3} = 0.9500$. The indexation parameter is estimated as $\hat{\varrho}_{S_t=1} = 0.0010,\ \hat{\varrho}_{S_t=2} = 0.7500 \text{ and } \hat{\varrho}_{S_t=3} = 0.0010$. Then, firms that do not receive the signal to optimize their prices have a weak and quasi-negligible tendency to index them on the past inflation. Finally, the parameter that measures the degree of substitution between goods is estimated as $\hat{\theta}_{S_t=k} \in [9.8580, 36.4040]$. This estimated degree of substitution between goods imply a mark-up of about $\vartheta_{S_t=k} = 11\%$.

The corresponding NKPC coefficients are computed for each inflation rate regimes and associated with each of the FI-VAR autoregressive coefficients. Figures 6a-6b show these Phillips curve coefficients $(b_1^{nkpc} \text{ and } \xi^{nkpc})$.

[24] Note that in this simplified version of the trade-off, the conditional expectations of the deviations of the two variables relative to their steady states are given by

$$E\left(\hat{\pi}_t \mid \tilde{Z}_{t-1}\right) = e'_{\pi} A_1 \tilde{Z}_{t-1}$$
$$E\left(\hat{\psi}_t \mid \tilde{Z}_{t-1}\right) = e'_{\psi} A_1 \tilde{Z}_{t-1}$$

and their empirical steady state equivalents are given by

$$\bar{\pi} = \exp\left(\bar{\pi}^{MSI}\right)$$
$$\bar{\psi} = \exp\left(\bar{\psi}^{Lin}\right)$$

Also, the resulting conditional expectations of each of these two variables are given by

$$\begin{aligned} E\left(\hat{\pi}_t \mid \tilde{Z}_{t-1}\right) &= e'_{\pi} A_1 \tilde{Z}_{t-1} \\ &= f\left(A_1, \Psi\right) \tilde{Z}_{t-1} \\ &= \bar{\rho}^{nkpc} e'_{\pi} \tilde{Z}_{t-1} + b_1^{nkpc} e'_{\pi} A_1^2 \tilde{Z}_{t-1} + \xi^{nkpc} e'_{\psi} A_1 \tilde{Z}_{t-1} \end{aligned}$$

and the unique set of moment conditions that capture the restrictions implied by the theoretical model on the set of parameters describing data via the reduced form will be written as

$$F_{S_t}\left(A_1, \Psi\right) = e'_{\pi} A_1 - f\left(\Psi\right)$$

In this NKPC framework, we then solve the following constrained minimization problem

$$\underset{\Psi}{Min} F_{S_t}\left(\Psi\right)' F_{S_t}\left(\Psi\right)$$

to obtain the estimated structural parameters.

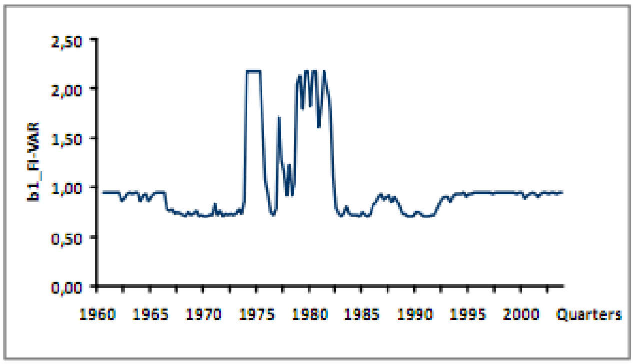


Figure 6a. Dynamic of the expected inflation rate impact based on the FI-VAR(1) estimates

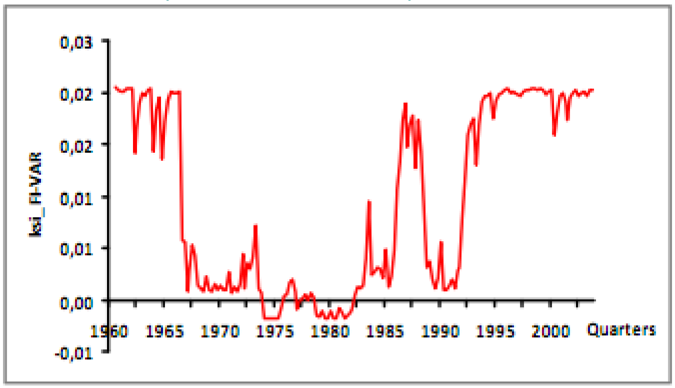


Figure 6b. Dynamic of the real marginal cost impact based on the FI-VAR(1) estimates

We note that during episodes of oil and monetary shocks (1973-1976 and 1979-1982), $\xi^{nkpc} < 0$ so that the Phillips curve seems to disappear. This reversal of the Inflation - Real activity arbitration clearly marks the renewal of the arbitration vision initiated by authors such as Phelps (1967), Friedman (1968) and Lucas (1972a). The impact of the expected inflation rate is high $(b_1^{nkpc} > 1)$ during these years.

4.3.4 FI-VAR Estimation of the NKPC-PI Model

In this New Keynesian Phillips Curve with Positive steady state Inflation framework, the FI-VAR(1) estimation procedure starts with the univariate estimated differencing parameters (Tables 10a-10c) for the all variables and setting all the initial off-diagonal elements of A_{11} and Σ to zero.

Table 10a. FI-AR(1) estimates based on the first inflation rate regime

	Inflation	Real Marg Cost	Discount	Output growth
ϕ_1	0.1489 $(1.04e^{-1})$	0.6118 $(7.61e^{-2})$	0.5903 $(2.94e^{-2})$	0.2635 $(2.27e^{-1})$
σ_ε^2	0.0005 $(1.89e^{-8})$	$1.0 \cdot e^{-4}$ $(2.43e^{-8})$	0.0001 $(1.40e^{-9})$	0.0001 $(1.77e^{-9})$
d	0.2819 $(4.47e^{-2})$	0.3496 $(4.77e^{-2})$	0.4636 $(3.21e^{-3})$	0.0480 $(1.66e^{-1})$
ln L	110.5020	145.8362	133.6091	132.6649

Table 10b. FI-AR(1) estimates based on the second inflation rate regime

	Inflation	Real Marg Cost	Discount	Output growth
ϕ_1	0.7977 (2.92e−2)	0.8050 (1.95e−2)	0.8547 (3.90e−4)	0.7643 (9.29e−2)
σ_ε^2	0.0002 (1.43e−9)	$1.0 \cdot e^{-4}$ (6.30e−10)	$1.0 \cdot e^{-4}$ (8.45e−10)	$1.0 \cdot e^{-4}$ (5.47e−9)
d	0.4199 (4.26e−2)	0.2001 (4.49e−2)	0.2477 (2.57e−4)	0.4900 (1.40e−1)
ln L	261.7809	312.5373	309.6323	303.5580

Table 10c. FI-AR(1) estimates based on the third inflation rate regime

	Inflation	Real Marg Cost	Discount	Output growth
ϕ_1	0.0017 (5.33e−2)	0.9000 (8.85e−4)	0.8462 (1.27e−5)	0.0444 (1.25e−1)
σ_ε^2	1.00e−4 (7.06e−10)	$1.0 \cdot e^{-4}$ (6.06e−9)	$1.0 \cdot e^{-4}$ (4.29e−10)	$1.0 \cdot e^{-4}$ (3.27e−9)
d	0.1075 (3.42e−2)	0.1157 (1.34e−3)	0.2525 (6.11e−5)	0.1499 (6.85e−2)
ln L	302.5667	311.8143	325.8422	313.7856

The multivariate results are reported in Tables 11a-11c.

Table 11a. FI-VAR(1) estimates in the first inflation rate regime

	Maximum Likelihood with regression approximation			
A_1	0.0286 (4.13e−2)	0.1783 (8.38e−2)	−0.0199 (2.17e−2)	−0.2081 (8.31e−2)
	0.0959 (1.22e−1)	0.5498 (6.76e−1)	0.0640 (1.74e−1)	0.2777 (2.28e−1)
	0.2251 (8.00e−2)	0.0017 (9.41e−4)	0.5162 (9.16e−2)	−0.1748 (1.63e−1)
	0.0202 (5.57e−2)	−0.1421 (3.63e−2)	0.0256 (1.71e−1)	0.2928 (1.05e−1)
Σ	$4.8 \cdot e^{-4}$ (8.88e−6)	$2.7 \cdot e^{-5}$ (4.13e−6)	$7.1 \cdot e^{-5}$ (1.77e−6)	$4.3 \cdot e^{-6}$ (4.67e−6)
	$2.7 \cdot e^{-5}$ (4.13e−6)	$1.0 \cdot e^{-4}$ (3.73e−6)	$-4.0 \cdot e^{-5}$ (3.88e−6)	$-1.1 \cdot e^{-4}$ (8.66e−6)
	$7.1 \cdot e^{-5}$ (1.77e−6)	$-4.0 \cdot e^{-5}$ (3.88e−6)	$1.3 \cdot e^{-4}$ (9.67e−7)	$3.4 \cdot e^{-5}$ (1.08e−5)
	$4.3 \cdot e^{-6}$ (4.67e−6)	$-1.1 \cdot e^{-4}$ (8.66e−6)	$3.4 \cdot e^{-5}$ (1.08e−5)	$2.3 \cdot e^{-4}$ (1.76e−6)
d	0.4663 (1.30e−2)	0.4299 (1.64e−1)	0.4872 (2.77e−4)	0.0788 (2.58e−2)
$\ln L$	541.9903			

Table 11b. FI-VAR(1) estimates in the second inflation rate regime

	Maximum Likelihood with regression approximation			
A_1	0.8020 (4.40e−1)	0.0203 (5.6173)	0.0718 (1.2587)	−0.0002 (1.1825)
	−0.0101 (1.7037)	0.7967 (3.9251)	−0.0169 (2.1861)	0.1267 (3.0456)
	−0.0607 (1.05e−1)	0.0098 (1.73e−2)	0.8506 (6.8901)	0.0103 (3.6321)
	0.0009 (2.60e−1)	−0.0828 (3.25e−2)	−0.0068 (4.04e−1)	0.7581 (1.4853)
Σ	$2.0 \cdot e^{-4}$ (1.18e−6)	$-1.5 \cdot e^{-6}$ (9.91e−7)	$-5.2 \cdot e^{-6}$ (4.32e−6)	$-1.8 \cdot e^{-5}$ (1.45e−6)
	$-1.5 \cdot e^{-6}$ (9.91e−7)	$1.0 \cdot e^{-4}$ (4.35e−6)	$9.6 \cdot e^{-7}$ (1.22e−4)	$-3.0 \cdot e^{-5}$ (1.68e−6)
	$-5.2 \cdot e^{-6}$ (4.32e−6)	$9.6 \cdot e^{-7}$ (1.22e−4)	$1.0 \cdot e^{-4}$ (7.71e−6)	$-5.9 \cdot e^{-5}$ (4.26e−5)
	$-1.8 \cdot e^{-5}$ (1.45e−6)	$-3.0 \cdot e^{-5}$ (1.68e−6)	$-5.9 \cdot e^{-5}$ (4.26e−5)	$1.4 \cdot e^{-4}$ (4.93e−6)
d	0.4330 (1.04e−2)	0.1869 (1.46e−1)	0.2221 (1.95e−2)	0.4900 (2.58e−1)
$\ln L$	1195.1250			

Table 11c. FI-VAR(1) estimates in the third inflation rate regime

	Maximum Likelihood with regression approximation			
A_1	-0.1010 $(-5.54e^{-2})$	0.0943 $(-1.01e^{-1})$	0.2712 $(-7.84e^{-2})$	0.0897 $(-3.72e^{-2})$
	-0.0585 $(-9.99e^{-3})$	0.8867 $(-3.05e^{-2})$	-0.0725 $(-1.08e^{-1})$	0.0782 $(4.13e^{-2})$
	-0.0007 $(-1.11e^{-1})$	0.0356 $(-1.92e^{-2})$	0.8042 $(6.26e^{-2})$	-0.0846 $(1.72e^{-1})$
	-0.0395 $(-1.03e^{-1})$	-0.0853 $(1.59e^{-2})$	0.0344 $(-2.55e^{-1})$	0.1549 $(-7.16e^{-2})$
Σ	$1.0 \cdot e^{-4}$ $(-1.09e^{-6})$	$-1.7 \cdot e^{-5}$ $(-1.99e^{-6})$	$2.6 \cdot e^{-6}$ $(-2.95e^{-6})$	$-1.5 \cdot e^{-5}$ $(-1.94e^{-6})$
	$-1.7 \cdot e^{-5}$ $(-1.99e^{-6})$	$1.0 \cdot e^{-4}$ $(-2.99e^{-6})$	$-1.4 \cdot e^{-5}$ $(1.99e^{-6})$	$-2.4 \cdot e^{-5}$ $(-2.99e^{-6})$
	$2.6 \cdot e^{-6}$ $(-2.95e^{-6})$	$-1.4 \cdot e^{-5}$ $(1.99e^{-6})$	$1.0 \cdot e^{-4}$ $(-4.49e^{-6})$	$-6.0 \cdot e^{-5}$ $(-1.43e^{-5})$
	$-1.5 \cdot e^{-5}$ $(-1.94e^{-6})$	$-2.4 \cdot e^{-5}$ $(-2.99e^{-6})$	$-6.0 \cdot e^{-5}$ $(-1.43e^{-5})$	$1.5 \cdot e^{-4}$ $(5.78e^{-6})$
d	0.1103 $(-1.81e^{-2})$	0.1216 $(-1.24e^{-2})$	0.1899 $(-7.82e^{-2})$	0.0043 $(7.63e^{-3})$
$\ln L$	1262.732			

Considering the univariate results, we note that, like in the NKPC case, the differencing parameters are low in the optimism regime indicating that all the series have short memories. When the economy enter in the skepticism regime, we note that the differencing parameters of the inflation and the output growth rates increase to reach values close to 0.5 indicating that these series have long memories. In this second regime where the expected inflation rate is at an intermediate level, agents seem to be extremely concerned by the output growth dynamics as $\hat{d}_{\gamma_{y_t}} = 0.49$. This last result could indicate that agents are questioning the monetary authority's credibility in its fight against the inflation. In the pessimism regime, firms seem to be almost attentive to the real marginal cost evolution and extremely concerned by the dynamics of the discount rate. The differencing parameters of these two last series are $\hat{d}_{\hat{\psi}_t} = 0.3496$ and $\hat{d}_{\hat{R}_{t,t+1}} = 0.46$. Clearly, the agents are examining the monetary authority's decisions in these medium and high inflation rate regimes.

Considering the multivariate case, the results are globally the same as in the univariate one. However, in the pessimism regime, in addition to the real marginal cost and the discount rate, we note that firms continue to attentively look at the inflation rate dynamics as its estimated differencing parameter is $\hat{d}_{\hat{\pi}_t} = 0.47$. Note that we consider a measure of an inflation gap so that our estimated differencing parameters measure the persistence of this inflation gap. As suggested by Cogley, Primiceri, and Sargent (2010): "this inflation gap is weakly persistent when the effects of shocks decay quickly and that it is strongly persistent when they decay slowly. When the effects of past shocks die out quickly, future shocks account for most of the variations in the inflation gap, pushing our measure (of the differencing parameter) close to zero. But when the effects of decay slowly, they account for a higher proportion of the near-term movements, pushing our measure of persistence closer to" 0.5. Our results then suggest that the inflation gap's persistence has changed over time.

The results of the distance minimization, presented in Table 12, indicate instability in the price stickiness through the estimated inflation rate regimes. This parameter is estimated as $\hat{\alpha}_{S_t=1} = 0.0010$, $\hat{\alpha}_{S_t=2} = 0.6004$ and $\hat{\alpha}_{S_t=3} = 0.0509$.

Table 12. Structural parameters based on the FI-VAR(1) estimates

	α	ρ	θ
$S_t = 1$	0.0010	0.0010	57.0505
$S_t = 2$	0.6004	0.8002	12.8080
$S_t = 3$	0.0509	0.0010	57.0505

As reported by Groen, and Mumtaz (2008), a week instability tendency can be associated to this parameter. The estimated probabilities of prices non-adjustment are then much more varying when one considers both the presence of long memory and regimes switching in the inflation rate dynamics. We observe that in the second regime, the prices are much more rigid than in the other two regimes illustrating the skepticism in the economic environment.

The indexation parameter is estimated as $\hat{\varrho}_{S_t=1} = 0.0010$, $\hat{\varrho}_{S_t=2} = 0.8002$ and $\hat{\varrho}_{S_t=3} = 0.0010$. Globally, firms that do not receive the signal to re-optimize their prices have a weak and quasi-negligible tendency to index those prices on the past inflation except in the second regime.

