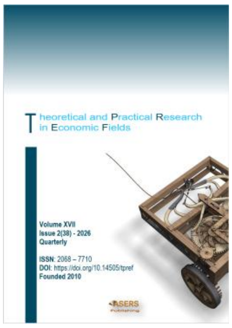
Dynamics at an Intermediate-Macroeconomics Level: A Keynesian Macroeconomic Model with Inventory and Production-Consumption Delays



Aleksandar Vasilev

Faculty of Economics and Business Administration,
University of Sofia, Bulgaria
Alvasilev@feb.uni-sofia.bg, alvasilev@yahoo.com

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Abstract: We present a dynamic Keynesian model at an intermediate macroeconomic level, a la Metzler (1941) and Leslie (1993), where the utilized propagation mechanism is firms' inventory management, and producers following adaptive expectations in order to meet stochastic demand. In addition, there is a delay between production, and subsequent delivery and consumption from the shelves. The discrete-time model generates a second-order linear difference equation, which exhibits endogenous cycles, with fluctuations (oscillations), which - for a plausible range of model parameters – are stable, and die out over time. Pedagogically, this is the simplest model that is able to generate interesting non-linear dynamics, and thus of interest to both undergraduate economics students, and instructors teaching intermediate macroeconomics.

Keywords: inventory model; business cycles; dynamics; oscillations.

JEL Classification: E32; E12; C62.

Introduction

Modern macroeconomics is inherently dynamic. However, classical intermediate macroeconomics books, *e.g.* Blanchard *et al.* (2024), generally shy away from explicitly introducing dynamics, and working out time-dependent solutions. We show that introducing dynamics in discrete time is something that the second-year undergraduate student in economics, who has seen some calculus, can handle without much difficulty. The particular case to be discussed has to do with the underlying basis of the Keynesian model - the assumption that producers are acting as "satisfiers," and just try to satisfy a stochastic demand in each time period (rather than acting as profit-maximizers, as stipulated in the neoclassical theory). Being able to meet uncertain demand is achieved by coupling production decisions with inventory management in the spirit of Vasilev (2021), though not in a micro-founded setup here.

In this paper, we extend the static Keynesian cross model a la Metzler (1941) and Leslie (1993), by factoring in also possible delays between production, and subsequent delivery, and more importantly - final consumption from the shelves. The discrete-time dynamic model generates a second-order linear difference equation, which exhibits endogenous cycles, with fluctuations (oscillations), which - for a plausible range of model parameters - are stable, and die out over time. Note that first-order dynamics will not be able to produce those interesting non-linearities, as such equations will only generate monotone paths. So, in this sense both model ingredients are crucial to generate the interesting result and serve as a pedagogical tool alongside Leslie (1993).

The limitations of the paper should be also properly acknowledged. After all, the conclusions of the paper should be taken with a grain of salt, as the model lacks proper micro-foundations, and features no stochasticity. Furthermore, the fluctuations are periodic, which comes at odds with data, where fluctuations in developed economies, such as the US, are irregular in both length and amplitude (as shown in Prescott 1986). Having said that, the deterministic setup should not be taken as a shortcoming; instead, it should be appreciated as a controlled environment used to illustrate some interesting dynamics emerging in a relatively simple Keynesian model. We believe extensions should be left for future research, as otherwise the treatment would be beyond the level of a second-year undergraduate economics students, who has just taken an intermediate-level course in macroeconomics, and only with a basic calculus course under their belt from the first year. It is thus important that the material covered is served in the optimal bite size – not too large, so that it can be still digestible, and not that small, so that it makes the student learn new methods, which are the norm in the profession.

Next section presents the model in detail, together with the solution method, and analyses in depth the dynamics followed the time series process, and the relevance for business cycle fluctuations. Section 3 concludes.

The Model

The model is a simple no-government, and closed-economy setup. In particular, final demand is represented by consumption and investment, with a certain twist. Final demand is represented by

$$Y_t = u_t + s_t + v_{0,} \tag{1}$$

where Y_t denotes real output/GDP/income/sales, u_t is the production for sale, *i.e.*, final consumption, s_t is production for inventory, i.e, inventory investment, and $v_0 > 0$ is fixed (non-inventory) investment, which is exogenous and held constant over time. We keep it fixed for the sake of simplicity. Without loss of generality, this component could be stochastic as well, in order to offset the stochastic demand component. This extension does not change the results in any major way.

