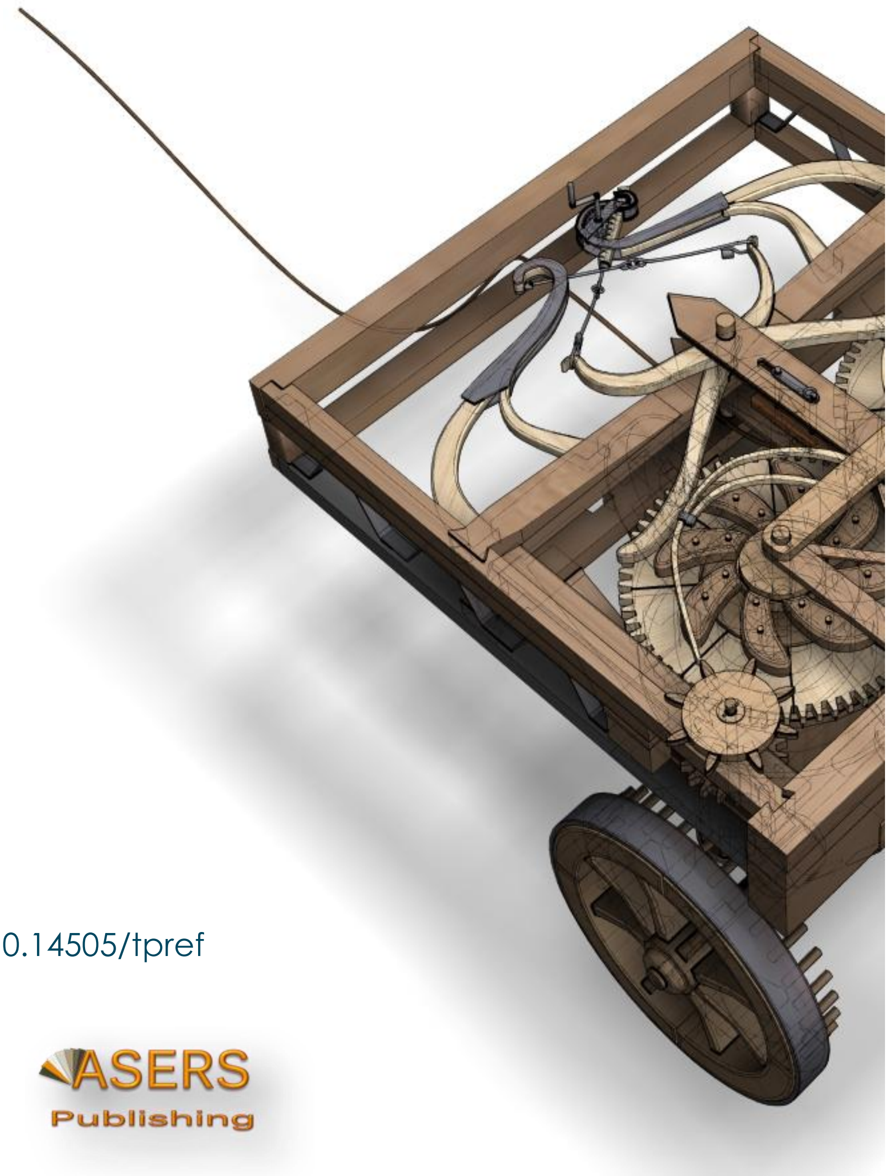
Theoretical and Practical Research in Economic Fields



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An Application of the Generalized Method of Moments on the Mankiw-Romer-Weil Model

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Abstract: This paper develops an algorithm for the restricted Generalized Method of Moments (RGMM) to re-evaluate the empirical findings of Mankiw, Romer, and Weil (1992), who emphasized the critical role of human capital accumulation in explaining cross-country income differences. Despite the influence of their results, subsequent literature has raised concerns about potential endogeneity arising from omitted variable bias. To address these concerns, we employ a novel instrumental variable strategy - using the algorithm of the restricted Generalized Method of Moments (RGMM). Using the restricted GMM, we find that the original Mankiw, Romer, and Weil (1992) results are robust for the method of estimation and survive the endogeneity criticisms that are present in the literature.

Keywords: omitted variable bias; restricted GMM; endogeneity; instrumental variables.

JEL Classification: O24; C26; E24.

Introduction

Income disparity across countries continues to be an intriguing research question. In this context, the paper by Mankiw, Romer, and Weil (1992) henceforth MRW, where they augmented the Solow growth model with human capital, is an oft-cited work in the empirical growth literature. Employing the least square and the restricted least square estimation method, they showed that the combined contribution of technology and investment in physical capital and human capital, can explain over three-fourths of the variation in long-run per-capita income across countries, and the implied model-based factor shares are consistent with the national accounts data.

However, Acemoglu (2009) pointed out that the MRW estimates could be biased due to potential endogeneity arising from the correlation between technology and investment in physical and human capital. Such correlation arises because countries with better institutions (a potential omitted variable) have better technology, and they also invest more in physical and human capital accumulation. Romer, while reviewing the work of Bernanke and Gürkaynak (2001), had also identified the same and suggested an instrumental variable estimation.

In this backdrop, we re-estimate the MRW model through GMM by using the future-time reference (FTR) of languages as an instrument for investment in physical capital (savings rate); and the proportion of the Protestant missionaries in 1923, and the national primary school enrollment in 1900, as instruments for human capital. Our findings indicate that the original MRW results are robust to the method of estimation, as well as for alternate datasets of human capital.

Our paper has two important contributions: 1. We develop an algorithm of restricted GMM and employ it to estimate the MRW model[1]. 2. We address the endogeneity issue in MRW by employing potential instruments for physical and human capital accumulation based on the extant literature.

1. Data

The period considered in our study is from 1970 to 2019. To keep the data consistent with that of MRW, we obtain the real GDP and the private investment data from the Penn World Table 10.0. For a country j, the average per capita income is y_j and the average investment to GDP ratio is s_{kj}. We use the average of the mean years of schooling (for the population aged at least 25 years) from UNESCO as a measure for human capital. We use it to construct the measure of human capital s_{hj}. Moreover, we also calculate the population growth rate n_j respectively using the Penn World Table 10.0 data. Additionally, following MRW, we set the sum of the average technology growth rate and depreciation of capital, i.e. $(g + \delta)$, equal to 5%.

For robustness check, we have also obtained alternate data for human capital. We have obtained the data of the mean years of schooling (secondary) from Barro and Lee (2013). The dataset considers the total population aged 15 and above for the period 1970-2010.