This results confirm those of Cogley, and Sbordone (2005) who estimate $\bar{\varrho}^{CS} = 0$. However, in the skepticism regime, where the differencing parameters of the inflation and the output growth rates are the highest, firms that do not have the opportunity to re-optimize their prices are much more backward looking than in the other two regimes. The fact that this parameter can be non-zero seems to confirm results obtained by many other studies performed in the NKPC with a zero steady state inflation rate (Gali, and Gertler (1999), Giannoni, and Woodford (2003)). For most of these studies, this indexation parameter is significant and estimated between 0.2 and 1. The existence of a non-zero indexation degree can capture the observed persistence of the inflation rate additionally to what is detected by the FI-VAR model. This result is also highlighted by Groen, and Mumtaz (2008) who estimate $\varrho^{GM} \in [0.65, 0.95]$.

Finally, the parameter that measures the degree of substitution between the goods is estimated as $\hat{\theta}_{S_t=k} \in [12.8080, 57.0505]$. These results remain fairly close to the values estimated by Cogley, and Sbordone (2005) and those of Groen, and Mumtaz (2008).

The NKPC-PI coefficients are derived from these estimated structural parameters computed for each inflation rate regimes and associated with each of the FI-VAR autoregressive coefficients. Figures 7a-7b show the Phillips curve coefficients $b_1^{nkpc-\bar{pi}}$ and $\xi^{nkpc-pi}$, reflecting the evolution of the expected inflation rate impact and the effectiveness of the Inflation-Real activity trade-off respectively.

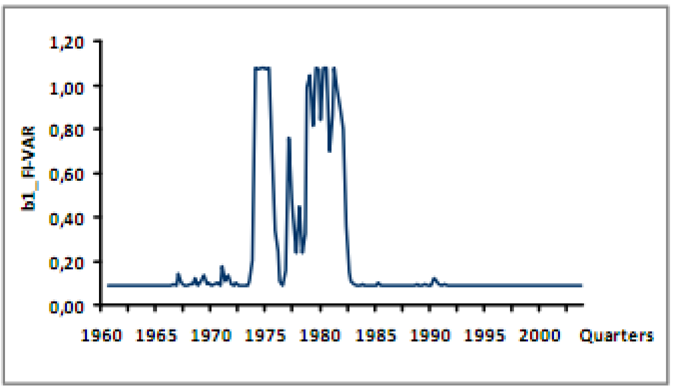


Figure 7a. Dynamic of the expected inflation rate impact based on the FI-VAR(1) estimates for the NKPC-PI model

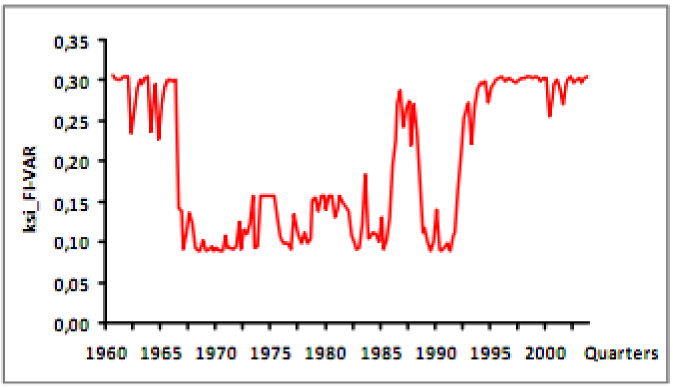


Figure 7b. Dynamic of the real marginal cost impact based on the FI-VAR(1) estimates for the NKPC-PI model

We note that during episodes of oil and monetary shocks, $\xi^{nkpc-pi} \rightarrow 0$ challenging the Phillips curve in these periods but less consistently than in the NKPC case. Also, some non-negligible challenges (possibly associated to the NBER recessions episodes and the September 11 events) are highlighted and, as one can expect, the impact of the expected inflation rate is high $(b_1^{nkpc-pi} > 1)$ during these years.

5. Conclusion

In this paper, we undertake some econometric inquiries into the dynamics of the main variables involved in modeling the New Keynesian Phillips Curve. We have shown that the inflation rate appears to be generated by an autoregressive process of the second order with constant lag coefficients and an unconditional mean, oscillating between three different regimes that could be assimilated to three expected targets of the U.S. inflation rate. According to this MSIH(3)-AR(2) specification, the expected inflation rate evolves between these regimes controlled by a Markov chain. This latter could be perceived as a system of beliefs formed by the agents on the three presumed fulfillments of their inflationist expectations. These beliefs can be qualified as adaptive since the probabilities of switching from one regime to another are conditional on the previous states of the economy. In the real marginal cost case, the MSIH(2)-AR(2) specification does not appear to be the most adequate one to characterize its dynamics. The linear model seems to be the most adequate specification to characterize this variable dynamics. For the discount rate and the output growth rate, the expected rates estimated by the MSIH(2)-AR(2) are almost constant so that the best specification to characterize these variables dynamics appears to be the linear one.

From this first empirical stage findings, we conducted empirical analysis around the famous bridge between the nominal and the real economic spheres associated to the Phillips curve. Our results are supportive of regimes' persistence inflation hypothesis, implying that shocks have a permanent effect in some of the regimes (like the skepticism or pessimism ones), but have finite lives in the optimism regime.

The results of this study show how the introduction of agents' expectations in the different versions of the trade-off has affected its empirical effectiveness and helped highlight the nuances between the three main visions of the evolution of the economic system. Globally, the results schematize some of the main aspects of the divergence between the classical, the monetarist and the Keynesian views on the theoretical validity of the Phillips curve.

References

[1] Alogoskoufis, G. S. and Smith, R. 1999. The Phillips Curve, The Persistence of Inflation and the Lucas Critique: Evidence from Exchange Rate Regimes, *American Economic Review,* 81: 1254-1275.

[2] Altig, D., Christiano, L., Eichenbaum, M. and Linde, J. 2011. Firm-Specific Capital, Nominal Rigidities and the Business Cycle, *Review of Economic Dynamics, Elsevier for the Society for Economic Dynamics,* 14(2): 225-247.

[3] Anders, W. 2000, Causality and Regime Inference in a Markov Switching VAR, *Manuscript, Research Department, Sveriges Riksbank*.

[4] Andrews, D.W.K. 1993. Tests for Parameter Instability and Structural Change with Unknown Change Point, *Econometrica,* 61: 821-858.

[5] Ascari, G. 2004. Staggered prices and trend inflation: some nuisances, *Review of Economic Dynamics,* 7(3): 642-667.

[6] Ascari, G. and Ropele, T. 2007. Optimal Monetary Policy under Low Trend Inflation, *Journal of Monetary Economics,* 54(8): 2568-2583.

[7] Bakhshi, H., Burriel, P., Khan, H. and Rudolf, B. 2007. The New Keynesian Phillips curve under positive trend inflation and strategic complementarity, *Journal of Macroeconomics,* 29(1): 37-59.

[8] Bakhshi, H., Khan, H. and Rudolf, B. 2008. The Phillips Curve under State-Dependent Pricing, *Journal of Monetary Economics,* 54(8): 2321-2345.

[9] Ball, L. 1994a. Credible Disinflation with Staggered Price Setting, *American Economic Review,* 84: 282-289.

[10] Ball, L. 1994b. What Determines the Sacrifice Ratio? *Monetary Policy, Edit by G. Mankiw, University of Chicago Press, Chicago*.

[11] Ball, L. 2008. Hysteresis in Unemployment, *Conference Series 53: Understanding Inflation and the Implications for Monetary Policy: A Phillips Curve Retrospective, Wequassett Inn Chatham, Massachusetts*.

[12] Ball, L., Mankiw, G. and Romer, D. 1988. The New Keynesian Economics and the Output-Inflation Trade-off, *Brookings Papers on Economic Activity,* 1: 1-65.

[13] Ball, L. and Croushore, D. 1998. Expectations and the effects of monetary policy, *Working Papers 98-13, Federal Reserve Bank of Philadelphia*.

[14] Barro, R. J. and Gordon, D. B. 1983. A Positive Theory of Monetary Policy in a Natural Rate Model, *Journal of Political Economy,* 91: 589-610.

[15] Benati, L. 2007. The Time-Varying Phillips Correlation, *Journal of Money, Credit and Banking,* 39(5): 1275-1283.

[16] Benati, L. 2008. Investigating Inflation Persistence across Monetary Regimes, *The Quarterly Journal of Economics, MIT Press,* 123(3): 1005-1060.

[17] Bernanke, B. and Gertler, M. 1995. Inside the Black Box: The Credit Channel of Monetary Policy Transmission, *Journal of Economic Perspectives,* 9(4): 27-48.

[18] Beyer, A. and Roger, E.A.F. 2007. Testing for Indeterminacy: An Application to U.S. Monetary Policy: Comments, *American Economic Review,* 97(1): 524-529.

[19] Bils, M. and Klenow, P. 2004. Some Evidence on the Importance of Sticky Prices, *Journal of Political Economy,* 112: 947-985.

[20] Bils, M. and Klenow, P. 2007. A New Keynesian model with unemployment, *CFS working paper 2007/08, Center for Financial Studies, Goethe University, Frankfurt*.

[21] Blanchard, O. and Fischer, S. 2007. A New Keynesian model with unemployment, *CFS working paper 2007/08, Center for Financial Studies, Goethe University, Frankfurt*.

[22] Blanchard, O. and Gali, J. 2007. Real Wage Rigidities and the New Keynesian Model, *Journal of Money, Credit and Banking,* 39(1): 35-65.

[23] Blanchard, O. and Gali, J. 2010. Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment, *American Economic Journal: Macroeconomics, American Economic Association,* 2(2): 1-30.

[24] Blinder, A.S. 1982. The Anatomy of Double-Digit Inflation in the 1970s, *in Inflation: Causes and Effects, edited by Robert E. Hall, pp. 261-82. Chicago: University of Chicago Press*.

[25] Bomfim, A.N. and Rudebusch, G.D. 2000. Opportunistic and Deliberate Disinflation under Imperfect Credibility, *Journal of Money, Credit and Banking,* 32: 707-21.

[26] Boutahar, M and Gbaguidi, D. 2009. Which Econometric Specification to Characterize the U. S. Inflation Rate Process, *Computational Economics,* 34(2): 145-172.

[27] Calvo, G. 1983. Staggered Prices in a Utility-Maximizing Framework, *Journal of Monetary Economics,* 12: 383-398.

[28] Caporale, G. M. and Gil-Alanna, L. A. 2008. Fractional integration and data frequency, *Faculty Working Papers 10/08, School of Economics and Business Administration, University of Navarra*.

[29] Caporale, G. M. and Gil-Alanna, L. A. 2008. Modelling the US, the UK and Japanese unemployment rates: Fractional integration and structural breaks, *Faculty Working Papers 11/08, School of Economics and Business Administration, University of Navarra*.

[30] Chappell, H. W. and McGregor, R. R. 2004. Did Time Inconsistency Contribute to the Great Inflation? Evidence from the FOMC Transcripts, *Economics and Politics, 16, pp. 233-51*.

[31] Christiano, L.J., Eichenbaum, M. and Evans, C.L. 2005. Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy, *Journal of Political Economy,* 113(1): 1-45.

[32] Cogley, T. and Sargent, T.J. 2001. Evolving post-World War II U.S. inflation dynamics, *NBER Macroeconomics Annual, 16*.

[33] Cogley, T. and Sargent, T.J. 2005. Drifts and volatilities: monetary policies and outcomes in the post WWII U.S., *Review of Economic Dynamics,* 8(2): 262-302.

[34] Cogley, T. and Sbordone, A. 2005. A search for a structural Phillips curve, *Manuscript Federal Reserve Bank of New York*.

[35] Cogley, T. and Sbordone, A. 2008. Trend Inflation and Inflation Persistence in the New Keynesian Phillips Curve, *American Economic Review,* 98(5): 2101-2126.

[36] Cogley, T., Primiceri, G. and Sargent, T.J. 2010. Inflation - Gap Persistence in the U.S., *American Economic Journal: Macroeconomics* 2: 43-69.

[37] Cooper, R. and Andrew, J. 1988. Coordinating coordination failures in Keynesian models, *Quarterly Journal of Economics,* 103(3):441-463.

[38] Davidson, J. and Teräsvirta, T. 2002, Long memory and nonlinear time series, *Journal of Econometrics,* 110(2): 105-112.

[39] Del Negro, M. and Schorfheide, F. 2009. Monetary Policy Analysis with Potentially Misspecified Models, *American Economic Review,* 99(4): 1415-1450.

[40] Diebold, F. and Inoue, A. 2001. Long memory and regime switching, *Journal of Econometrics,* 105(1): 131-159.

[41] Dotsey, M., King, R. G. and Wolman, A. 1999. State Dependent Pricing and the General Equilibrium Dynamics of Money and Output, *The Quarterly Journal of Economics,* 114(2): 655-690.

[42] Dufour, J. M., Khalaf, L. and Kichian, M. 2010. On the Precision of Calvo Parameter Estimates in Structural NKPC Models, *Journal of Economic Dynamics and Control,* 34(9): 1582-95.

[43] Eichenbaum, M. and Fisher, D. M. 2007. Estimating the Frequency of Price Re-Optimization in Calvo-Style Models, *Journal of Monetary Economics,* 54(7), 2: 32-47.

[44] Erceg, C. J. and Levin, A. T. 2003. Imperfect credibility and inflation persistence, *Journal of Monetary Economics,* 50(4): 915-944.

[45] Fauvel, Y., Guay, A. and Paquet, A. 2006. What Has the U.S. Phillips Curve Been Up To?, *Working Paper, CIRPEE*.

[46] Fernandez-Villaverde, J. and Rubio-Ramirez, J. 2007. How Structural Are Structural Parameters?, *NBER Macroeconomics Annual, Vol. 22, ed. Daron Acemoglu, Kenneth Rogoff, and Michael Woodford, 83-137, Cambridge University Press*.

[47] Fischer, S. 1977. Long-term Contracts, Rational Expectations and the Optimal Money Supply Rule, *Journal of Political Economy,* 85: 191-205.

[48] Friedman, M. 1968. The Role of Monetary Policy, *American Economic Review,* 58: 1-17.

[49] Friedman, M. 1968, Inflation: Causes and Consequences, *in Dollars and Deficits: Living with America's Economic Problems, pp. 21-60. Englewood Cliffs, NJ: Prentice-Hall*.

[50] Friedman, M. 1977. Inflation and Unemployment, *Journal of Political Economy,* 85: 451-472.

[51] Fuhrer. J. and Moore, G. 1995a. Inflation Persistence, *The Quarterly Journal of Economics,* 110(1): 127-160.

[52] Fuhrer. J. and Moore, G. 1995b. Monetary Policy Trade-offs and the Correlation between Nominal Interest Rates and Real Putput, *American Economic Review,* 85: 219-239.

[53] Fuhrer, J.C. 2006. Intrinsic and Inherited Inflation Persistence, *International Journal of Central Banking,* 2(3): 49-86.

[54] Fuhrer, J. C., Olivei, G. and Tootell, G. M. B. 2009. Empirical Estimates of Changing Inflation Dynamics, *Federal Reserve Bank of Boston Working Paper No. 2009-04*.

[55] Gali, C. and Gertler, M. 1999. Inflation dynamics: A structural econometric analysis, *Journal of Monetary Economics,* 44: 195-222.

[56] Galí, J., Gertler, M. and López-Salido, D. 1999. Inflation dynamics: A structural econometric analysis, *Journal of Monetary Economics,* 44: 195-222.

[57] Gbaguidi, D. 2011. La Courbe de Phillips: le temps d'un arbitrage et/ou l'arbitrage d'un temps, *Forthcomming l'Actualité économique*.

[58] Golosov, M. and Lucas, R. E. 2007. Menu Costs and Phillips Curves, *Journal of Political Economy, University of Chicago Press,* 115: 171-199.

[59] Goodfriend, M. and King, R.G. 1998. The New Neoclassical Synthesis and the Role of Monetary Policy, *NBER Macroeconomics Annual, Cambridge: MIT Press*.

[60] Groen, J. and Mumtaz, H. 2008. Investigating the structural stability of the Phillips curve relationship, *Working Paper No. 350, Bank of England*.

[61] Guay, A. and Pelgrin, F. 2004. The U.S. New Keynesian Phillips Curve: an empirical assessment, *Bank of Canada*.

[62] Guay, A. and Pelgrin, F. 2007. Using Implied Probabilities to Improve Estimation with Unconditional Moment Restrictions, *Cahier de recherché /Working Paper 07-47*.

[63] Guay, A. and Guerre, E. 2006. A Data-Driven Nonparametric Specification Test for Dynamic Regression Model, *Econometric Theory,* 22: 543-586.

[64] Giannoni, M. P. and Woodford, M. 2003. Optimal Inflation-Targeting Rules, in The Inflation-Targeting Debate, *ed. Ben S. Bernanke and Michael Woodford, 93-162. Chicago, IL: University of Chicago Press*.