As pointed out earlier, in reality, there is a lag between production for sale, delivery, actual purchase from the shelf, and final consumption. Hence, it will be assumed that in the model

$$u_t = \beta Y_{t-1}, \tag{2}$$

where $0 < \beta < 1$ denotes the marginal propensity to consume. This is reminiscent of the Keynesian consumption function, though with a time delay. (Note that (i) in the special case of no delay, and no inventory, we are back to the standard static Keynesian model; (ii) in the version of no delay, but with inventories, the model generates first-order monotone solution path. Solving those setups is left to the reader.) As we pointed above, the delay is due to the fact that production and consumption decisions may take place in different time periods. Goods need to be produced first, and put on the shelves, before they could be purchased for consumption. Certain (monetary-search) models, *i.e*, Nosal and Rocheteau (2011), use that timing structure by explicitly modeling that consumption markets open only after production markets have already closed.

Next, by definition, inventory stocks in the model are the difference between actual and expected sales, or

$$s_t = \beta Y_{t-1} - \beta Y_{t-2}. \tag{3}$$

Plug the expressions for s_t and u_t back into the main equation to obtain

$$Y_t = \beta Y_{t-1} + \beta Y_{t-1} - \beta Y_{t-2} + v_0. \tag{4}$$

Next, rearrange to obtain

$$Y_t - 2\beta Y_{t-1} + \beta Y_{t-2} = v_0. \tag{5}$$

Shift time subscript two periods forward to obtain

$$Y_{t+2} - 2\beta Y_{t+1} + \beta Y_t = v_0. \tag{6}$$

This is a standard second-order linear difference equation, in the form presented in Chiang and Wainwright (2005). To address it, first we solve for the particular solution (steady-state), i.e., we impose $Y^* = Y_{t+2} = Y_{t+1} = Y_t$. Then

$$Y^* - 2\beta Y^* + \beta Y^* = v_0$$

or

$$Y^* = \frac{v_0}{1-\beta} > 0. \tag{7}$$

Note that indeed in this case the steady-state is a fixed point, and not a (non-)linear trend.

Next, to obtain the complementary solution (function), we solve

$$Y_{t+2} - 2\beta Y_{t+1} + \beta Y_t = 0, \tag{8}$$

which is the associated homogeneous equation (as we set the right hand side to Eq. 8 to zero) to the original non-homogeneous equation (6) specified above.

We guess that the solution is of the form

$$Y_t^c = Ab^t, \tag{9}$$

where the superscript "c" stands for "complementary", and parameters *A, b* are to be determined. The form of the guess comes from the differential equations literature, i.e., from linear dynamic systems in continuous time, where the solution is a certain exponential process. In what follows, we will utilize an approach known as "guess and verify," which is a well-accepted approach in the literature

Plug in the guess and solve the equation using the undetermined coefficients method (note that $Y_{t+1} = Ab^{t+1}$ and $Y_{t+2} = Ab^{t+2}$):

$Ab^{t+2} - 2\beta Ab^{t+1} + \beta Ab^t = 0,$ or $Ab^t(b^2 - 2\beta b + \beta) = 0$, hence

$$b^2 - 2\beta b + \beta = 0. \tag{10}$$

Solving the quadratic equation above for *b*, and after some algebra, the roots of the equation are computed to be

$$b_{1,2} = \beta \pm \sqrt{\beta(\beta - 1)} \tag{11}$$

Given that $0 < \beta < 1$, due to the fact that it represents the marginal propensity to consume, the value of the discriminant (the expression under the square root operator) is then negative. This means that there are two distinct (conjugate), imaginary roots that solve the homogeneous equation.

Since both of the complex conjugate roots feature a real part β, which is a positive fraction, the exponential solution process will be convergent, but with dampened oscillations (cycles) - due to the imaginary part of the root - which will produce dynamics akin to one exhibited by certain trigonometric functions, though with a twist.

In particular, the characteristic roots of the process are given by $b_{1,2} = \beta \pm vi$, where we have denoted $v = \sqrt{\beta(1-\beta)}$ and $i = \sqrt{-1}$. The purpose of v, (not to be confused with v_0) will become clear below.

