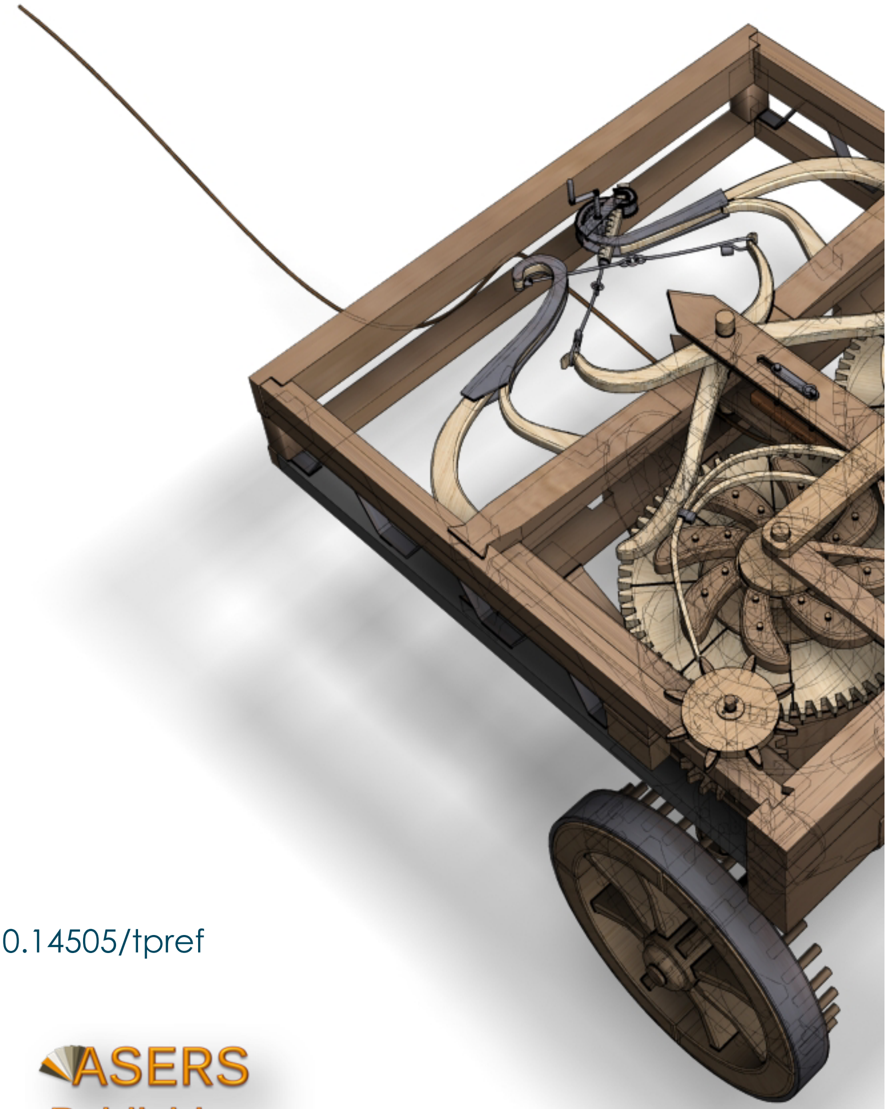
Theoretical and Practical Research in Economic Fields



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The Credit Spread: Risk-Free Rate in the Model

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Abstract: This paper proposes a parsimonious credit spread estimation model for valuation of corporate bonds in data-scarce markets. We emphasize the importance of incorporating the risk-free rate directly into credit spread determination. Our model aligns with established literature and demonstrates the ability to capture the observed influence of risk-free rates on credit spreads across economies. We posit that models omitting the risk-free rate component may underestimate credit spreads, particularly impactful in emerging markets with elevated default probabilities and high risk-free rates. Finally, we discuss practical applications of the model, including exchange rate premium calculations, policy analysis, and negative yield spread analysis.

Keywords: default probability; emerging markets; risk free rate; scarce data; valuation; yield spread.

JEL Classification: C25; G12; G32; E44; E51.

Introduction

The ongoing evolution of financial markets presents new challenges and opportunities for investors. One key area of focus is the valuation of corporate bonds[3], which offers investors the potential for higher returns compared to traditional risk-free assets. This increased demand for corporate bonds highlights the critical need for accurate valuation methodologies for both investment and risk management professionals. (Schwarz 2019) emphasizes this importance, suggesting that investors with long time horizons may favor assets with higher yields, particularly if those yields reflect a shift in market dynamics rather than an increased risk of default. This perspective underscores the significance of understanding the yield spread structure, which refers to the difference between a corporate bond's yield and the yield of a risk-free bond with the same maturity (G-spread).