[65] Hamilton, J. 1989. A New Approach to the Economic Analysis of Nonstationaty Time Series and the Business Cycle, *Econometrica,* 57: 357-84.

[66] Hamilton, J. 1996. Specification Testing in Markov Switching Time-Series Models, *Journal of Econometrics,* 70: 127-157.

[67] Hansen, B.E. 1992. Testing for Parameter Instability in Linear Models, *Journal of Policy Modeling,* 14(4): 517-533.

[68] Hetzel, R-L. 1998. Arthur Burns and Inflation, *Federal Reserve Bank of Richmond Economic Quarterly,* 84(1).

[69] Hornstein, A. and Wolman, A. L. 2005. Trend Inflation, Firm-Specific Capital and Sticky Prices, *Economic Quarterly, Federal Reserve Bank of Richmond,* Issue Fall*: 57-83*.

[70] Hornstein, A. 2007. Evolving Inflation Dynamics and the New Keynesian Phillips Curve, *Economic Quarterly - Federal Reserve Bank of Richmond*.

[71] Ireland, P. N. 1999. Does the Time-Consistency Problem Explain the Behavior of Inflation in the United States? *Journal of Monetary Economics,* 44: 279-91.

[72] Ireland, P. N. 2004a. Technology Shocks in the New Keynesian Model, *Review of Economics and Statistics,* 86: 923-36.

[73] Ireland, P. N. 2004b. Money's Role in the Monetary Business Cycle, *Journal of Money, Credit and Banking,* 36: 969-83.

[74] Ireland, P. N. 2007. Changes in the Federal Reserve's Inflation Target: Causes and Consequences, *Journal of Money, Credit and Banking,* 39(8):1851-1882.

[75] Jondeau, E. and Le Bihan, H. 2001. Testing for a Forward-Looking Phillips Curve Additional Evidence from European and US Data, *Notes d'Études et de Recherches N.86, Banque de France*.

[76] Justiniano, A. and Primiceri, G-E. 2008. The Time-Varying Volatility of Macroeconomic Fluctuations, *American Economic Review, American Economic Association,* 98(3): 604-41.

[77] Kang, K. H., Kim, C. J., and Morley, J. 2009. Changes in U.S. Inflation Persistence, *Studies in Nonlinear Dynamics and Econometrics, 3(4), Article 1*.

[78] Khan, H. and Moessner, R. 2004. Competitiveness, inflation, and monetary policy, *Bank of England working paper no. 246*.

[79] Kiley, M. 2007. Is moderate-to-high inflation inherently unstable? *International Journal of Central Banking,* 3(2): 173-201.

[80] Kim, C. J., and Nelson, C. R. 1989. The Time-Varying-Parameter Model for Modeling Changing Conditional Variance: The Case of the Lucas Hypothesis, *Journal of Business and Economic Statistics,* 7(4): 433-440.

[81] King, R. G. and Watson, M. W. 1994. The post-war U.S. Phillips curve: a revisionist econometric history, *Carnegie-Rochester Conference Series on Public Policy* 41: 157-219.

[82] King, R. G. and Wolman, A. L. 1996. Inflation Targeting in a St-Louis model of the 21st century, *Federal Reserve Bank of St. Louis Quarterly Review,* 83-107.

[83] King, R. G. 2000. The New IS -LM Model: Language, Logic, and Limits, *Federal Reserve Bank of Richmond, Economic Quarterly,* 86(3).

[84] Klenow, P. and Kryvtsov, O. 2008. State-Dependant or Time-Dependant Pricing: does it matter for recent U.S. inflation? *Quarterly Journal of Business and Economic Statistics,* 27: 293-311.

[85] Kozicki, S. and Tinsley, P. A. 2003. Alternative Sources of the Lag Dynamics of Inflation, *in Price Adjustment and Monetary Policy, 3-47. Bank of Canada Conference Proceedings. Ottawa: Bank of Canada*.

[86] Kozicki, S. and Tinsley, P. A. 2005. Permanent and Transitory Policy Shocks in an Empirical Macro Model with Asymmetric Information, *Journal of Economic Dynamics and Control,* 29:1985-2015.

[87] Kydland, F.E. and Prescott, E. C. 1977. Rules Rather than Discretion: The Inconsistency of Optimal Plans, *Journal of Political Economy,* 85: 473-91.

[88] Levin, A. and Piger, J. 2003. Is Inflation Persistence Intrinsic in Industrialized Economies?, *Federal Reserve Bank of St. Louis Working Paper No. 2002-023E*.

[89] Lucas, R. 1972a. Econometric Testing of the Natural Rate Hypothesis, *in Otto Eckstein (ed.), The Econometrics of Price Determination Conference, reprinted in Studies in Business Cycle Theory, 1981, Cambridge: MIT Press*.

[90] Lucas, R. 1972b. Expectations and the Neutrality of Money, *Journal of Economic Theory,* 4: 103-124.

[91] Lucas, R. 1973. Some International Evidence on Output-Inflation Tradeoffs, *American Economic Review,* 63: 326-334.

[92] Lucas, R. 1976. Econometric Policy Evaluation: a Critique, *Journal of Monetary Economics:* 19-46.

[93] Lucas, R. 1977. Understanding Business Cycles, *Journal of Monetary Economics:* 7-30.

[94] Mankiw, N. G. 1990. A Quick Refresher Course in Macroeconomics, *Journal of Economic Literature,* Vol. XXVIII: 1645-1660.

[95] Mankiw, N. G. 2001. The Inexorable and Mysterious Tradeoff Between Inflation and Unemployment, *Economic Journal,* 111: C45-C61.

[96] Mankiw, G. and Reis, R. 2002. Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve, *Quarterly Journal of Economics,* 117: 1295-1328.

[97] Mayer, T. 1998. Monetary Policy and the Great Inflation in the United States: The Federal Reserve and the Failure of Macroeconomic Policy, 1965-79, *Cheltenham: Edward Elgar*.

[98] Milani, F. 2007. Expectations, learning and macroeconomic persistence, *Journal of Monetary Economics,* 54(7): 2065-2082.

[99] Milani, F. 2010. Expectation Shocks and Learning as Drivers of the Business Cycle, *CEPR Discussion Papers 7743, C.E.P.R. Discussion Papers*.

[100] Mourelle, E., Cuestas, J. C. and Gil-Alana, L. A. 2011. Is There an Asymetric Behavior in African Inflation? A Non-Linear Approach, *South African Journal of Economics,* 79: 68-90.

[101] Muth, J. 1961. Rational Expectations and the Theory of Price Movements, *Econometrica, 29(3), pp. 315-335 (Reprinted in The new classical macroeconomics. Vol 1. (1992): 3-23, International Library of Critical Writings in Economics, vol. 19. Aldershot, UK: Elgar*.

[102] Nakamura, E. and Steinsson, J. 2007. Five Facts about Prices: A Reevaluation of Menu-Cost Models, *Unpublished*.

[103] Nason, J. 2006. Instability in U.S. Inflation: 1967-2005, *Federal Reserve Bank of Atlanta Economic Review,* 91(2): 39-59.

[104] Nyblom, J. 1989. Testing for the Constancy of Parameters Over Time, *Journal of the American Statistical Association,* 84(405): 223-230.

[105] Orleans, A. 2006. The Cognitive Turning Point in Economics: Social Beliefs and Conventions, *in R. Arena et A. Festré, Knowledge, Beliefs and Economics, Cheltenham (RU) et Northampton (MA, USA), Edward Elgar,* chapter 9: 181-202.

[106] Orleans, A. 2008. Les croyances et représentations collectives en économie, *in Bernard Walliser (éd.), Économie et Cognition, Éditions Ophrys et Maison des Sciences de l'Homme, coll. "Cogniprismes"*.

[107] Orleans, A. 2008. Les croyances monétaires et le pouvoir des banques centrales, *in Jean-Philippe Touffut (éd.), Les banques centrales sont-elles légitimes ? Albin Michel, pp. 17-35*.

[108] Orleans, A. *et al.* 2006., Valeurs, coordination et rationalité, *in François Eymard-Duvernay (éd.), L'économie des conventions, méthodes et résultats. Tome 1: Débats, Paris, La Découverte:* 23-44.

[109] Ormeño, A. 2009. Disciplining Expectations: Using Survey Data in Learning Models, *Unpublished*.

[110] Orphanides, A. 2002. Monetary Policy Rules and the Great Inflation, *American Economic Review,* 92: 115-20.

[111] Orphanides, A. and Wilcox, D. W. 2002. The Opportunistic Approach to Disinflation, *International Finance,* 5: 47-71.

[112] Otranto, E. 2005. The multi-chain Markov switching model, *Journal of Forecasting,* 24(7): 523-537.

[113] Peel, D. A. and Speight, A. H. 1996. Is the US Business Cycle Asymmetric? Some Further Evidence, *Applied Economics, Taylor and Francis Journals,* 28(4): 405-15.

[114] Phelps, E. S. 1967. Phillips Curves, Expectations of Inflation, and Optimal Inflation over Time, *Economica,* 135: 254-281.

[115] Phelps, E. S. 1970. Introduction: The New Microeconomics in Employment and Inflation Theory, *in E. Phelps et al., Microeconomic Foundations of Employment and Inflation Theory, Norton, New York:* 1-23.

[116] Phelps, E. S. and Taylor, J. B. 1977. Stabilizing Powers of Monetary Policy under Rational Expectations, *Journal of Political Economy,* 85: 163-190.

[117] Phillips, A. W. 1958. The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957, *Economica,* 25: 283-99.

[118] Pivetta, F. and Reis, R. 2007. The Persistence of Inflation in the United States, *Journal of Economics Dynamics and Control,* 31(4): 1326-1358.

[119] Primiceri, G.E. 2006. Why Inflation Rose and Fell: Policy-Makers' Beliefs and U.S. Postwar Stabilization Policy, *Quarterly Journal of Economics,* 121(3): 867-901.

[120] Roberts, J.M. 2006. Monetary Policy and Inflation Dynamics, *International Journal of Central Banking,* 2(3): 193-230.

[121] Romer, D. and Romer, C. 1989. Does Monetary Policy Matter? A New Test in the Spirit of Friedman and Schwartz, *NBER Macroeconomics Annual,* 4: 121-170.

[122] Romer, D. 2000. Macroéconomie approfondie, *Ediscience International*.

[123] Rotemberg, J. 2003. Customer anger at price increases, time variation in the frequency of price changes and monetary policy, *NBER working paper no. 9320*.

[124] Rothman, P. 1991. Further Evidence on the Asymmetric Behavior of Unemployment rates over the Business Cycle, *Journal of Macroeconomics,* 13(2): 291-298.

[125] Sahuc, J. G. 2006. Partial indexation, trend inflation, and the hybrid Phillips curve, *Economics Letters,* 90(1): 42-50.

[126] Salemi, M. K. 2006. Econometric Policy Evaluation and Inverse Control, *Journal of Money, Credit and Banking,* 38: 1737-64.

[127] Samuelson, P.A. 2008. Thoughts about the Phillips Curve, *Conference Series 53: Understanding Inflation and the Implications for Monetary Policy: A Phillips Curve Retrospective, Wequassett Inn Chatham, Massachusetts*.

[128] Sargent, T. J. and Wallace, N. 1973. Rational Expectations and the Dynamics of Hyperinflation, *International Economic Review, Department of Economics, University of Pennsylvania and Osaka University Institute of Social and Economic Research Association,* 14(2): 328-50.

[129] Sargent, T. J. 1987. Rational expectations, *The New Palgrave: A Dictionary of Economics,* 4: 76-79.

[130] Sargent, T. J. 1987. Macroeconomic Theory, *Academic Press, New York*.

[131] Sargent, T., Williams, N. and Zha, T. 2006. Shocks and Government Beliefs: The Rise and Fall of American Inflation, *American Economic Review,* 96(4): 1193-1224.

[132] Savin, N. E. 1987. Rational expectations: econometric implications, *The New Palgrave: A Dictionary of Economics,* 4: 79-85.

[133] Sbordone, A. 2002. Prices and Unit Labor Costs: A New Test of Price Stickiness, *Journal of Monetary Economics, 49(2), pp. 265-292*.

[134] Schorfheide, F. 2005. Learning and Monetary Policy Shifts, *Review of Economic Dynamics,* 8(2): 392-419.

[135] Sela, R. J and Hurvich, C. M. 2009. Computationally Efficient Methods for Two Multivariate Fractionally Integrated Models, *Journal of Time Series Analysis,* 30(6): 631-651.

[136] Sims, C. A. 2001. A Review of Monetary Policy Rules, *Journal of Economic Literature, American Economic Association,* 39(2): 562-566.

[137] Sims, C. A. 2008. Inflation Expectations, Uncertainty, The Phillips Curve and Monetary Policy, Conference Series 53: Understanding Inflation and the Implications for Monetary Policy: A Phillips Curve Retrospective, Wequassett Inn Chatham, Massachusetts.

[138] Sims, C. and Zha, T. 2006. Were there regime switches in U.S. monetary policy?, American Economic Review, American Economic Association, 96(1): 54-81.

[139] Smets, F. and Wouters, R. 2003. An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area, *Journal of the European Economic Association,* 1(5): 1123-75.

[140] Smets, F. and Wouters, R. 2007. Shocks and Frictions in U.S. Business Cycles: A Bayesian DSGE Approach, *American Economic Review,* 97(3): 586-606.

[141] Solow, R. 1968. Recent Controversy on the Theory of Inflation: An Eclectic View, *in Inflation: Its Causes, Consequences, and Control, edited by Stephen W. Rousseas. New York: New York University*.

[142] Steinsson, J. 2003. Optimal Monetary Policy in an Economy with Inflation Persistence, *Journal of Monetary Economics,* 50: 1425-56.

[143] Stock, J. H. 2001. Macro-econometrics, *Journal of Econometrics,* 100(1): 29-32.

[144] Stock, J. H. and Watson, M. W. 2007. Why Has U.S. Inflation Become Harder to Forecast?, *NBER Working Paper 12324, National Bureau of Economic Research, Inc*.

[145] Taylor, J. 1980. Aggregate Dynamics in Staggered Contracts, *Journal of Political Economy,* 88: 1-22.

[146] Taylor, J. 1999. Staggered Price and Wage Setting in Macroeconomics, *in J. Taylor et M. Woodford (eds.), Handbook of Macroeconomics, Elsevier, Amsterdam*.

[147] Tinsley, P.A. 2002. Rational Error Correction, *Computational Economics, Springer,* 19(2): 197-225.

[148] Wallace, N. 1975. Rational Expectations, the Optimal Monetary Instrument, and the Optimal Monetary Supply Rule, *Journal of Political Economic, University of Chicago Press,* 83(2): 241-54.

[149] Watson, A. 2010. The Impact of Trade Integration and Competition on Real and Nominal Price Rigidities: Insights from a New-Keynesian DSGE Model, *Unpublished*.

[150] Williams, J.C. 2006. Inflation Persistence in an Era of Well-Anchored Inflation Expectations, *Federal Reserve Bank of San Francisco Economic Letter No. 2006-27*.

[151] Woodford, M. 2003. Interest and Prices: Foundations of a Theory of Monetary Policy, *Princeton University Press: Princeton NJ*.

[152] Woodford, M. 2005. Firm-Specific Capital and the NEW Keynesian Phillips Curve, *International Journal of Central Banking,* 1(2): 1-46.

RATIONALITY AND CHOICES IN ECONOMICS: BEHAVIORAL AND EVOLUTIONARY APPROACHES

Mario GRAZIANO
Università di Messina, **Italy**
Dipartimento di Scienze Cognitive, della Formazione e degli Studi Culturali
mgraziano@unime.it
Daniele SCHILIRÒ
University of Messina, **Italy**
DESMaS "V.Pareto"
schi.unime@katamail.com

Abstract:

The paper critically discusses the issue of rationality and choices in economics in both the behavioural and evolutionary approaches. Our study aims, on the one hand, to highlight the scientific contributions of psychology in economics, since psychology, and with it the theoretical approach of the behavioral economics, has made more complex and problematic the analysis of economic choices, showing the limits of rationality. On the other hand, the work offers a reinterpretation of the theory of Alfred Marshall in a biological-evolutionary perspective. The reinterpretation of Marshall's theory in a evolutionary perspective aims to show that, historically, economics has not been a discipline aligned in a homogenous way to a single and undifferentiated thought, locked into the idea of perfect rationality, but, on the opposite, is a discipline that has enriched itself and continually is enriching by contributions and significant contaminations with other research fields.

Keywords: rationality, choice, behavioral economics, evolutionary theories, biology

JEL Classification: B13, B52, D01, D03, D81, D90.

1. Introduction

This paper critically discusses the issue of rationality and choices in economics by examining both the behavioral and evolutionary approaches.