1.1 Instrument for Physical Capital Accumulation

We utilize the future-time reference (FTR) characteristic of languages as an instrument to account for physical capital accumulation. In languages classified as strong-FTR - such as English and French - the grammatical structure mandates a clear distinction between present and future tense. Unlike speakers of weak-FTR languages, who are not required to make this separation, strong-FTR language users must explicitly mark temporal differences. Since Chen (2013) initiated the discourse, numerous studies have consistently drawn connections between the FTR structure of languages and various economic behaviors. Chen's (2013) findings revealed that countries with weak-FTR languages tend to exhibit greater future-oriented behavior - manifested through elevated saving patterns - both at individual and national levels. While some critiques targeted Chen's (2013) assumption of linguistic independence, Roberts *et al.* (2015) addressed this limitation by incorporating linguistic interdependence, ultimately affirming the robustness of Chen's (2013) original conclusions. Building on this linguistic framework, we interpret the FTR's correlation with saving behavior as a theoretical foundation for treating it as a valid instrument. Given that FTR has a well-established relationship with intertemporal decision-making and lacks any empirically supported connection to technology, we argue that it serves as a credible instrument for capturing variation in physical capital accumulation.

Data for FTR of languages is from Chen (2013). Specifically, we use an instrument $z_{kj} = ftr_j - ln\,(n_j + g + \delta)$ for $x_{kj} = ln\,(s_{kj}) - ln\,(n_j + g + \delta)$; where, $ftr_j = 1$ if the official language of country j has weak FTR, and 0 otherwise.

1.2 Instruments for Human Capital Accumulation

Acemoglu *et al.* (2014), along with the references[2] therein, posited that: (i) Protestant missionaries' activities, partly motivated by encouraging readings of the scriptures, played an important role in setting up schools in different countries, thereby leading to a lasting impact on the evolution of human capital, (ii) Country-wise variation of the Protestant missionaries' activities, after controlling for variations in continent dummies, the identity of the colonial power, and institutions, have largely been determined by idiosyncratic factors and need not be correlated with potential future economic prosperity of a country.

[1] It produces consistent, and efficient parameter estimates that matches the data, and it can be applied on any data in the future.

[2] Please see Benavot and Riddle (1988) for the data on the secondary enrollment rate of the 1900s. Also, please refer to Nunn (2014), Becker and Woessmann (2009), and Woodberry (2004, 2012) for the data on Protestant missionary activities.

Following Acemoglu *et al.* (2014), we use the share of Protestant missionaries in 1923 per 10,000 population (pm_j), and the national primary enrollment in 1900 $(enrol_j)$ for country j as instruments for human capital. Specifically, we use the vector[3],

$$z_{hj} = [pm_j - ln\ (n_j + g + \delta), ln\ (enrol_j) - ln\ (n_j + g + \delta)]$$

as an instrument for $x_{hj} = ln\ (s_{hj}) - ln\ (n_j + g + \delta)$

2. Methodology

Starting with the benchmark Solow model and assuming s_{kj} and s_{hj} as the fraction of income saved by the households to accumulate physical and human capital respectively by country j, MRW (1992) derived equation (1) by taking the logarithmic transformation of a production function with labour augmenting technological progress. The production function in MRW (1992) follows CRS in physical capital, human capital and effective labor force with $0 < \alpha < 1$ being the share of capital in national income, $0 < \beta < 1$ being the same for human capital, and $\alpha + \beta < 1$.

$$ln\ (y_j) = \gamma_0 + \gamma_1 x_{kj} + \gamma_2 x_{hj} + u_j, \qquad (1)$$

where, $\gamma_1 = \frac{\alpha}{1-(\alpha+\beta)}$;$\gamma_2 = \frac{\beta}{1-(\alpha+\beta)}$; and $j = 1,2, \dots, N$ is the number of countries, u_j represents the technology level of country j in MRW (1992). Acemoglu (2009) points out that countries with better technology often save more on physical and human capital. This implies, both x_{kj} and x_{hj} are endogenous with cov$(x_{kj}, u_j)\neq 0$, and cov$(x_{hj}, u_j)\neq 0$. As a result, OLS estimates of equation (1) are inconsistent, and we need an IV estimation to achieve consistency. We have estimated equation (1) through GMM and restricted GMM by using z_{kj} as instrument of x_{kj}, and z_{hj} as instrument of x_{hj}. The algorithm of the GMM and restricted GMM; the calculation of $\bar{R}^2$, AIC and SBC are given in the appendix. Please note, the distribution of imputed $\widehat{\alpha}$ and $\widehat{\beta}$ are calculated using the Delta method.

3. Results

MRW (1992) estimated equation (1) by OLS for 98 non-oil producing countries for the period 1965-1980 using secondary school enrolment as a measure of human capital. They found that both physical and human capital accumulation are important for long-term prosperity of a country, and the two factors jointly explained 78% of the variation of long-run per-capita income.

Table 1. OLS, GMM and Restricted GMM estimates of the MRW (1992) equation

AYS				
	OLS	GMM	Restricted GMM (r = –0.8)	Restricted GMM (r = –0.1)
$\widehat{\gamma}_0$	8.59*** (0.19)	9.11*** (0.50)	9.13*** (0.09)	8.57*** (0.09)
$\widehat{\gamma}_1$	0.82*** (0.17)	0.42 (0.47)	0.40*** (0.09)	0.91*** (0.09)
$\widehat{\gamma}_2$	0.86*** (0.09)	1.19*** (0.17)	1.20*** (0.09)	1.01*** (0.09)
$\widehat{\alpha}_{imputed}$	0.31*** (0.05)	0.16 (0.16)	0.15*** (0.02)	0.31*** (0.01)
$\widehat{\beta}_{imputed}$	0.32*** (0.04)	0.46*** (0.11)	0.46*** (0.002)	0.35*** (0.009)
N	74	74	74	74
J-statistic	–	0.098 [0.75]	0.095 [0.95]	1.66 [0.44]
$\bar{R}^2$	0.71	0.65	0.65	0.69

Note: ***; **; and * indicate significance at the 1%, 5%, and 10% levels, respectively. Standard errors are in parentheses; p-values are in square brackets. The J-statistics follows a chi-square distribution with 1 and 2 degrees of freedom for the unrestricted and restricted GMM, respectively. Restricted GMM is estimated for Average Years of Schooling (AYS) as a measure of human capital. Source: Calculated by the authors.