Passing to polar coordinates, one can obtain

$$\beta \pm vi = R(\cos\theta \pm i\sin\theta), \tag{12}$$

Where $\sin\theta = \frac{v}{R}$, and $\cos\theta = \beta/R$, with v and R to be determined from the polar coordinate relations.

Using Pyhagoras theorem, it follows that

$$R = \sqrt{\beta^2 + v^2}. \tag{13}$$

Next, since

$$v^2 = \beta(1-\beta) = \beta - \beta^2, \tag{14}$$

It follows (after some easy algebra) that

$$R = \sqrt{\beta}. \tag{15}$$

Next, to determine θ, we will make use of the trigonometric relationship, which states that

$$\tan\theta = v/\beta. \tag{16}$$

Thus

$$\theta = tan^{-1}\left(\frac{v}{\beta}\right) = tan^{-1}(\sqrt{\frac{1-\beta}{\beta}}). \tag{17}$$

Next, from De Moivre's theorem (the interested reader is referred to Chiang and Wainwright (2005) for more detail), which states that

$$(\beta \pm iv)^t = R^t(\cos\theta t \pm i\sin\theta t) \quad . \tag{18}$$

Now, split the two cases as

$$(\beta + iv)^t = R^t(\cos\theta t + i\sin\theta t) \tag{19}$$

And

$$(\beta - iv)^t = R^t(\cos\theta t - i\sin\theta t)\ . \tag{20}$$

Combining the particular solution and the complementary function, the equilibrium process is then of the form

$$Y_t = Y^* + c_1 R^t(\cos\theta t + i\sin\theta t) + c_2 R^t(\cos\theta t - i\sin\theta t), \tag{21}$$

where c_1 and c_2 are some scaling parameters, which are yet to be determined.
Next, simplify by grouping terms to obtain

$$Y_t = Y^* + A_1 R^t \cos\theta t - A_2 R^t \sin\theta t, \tag{22}$$

where $A_1 = c_1 + c_2$ and $A_2 = i(c_2 - c_1)$. Next, write A_1 and A_2 in polar coordinates as

$$A_1 = A\cos\epsilon \tag{23}$$

$$A_2 = A\sin\epsilon \tag{24}$$

Substitute with the expressions above for the scaling parameters to obtain

$$Y_t = Y^* + AR^t(\cos\theta t\cos\epsilon - \sin\theta t\sin\epsilon\ ). \tag{25}$$

The above can be written more compactly as

$$Y_t = Y^* + AR^t\cos(\theta t + \epsilon). \tag{26}$$

As pointed earlier, the crucial parameter is R, which determines whether oscillations generated by the equilibrium process are increasing, decreasing, or constant over time.

Since $R = \sqrt{\beta < 1}$, oscillations will "die out" over time, and the process will converge to Y^*. The role of the other parameters, A and ϵ, are of secondary importance. In particular, scale parameter A only determines the height of the amplitude/fluctuations, while ϵ captures the "phase" of these endogenously-generated "business cycles". In particular, the period of the "cycle" (from peak to peak) is $\frac{2\pi}{\theta}$. Yes, in this simple model, cyclical fluctuations are periodic - while in reality business cycles are irregular, as documented in Prescott (1986). In addition, these endogenous business cycles are not derived from general equilibrium, and are not micro-founded, thus they are subject to the Lucas' (1976) critique. In the model, this periodicity result follows from the DeMoivre's transformation involved, which re-cast the exponential process into a combination of sin and cosine functions.

Finally, note that a higher value of θ (computed earlier as $= tan^{-1}(\sqrt{\frac{1-\beta}{\beta}})$), shortens the time requirement, or leads to higher frequency in time series language. In other words, cycles will happen faster (more frequently), so the oscillations will appear more "squeezed" when plotted over time.

Conclusions and Further Research

In this paper we presented a dynamic Keynesian model a la Metzler (1941) and Leslie (1993), where the utilized propagation mechanism is rms' inventory management, and producers following adaptive expectations in order to meet stochastic demand. In addition, there is a delay between production, and subsequent delivery and consumption from the shelves. The discrete-time model generates a second-order linear difference equation, which exhibits endogenous cycles, with fluctuations (oscillations), which - for a plausible range of model parameters - are stable, and die out over time.

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Declarations

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Aleksandar Vasilev: Conceptualization, Investigation, Methodology, Formal analysis, Writing – original draft, Writing – review and editing, Visualization, Funding acquisition.

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