Economics has pointed out, with Leon Walras and Vilfredo Pareto, its independence from the other disciplines following a deductive-nomological approach and treating the non-economic behavior as "irrational". This has led to exclude all links or contamination of economics with disciplines such as philosophy, psychology and biology. Not by chance that Lionel Robbins expressed a concept of choice entirely contained in the economic dimension. He claimed in his book *Essay on the Nature and Significance of Economic Science* in 1932 that economic theory is a deductive science, by which can be calculated the behavior of individuals, if are known the purposes, the means available and the preferences. Robbins then postulates the concept of choice in economics as a result of a coherent and logical reasoning, which implies and emphasizes the role of rationality.

Research in psychology, instead, has systematically depth the study of topics such as human judgment and behavior (also economic behavior) of individuals, challenging the traditional concept of perfect rationality of economic science. In addition, psychological research in economics, known as cognitive-behavioral approach and the biological and evolutionary line of research have highlighted some shortcomings and inconsistencies of economic science, showing, as in the case of behavioral economics, a systematic discrepancy between economic theory and reality. This implies the inability of economics and its theoretical system known as the theory of rational choice to explain and describe the complexity of reality.

This contribution argues the issue of choice and rationality in economics looking at different approaches with respect to the traditional analysis. Our investigation aims, on the one hand, to highlight the scientific contributions of psychology in economics, since psychology, and with it the theoretical approach of the behavioral economics, has made more complex and problematic the analysis of economic choices, showing the limits of rationality. On the other hand, the work offers a reinterpretation of the theory of Alfred Marshall in a biological-evolutionary perspective. His theory showed the dynamics and the evolution of the economic system. In Marshall's works, there is the clear belief that economic and biological phenomena share a large number of similarities concerning the organic and complex nature, but also the involvement in a constantly changing world. The reinterpretation of Marshall's theory in a evolutionary perspective aims to show that, historically, economics has not been a discipline aligned in a homogenous way to a single and undifferentiated thought, locked into the idea of perfect rationality, but, on the opposite, is a discipline that has enriched itself and continually is enriching by contributions and significant contaminations with other research fields.

2. Choice and rationality, between economics and psychology *

2.1. Expected utility and choices under uncertainty

Neoclassical economics, by adopting a system known as the "Theory of rational choice", has described the decision as a rational process conducted by a single cognitive process. In such a process it is assumed that each individual has stable and consistent preferences and make decisions based on the principle of maximization of the subjective expected utility. So given a set of options and beliefs expressed in probabilistic terms, it is assumed that the individual maximizes the expected value of a utility function U (x). This means that - according to the Theory of rational choice - an agent is rational if it maximizes the value of his expected utility function, where this function exists if are met the three axioms (*transitivity, continuity, independence*) on which are based the preferences. To maximize his expected utility function, the individual uses probability estimates and utility values as elements of the calculation, assessing the probabilities and significant utilities on the basis of his personal opinion but also using all relevant information available. In practice, this process is based on the assumption that the utility of an individual under conditions of uncertainty can be calculated as a weighted average of the utilities in each possible state, adopting as weights the probability of occurrence of each alternative. In this complex theoretical model the actions of individuals are the result of a choice and the rationality of actions is, therefore, the manifestation of the criterion of rationality of the agents.

The complexity of the system of choice under uncertainty manifests itself in the fact that one must take into account several conditions of rationality, such as the existence of a regular system of preferences on the consequences; the rationality of expectations about the consequences of actions; the rationality of the function that determines the system of preference on the actions relative to the expectations about the consequences of actions (Schilirò, 2011).

The expected utility theory has been generally accepted as a normative model of rational choice, by defining what decisions are rational. In fact, if an individual does not maximize his expected utility is intended to violate in his choices some precise axiomatic principles, which are also rationally binding. This theory has also been applied as a descriptive model of economic behavior (Friedman, Savage, 1948; Arrow, 1971) so as to constitute an important reference model in economics. However, the expected utility theory has proved unable to adequately describe the behavior of individuals, as Maurice Allais has been shown in Paris in 1952 with his experiment on gambling, which he published in Econometrica in 1953 and known as the "Allais' Paradox" [25]. The purpose of Allais was to show that the axiomatic method of subjective expected utility theory, proposed by von Neumann and Morgenstern in their book *Theory of Games and Economic Behavior* (1944), did not constitute a proper descriptive theory of choices under uncertainty, since he had met in several experiments the violation of the axiom of *independence*. Individuals, in fact, behaved in an ambiguous way in the choice between almost

*Paragraph 2. is written by Daniele Schilirò

[25] In 1952 in Paris, Allais presented his famous "Paradox" to an audience composed of the best economists of his generation: among others, Kenneth Arrow, Paul Samuelson, Milton Friedman, Jacob Marschak, Oskar Morgenstern and Leonard Savage. The results of laboratory experiments conducted by Allais showed that people chose inconsistently and preferred solutions that do not maximize expected utility, thus demonstrating that the axiomatic definition of rationality did not allow describing and even predicting the economic decisions.

certain events and probable events. Allais then expressed the need to change the formal criteria of the theory.

2.2. Rationality and economic choices in the behavioral approach

The "Allais Paradox" is a first important critique of the theory of expected utility because it highlights how individuals are "inconsistent" in their choices, as they are driven by motivations that the Theory of rational choice does not explain and cannot justify, due to its axiomatic definition of rationality. Another "Paradox" is the one developed by Ellsberg (1961), who demonstrated experimentally another type of inconsistency in preferences, showing that individuals prefer to bet on a lottery with known probabilities of winning rather than on a lottery with ambiguous results. Individuals therefore show an aversion to ambiguity. This aversion to uncertainty (ambiguity) is subjective but completely ignored in the model of expected utility from a descriptive point of view, while it is not considered eligible from a normative point of view.

But the most interesting critical contributions to the axiomatic approach of the Theory of rational choice theory come from the cognitive-behavioral theoretical strand that has emerged in the seventies. The results of research in cognitive psychology reveal different distortions (biases) in decision making under uncertainty that the economic theory of rational choice did not consider at all. These results highlight especially a number of anomalies in conceptualizing a model where individuals maximize their utility function, which, by hypothesis, is perceived as conistent, accurate and also stable over time. It follows that they often make systematic errors in predicting their future experience of results and, therefore, fail to maximize their utility (Kahneman, Thaler, 2006). This occurs because individuals face in their acting real difficulty in assessing their preferences. Therefore, they prefer the pursuit of instant gratification, which, however, are often inconsistent with their long-term preferences (Rabin, 1998).

Behavioral economics is then really a return to reality from an untenable position that the rational optimizing model is the only framework for economics (Shiller, 2005). Thus, behavioral economics, through his experiments, tries to account for constant deviations that individuals make in real life with respect to the theory of choice in its standard version, at the same time postulating a system of heuristic evaluation. Kahneman and Tversky were the promoters and the protagonists of the cognitive-behavioral turn. In one of their contributions Tversky and Kahneman (1974) documented that individuals made systematic deviations from perfect rationality when they express a judgement under uncertainty. Tversky e Kahneman provide a theoretical explanation about the observed deviations from perfect rationality, noting that people rely on «heuristic principles which reduce the complex tasks of assessing probabilities and predicting values to simpler judgmental operations» (1974, p.1124). Tversky e Kahneman, therefore, do not abandon the assumption that individuals are intelligent and intentional in making decisions, but they assume just systematic and specific distortions that move away the individuals' judgements from perfect rationality.

Rabin (1998, pp. 24-32) mentions among the various distortions highlighted by Tversky and Kahneman, and other scholars – e.g., Gilovich, Vallone and Tversky (1985), Bruner and Potter (1964), Keren (1987) and others, who contributed to this line of researches in behavioral economics: the "law of small numbers", or the tendency of individuals to overestimate by a rather small sample the similarities with the entire population from which the sample came from, and the "confirmatory bias". This stems from the fact (as documented by the results of research in cognitive psychology) that people tend to incorrectly read the evidence of the data, so as to constitute further evidence in support of their hypothesis, which explains the occurrence of "confirmatory bias".

Other violations of the expected utility paradigm that have a psychological motivation and which are important in the choice of a financial nature are risk aversion and, above all, loss aversion (Kahneman and Tversky, 1984). For most individuals, in fact, the motivation to avoid a loss is greater than the motivation to make a profit. This general psychological principle, which is connected to a kind of survival instinct, means that the same decision may give rise to opposite choices depending on whether the results are presented to the subject as losses rather than such as loss of earnings. This type of evidence has led Kahneman and Tversky (1979, 1984), but also Thaler (1980) to develop the "prospect theory" as part of their cognitive-behavioral approach. This theory starts from the observation that the probability distributions perceived by individuals, who make decisions under uncertainty, are not invariant with respect to environmental contexts. Hence one of the cardinal principles of behavioral economics is framing, that is, human actions are heavily influenced by frames of reference. The institutional structure that individuals have is the basic framework for all of their economic decisions[26]. Kahneman e Tversky have shown, for example, that many of the risks of little importance are given disproportionate weight, but also that the losses and future earnings are not treated symmetrically. The uncertain gains and losses are evaluated in this case in relation to the wealth possessed by the individual (endowment effect)[27]. The expected utility of the subjects is not calculated according to the monetary values that make the probability distribution, but rather on the deviation of these values from the *status quo*, which corresponds to the wealth of the individual. In this way, at different levels of wealth can match order of preference on the same pair, contradictory to each other, and this behavior is attributed to the perceptual processes that process information about uncertain events. Kahneman and Tversky attribute this behavior to the perceptual processes that process information about uncertain events taking as reference the situation to which the individual has been previously adapted. The authors link the description and understanding of economic decision to the analysis of the functioning of mental processes, thus emphasizing the psychological motivations of this perceptual distorsion. Kahneman and Tversky have, thus, questioned the assumptions of rationality of neoclassical economics. In fact, most of the economic analysis assumes perfect rationality in agents' decisions, but also the rationality of the judgments and predictions on which those decisions are based.

The approach of behavioral economics goes therefore beyond the rationality postulated by the neoclassical economics and formalized in the Theory of rational choice; it does indeed point to a different concept of rationality: i.e. that of bounded rationality. This concept, introduced by Herbert Simon (1955, 1956, 1978)[28] indicates that rational choice that takes into account the cognitive limits of the agents.

According to Simon, even though individuals try to make decisions as rationally as possible, they can not be entirely rational due to some factors or limits such as: the complexity of the problems, the availability of incomplete information, the limited ability of agents to process information, the limited time available to them, the conflicting preferences of decision makers with respect to objectives. Simon's approach based on bounded rationality has focused primarily on so-called process of finding a satisfactory level ("satisficing search and process") through which individuals, because of limitations in cognitive processes, unlike the process of maximizing, seek solutions to achieve a sufficiently good level of aspiration that they have set. This mechanism, validated by a large amount of empirical results, leads individuals, in the case where it is difficult for them to find "good enough" solutions to set an aspiration level, to diminish and/or increase the search activity. This rationality of the process of looking for alternatives, according to Simon (1987), is more procedural than substantive, rationality is in fact a feature / quality of the research process - as is highlighted by Novarese, Castellani, Di Giovinazzo (2009) - and is a psychological form of rationality, as opposed to economic rationality. So in the Simon's vision, bounded rationality is more related to the procedural level of decision making. Bounded rationality is consequently a complex mechanism, which takes into account many aspects of the mental process of the human person: both aspects of the research activity and/or ongoing adjustment of the level of aspiration and of post-decision assessment. Both these phases of decision-making are closely linked to personal experience and to the ability to "frame" a situation, putting it in relation to previous similar situations through a kind of analogical reasoning.

[26] Shiller (2005) argues that behavioral economics yielded important institutional innovation and in any case maintained an environment friendly to institutional innovation.
[27] Kahneman, Knetsch, Thaler (1990).
[28] See also Cyert, Simon, Trow, (1956), Simon (2000).

Simon's ideas on bounded rationality mean that, as regards the decision analysis, the focus shifts - as stated by Egidi (2005)[29] – from decision to the representation of the alternatives, thus paving the way for an extremely wide range of empirical studies on the building of strategies, problem-solving and learning.

Kahneman and Tversky are broadly in agreement with Simon in arguing that the analysis of cognitive process should be put at the center of the investigation of economic behavior, though without calling into question the normative force of rational choice. Their theory does not seek to falsify the expected utility theory as a normative theory, but will demonstrate its inadequacy at empirical level and, consequently, the poor predictive ability of economic theory on which it rests. Kahneman and Tversky distinguish two stages in the process of choice under uncertainty: first, the mental representation of events, second, the phase of evaluation (Tversky, Kahneman, 1986). In particular, the mental representation of events constitutes, in their view, the key element of the decision. The attitude to risk varies according to the different individuals, depending on the frame within which lies the choice (Kahneman and Tversky, 1979). It follows that the frame, or the context of choice, *coeteris paribus*, helps to determine a different behavior. In conclusion, Tversky and Kahneman, with their theory, certainly did not want to support the thesis of human irrationality, but they indicated the way to build models or "maps" of bounded rationality (Kahneman, 2002), thereby following an approach not much far from Simon's.

There is, however, another perspective, different from behavioral economics: the experimental economics. This latter approach justifies, by means of laboratory experiments, its capacity of analysis and verification of theories. For instance, in the event you want to test the empirical plausibility of an economic theory, the experimental analysis may be useful to check its generality; the laboratory represents, in fact, a special case in which a theory, that claims to be general, must find a positive response. The experimental evidence may also provides useful suggestions to the refinement of a theory, showing, in front of the empirical evidence produced in the laboratory, where it will highlight its greatest weaknesses. This theoretical strand of experimental economics identified weaknesses and strengths in the approach of behavioral economics. Vernon Smith (2000) – the most influential exponent of experimental economics – critiques several aspects of the theory of Kahneman e Tversky, but he shares with them some conceptualizations, he also considers Herbert Simon as one of the major economists for his theory and his experimental method. With regard to neoclassical theory, Smith (1982, 2000) recognizes in it some elements of great explanatory power, but also significant weaknesses. He argues that neoclassical economics in its standard formulation is a good first approximation to understand and predict the behavior of markets[30], but this theory - according to Smith - is incomplete, especially in the definition of the convergence processes over time.

The economic literature, which documents paradoxes and inconsistencies of rationality is rich and certainly goes beyond the above mentioned contributions from Kahenamann and Tversky, Thaler and other protagonists of the behavioral approach. An important issue about this concerns intertemporal choice, namely a choice between options the consequences of which occur at different points in time. It is good to mention in this regard the empirical paradoxes that document the failure of the traditional discounted utility model. As is well known, in the literature the dominant normative model is the model of Samuelson (1937), in which the utility of the various alternatives over time, measured at the time the investment decision is taken, is the weighted sum of the values of utility of all the alternatives, discounted as a function of time in which they arise. According to this model, the behavior of an investor is rational in dynamic terms when he realizes his investment plan consistenly as formulated in the present.

The discounted utility model provides a way to evaluate the intertemporal choice. First, there is the stationarity of preferences, whose main implication is that if A is preferred to B at time *t*, it will be preferred at any other time in the future. Secondly, the main component of the model is a function of

[29]Egidi, *Prefazione* in Motterlini, Guala (2005, p.XV).

[30] Experimental economics is concerned with markets and institutions rather than individuals.

discount that is used to calculate the present value of future utility. In many applications of the model are being used exponential discount functions. These functions generally have an exponential discount rate that is constant through time and equal for all goods. The exponential discount functions also have the convenient property of not generating inversions of preference. Strotz (1956)[31] was the first economist to formalize a theory of commitment and to demonstrate that the mechanisms of commitment may be important determinants of economic performance. He has shown that when the discount functions of individuals are not of exponential type, individuals prefer to limit their future choices. The mechanism of commitment is usually undertaken by individuals-decision makers when their preferences are dynamically inconsistent.

Within the cognitive social psychology and behavioral economics, the research on intertemporal choice (Ainslie, 1991, Akerlof, 1991; Thaler, 1981) has produced abundant evidence of the pervasive devaluation of the future by the economic agents. These empirical studies have led to affirm that the discount functions are usually hyperbolic (Mazur, 1987, 1988). The hyperbolic discount functions are characterized by a relatively high discount rate on the short horizons and a relatively low discount rate on the long periods of time. This complex structure of discount rates creates a conflict between today's preferences and preferences to be held in the future. In addition to the hyperbolic functions, appeared in the literature other discount models that contain functions that try to explain this "bias towards the present" in the evaluation of monetary sums. These models make use of the quasi-hyperbolic discount function (Laibson, 1997). The empirical evidence is therefore generally inconsistent with the property of a constant discount rate as postulated in discounted utility model.Thus, these different discount structures are relevant in that they may play a role not only in economic decisions, but also in the generation of self-control problems.In conclusion, the models that are following the cognitive-behavioral approach represent, according to some scholars, a collection of tools and/or ideas rather than a unified theory, so this approach is not yet a theoretical paradigm independent and structured (Motterlini, Guala, 2005, p. 26). The fact remains that the “psycho-economic” models, if formally founded and empirically tractable, may contribute to the development of a more general theory of rationality based on cognitive-behavioral bases. The behavioral economics with its research program and its models seeks to balance the relationship between theory and evidence, using a broader but, at the same time, less rigid concept of rationality, where the psychological dimension of individuals is not excluded in the economic choices.