[3] We use identical data sources of Acemoglu et al. (2014) to collect data and also followed them to address the problem of missing data for the instruments of human capital. Following Acemoglu et al. (2014), we obtain the missing data about the arrival of Protestant missionary in five different countries from Dennis et al. (1911).

Further, they found that the share of physical capital and human capital in the national income are 33% and 28% respectively.

Among these 98 non-oils producing countries of MRW (1992), we have data of all control variables and instruments for 74 countries where average years of schooling is considered an indicator of human capital. Columns 2 of Table 1 present the OLS results of equation (1) for AYS as a measure of human capital. The findings indicate that:

1. Physical capital and human capital accumulation are important for long-run prosperity, and they jointly explain 71% of the variation of the long-run per-capita income for the indicator of human capital.

2.The share of physical capital in national income is 31% (column 2), and that of human capital in national income is 32% (column 2) respectively.

Post the OLS estimation of the benchmark MRW (1992) equation, to address the endogeneity concerns explained earlier, we now estimate equation (1) through GMM by using z_{kj} and z_{hj} as instruments for x_{kj} and x_{hj} respectively. It is important to mention here that, unlike least square estimates, GMM estimates do not have criteria for goodness of fit to determine the best fitted model. But we can test the validity of instruments used in the GMM estimation based on the over identification restrictions through the J-statistics. We must discard a GMM estimate when the instruments tested through the J-statistics are not valid.

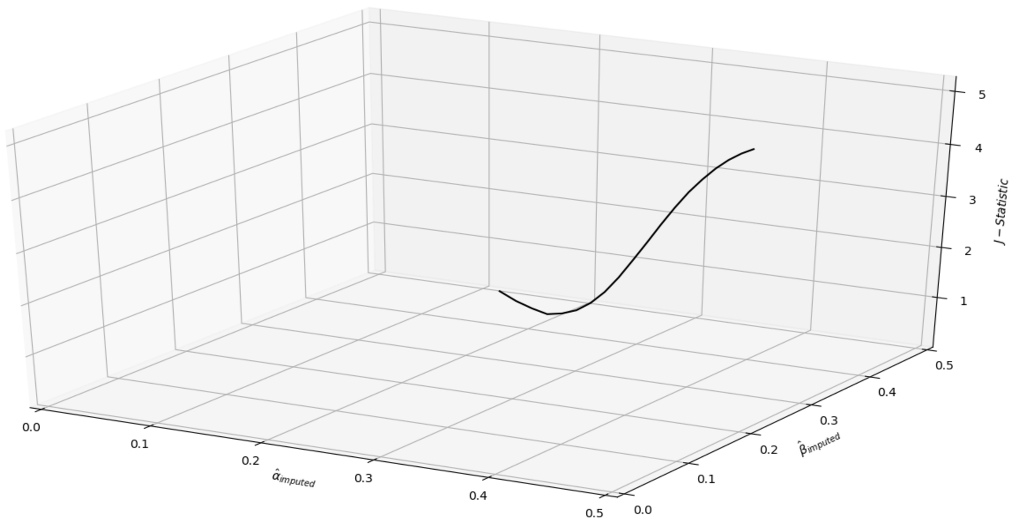
Next, we estimate equation (1) for the 74 non-oil producing countries through GMM by using average years of schooling (AYS) as a measure of human capital; and the results are reported in column 3 of Table 1. The GMM estimation yields correct signs for all the control variables but $\hat{\gamma}_1$ and the share of physical capital $(\hat{\alpha}_{imputed})$ are not significant. As a result, we estimate a restricted version of the GMM model. To do that, we impose the restriction $\gamma_1 - \gamma_2 = r$ by keeping the following parametric restrictions of our model in mind:

1. $\alpha > 0$;
2. $\beta > 0$;
3. $\alpha + \beta < 1$.

The algorithm of the restricted GMM estimation developed by us is given in the appendix section. Following the algorithm of the restricted GMM, we estimate equation (1) for $r = -0.9, -0.8, \dots, 0, \dots, 0.8, 0.9$ and calculate the corresponding $\hat{\alpha}_{imputed}$ and $\hat{\beta}_{imputed}$ from $\hat{\gamma}_1$ and $\hat{\gamma}_2$ using the equation:

$$\gamma_1 = \frac{\alpha}{1-(\alpha+\beta)}, \gamma_2 = \frac{\beta}{1-(\alpha+\beta)}.$$

Figure 1. Model Selection for AYS as a measure of Human Capital Using J-statistics



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We also calculate the J-statistic for each model. Figure 1 plots $\hat{\alpha}_{imputed}$, $\hat{\beta}_{imputed}$, and the J-statistic for each of the 19 models. It shows that the J-statistic is minimum for $r = -0.8$ when $\hat{\alpha}_{imputed} = 0.15$, and $\hat{\beta}_{imputed} = 0.46$. We have reported the result of the restricted GMM for $r = -0.8$ in Column 4 of Table 1. The p-value of the J-statistics reported in Column 4 of Table 2 confirms that the instruments of physical capital and human capital used in our analysis are valid. However, we find that the share of physical capital ($\hat{\alpha}_{imputed} = 15\%$) is too low; and that of human capital ($\hat{\beta}_{imputed} = 46\%$)is too high and not matching with the data of national accounts. To address this issue, we also calculate the $\bar{R}^2$ of the 19 restricted GMM models to identify a model whose fit is closest to the benchmark OLS estimation of equation (1). We plot $\hat{\alpha}_{imputed,}$ $\widehat{\beta}_{imputed}$, and $\bar{R}^2$ for each of the 19 models in Figure 2. Figure 2 shows that the maximum $\bar{R}^2 = 0.69$, which is closest to the corresponding benchmark OLS estimation of equation (1), is achieved for $(r, \hat{\alpha}_{imputed}, \hat{\beta}_{imputed}) = (-0.1, 0.31, 0.35)$. We report the result of restricted GMM for $r = -0.1$ in Column 5 of Table 2. The p-value of the J-statistic reported in Column 5 of Table 2 shows that the instruments of physical capital and human capital used in our estimation remain valid even if the J-statistic rises from 0.095 to 1.66 and its p-value changes from 0.95 to 0.44 when r changes from -0.8 to -0.1[4].