3. Evolutionism and biology in economic models**

3.1 Economics and biology in the theories of Alfred Marshall

One of the major merits of behavioral economics has been to show experimentally that, in everyday life, there are numerous possibilities that arise when we make an important decision; thus, the consequences of these choices are not clearly predictable or known *a priori.* In this complex and uncertain environment, our choices have consequences which can be more or less favorable over time. Experiencing the different consequences permits the knowledge of various options and allows to elaborate a preference for some of them. If we then verify the consequences associated with each choice, our uncertainty decreases and, therefore, decisions will be driven, according to our inclinations, whether or not to run risks. As a consequence, we can speak of the decision-making as an adaptive process that manifests itself in fundamental steps that an individual adopts in relation to a complex and dynamic environment. It allows us to control and direct the choices to the options that are more profitable. A '"right" decision-making allows us to learn of new situations, to move forward taking into account the errors and also to modify the actions that have proven inappropriate. On the opposite, when behaviors are no longer suitable, that is, when the choices of individuals have systematically unfavorable consequences for themselves and for their group, than the decision is called "inappropriate" (or pathological). The most complicated aspect of decision-making is that not only the choices are at stake, but rather the values that we assign to them. The preference for an action over another depends,

[31] For a survey: Shane, Loewnstein, O'Donoghue, (2002).

in fact, by the way our brain interfaces with the internal and external world. It is these same mechanisms that constrain our choices, anchoring them to our biological needs. The ability to assign a value to the different options of choice is becoming more sophisticated over time, because it represents a valid strategy for solving problems that affect our survival and also for a requirement of flexibility in dealing with the uncertainties of the surrounding environment (Montague, 2008). The living beings also tend to develop increasingly complex structures, accumulating changes due to various adjustments or reorganizing their previous configurations. In this sense, the standard economic models have had the great limitation of not taking into account that the phenomena and processes of change must be historicized and thus can not be caged into rigid and outdated mathematical models.

In this context of reform of contemporary economic theory, the discipline that has had the merit of capturing the evolution of economic realities is biology (Hodgson, 1993). However, as Theodosius Dobzhansky (1973) had already realized, you can not grasp something important in biology without the idea of Darwinian evolution by natural selection. It is the latter, in fact, to have offered to biological data a very powerful overall consistency from the explanatory standpoint and, in some cases, also from the predictive point of view, bringing together the great amount of experimental data within a single theoretical framework by which we can interpret the multiformity of living being and his many transformations (Pievani, 2005). However, before the theory of evolution and economics could establish a fruitful interdisciplinary dialogue, there was the need that both reached a certain level of maturity. This dialogue was possible given certain assumptions that the two sciences share, i.e.: 1. Rarity. The economic/biological individual search limited resources. 2. Competition. The individuals are competing for the achievement of the limited resources. 3. Maximisation. The individual is represented as the one that maximizes a value (utility / attitude) in the attainment of resources. 4. Joint Emergency. The processes (market / evolution) are not directed by the agents, but derive from the competitive interaction between them. Although several authors have referred to the economy as a biological system, Alfred Marshall is the economist who looks at an advanced use of the evolutionary paradigm. In fact, he provides in his work, *Principles of Economics*[32], the argument in favor of "organic economy", based on the assumption that the economic and biological phenomena share a large number of affinities, from the complex and organic nature to the involvement in a changing world, to the qualititive and quantitative influences that imply that future events do not ever reproduce the same conditions.

Apart from the *Principles,* also the later work of Marshall, i.e. *Industry and Trade* (1919), is crossed by a chronic dissatisfaction that is revealed every time he found not so much the logical inconsistency of the successful economic analytical tools, but rather their lack of immediate applicability to practical situations. The originality of his idea of economic dynamics, or rather of economic evolution, derived from a biological model that has been shelved for a long time until many areas of knowledge have been permeated by evolutionary theories. In economics, however, the term 'evolution' (or evolutionism) is used in different and sometimes opposite ways. There is indeed a sense of usage of the term 'evolution' that does not go beyond an explanation in analogical terms. In this case the use of biological terms is simply to criticize or justify the standard economic theories or notions as, for example, the equilibrium of the markets. Another point of view could be to take rather seriously the evolutionary paradigm, specify its characteristics, and then to reason in terms of reproduction and selection. To construct a satisfying theory of economic evolution is not enough then, to look for similarities with the notions of natural selection, change, unit of selection, but it should be necessary to supplement, beyond the simple conceptual marks, what brings together the economic approach with the evolutionary one. In fact, because the principle of natural selection, applied to the evolution of the patterns of social and industrial organization, is theoretically relevant, it requires, in origin, an integration and understanding of Darwinian evolutionism as part of its global issue. Alfred Marshall has worked following this perspective, that is, questioning about the way that

** Paragraph 3. is written by Mario Graziano.

[32] First Edition 1890.

Darwinian evolution could be used for the study of economic phenomena. The peculiarity of Alfred Marshall is, in fact, that of being one of the first economists to have explicitly claimed the use of a double approach, static and dynamic, for the study of economic phenomena. Both approaches are structured around systems of reference, based on analogy, of different nature, which are respectively the physical model and the biological model. Marshall argues that there is a relative strong analogy between the first stages of an economic reasoning and the static step in the physical sense. Then he asks if there is a step so profitable in the last stages of an economic reasoning and the dynamic methods, and his answer is negative. He, in fact, thinks that in advanced stages of reasoning, biological analogies are more appropriate than physical ones. (Marshall**,** 1961). In his argument, the two reference systems, based on analogy, (i.e., the physical and biological) do not underlie the same kind of theoretical reasoning. According to Marshall, in fact, the formal analogies set in physics or mathematics have the advantage of providing static solutions, in terms of equilibrium, with an emphasis on some economic aspects. In this case, a formal correspondence is established between two fields of knowledge (physics and economics) in which their contribution to the economic analysis is exhausted in providing a series of arguments that do not lead to useful conclusions. In contrast, Marshallian biological analogies establish a substantial correspondence between a field of knowledge and another, through a coherent network of similar relationships between objects and properties of both domains. Differently from the formal analogy, which is an integral part of the logical construction of a theory, the place of predilection of the substantial analogy is therefore that where you work in the selection of pre-theoretical assumptions. According to this perspective, the substantial analogies are often used to compensate for the limitations of an emerging science through the use of scientific paradigms of another science which is more consolidated or 'classical'. In the case of Marshall this may seem paradoxical given that biology as a discipline was born in the nineteenth century (with Lamarck), while economics goes back much further. Actually, Marshall uses biological analogies to treat, according to a different point of view, the already old question of economic dynamics. However, the real question is whether the use of biological analogies is able to overcome the simple imitation or may become a useful learning tool for the conceptualization stage of a new knowledge. Our analysis aims to demonstrate how Marshall sees the study of economic life in the Darwinian evolutionary spectrum, thus identifying the economic forces as living and moving forces, dropped in a changing world.

In the era of Marshall, most of the theoretical constructs developed by the fathers of the equilibrium theory was based on an abstract notion of time borrowed from rational mechanics. For example, in the case of Walras, the mathematical conditions on which it was based its concept of equilibrium canceled out any time horizon. As highlighted by Claude Ménard (1979): «the action and the consequences are mixed, the dimensions are perfectly continuous; [...] we are in a timeless world and without costs». From a technical point of view, «the image of time is copied on the kinematics of the machines without frictions [...] and the global time is constituted only by juxtaposed moments (of stops over time)» (Menard, 1979, p.3). Thus, being the time materialized in the form of a continuous and uniform usage, the economist could afford to make abstraction of this variable in the determination of economic laws.

By considering, instead, the notion of time as one of the most burdensome of any construction of economic models, Marshall's analysis differs from timeless economic analysis. In the Darwinian version of evolution, the time is inseparable from the origin of the living world and its evolution and it is also associated with a certain idea of continuity, instability and contingency. According to Bergson, the time of evolution is «a real-time envisioned as a stream, or in other words, as the mobility itself of the being» and it is opposed to the abstract time «time that intervenes in our speculations on artificial systems, that die and reborn forever» (Bergson, 1996, p.336). Similarly, Alfred Marshall defined economic progress as the organic growth, which is limited, restricted and sometimes opposed by a host of factors that affect each other and whose effect varies depending on the state of growth already achieved by each of them (Marshall, 1961). His equilibrium analysis has different basis and detaches itself from that of other equilibrium theorists (like Walras or Jevons) mainly because he included in his model the appearance of the time period. The choice of the period, in fact, determines the point of view of the observer, and, therefore, the theoretical explanation. The two crucial Marshallian periods, the short-term and long-term, define deeply different modes of regulation. In the long term, for example, the determination of regulatory mechanisms calls upon temporal specificities of the market, since the behaviors depend on the duration considered. So, while for Walras, the "real" market plays rather the role of a logical construction, the equilibrium markets in the marshallian tradition are *in time*. For Claude Ménard, the fact that the very foundations of Marshallian equilibrium are different with respect to marginalist economists allows us to glimpse already a turn toward biological analogies: «the problems raised by the integration of decentralized markets and active units led him to seek on the side of the living beings most suitable models to the expression of the economic processes» (Ménard, 1979, p.51).

Marshall shows us how the whole organization is characterized by its transitory form, so that compromises any pretense of the economist to identify universal laws similar to those of physics. As an example, Marshall presents the principle of division of labor. This principle, at the time of Adam Smith, was identified as a "routine" that favored, due to standardization, the quality of the products. Later gradually it took the form of mechanization, a process by which man is gradually replaced by the machine. Marshall has shown that, regardless of its form, the principle of division of labor must its preservation at a proper adjustment to the purpose for which it is applied. Adam Smith had already explained the advantages of this method but, as Marshall points out, has always avoided making it a universal law that would guarantee the prosperity and welfare of the people as they did instead, according to Marshall, Smith's disciples. This form of industrial organization that simply meets the needs of its time, owes its success to temporary adavantages that outweigh the drawbacks (especially on social matters). The economic model of Marshall, by contrast, is inseparable from the social reality and, therefore, must first increase its knowledge with respect to the reality itself and, then, concentrate on the effects that is could have on the practical life.

The problem of Marshall is therefore to find a classification system that allows the economist, using a small number of terms of common use, to express a large number of subtle distinctions. The main difficulty will be to express all this in a language intelligible to the general public even by structuring a system of definition universally valid. The solution proposed by Marshall, is of utmost importance for the goal we have set ourselves, since it makes direct reference to Charles Darwin and his classification system. We quote a key sentence, in which Marshall, after adhering to the idea of Mill to develop a scientific classification for economic objects, precises exactly the nature of this undertaking: «We come up often against this difficulty: that the most important proposals in a phase of economic development may be among the least important for another. On the subject, economists have much to learn from recent experiences of biology and the depth discussion of the question that Darwin did, which throws a vivid light on the difficulties that present themselves to us. Darwin shows that the characters that determine the practical life of every living being in the economy of nature are not, normally, those elements that throw more light on his origin, but the ones that throw less. [...].

Similarly, for an economic institution, those peculiarities that contribute the most to make it work that should actually do, are probably, for this reason, even of recent date» (Marshall, 1961, II, I, IV). Applied to the economic object, this means that the real "affinity" (Darwin's definition) or the fundamental properties of a particular notion are those that determine its adaptation to the means, but those that are «the hereditary result of the community of descendents» (Darwin, 1967). Marshall wants to prove that the (current) social organization is the product of a slow development of many generations who owe their basic properties, not so much to those that allow it to adapt to their purposes, but to others, which were conveyed as a sort of common code, through the various forms that the same properties have taken over time. Such is the influence of heredity for Marshall that works both for living beings and for business organizations. He wants to show how most of the distinctions that are expressed in economic terms are based on differences of degree and not of nature.

3.2 Economic progress and natural evolution

Ever since the publication of Darwin's theory, there has been a misunderstanding that originates in the identification that was made between evolutionary tendencies and progress. This is partly explained by the influence of the writings of Herbert Spencer, philosopher, contemporary of Darwin, who, trying to unify under one principle: the law of evolution, the phenomena described by the natural and human sciences, it has distorted the original Darwinian sense. The law of evolution, according to Spencer, expresses a tendency inherent to the increasing complexity of the organization of living beings, which is always achieved in the sense of progress. None of this was ever said by Darwin, who, however, has contributed in some way, to ensure that this misunderstanding lasted, replacing, albeit with some reluctance, in the sixth edition of "The Origin of Species," the notion of natural selection with that of "survival of the most suitable", borrowed from Spencer. Some have interpreted this new formula as the quintessential example of the viable competition: the victory of the strong over the weak. In Darwin's key idea, however, natural selection has as its ultimate goal the ever increasing improvement, an improvement which inevitably leads to a gradual progress in the organizations and in most living beings. As Telmo Pievani explains: «It is important to remember, however, that Darwin was never tired to distance his theory firmly away from any social and political implications: the struggle for survival, for him, was a complex scenario of interrelations between organisms in of an ecosystem and had nothing to do with the metaphor of the survival of the more suitable that will suffer pernicious applications in the social and racial sphere» (Pievani, 2005, p. 8). But then, «what is called progress or adaptation is merely the necessary result of this game of interactions that inevitably take place between the system and its surroundings» (Jacob, 1980, p. 194). The idea of evolutionary progress, therefore, does not imply that of an internal principle of improvement. If many have misunderstood the mechanism of natural selection, it is also because they had not integrated the pattern that underlies it. Natural selection is not deterministic, it must be interpreted as a statistical concept: «to have a superior genotype does not guarantee the survival and abundant reproduction: this only gives a higher probability» (Mayr, 1982, p.653).

In the chapter of the *Principles* dedicated to the analysis of social and industrial organizations, Marshall gives the feeling of having perfectly integrated this dissociation between progress and development that exists in the theory of Darwinian evolution. He writes: «Every economic strength will constantly change its action under the influence of other forces acting around it incessantly. The changes that occur in the volume of production, in its methods and its cost of production will affect each other, without stopping» (Marshall, 1961, II, I, V). This definition incorporates, therefore, in any construction of economic models, two of the main philosophical parameters of Darwinian view of evolution: first, the integration of a historic time for the study of economic evolution that forces the scholars to take into account the irreversibility of phenomena; secondly, the economic evolution does not presuppose the idea of a linearity of change oriented towards the perfectibility. Evolution and progress are two separable notions. This separation explains how the different forms of social or industrial organizations are subjected to a process of natural selection in economic terms that, in a competitive world, gradually eliminate those organizations which do not have a proper adjustment to their means and simultaneously promote the formation of new organizational forms. Moreover in his view, the economic mechanism of natural selection, as the biological mechanism, is understood probabilistically. It can always practice outside influences preventing the emergence of a form of organization, which *a priori* would be in perfect adequacy with a given means and mutually, it can happen that the struggle for survival is not able to give birth to organisms which would however very advantageous for their purpose. In the business world, the need for a new industrial layout is certainly not enough to cause the supply. Finally, Marshall retains the uncertain and external aspect of the mechanism of natural selection. Progress in the division of labor, as a special form of organization, is mainly due to factors external to firms, as «the extension of markets, the increased demand for large quantities of goods of the same kind» (Marshall, 1961, IV, IX, III). He then proposes a use of the

Darwinian principle of divergence. Such a principle is derived from the physiological division of labor of Milne-Edwards, which provides to the evolution a preferred direction.

As pointed out by Jean Mathiot, this principle «is the conquest of new ecological suitable places that opens up cumulatively to new opportunities of benefits for innovative organisms. One advantage in an organism creates the ecological conditions of its amplification, correlatively to an increase divergence with respect to the initial conditions of existence of the group from which it is derived, this divergence will open up new possibilities of existence and reproduction» (Mathiot, 1998, p. 2). This principle, applied to the economy, explains the progressive disappearance of some firms in the same market that does not have a quite differentiated structure.

Marshall, raising the question of the various forms of organization, places his reasoning in a context in which the variability is the true norm. He does not start from an arbitrary definition of industrial enterprise to reject afterwards all the forms that deviate. On the contrary, he defines an enterprise through its life cycle as every kind of tree has its normal life in the same way the length of time during which an enterprise of any type is likely to retain its full strength is limited by the laws of nature combined with the circumstances of place and time, with the features and the degree of development of the industrial sector to which it belongs (Marshall, 1961). These laws of nature act on the company by limiting the length of life such that, after a period in which the firm will realize, if it is well suited, economies of scale, it will lose part of its strength, its flexibility and power of progress in the fight against its leading competitors.

The idea of a struggle for survival is, however, an old idea, the use of which dates back to the seventeenth and eighteenth centuries. But then it was considered a benign formula, which allowed the necessary corrections to equilibrium of the nature (as for example in Line, Cuvier, etc.). Instead, the Darwinian theory of struggle for existence calls into question the idea of a harmonious constancy of the world. Since Darwin, the adaptation of organisms must be seen as a dynamic process, and not as a *static status* over time, the organisms are devoted to extinction, unless there is a continuous change to perpetuate itself in a constantly changing physical and biotic environment. Returning to the economic context, the Darwinian principle of struggle for existence must express the similar idea of a competition between various agents and organizations for the existing resources. Marshall applies this idea in *Industry and Trade* to competition between the different organizational forms of the same branch of industry. In particular, as Keynes, which was a Marshall's pupil, said: «The volume as a whole also serves to illustrate what Marshall was always concerned to emphasise, namely the tran-sitory and changing character of the forms of business organisa-tion and of the shapes in which economic activities embody themselves. He calls particular attention to the precarious and impermanent nature of the foundations on which England's industrial leadership had been built up» (Keynes, 1924, p.370).

Specifically, Marshall gives top priority to the advantage that large companies have, which have evolved methods of mechanization, that allow them to take advantage of external economies, related to the overall development of the industry, and of internal economies at the same time, which are linked to their resources.

4. Conclusion

This work, critically arguing the issue of rationality and choices in both the behavioral and evolutionary approaches, has tried to highlight the limits of the theory of rational choice of neoclassical economics. This theory, over the years, aimed at increasing its mathematical precision trying to gain the same level of prestige of the physical sciences in the Victorian era. If its methodological framework has been able to hold, though with some difficulty, on the legal front, so it was not at descriptive and predictive level, since manifestly far from reality as the empirical evidence and expriments of cognitive-behavioral economics have demonstrated more in detail from the seventies onwards. So to the conception of *homo economicus*, perfectly rational, that has a complete knowledge, is replaced that of a different king of individual endowed with a bounded rationality. According to behavioral economics, in fact, people in everyday life rarely apply expensive choice procedures that lead to the selection of the option capable of maximizing their expected utility; they, instead, use much simpler procedures that do not provide the best choice, but that lead to results at least “satisfying”. A similar argument in terms of critique of the neoclassical economic theory in the tradition of Walras and Pareto can be done by following the evolutionary-biological approach. The extension of Darwin's theory to economics (as well as other branches of knowledge) is based on the ability to identify a substrate that, although it is not genetic, it is capable to replicate itself, passing on to descendants parts of their own characteristics, and admitting also a variety of possible outcomes for the evolutionary process. The models and theories by Marshall, discussed in this paper, carried away a vision of the economy in dynamic-evolutionary terms different from standard traditional economics, which is formally abstract, where words such as law, normal, trend, average, strength, cause, do not express what really happens, but what might happen on the basis of certain assumptions that are never exactly realized. The true novelty from Marshall was to have exceeded this limit, through a double identification of biological and economic laws, on the one hand, and the economic object and the living matter, on the other.

References

[1] Ainslie, G. 1991. Intertemporal choice. Derivation of “rational” economic behavior from hyperbolic discount curves, American Economic Review 81(2), pp. 334-340.

[2] Akerlof, G. A. 1991. Procrastination and obedience, American Economic Review, 81(2), pp. 1-19.

[3] Allais, M. 1953. Le comportement de l’homme rationnel devant le risque: critique des postulats et axiomes de l’école américaine, *Econometrica*, 21, pp. 503-546.

[4] Arrow, K. J. 1971. *Essay in the theory of risk-bearing*, Amsterdam, North Holland Pub. Co.

[5] Bergson, H. 1996. *L’évolution créatrice*, Paris, PUF.

[6] Bruner, J. S., Potter, M.C. 1964. Interference in visual recognition, *Science*, 144, pp. 424-425, April.

[7] Cournot, A. 1838. *Recherches sur les principes mathematiques de la theorie des richesses*, Paris, L.Hachette.

[8] Cyert, R. M., Simon, H. A., Trow, D.B. 1956. Observation of a business decision, *Journal of Business*, 29 (4): 237-48.

[9] Darwin, C. 1967. *L'origine della specie*, Torino, Bollati Boringhieri.

[10] Dobzhansky, T. 1973. Nothing in biology makes sense except in the light of evolution. In The *American Biology Teacher*, 35, pp. 125-129.

[11] Ellsberg, D. 1961. Risk, ambiguity, and the Savage axioms, *Quarterly Journal of Economics*, 75 (4), pp. 643–669.

[12] Friedman, M., Savage, L. 1948. Utility analysis of choices involving risk, *Journal of Political Economy*, 56 (4), pp. 279-304.

[13] Gilovich, T., Vallone, R., Tversky, A. 1985. The hot hand in basketball: on the misperception of random sequences, *Cognitive Psychology*, 17 (3), pp. 295 - 314, July.

[14] Hogdson, T. 1993. Economics and evolution, bringing life back into economics, Cambridge, Polity Press.

[15] Jacob, F. 1980. La logique du vivant, une histoire de l’hérédité, Paris, Gallimard.

[16] Kahneman, D. 2002. Maps of bounded rationality: a perspective on intuitive judgment and choice, *Nobel Lecture*, Princeton.

[17] Kahneman, D., Thaler, R.H. 2006. Anomalies: utility maximization and experienced utility, *Journal of Economic Perspectives,* 20 (1), pp. 221 - 234.

[18] Kahneman, D., Tversky, A. 1979. Prospect theory. An analysis of decision under risk, *Econometrica*, 47 (2), pp. 263 - 292.

[19] Kahneman, D., Tversky, A. 1984. Choices, values and frames, *American Psychologist*, 39, pp. 341 - 350.

[20] Kahneman, D., Knetsch, J. L., Thaler R. H. 1990. Experimental tests oft he endowment effect and the Coase theorem, *Journal of Political Economy*, 98 (6), pp. 1325 - 1348.

[21] Keynes, J. M. 1924. Alfred Marshall, 1842-1924, *Economic Journal*, 34, pp. 311- 372, September.

[22] Keren, G. 1987. Facing uncertainty in the game of bridge: a calibration study, *Organizational behaviour and human decision processes*, 39 (1), pp. 98-114, February.

[23] Laibson, D. 1997. Golden eggs and hyperbolic discounting, *Quarterly Journal of Economics,* 112(2), pp. 443 - 477.

[24] Marshall, A. 1919. *Industry and Trade,* London, Macmillan & Co.

[25] Marshall, A. 1961, 1890 (1st edition). *Principles of Economics*, London, Macmillan.

[26] Marshall, A. 1991. Analogies mécaniques et biologiques en économie (1898), *Revue Française d'Economie*, 6 (1), pp. 103-113.

[27] Mathiot, J. 1998. Evolution, sélection, information. La question de la convergence, *Philosophiques*, Département de philosophie de l'UQAM.

[28] Mayr E. 1982. *Storia del pensiero biologico. Diversità, evoluzione, eredità*. Torino, Bollati Boringhieri, *in Italian*

[29] Mazur, J. E. 1987. An adjusting procedure for studying delayed reinforcement. In *Quantitative analyses of behavior: V. The effect of delay and of intervening events on reinforcement value.*

[29] M. L. Commons, J. E. Mazur, J. A. Nevin and H. Rachlin, eds., Hillsdale, NJ: Lawrence Erlbaum, pp. 55 – 73.

[30] Mazur, J. E. 1988. Estimation of indifference points with an adjusting delay procedure. *Journal of the Experimental Analysis of Behavior*, 49 (1), pp. 37– 47.

[31] Ménard, C.L. 1979. Equilibre, déséquilibre, temps : un peu d'histoire, Economie Appliquée, 32 (2-3), pp. 229 - 251.

[32] Montague, R. 2008. Perchè l'hai fatto? Come prendiamo le nostre decisioni, Milano, Raffello Cortina.

[33] Motterlini, M., Guala, F., Eds. 2005. *Economia cognitiva e sperimentale*, Milano, Egea.

[34] Novarese, M., Castellani, M., Di Giovinazzo, V. 2009. Procedural rationality and happiness, *MPRA Paper*, n.18290, October.

[35] Pievani, T. 2005. *Introduzione alla filosofia della biologia*, Bari, Laterza.

[36] Rabin, M. 1998. Psychology and economics, *Journal of Economic Literature*, 36 (1), pp. 11- 46.

[37] Robbins, L.C. 1932. 1935 (2nd edition). *An essay on the nature and significance of economic science*, London, Macmillan.

[38] Samuelson, P. 1937. A note on measurement of utility*,* *Review of Economic Studies*, 4, pp. 155 – 161.

[39] Schilirò, D. 2011. *Decisioni e razionalità in economia,* MPRA Paper *n.29477. March.*

[40] Schumpeter, J. A. 1954. *History of economic analysis*, New York, Oxford University Press.

[41] Shane, F., Loewnstein, G., O'Donoghue, T. 2002. Time discounting and time preference. A critical review, *Journal of Economic Literature,* XL (2), pp. 351 - 401.

[42] Shiller, R.J. 2005. Behavioral economics and instituional innovation, *Cowles Foundation Discussion Paper* no. 1499.

[43] Simon, H. A. 1955. A behavioral model of rational choice, *Quarterly Journal of Economics,* 69 (1), pp. 99 - 118.

[44] Simon, H. A. 1956. Rational choice and the structure of the environment, *Psychological Review*, 63 (2), March, pp. 129 - 138.

[45] Simon, H. A. 1978. Rational decision-making in business organizations. *Nobel Memorial Lecture, 8 December 1978, Economic Sciences*, pp. 343-371.

[46] Simon, H. A. 1987. Rationality in psychology and economics, in *Rational choice*, R.M. Hogarth, Reder, R.W., eds., Chicago, University of Chicago Press, pp. 25 - 40.

[47] Simon, H. A. 2000. Bounded rationality in social science: today and tomorrow. *Mind & Society*, 1, pp. 25 - 39.

[48] Smith, V. L. 1982. *Microeconomic system as an experimental science*, *American Economic Review*, 72, pp. 923 - 955.

[49] Smith, V. L. 2000. *Bargaining and market behavior: essays in experimental economics*, Cambridge, Cambridge University Press.

[50] Strotz, R. H. 1955-1956. Myopia and inconsistency in dynamic utility maximization. *Review of Economic Studies*, 23 (3), pp. 165 - 180.

[51] Thaler, R. H. 1980. Towards a positive theory of consumer choice. *Journal of Economic Behavior and Organization*, 1, pp. 39-60.

[52] Thaler, R. H. 1981. Some Empirical Evidence on Dynamic Inconsistency," *Economic Letters*, 8, pp. 201-207.

[53] Tversky, A., Kahneman, D. 1974. Judgment under uncertainty: heuristics and biases, *Science*, 185, pp. 1124-31, September.

[54] Tversky, A., Kahneman D. 1986. Rational choice and the framing of decisions, *Journal of Business*, 59 (4) Part 2, pp. S251-S278.

A MATHEMATICAL MODEL FOR A COMPANY'S ADVERTISING STRATEGY

Laura UNGUREANU
Spiru Haret University, **Romania**
laura.ungureanu@spiruharet.ro

Abstract:

Models are better means of approximating reality, suitable for most economic phenomena which are generally represented by dynamical processes. Economic mathematicians have begun their study of this type of processes and have reached so far that today they are able to elaborate dynamical bifurcation diagrams that include all mathematical phenomena and Hopf bifurcation, in particular. In this paper, we have explained the behavior of an advertising model that consists out of a Cauchy problem made for/from a system of ordinary differential equations.

An advertising model is written in the form a Cauchy problem for a system of two first order ordinary differential equations involving two real parameters. For two particular values of them it is shown that a degenerate Bogdanov-Takens bifurcation phenomenon occurs. This implies an extremely complex behavior of the economic model for advertising.

Keywords: dynamical model, advertising strategy, limit cycle, advertising spiral.

JEL Classification: C02, C32, M37.

1. Mathematic model

This advertising model firstly elaborated in 1950 has been the subject of a more rigorous analysis in 1992, by Feichtinger. It was afterwards associated with one of the epidemic theories - as advertising can be compared with "spreading germs". Potential buyers (*X*) are „infected" by these germs through advertising and thus they get in contact with brand consumers (*Y*).

$$\begin{cases} \dot{X} = k - aXY + \beta Y \\ \dot{Y} = aXY - \delta Y \end{cases}$$

where $a(t) = \alpha Y$ is the contact rate of advertising in a given time *t*, presumed to be proportional with the number of regular buyers; β represents the distribution rate of competing brands; ε stands as the rate of migration, mortality or oblivion; $\delta = \varepsilon + \beta$. Thus, the system becomes (Tu, 1994):

$$\begin{cases} \dot{X} = k - aXY^2 + \beta Y \\ \dot{Y} = aXY^2 - \delta Y \end{cases} \quad (1.1)$$

1.2. Case scenario - $k = 0$

In this particular situation, the system becomes $\begin{cases} \dot{X} = -aXY^2 + \beta Y \\ \dot{Y} = aXY^2 - \delta Y \end{cases}$ and allows an infinite number of equilibrium points of type $(X,0)$. The linearized approximation of such a point is given by $\begin{cases} \dot{X} = \beta Y \\ \dot{Y} = -\delta Y \end{cases}$ and defined by the Jacobian matrix $\begin{pmatrix} 0 & \beta \\ 0 & -\delta \end{pmatrix}$. The equation that asserts this matrix is $s^2 + \delta s = 0$ and allows its own values, $s_1 = 0$ and $s_2 = -\delta$, which makes the equilibrium point non-hyperbolic. Its non-hyperbolicity makes it impossible to apply a Hartman-Grobman theorem. This is why we cannot link the portrait phase of the nonlinear dynamical system to the portrait phase of the corresponding linear dynamical system, in the sense of topological equivalence. The eigen vectors corresponding are $v_1(-\beta,\delta)$ and $v_2(1,0)$. The set of equilibrium points of the linear system are situated on the line $Y=0$.

1.3. Case scenario $k \neq 0$

In this case, the following transformations can be made $x=\frac{\alpha k}{\delta\varepsilon}X$, $y=\frac{\varepsilon}{k}Y$, $\gamma=\frac{\alpha k^2}{\delta\varepsilon^2}$, $\theta=\frac{\beta}{\delta}$, $u=x-1$, $v=y-1$ and changing the time variable $r=\delta t$, this is the result:

$$\begin{cases} \dot{u} = -\gamma\left(u+\psi v+2uv+v^2+uv^2\right) \\ \dot{v} = u+v+2uv+v^2+uv^2 \end{cases} \tag{1.2}$$

where $\psi = 2-\phi$.

The above system (1.2) can also be written as:

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} -\gamma & -\gamma\psi \\ 1 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} + g(u,v)\begin{pmatrix} -\gamma \\ 1 \end{pmatrix}$$

where $g(u,v)=2uv+v^2+uv^2$

2. Equilibrium points

Taking into account α 's economic significance, the result is generally $\gamma \neq 0$. Also, because $\varepsilon \neq 0$ then $\psi \neq 1$. Under these circumstances, the system allows only a single equilibrium point $r_0(0,0)$, the origin. The linearized approximation is the system:

$$\begin{cases} \dot{U} = -\gamma U - \gamma\psi V \\ \dot{V} = U + V \end{cases} \tag{2.1}$$

and the Jacobian matrix $A=\begin{pmatrix} -\gamma & -\gamma\psi \\ 1 & 1 \end{pmatrix}$ with the characteristic polynomial of $P(\lambda)=\lambda^2+(\gamma-1)\lambda+\gamma\psi-\gamma=0$ which has the discriminant $\Delta=(\gamma-1)^2-4\gamma(\psi-1)=\gamma^2-4\gamma\psi+2\gamma+1$ and values of $\lambda_{1,2}=\frac{1}{2}\left(1-\gamma\pm\sqrt{\Delta}\right)=\frac{1-\gamma}{2}\pm\frac{1}{2}\sqrt{(\gamma+1)^2-4\gamma\psi}$.