Figure 2. Model Selection for AYS as a measure of Human Capital Using $\bar{R}^2$

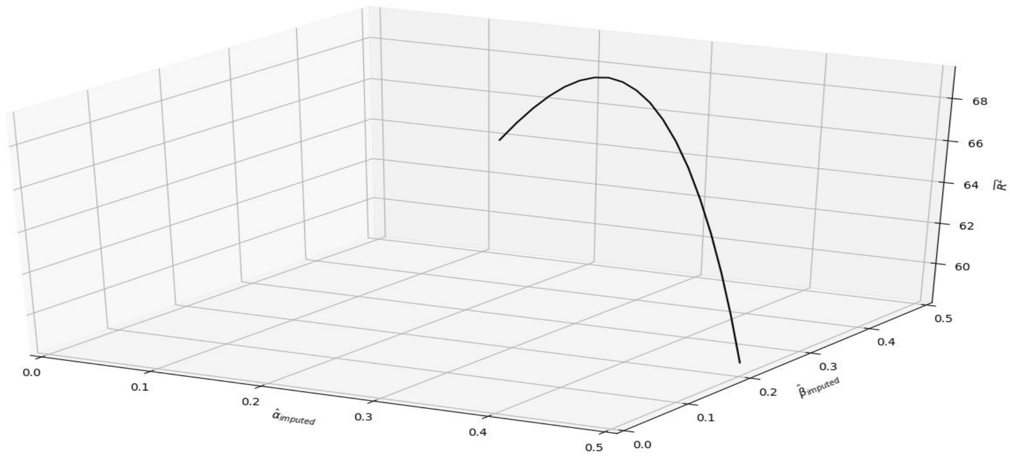

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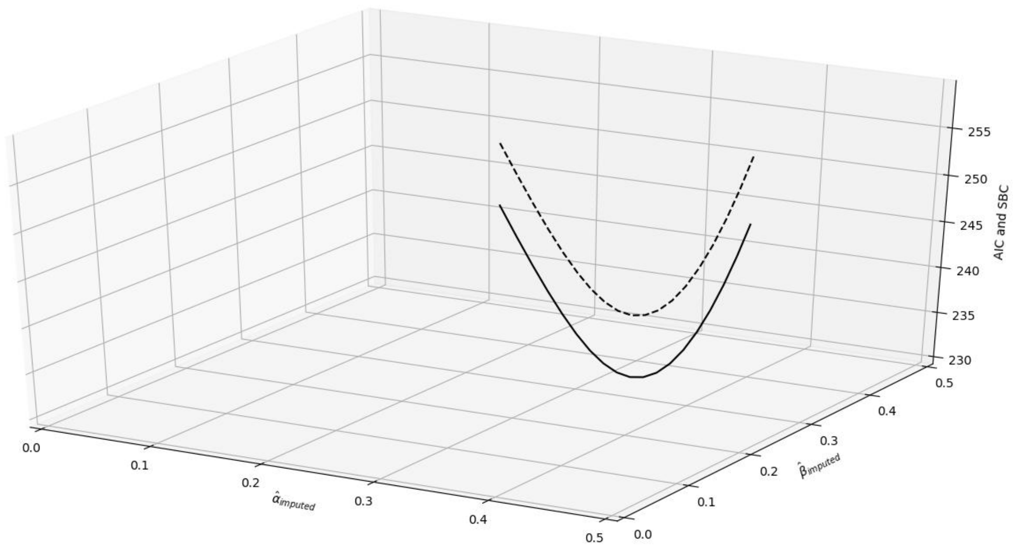
Figure 3. Model Selection for AYS as a measure of Human Capital Using AIC and SBC



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Moreover, the restricted GMM estimates reported in Column 5 of Table 1 also yield the share of physical capital and human capital in national income (31% and 35% respectively) that belong to the range of the data supported by national accounts statistics.

[4] We also plot the AIC and SBC of the 19 restricted GMM model against $\widehat{\alpha}_{imputed}$ and $\hat{\beta}_{imputed}$ in Figure 3. Figure 3 shows that, the AIC (solid line) and SBC (dotted line) are also minimum at $(r, \hat{\alpha}_{imputed}, \hat{\beta}_{imputed}) = (-0.1, 0.31, 0.35)$.

4. Robustness Check

We have estimated equations (1) by OLS, GMM, and restricted GMM using secondary AYS obtained from Barro and Lee (2013) for 71 countries from 1970-2010 as a measure of human capital to check the robustness of our results. We have also estimated the same model by GMM using z_{kj} and z_{hj} as measures of physical capital and human capital respectively for country j. The result of the OLS estimation is reported in Column 2 of Table 2. The OLS estimation yields, $\hat{\alpha}_{imputed} = 23\%$, $\hat{\beta}_{imputed} = 33\%$ and $\bar{R}^2 = 0.72$. The result of the GMM estimation is reported in Column 3 of Table 2. The p-value of the J-statistic of the GMM estimation shows that our instruments are valid as they satisfy the overidentification restrictions. The GMM estimation yields correct signs for all the control variables, but $\hat{\gamma}_1$ is not significant. As a result, we estimate a restricted version of the GMM model.

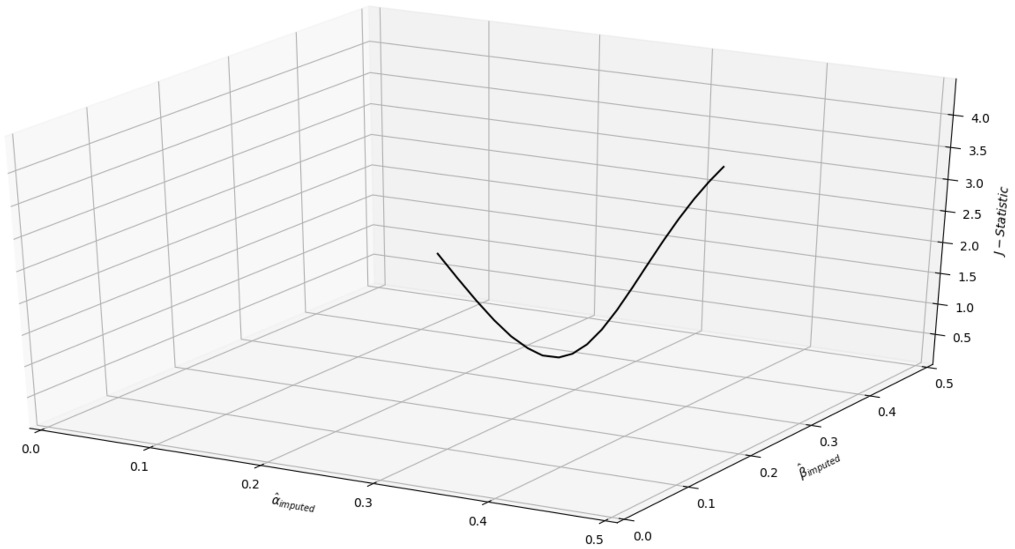
Following the methodology already discussed in the results section, we estimate the model by restricted GMM with restrictions, $\gamma_1 - \gamma_2 = r$; $r = -0.9, -0.8, \dots, 0.8, 0.9$, and calculate the J-statistic along with $\hat{\alpha}_{imputed}$ and $\hat{\beta}_{imputed}$. We plot the J-statistic against $\hat{\alpha}_{imputed}$ and $\hat{\beta}_{imputed}$ in Figure 4. Figure 4 shows that the J-statistic is minimum when $(r, \hat{\alpha}_{imputed}, \hat{\beta}_{imputed}) = (-0.4, 0.21, 0.38)$. We have reported the result of restricted GMM for $r = -0.4$ in Column 4 of Table 2. Note, the result of the GMM estimation reported in Column 4 of Table 2 closely resembles the result of the OLS estimation reported in Column 2 of Table 2. Moreover, the p-value of the J-statistics shows that the overidentification restrictions are satisfied and instruments used in our estimation are valid.

Table 2. OLS, GMM and restricted GMM estimates for Robustness Check

Secondary AYS				
	OLS	GMM	Restricted (r = –0.4)	Restricted (r = –0.2)
$\hat{\gamma}_0$	9.85*** (0.28)	10.14*** (0.53)	10.17*** (0.07)	9.93*** (0.07)
$\hat{\gamma}_1$	0.54** (0.17)	0.51 (0.33)	0.52*** (0.07)	0.66*** (0.07)
$\hat{\gamma}_2$	0.76*** (0.08)	0.93*** (0.16)	0.92*** (0.07)	0.86*** (0.07)
$\hat{\alpha}_{imputed}$	0.23*** (0.06)	0.21* (0.12)	0.21*** (0.02)	0.26*** (0.01)
$\hat{\beta}_{imputed}$	0.33*** (0.04)	0.38*** (0.09)	0.38*** (0.007)	0.34*** (0.008)
N	71	71	71	71
J-statistic	–	0.09 [0.76]	0.09 [0.96]	0.24 [0.89]
$\bar{R}^2$	0.72	0.70	0.70	0.71

Note: ***; **; and * indicate significance at the 1%, 5%, and 10% levels, respectively. Standard errors are in parentheses; p-values are in square brackets. The J-statistic follows a chi-square distribution with 1 and 2 degrees of freedom for the unrestricted and restricted GMM, respectively. Restricted GMM is estimated for the Secondary Average Years of Schooling obtained from Barro and Lee (2013) as a measure of human capital. Source: Calculated by the authors.

Figure 4. Model Selection for Secondary AYS as a measure of Human Capital Using J-Statistics



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Next, we plot the $\bar{R}^2$ against the corresponding $\hat{\alpha}_{imputed}$ and $\hat{\beta}_{imputed}$ for 19 restricted GMM models in Figure 5 to identify a model whose fit is closest to the same of the benchmark OLS estimation of equation (1). We find that the model with $(r, \hat{\alpha}_{imputed}, \hat{\beta}_{imputed}) = (-0.2, 0.26, 0.34)$ yields maximum $\bar{R}^2 = 0.71$, which is closest to the benchmark OLS estimation of equation (1) reported in Column 2 of Table 2. We also plot the AIC and SBC of the 19 restricted GMM models against $\hat{\alpha}_{imputed}$ and $\hat{\beta}_{imputed}$ in Figure 6. Figure 6 shows that the AIC (solid line) and SBC (dotted line) are also minimum at, $(r, \hat{\alpha}_{imputed}, \hat{\beta}_{imputed}) = (-0.2, 0.26, 0.34)$. We report the result of our restricted GMM estimation for $r = -0.2$ in Column 5 of Table 2. Note, the result of the GMM estimation reported in Column 5 of Table 2 also closely resembles the result of the OLS estimation reported in Column 2 of Table 2. Moreover, the p-value of the J-statistic shows that the overidentification restrictions are still satisfied and instruments used in our estimation remain valid when we choose a model whose fit is closest to the same of the benchmark OLS estimation of equation (1).

Figure 5. Model Selection for Secondary AYS as a measure of Human Capital Using $\bar{R}^2$