We notice that $\Delta=0$ represents – in the parameter space (γ,ψ) - a hyperbola whose center is given by $C\left(0,\frac{1}{2}\right)$ and whose asymptote equations are $\gamma=0$ and $\gamma-4\psi+2=0$. In the domain held between the bows of the hyperbola which are the origin of the landmark, we have $\Delta>0$, and beyond this point, the discriminant is negative. Concerning the values, the following situation arises (Figure 2.1):

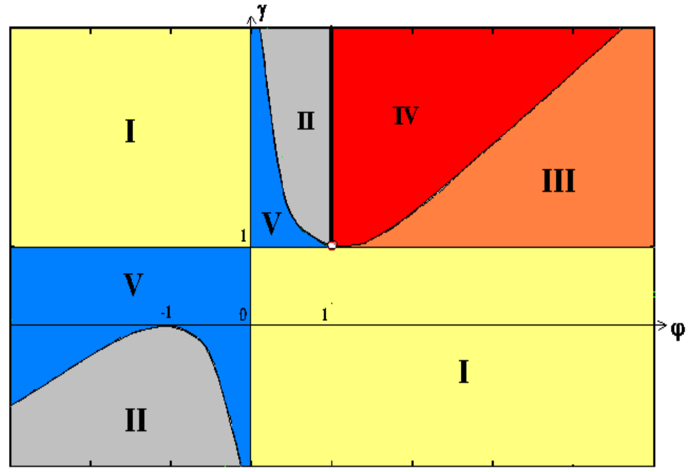


Figure 2.1. The dynamical bifurcation diagram

In the domain **I**, the origin is a saddle point; in domain **II** – it is a repulsive focus; in **III** – an attracting knot; in domain **IV –** an attracting focus; in domain **V –** a repulsive knot, on the semi-line $\gamma = 1$, where $\psi > 1$ is the center, and **O** is a non-hyperbolic point (double zero).

The only points from the parameter space for which the equilibrium point isn't a hyperbolic point are those situated on the semi-line $\gamma = 1$ $\psi \geq 1$. For all the other values taken by the γ and ψ parameters, the origin is a hyperbolic point and – based on the Hartman-Grobman theorem – the phase portraits from the nonlinear case are topological equivalents to those of the linearized. Hence, we deal with the non-hyperbolic case in which the information provided by the associated linearized system will have to be added. In the case $\gamma = 1$ the initial system becomes:

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} -1 & -\psi \\ 1 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} + g(u,v) \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad (2.2)$$

The matrix of the linearized system is $A = \begin{pmatrix} -1 & -\psi \\ 1 & 1 \end{pmatrix}$ and its values are purely imaginary $\lambda_{1,2} = \pm i\omega$ cu $\omega = \sqrt{\psi - 1}$ and $\left.\frac{\partial \mathrm{Re}\,\lambda_1}{\partial \gamma}\right|_{\gamma=1} = -\frac{1}{2} \neq 0$. So, for $\psi > 1$ (invariant) and $\gamma = 1$, a Hopf bifurcation is revealed. For $\lambda_1 = i\omega$, the complex vector is:

$$v = \begin{pmatrix} i\omega - 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} + i \begin{pmatrix} \omega \\ 0 \end{pmatrix}.$$

The real vector subspace of (1.1) for a base of $B = \{v_1(-1,1), v_2(\omega,0)\}$; the crossing matrix from the initial base to base B is $P = \begin{pmatrix} -1 & \omega \\ 1 & 0 \end{pmatrix}$ and so the formula that allows the changing of the coordinates of a vector is $\begin{pmatrix} u \\ v \end{pmatrix} = P \begin{pmatrix} x \\ y \end{pmatrix}$. In base B, the A matrix assumes the canonical shape $A' = P^{-1}AP = \begin{pmatrix} 0 & -\omega \\ \omega & 0 \end{pmatrix}$ and it yields:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 0 & -\omega \\ \omega & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} h(x,y) \\ k(x,y) \end{pmatrix} \tag{2.3}$$

where $h(x,y) = 2(-x+\omega y)x + x^2 + (-x+\omega y)x^2$

$k(x,y) = 0$

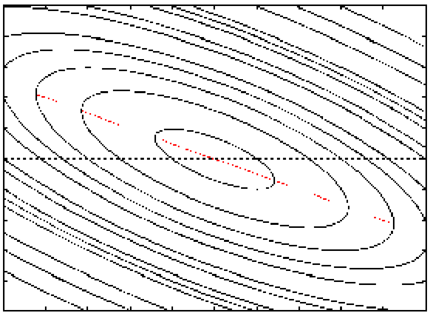


Figure 2.2a. Linear case (center)

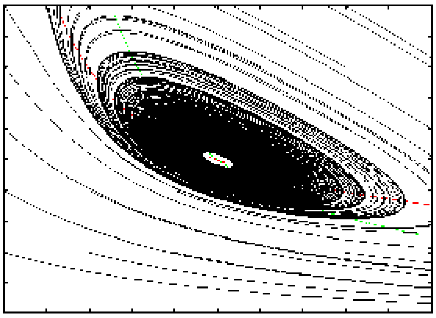


Figure 2.2b. Non-linear case (Hopf bifurcation point)

This is the normal shape and the a coefficient, introduced by Guckenheimer, gives stability to the orbit; evaluated by $x = y = 0$ it yields:

$$a = \frac{1}{16}(h_{xxx} + h_{xyy}) + \frac{1}{16\omega} h_{xy}(h_{xx} + h_{yy}) = \frac{1}{16}(-6-4) = -\frac{5}{8} < 0$$

Because $a < 0$ - according to the Hopf bifurcation theorem – the result is that the system has a stable periodic orbit (Figure 2.2). In this case, the complex conjugate values are purely imaginary; hence, the origin is a repulsive subcritical Hopf bifurcation point (Figure 2.3). The equilibrium point is non-hyperbolic, defined as a condition of involution ($trA = 0$), so that it is situated in co-dimension one.

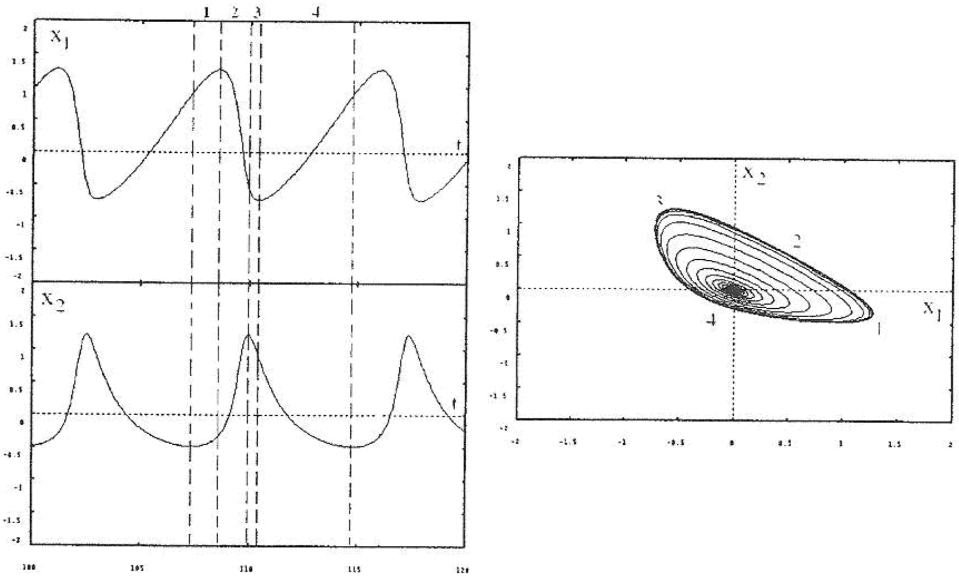


Figure 2.3. Limit cycle, an inside trajectory and the oscillations for $\gamma = 0.9$ and $\psi = 2$

3. The case of $\gamma = \psi = 1$

In the particular case that $\gamma = \psi = 1$, the system becomes $\begin{cases} \dot{u} = -(u + v + 2uv + v^2 + uv^2) \\ \dot{v} = u + v + 2uv + v^2 + uv^2 \end{cases}$. Its equilibrium points are situated on the equation curve of $u + v + 2uv + v^2 + uv^2 = 0$, which is the equivalent of the algebraic system $\begin{cases} v+1=0 \\ u+v+uv=0 \end{cases}$. So, the equilibrium points describe the line $v=-1$ and the hyperbola $u+v+uv=0$. Furthermore, there is also the fixed variety given by $v=-u$, tangent at the origin of the hyperbola $u+v+uv=0$ (Figure 3.1). The sense of this variety is given by the sign of the derivative $\dot{u}=v^2(1+v)$.

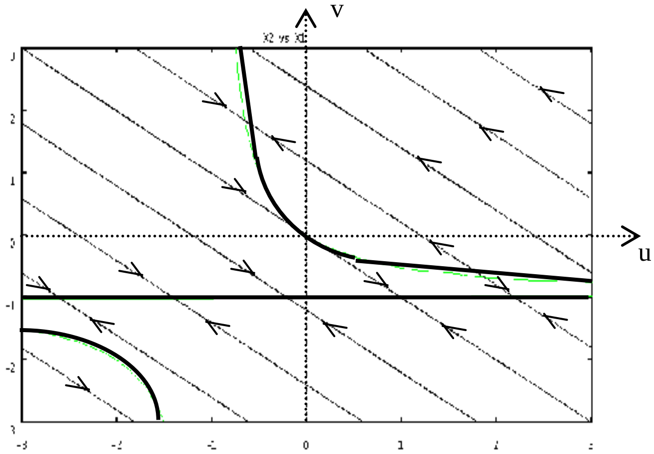


Figure 3.1. The set of equilibrium points

At the origin, the Jacobian matrix resembles $A=\begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix}$ and we obtain $\det A=0$, $trA=0$, $A\neq O_2$, being situated in the conditions of the Bogdanov-Takens bifurcation, which means co-dimension two, with two involution conditions.

For a given equilibrium point M_0 on the hyperbola $H: u+v+uv-0$, $M_0(u_0,v_0)$, the Jacobian matrix of the linearized system is $B=\begin{pmatrix} -1-2v_0-v_0^2 & -1 \\ 1+2v_0+v_0^2 & 1 \end{pmatrix}$ and the corresponding equation has its roots in $\lambda_1=0$, $\lambda_2=-2v_0-v_0^2$. For $\lambda_1=0$, the main direction is given by the vector $\overline{w_1}(1+u_0,-1-v_0)$, and for $\lambda_2=-2v_0-v_0^2$ the main direction is given by $\overline{w_2}(1,-1)$.

The tangent at M_0 on H is given by the right side of the equation $(v_0+1)u+(u_0+1)v+u_0+v_0=0$. And what is interesting enough to observe is that $\overline{w_1}$ is a direction vector for this tangent.

3.1. The analysis around the equilibrium $(0,0)$

In order to have this equilibrium for the zero values of the parameters, we performed the transformation $\gamma=1+\alpha_1$, $\psi=1+\alpha_2$. Thus (1.2) becomes:

$$\begin{cases} \dot{u}=-(1+\alpha_1)[u+(1+\alpha_2)v+v^2+uv^2+2uv] \\ \dot{v}=u+v+2uv+v^2+uv^2 \end{cases} \quad (3.1)$$

corresponding to the matrix

$$\overline{A}(\alpha)=\begin{pmatrix} -(1+\alpha_1) & -(1+\alpha_1)(1+\alpha_2) \\ 1 & 1 \end{pmatrix} \quad (3.2)$$

For $\overline{\alpha}=0$, it has the eigenvalues $\lambda_1(0)=\lambda_2(0)=0$. The corresponding eigenvectors are $\overline{v}_0=(-1.1)^T$, $\overline{v}_1=(0.1)^T$ and the generalized eigenvectors are $\overline{\omega}_0=(-1.0)^T$, $\overline{\omega}_1=(1.1)^T$.
Let us use the linear transformation

$$\begin{cases} u=-u_1 \\ v=u_1+v_1 \end{cases} \tag{3.3}$$

to turn (3.1) into:

$$\begin{cases} \dot{u}_1=(1+u_1)(\alpha_2 u_1+v_1+\alpha_2 v_1+u_1^2-v_1^2-u_1^3-2u_1^2 v_1-u_1 v_1^2) \\ \dot{v}_1=-\alpha_2(1+\alpha_1)u_1-(\alpha_1+\alpha_2+\alpha_1\alpha_2)v_1-\alpha_1 u_1^2+\alpha_1 v_1^2+\alpha_1 u_1^3+ \\ \qquad +2\alpha_1 u_1^2 v_1+\alpha_1 u_1 v_1^2 \end{cases} \tag{3.4}$$

Writing (3.4) in the form:

$$\begin{cases} \dot{u}_1=v_1+a_{00}(\alpha)+a_{10}(\alpha)u_1+a_{01}(\alpha)v_1+\frac{1}{2}a_{20}(\alpha)u_1^2+a_{11}(\alpha)u_1 v_1+ \\ \qquad +\frac{1}{2}a_{02}(\alpha)v_1^2+P_1(u_1,v_1,\alpha) \\ \dot{v}_1=b_{00}(\alpha)+b_{10}(\alpha)u_1+b_{01}(\alpha)v_1+\frac{1}{2}b_{20}(\alpha)u_1^2+b_{11}(\alpha)u_1 v_1+ \\ \qquad +\frac{1}{2}b_{02}(\alpha)v_1^2+P_2(u_1,v_1,\alpha) \end{cases} \tag{3.5}$$

where $a_{kl}(\alpha)$ and $P_{1,2}(u_1,v_1,\alpha)$ are smooth functions of their arguments. We have $a_{00}=0$; $a_{10}=\alpha_2(1+\alpha_1)$; $a_{01}=(1+\alpha_1)(1+\alpha_2)-1$; $a_{20}=2(1+\alpha_1)$; $a_{02}=-2(1+\alpha_1)$; $a_{11}=0$ and $b_{00}=0$; $b_{10}=-\alpha_2(1+\alpha_1)$; $b_{01}=-(\alpha_1+\alpha_2+\alpha_1\alpha_2)$; $b_{20}=-2\alpha_1$; $b_{02}=2\alpha_1$; $b_{11}=0$.

With the transformation:

$$\begin{cases} y_1=u_1 \\ y_2=v_1+a_{00}+a_{10}u_1+a_{01}v_1+\frac{1}{2}a_{20}u_1^2+a_{11}u_1 v_1+\frac{1}{2}a_{02}v_1^2+P(u,\cdot) \end{cases}$$

(3.4) becomes:

$$\begin{cases} \dot{y}_1=y_2 \\ \dot{y}_2=g_{00}(\alpha)+g_{11}(\alpha)y_1+g_{01}(\alpha)y_2+\frac{1}{2}g_{20}(\alpha)y_1^2+g_{11}(\alpha)y_1 y_2+ \\ \qquad +\frac{1}{2}g_{02}(\alpha)y_2^2+Q(y,\alpha) \end{cases} \tag{3.6}$$

for certain smooth functions $g_{kl}(\alpha)$, $g_{00}(0)=g_{10}(0)=g_{01}(0)=0$ and a smooth functions $Q(y,\alpha)=O(\|y\|^3)$. One can verify that $g_{20}(0)=b_{20}(0)$, $g_{11}(0)=a_{20}(0)+b_{11}(0)$, $g_{02}(0)=b_{02}(0)+2a_{11}(0)$.
With the transformation:

$$\begin{cases} y_1 = z_1 + \delta(\alpha) \\ y_2 = z_2 \end{cases}$$

(3.5) becomes (3.6):

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = g_{00} + g_{10}\delta + O(\delta^2) + (g_{10} + g_{20}\delta + O(\delta^2))z_1 + (g_{01} + g_{11}\delta + O(\delta^2))z_2 + \\ \qquad + \frac{1}{2}(g_{20} + O(\delta))z_1^2 + (g_{11} + O(\delta))z_1 z_2 + \frac{1}{2}(g_{02} + O(\delta))z_2^2 + O(\|z^3\|) \end{cases}$$

where $g_{11}(0) = a_{20}(0) + b_{11}(0) = 2 \neq 0$, there $\delta(\alpha) = -\frac{g_{01}(\alpha)}{g_{11}(0)}$ is completed by annihilating the coefficient of the term in z_2, in the equation (3.6). There (3.6) becomes:

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = h_{00}(\alpha) + h_{10}(\alpha)z_1 + \frac{1}{2}h_{20}(\alpha)z_1^2 + h_{11}(\alpha)z_1 z_2 + \frac{1}{2}h_{02}(\alpha)z_2^2 + R(z,\alpha) \end{cases} \quad (3.7)$$

Where:

$h_{00}(\alpha) = g_{00}(\alpha) + \ldots$, $h_{10}(\alpha) = g_{10}(\alpha) - \frac{g_{20}(0)}{g_{11}(0)} g_{01}(\alpha) + \ldots$, $h_{20}(0) = g_{20}(0)$, $h_{11}(0) = g_{11}(0)$, $h_{02}(0) = g_{02}(0)$.