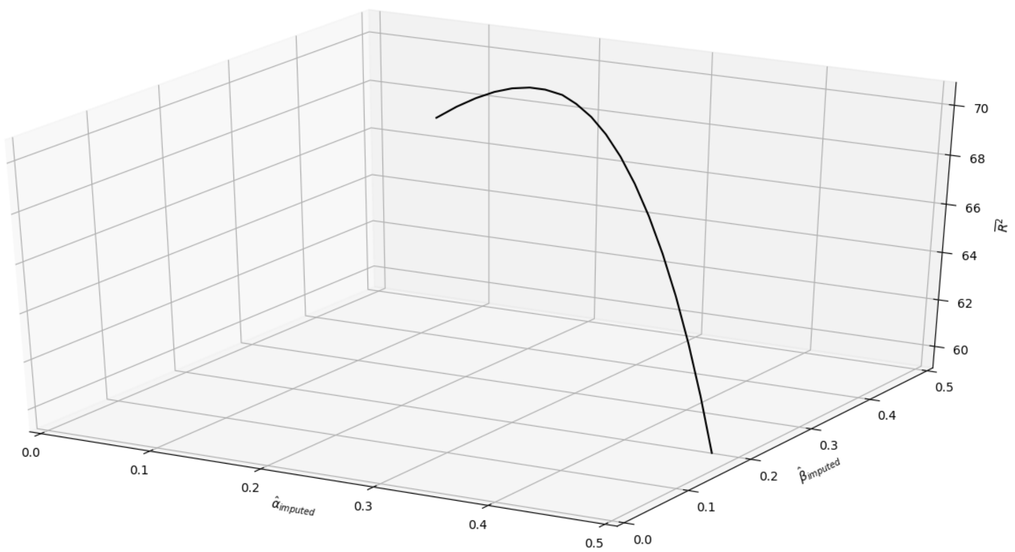

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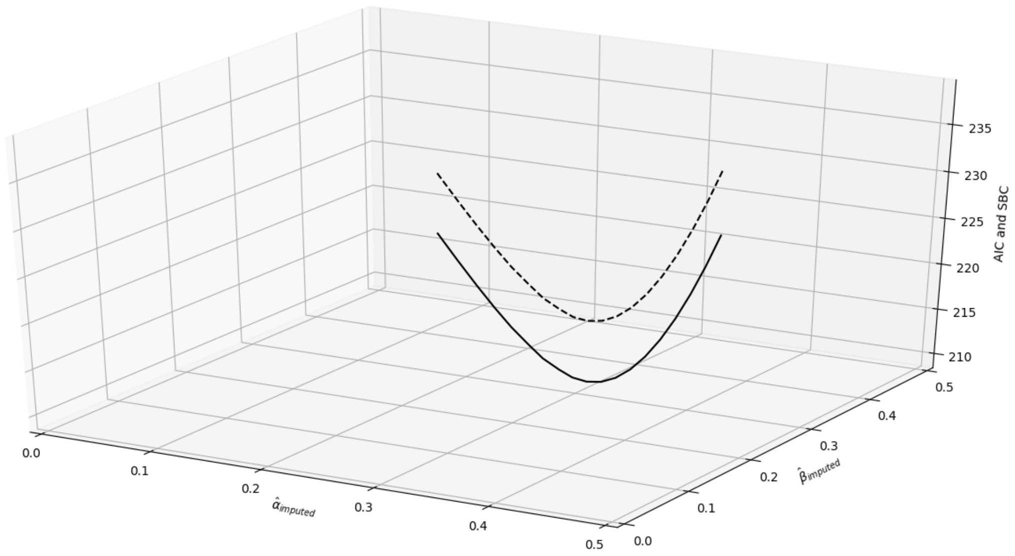
Figure 6. Model Selection for Secondary AYS as a measure of Human Capital Using AIC and SBC



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Conclusion

This study revisits the human capital-augmented Solow growth model of Mankiw, Romer, and Weil (1992) with a focus on addressing the endogeneity issues that have challenged the validity of their empirical estimates. By employing the restricted generalized method of moments (RGMM) framework, we provide a rigorous reassessment of the original model's robustness.

The findings reveal that the key results of MRW (1992) remain robust across datasets and estimation techniques, including under the more stringent restrictions imposed by the restricted GMM. This highlights the strength of their original conclusions and demonstrates that, when properly instrumented the model is resilient to critiques of endogeneity.

By estimating the model using the restricted GMM, we not only validate the empirical relevance of human capital in growth regressions but also contribute a robust econometric method that can be applied to future research on macro-development and other subjects.

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Credit Authorship Contribution Statement

Arpan Chakraborty: Corresponding author, Conceptualization, Investigation, Methodology, Software, Formal analysis, Writing – original draft, Data curation, Validation, Writing – review and editing, Visualization.
Siddhartha Chattopadhyay: Supervision, Conceptualization, Investigation, Methodology, Formal analysis, Writing, Validation, Writing – review and editing, Visualization.
Sohini Sahu: Conceptualization, Investigation, Formal analysis, Writing – original draft, Validation, Writing – review and editing, Visualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Declaration of Use of Generative AI and AI-Assisted Technologies

The authors declare that they have not used generative AI and AI-assisted technologies in the writing process before submission.

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Appendix

In this section we present the general model which can be used for any datasets in the future. The R code will be made available on request. Suppose our model is:

$$y = X\gamma + u \tag{A1}$$

where,

$$y = (y_1 \quad y_2 \quad \dots \quad y_N)'$$

is a $(NG \times 1)$vector; where, y_i is a $(G \times 1)$ vector for $i = 1,2,\dots,N$. The matrix of control variables,

$$X = (X_1 \quad X_2 \quad \dots \quad X_G)'$$

is a $(NG \times K)$ matrix; where X_i is a$(G \times K)$ matrix, and γ is a$(K \times 1)$ vector of parameters. The random error term

$$u = (u_1 \quad u_2 \quad \dots \quad u_G)'$$

is a $(NG \times 1)$ vector; where, u_i is a $(G \times 1)$ vector for $i = 1,2,\dots,N$. Also assume that the matrix of instruments,

$$Z = (Z_1 \quad Z_2 \quad \dots \quad Z_N)'$$

is a $(NG \times L)$ matrix with $L \geq K$; where, Z_i is a $(G \times L_i)$ matrix with $\sum_{i=1}^{N} L_i = L$.

Our objective is to minimize the quadratic form,

$$(Z'u)'W(Z'u) = (y - X\gamma)'ZWZ'(y - X\gamma)$$

subject to the linear restrictions $R\gamma = r$ by choosing γ, where W is a $(L \times L)$ symmetric and positive semidefinite weight matrix. Here, R is a $(q \times K)$; and r is a $(q \times 1)$matrix, where q represents the number of restrictions. Suppose, λ is a $(q \times 1)$matrix of Lagrange multipliers. Then, the relevant Lagrangian of our problem is,

$$\mathrm{L} = (y - X\gamma)'ZWZ'(y - X\gamma) - 2\lambda'(R\gamma - r)$$

FOCs:

$$\frac{\partial \mathrm{L}}{\partial \gamma} = -X'ZWZ'y + X'ZWZ'X\gamma - R'\lambda = 0, \tag{A2}$$

$$\frac{\partial \mathrm{L}}{\partial \lambda} = R\gamma - r = 0. \tag{A3}$$

Pre-multiplying equation (A2) by $R(X'ZWZ'X)^{-1}$, and using equation (A3) we get,

$$R'\lambda = R'[R(X'ZWZ'X)^{-1}R']^{-1}(r - R\hat{\gamma}). \tag{A4}$$

where,

$$\hat{\gamma} = (X'ZWZ'X)^{-1}(X'ZWZ'y). \tag{A5}$$

Substituting equation (A4) into equation (A2) and pre-multiplying the equation by $(X'ZWZ'X)^{-1}$gives,

$$\hat{\gamma}_R = \hat{\gamma} + (X'ZWZ'X)^{-1}R'[R(X'ZWZ'X)^{-1}R']^{-1}(r - R\hat{\gamma}). \tag{A6}$$

We have used the following algorithm to obtain the consistent and efficient restricted GMM estimator; its asymptotic variance-covariance matrix, and its asymptotic distribution. Set $W = (Z'Z)^{-1}$and calculate the 2SLS estimator from equation (A5) as follows,

$$\hat{\gamma}_{2SLS} = (X'Z(Z'Z)^{-1}Z'X)^{-1}(X'Z(Z'Z)^{-1}Z'y).$$

1.Using $\hat{\gamma}_{2SLS}$, obtain the corresponding estimated vector of random errors,

$$\hat{u}_{2SLS} = (\hat{u}_{1,2SLS} \quad \hat{u}_{2,2SLS} \quad \dots \quad \hat{u}_{N,2SLS})',$$

and calculate,

$$\hat{\Lambda}_{2SLS} = \frac{1}{N}\sum_{i=1}^{N} Z_i'\hat{u}_{i,2SLS}\hat{u}_{i,2SLS}'Z_i\,.$$

2.GMM estimator: Set $W = (\hat{\Lambda}_{2SLS})^{-1}$, and calculate the GMM estimator from equation (A5) as follows,

$$\hat{\gamma}_{GMM} = (X'Z(\hat{\Lambda}_{2SLS})^{-1}Z'X)^{-1}(X'Z(\hat{\Lambda}_{2SLS})^{-1}Z'y).$$

3.Using $\hat{\gamma}_{GMM}$, obtain the corresponding estimated vector of random errors,

$$\hat{u}_{GMM} = (\hat{u}_{1,GMM} \quad \hat{u}_{2,GMM} \quad \dots \quad \hat{u}_{N,GMM})',$$

and calculate,

$$\hat{\Lambda}_{GMM} = \frac{1}{N}\sum_{i=1}^{N} Z_i'\hat{u}_{i,GMM}\hat{u}_{i,GMM}'Z_i\,.$$

4.Distribution of $\hat{\gamma}_{GMM}$: Following Wooldridge (2010), we get,

$$\sqrt{N}(\hat{\gamma}_{GMM}-\gamma) \sim AN(0,\Sigma_{GMM}),$$

where,

$$C=(\frac{Z'X}{N}), \Sigma_{GMM}=(C'\hat{\Lambda}_{GMM}^{-1}C)^{-1}.$$

The estimated asymptotic variance-covariance matrix of $\hat{\gamma}_{GMM}$ is,

$$\hat{V}(\hat{\gamma}_{GMM})=[(X'Z)(\sum_{i=1}^{N} Z_i'\hat{u}_{i,GMM}\hat{u}_{i,GMM}'Z_i)^{-1}(Z'X)]^{-1}.$$

5.Set $W=(Z'Z)^{-1}$and $\hat{\gamma}=\hat{\gamma}_{GMM}$ in equation (A6) and calculate,

$$\hat{\gamma}_R=\hat{\gamma}_{GMM}+(X'Z(Z'Z)^{-1}Z'X)^{-1}R'[R(X'Z(Z'Z)^{-1}Z'X)^{-1}R']^{-1}(r-R\hat{\gamma}_{GMM}).$$

6.Using $\hat{\gamma}_R$, obtain the corresponding estimated vector of random errors,

$$\hat{u}_{R2SLS}=(\hat{u}_{1,R2SLS} \quad \hat{u}_{2,R2SLS} \quad \cdots \quad \hat{u}_{N,R2SLS})',$$

and calculate,

$$\hat{\Lambda}_{R2SLS}=\frac{1}{N}\sum_{i=1}^{N} Z_i'\hat{u}_{i,R2SLS}\hat{u}_{i,R2SLS}'Z_i .$$

7.Restricted GMM estimator: Using $W=(\hat{\Lambda}_{R2SLS})^{-1}$, calculate the restricted GMM estimator from equation (A6) as follows,

$$\hat{\gamma}_{RGMM}=\hat{\gamma}_{GMM}+((X'Z)(\hat{\Lambda}_{R2SLS})^{-1}(Z'X))^{-1}R'[R((X'Z)(\hat{\Lambda}_{R2SLS})^{-1}(Z'X))^{-1}R']^{-1}(r-R\hat{\gamma}_{GMM}).$$

8.Using $\hat{\gamma}_{RGMM}$, obtain the corresponding estimated vector of random errors,

$$\hat{u}_{RGMM}=(\hat{u}_{1,RGMM} \quad \hat{u}_{2,RGMM} \quad \cdots \quad \hat{u}_{N,RGMM})',$$

and calculate,

$$\hat{\Lambda}_{RGMM}=\frac{1}{N}\sum_{i=1}^{N} Z_i'\hat{u}_{i,RGMM}\hat{u}_{i,RGMM}'Z_i .$$

$$\Psi=I-(C'(\hat{\Lambda}_{RGMM})^{-1}C)^{-1}R'[R(\hat{\Lambda}_{RGMM})^{-1}R']^{-1}R.$$

9. Distribution of $\hat{\gamma}_{RGMM}$:

$$\sqrt{N}(\hat{\gamma}_{RGMM}-\gamma) \sim AN(0,\Psi\Sigma_{GMM}\Psi').$$

The estimated asymptotic variance-covariance matrix of $\hat{\gamma}_{RGMM}$ is,

$$\hat{V}(\hat{\gamma}_{RGMM})=\Psi\hat{V}(\hat{\gamma}_{GMM})\Psi'.$$

10. For GMM, using,

$$\hat{u}_{GMM}=(\hat{u}_{1,GMM} \quad \hat{u}_{2,GMM} \quad \cdots \quad \hat{u}_{N,GMM})',$$

Calculate

$$\bar{R}^2=1-\frac{\frac{\hat{u}_{GMM}'\hat{u}_{GMM}}{NG-K}}{\frac{y'M_i y}{NG-1}}$$

$$AIC=NGln\,(\hat{u}_{GMM}'\hat{u}_{GMM})+2K$$

$$SBC=\mathrm{NGln}(\hat{u}_{GMM}'\hat{u}_{GMM})+\mathrm{Kln}(NG)$$

where ı is a $(NG\times 1)$ vector of 1, and $M_\iota=\iota(\iota'\iota)^{-1}\iota'$.

11. For restricted GMM, by using $\hat{u}_{RGMM}=(\hat{u}_{1,RGMM} \quad \hat{u}_{2,RGMM} \quad \cdots \quad \hat{u}_{N,RGMM})'$, calculate:

$$\bar{R}^2=1-\frac{\frac{\hat{u}_{RGMM}'\hat{u}_{RGMM}}{NG-K}}{\frac{y'M_i y}{NG-1}}$$

$$AIC=NGln\,(\hat{u}_{RGMM}'\hat{u}_{RGMM})+2K$$

$$SBC=NGln\,(\hat{u}_{RGMM}'\hat{u}_{RGMM})+Kln\,(NG)$$

Similarly, we have calculated the $\bar{R}^2$, AIC, and SBC for the OLS estimator by using $\hat{u}_{OLS}=(\hat{u}_{1,OLS} \quad \hat{u}_{2,OLS} \quad \cdots \quad \hat{u}_{N,OLS})'$respectively.