By the time transformation $dt = (1 + \theta z_1)d\tau$, (3.7) reads:

$$\begin{cases} \dot{\xi}_1 = z_2 + \theta z_1 z_2 \\ \dot{\xi}_2 = h_{00} + (h_{10} + h_{00}\theta)z_1 + \frac{1}{2}(h_{20} + 2h_{10}\theta)z_1^2 + h_{11}z_1 z_2 + \frac{1}{2}h_{02}z_2^2 + O(\|z^3\|) \end{cases} \quad (3.8)$$

and by a coordinate transformation $\eta_1 = \xi_1$, $\eta_2 = \xi_2 + \theta\xi_1\xi_2$ (3.8) becomes:

$$\begin{cases} \dot{\eta}_1 = \eta_2 \\ \dot{\eta}_2 = f_{00}(\alpha) + f_{10}(\alpha)\eta_1 + \frac{1}{2}f_{20}(\alpha)\eta_1^2 + f_{11}(\alpha)\eta_1\eta_2 + \frac{1}{2}f_{02}(\alpha)\eta_2^2 + O(\|\eta^3\|) \end{cases} \quad (3.9)$$

where $f_{00}(\alpha) = h_{00}(\alpha)$, $f_{10}(\alpha) = h_{10}(\alpha) + h_{00}(\alpha)\theta(\alpha)$, $f_{20}(\alpha) = h_{20}(\alpha) + 2h_{10}(\alpha)\theta(\alpha)$, $f_{11}(\alpha) = h_{11}(\alpha)$, $f_{02}(\alpha) = h_{02}(\alpha) + 2\theta(\alpha)$

we choose θ such that the coefficient of η_2^2 vanishing, $\theta(\alpha) = -\frac{h_{02}(\alpha)}{2}$.

So far, we have:

$$\begin{cases} \dot{\eta}_1 = \eta_2 \\ \dot{\eta}_2 = \mu_1(\alpha) + \mu_2(\alpha)\eta_1 + A(\alpha)\eta_1^2 + B(\alpha)\eta_1\eta_2 + O(\|\eta^3\|) \end{cases} \quad (3.10)$$

where $\mu_1(\alpha) = h_{00}(\alpha)$, $\mu_2(\alpha) = h_{10}(\alpha) - \frac{1}{2}h_{00}(\alpha)h_{02}(\alpha)$ and $A(\alpha) = \frac{1}{2}(h_{20}(\alpha) - h_{10}(\alpha)h_{02}(\alpha))$, $B(\alpha) = h_{11}(\alpha)$.

Now, in the method from Kuznetsov (Kuznetsov,1995) the next time transformation is $t = \left[\frac{B(\bar{\alpha})}{A(\bar{\alpha})}\right]\tau$. Due to the fact that $b_{20} = 0$, it follows $A(\bar{0}) = 0$ and, so, we cannot continue the chain of transformations leading to the Bogdanov-Takens normal form. Hence, the point $(\gamma, \psi) = (1,1)$ corresponds to a degenerated Bogdanov-Takens bifurcation point. In order to obtain the corresponding normal form it necessary to apply other methods.

4. Economic interpretation of the results

From the economic point of view, and taking into account the significance of α, one can conclude that normally $\gamma \neq 0$. In the same manner, because $\varepsilon \neq 0$, then $\psi \neq 1$. In the equilibrium points that are situated on $\gamma = 1$, $\psi > 0$, the system allows a stable periodic orbit (with a stable limit cycle) consisting out of four phases (Figure 4.1):

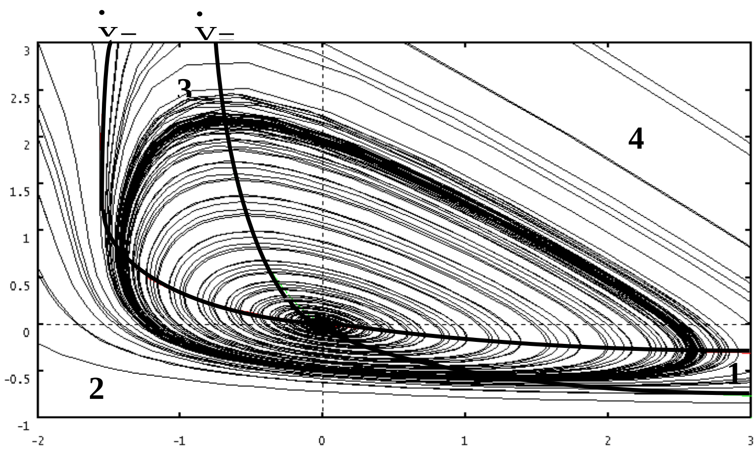


Figure 4.1. The stable periodic orbit, with four phases

1) prosperity - when *X* increases, Y increases; so, the number of potential buyers and the number of users both increase, hence it indicates a time of prosperity, when the product fills a gap in the market;

2) saturation - when X decreases, Y increases; this reveals the time when the product saturates the market, it is well-known and the main concern of the buyer is “which product should I buy?”;

3) free-fall - when X, Y both decrease; at this phase, the number of users decreases; it's a time of decrease, when the product is at a certain point of acceptance, its utility known but its place on the market is only due to its past reputation

4) come-back, recovery when X increases and Y decreases. In the recovery phase the buyer is reminded why the product exists, as it is relaunched by a new advertising campaign.

The purpose of advertising is as much to inform consumers regarding the existence of a new product (in terms of its functionality, usage, advantages compared with other similar products), as it is to orient them towards purchasing these products. Advertising campaigns must be correlated with the activities conducted in the market and with the actions of the launching – on a large scale – of the product.

The equations of the mathematical model proposed in this article can be extended taking into account that the estimation of parameters in such functions use statistic data that come from surveys, selective market research, family budget planning, research studies regarding the launch of a new product, studies concerning the life cycle of a certain long-term use product, yearly statistics, the degree of substitution and complementarily of certain products or the buyers' behavior.

5. Conclusion and prospects for further research

The apparition of the nonlinear dynamics had permitted the understanding and the development of some processes and methods that draw near the phenomenon to reality. The development of the theory of singularities and the theory of bifurcations had completed the multitude of instruments of analyzing and represents dynamics more and more complex giving the possibility of analyzing systems that were hard even impossible to use them from the traditional point of view. The study of nonlinear dynamics is very important for us because the economic systems are defined as nonlinear. Much of them contains multiple discontinuities and incorporates an inherent instability being permanently submitted to the actions of shocks and external and internal perturbations.

Generally, with the activities being more complex, with the need of planning, search for strategies and formal actions grows. The economic domain is a domain in which the uncertain grade and risk is very high and in which the planning plays an important role in trying to reduce this incertitude. In essence, the elaboration of strategies in this domain purposes a clear and systematic structure of the modulations in which the followed objectives can be touched by a judicious allocation of the resources by long or short term.

This study is only a starting point of economic dynamics. We are confronted with more difficult analytic problems: economic systems are described by unstable nonlinear dynamic equations of high dimensions with different adjustment speeds. The existence nonlinear theories have significant implications for economic forecasting, methodologies and so on.

References

[1] Arrowsmith, D. K., and Place, C. M. 1990. *An Introduction to Dynamical Systems*. Cambridge University Press.

[2] Beltrami, E. 1990. *Mathematics for Dynamic Modeling*. Academic Press, New York

[3] Braun, M. 1983. *Differential Equations and their Applications*. Third Edition, Springer, New York.

[4] Gandolfo, G. 1996. *Economic dynamics*, Springer, Berlin.

[5] Georgescu, A., Moroianu, M. and Oprea, I. 1999. *Teoria bifurcaţiei.* Piteşti University Publishing House, *in Romanian.*

[6] Hirsch, M. W. and Smale, S. 1996. *Differential Equations, Dynamical Systems and Linear Algebra.* Academic Press, New York.

[7] Kuznetsov, Y. A. 1995. *Elements of applied bifurcation theory.* Springer, New York.

[8] Tu, P.N.V. 1994. *Dynamical Systems*, Springer.

[9] Ungureanu, L. 2004. *Structural Stability and Bifurcation in two Models of Economic Dynamics*, Piteşti University Press, Piteşti (in Romanian).

[10] Zhang, W.B. 1990. *Economic Dynamics*. Springer-Verlag, Berlin.

ASERS

Association for Sustainable Education Research and Science

Programs and Publications

- *Journals ...*
- *Conferences Proceedings ...*
- *Books Collections ....*

Journals …

Journal of Advanced Research in Law and Economics

Editor in Chief: PhD Mădălina **Constantinescu**
Co-Editors: PhD Russell **Pittman** and PhD Eric **Langlais**

Journal of Advanced Research in Law and Economics provides readers with high quality and empirical research in law and economics. The *Journal* publishes analytical studies on the impact of legal interventions into economic processes by legislators, courts and regulatory agencies. Finally, important developments and topics in law and economics analysis will be documented and examined in special issues dedicated to that subject. The journal is edited for readability; lawyers and economists, scholars and specialized practitioners count among its readers.
Journal of Advanced Research in Law and Economics, starting with its first issue, is indexed in RePEC, IndexCopernicus, CEEOL and EBSCO databases.

Web: http://www.asers.eu/journals/jarle.html E-mail: jarle@asers.eu

Journal of Advanced Research in Management Biannually

Editor in Chief: PhD Andy **Ştefănescu**

Co-Editor: PhD Rajesh K. **Pillania**

The Journal aims to serve researchers, scholars through prompt publications of significant advances in any branch of management science, and to provide a forum for the reporting and discussion of news and issues concerning management science.

Journal of Advanced Research in Management starting with its first issue is indexed in RePEC, IndexCopernicus, and EBSCO databases.

Web: http://www.asers.eu/journals/jarm.html
E- mail: jarm@asers.eu

Journal of Advanced Studies in Finance – Biannually

Editor in Chief: PhD. Laura **Ştefănescu**
Co-Editor: PhD Rajmund **Mirdala**

The Journal aims to publish empirical or theoretical articles which make significant contributions in all areas of finance, such as: asset pricing, corporate finance, banking and market microstructure, but also newly developing fields such as law and finance, behavioural finance, and experimental finance. The Journal will serves as a focal point of communication and debates for its contributors for better dissemination of information and knowledge on a global scale.

Journal of Advanced Studies in Finance, starting with its first issue is indexed in IndexCopernicus, RePEC, CEEOL and EBSCO databases.

Web: http://www.asers.eu/journals/jasf.html E-mail: jasf@asers.eu

Journal of Environmental Management and Tourism – Biannually

Editor in Chief: PhD Cristina **Barbu**

Journal of Environmental Management and Tourism will publish original research and seeks to cover a wide range of topics regarding environmental management and engineering, environmental management and health, environmental chemistry, environmental protection technologies (water, air, soil), pollution reduction at source and waste minimization, energy and environment, modelling, simulation and optimization for environmental protection; environmental biotechnology, environmental education and sustainable development, environmental strategies and policies, etc.

Journal of Environmental Management and Tourism starting with its first issue is indexed in RePEC, IndexCopernicus and EBSCO databases.

Web: http://www.asers.eu/journals/jemt.html

E-mail: jemt@asers.eu

Journal of Research in Educational Sciences – Biannually

Editor in Chief: PhD Laura **Ungureanu**

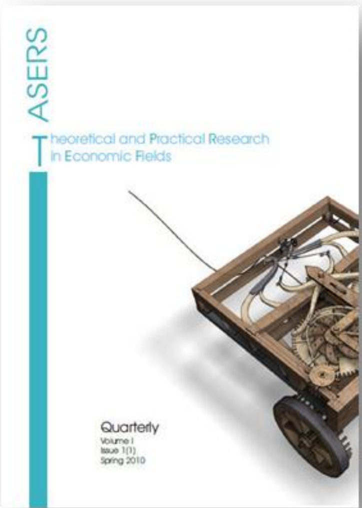
The Journal is design to promote scholars thought in the field of education with the clearly mission to provide an interdisciplinary forum for discussion and debate about education's most vital issues. We intend to publish papers that contribute to the expanding boundaries of knowledge in education and are focusing on research, theory, current issues and applied practice in this area.

Journal of Research in Educational Sciences starting with its first issue is indexed in RePEC, IndexCopernicus and EBSCO databases.

Web: http://www.asers.eu/journals/jres.html

E-mail: jres@asers.eu

Theoretical and Practical Research in Economic Fields – Biannually



Editor in Chief: PhD Laura **Ungureanu**

Co-Editor: PhD Ivan **Kitov**

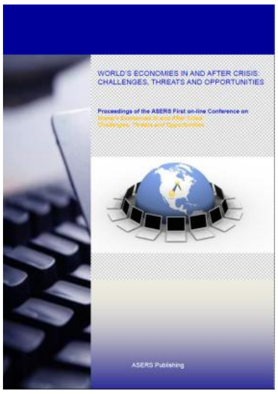
Theoretical and Practical Research in Economic Fields publishes original articles in all branches of economics - theoretical and empirical, abstract and applied, providing wide-ranging coverage across the subject area. Journal promotes research that aim at the unification of the theoretical-quantitative and the empirical-quantitative approach to economic problems and that are penetrated by constructive and rigorous thinking.

The Journal starting with its first issue will be indexed in RePEC, IndexCopernicus and EBSCO databases.

Web: http://www.asers.eu/journals/tpref.html

Email: tpref@asers.eu

Conferences Proceedings …



Proceedings of the ASERS First on-line Conference on
World's Economies in and after Crisis: Challenges, Threats and Opportunities

Coordinator: Laura ŞTEFĂNESCU

Format: 17cm x 24cm
ISBN: 978-606-92386-0-8

Proceedings of the ASERS First on-line Conference on
The Real Environmental Crisis –
Effects in Tourism Development, Conflicts and Sustainability

Coordinator: Cristina BARBU

Format: 17cm x 24cm
ISBN: 978-606-92386-3-9

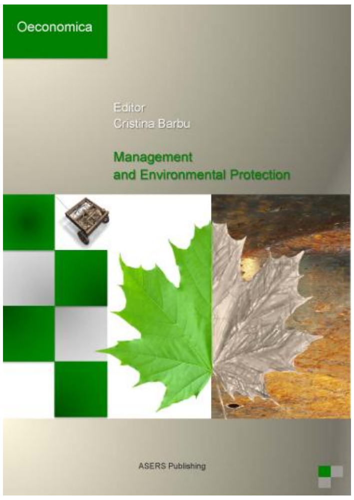
Proceedings of the ASERS First on-line Conference on
Competitiveness and Economic Development:
Challenges, Goals and Means in a Knowledge based Society

Coordinator: Andy ŞTEFĂNESCU

Format: 17cm x 24cm
ISBN: 978-606-92386-4-6

Books Collections ...

Management and Environmental Protection



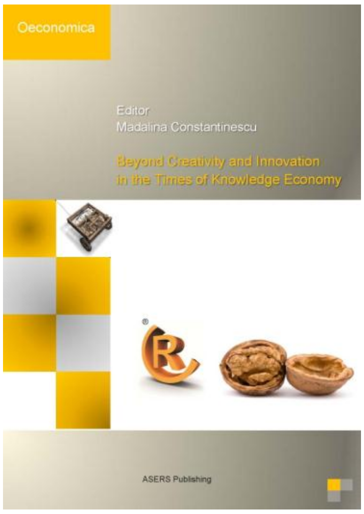
A book edited by PhD **Cristina Barbu**
European Research Centre for Managerial Studies in Business Administration
Spiru Haret University, Romania

cristina.barbu@spiruharet.ro
http://www.asers.eu/asers-publishing/books

To be published by ASERS Publishing in CD-ROM format with ISBN.

Submission: Open
Download Call for Book Chapters at:
http://asers.eu/asers_files/books/Call%20MEP.pdf

Beyond Creativity and Innovation in the Times of Knowledge Economy



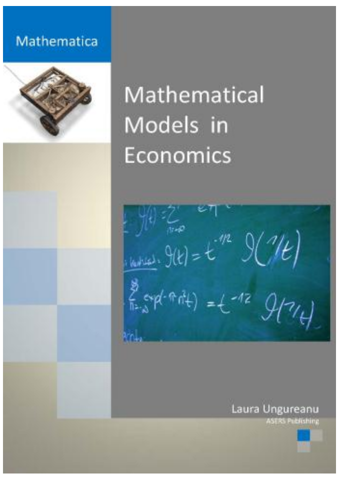
A book edited by PhD **Madalina Constantinescu**
European Research Centre for Managerial Studies in Business Administration
Spiru Haret University, Romania

constantinescu_madalina2002@yahoo.co.uk
http://www.asers.eu/asers-publishing/books

To be published by ASERS Publishing in CD-ROM format with ISBN.
Submission: Open

Download Call for Book Chapters at:
http://www.asers.eu/asers_files/books/Call%20BCI_KE.pdf

Mathematical Models in Economics



A book edited by PhD **Laura Ungureanu**

European Research Centre for Managerial Studies in Business Administration, Spiru Haret University, Romania

laura.ungureanu@spiruharet.ro
http://www.asers.eu/asers-publishing/books

To be published by ASERS Publishing in CD-ROM format with ISBN.
Submission: Open
Download Call for Book Chapters at:
http://www.asers.eu/asers_files/books/Call%20ASERS_Book%20MME_extended.pdf

ASERS Publishing
ASERS

ASERS Publishing
Web: www.asers.eu
URL: http://www.asers.eu/asers-publishing
ISSN 2068 – 7710