12. Using $\hat{u}_{GMM}$ and $W=(\hat{\Lambda}_{2SLS})^{-1}$, the J-statistic for GMM as follows,

$$J = (N^{-\frac{1}{2}}\sum_{i=1}^{N} Z_i^{'}u_{iGMM})^{'}W(N^{-\frac{1}{2}}\sum_{i=1}^{N} Z_i^{'}u_{iGMM}) \sim \chi_{L-K}^2.$$

13. Using $\hat{u}_{RGMM}$ and $W = (\hat{\Lambda}_{R2SLS})^{-1}$, calculate the J-statistic for the restricted GMM as follows,

$$J = (N^{-\frac{1}{2}}\sum_{i=1}^{N} Z_i^{'}u_{iRGMM})^{'}W(N^{-\frac{1}{2}}\sum_{i=1}^{N} Z_i^{'}u_{iRGMM}) \sim \chi_{L-(K-q)}^2.$$

Note, in our paper we have single equation estimation with, $G = 1; \gamma = (\gamma_0 \quad \gamma_1 \quad \gamma_2)^{'} \Rightarrow K = 3; L = 4$. Moreover, for restricted GMM we have, $R = (0 \quad 1 \quad -1) \Rightarrow q = 1$ in our paper. Therefore, we can easily calculate,

$$\hat{\Lambda}_{2SLS} = \frac{1}{N}\sum_{i=1}^{N} \hat{u}_{i,2SLS}^2 z_i^{'}z_i,$$

$$\hat{\Lambda}_{GMM} = \frac{1}{N}\sum_{i=1}^{N} \hat{u}_{i,GMM}^2 z_i^{'}z_i,$$

and

$$\hat{\Lambda}_{R2SLS} = \frac{1}{N}\sum_{i=1}^{N} \hat{u}_{i,R2SLS}^2 z_i^{'}z_i,$$

$\hat{\Lambda}_{RGMM} = \frac{1}{N}\sum_{i=1}^{N} \hat{u}_{i,RGMM}^2 z_i^{'}z_i$, where z_i is the i-th row of matrix Z. We have checked that, the GMM estimator $\hat{\gamma}_{GMM}$ its estimated asymptotic variance-covariance matrix $\hat{V}(\hat{\gamma}_{GMM})$; and the corresponding J-statistic calculated using our algorithm are identical with the same obtained from Stata 13.

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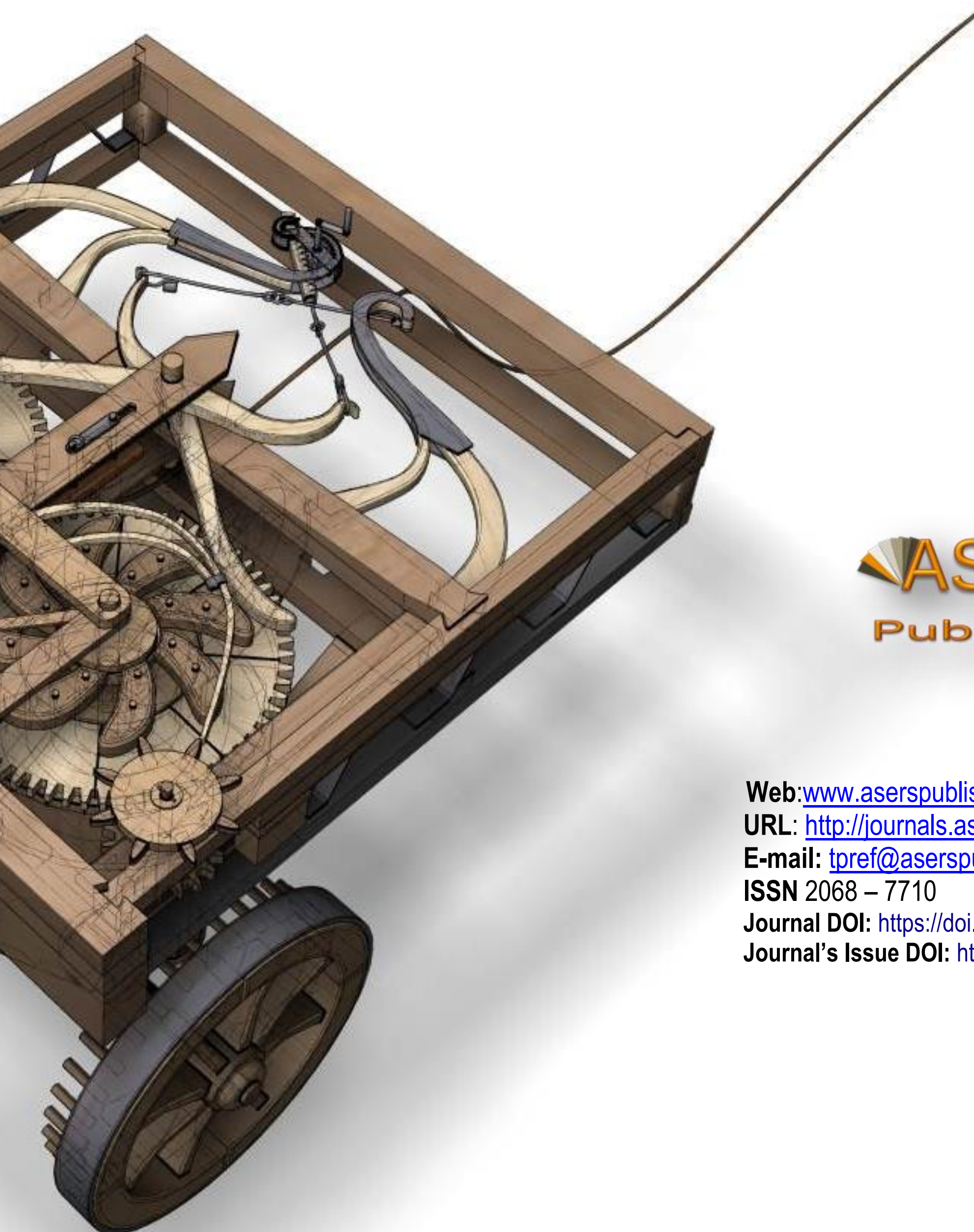

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