[3] For simplicity, we define bonds exposed to credit risk (non-government bonds and foreign currency sovereign bonds with lower ratings) as corporate bonds.

Yield spreads consist of two primary components: liquidity spread, which compensates investors for the potential costs of selling an illiquid asset, and credit spread, which compensates for potential default risk.[4] (Petitt, Pinto and Pirie 2015).

Our research focuses on credit spread estimation. This paper proposes a parsimonious credit spread estimation model tailored for valuation of corporate bonds in markets with limited data or depth. A key innovation of our model lies in its explicit incorporation of the risk-free rate.

Beyond credit spread estimation, our model has broader applicability. The model can be used to calculate implied default probabilities and exchange rate premiums for local currency bonds issued in emerging markets. We also showcase its utility in policy analysis by performing break-even analyses to determine the risk-free rate changes needed to offset increases in default probabilities.

The model presented here is a core framework that can be further enhanced. We refrain from imposing specific details regarding default probabilities and loss given default, but the model can be augmented with coefficients to account for business cycle fluctuations and incorporate the differences between historical and market-implied default probabilities.

The remainder of the paper is organized as follows:

- Section 1: Reviews the literature on the topic.
- Section 2: Discusses corporate bond valuation approaches and introduces a key formula used in our credit spread model derivation.
- Section 3: Addresses the use of default probabilities and recovery rates within the model, highlighting the connection between conditional and unconditional default probabilities.
- Section 4: Presents the derivation of the credit spread estimation model and explores its specific features.
- Section 5: Analyzes the modeling framework and discusses its applications in various contexts.
- Section 6: Concludes the paper.
- Appendices: Contain detailed derivations of the formulas.

1. Literature Review

As previously mentioned, yield spreads consist of credit and liquidity spreads. The relative importance of each component is a topic of research, with studies by (Chen, Lesmond and Wei 2007) and (Bao, Pan and Wang 2011), while (Longstaff, Mithal and Neis 2005) emphasize the importance of credit spread.

A vast body of literature explores the determinants of credit risk. For instance, (Boss, et al. 2009) analyze the drivers of default probabilities in economic sectors of Australia and (Castro 2012) investigates the credit risk of banking sector in GIPSI economies[5] by examining factors influencing non-performing loans. Another research strand focuses on deriving credit spreads from market data such as CDS spreads which are considered proxies for credit spreads (Longstaff, Mithal and Neis 2005), (Ericsson, Jacobs and Oviedo 2009), (Hull 2018) and (Specht 2023).

Our model falls within the reduced-form framework of credit spread modeling, prioritizing tractability and applicability in settings of limited data or depth. Existing literature offers two dominant credit risk-modeling approaches: structural models and reduced-form models. Structural models delve into the value determinants of a firm, subsequently linking them to credit spreads through firm-specific factors (Merton 1974), (Black and Cox 1976), (Leland 1994), (Longstaff and Schwartz 1995), (Maglione 2024), (Ben-Abdellatif, et al. 2024). Conversely, reduced-form models directly connect credit spreads to default probabilities and recovery rates (Pye 1974), (Duffie and Singleton 1999), (Driessen 2005). Our model aligns with the reduced-form approach, prioritizing tractability and applicability in markets with limited data availability.

While some reduced-form models acknowledge the indirect influence of the risk-free rate on credit spreads via its impact on default probabilities (Longstaff and Schwartz 1995), (Duffee 1998), we underline the direct effect as well. This aligns with the understanding that credit spreads reflect a confluence of both firm-specific factors and broader macroeconomic forces, with the risk-free rate serving as a crucial indicator of the overall investment environment (Fabozzi and Mann 2005). Excluding the risk-free rate from spread calculations

[4] Combined effect of credit and liquidity premiums is also discussed in the literature, for example (He and Xiong 2012), (Chen, *et al.* 2014). However, in our derivations we assume no combined effects.

[5] Abbreviation in (Castro 2012) meaning Greece, Ireland, Portugal, Spain and Italy

implies, theoretically, that credit spreads would remain constant irrespective of the risk-free rate environment[6]. However, empirical evidence suggests a clear relationship between the two (Ohyama and Sugimoto 2007), (Lepone and Wong 2009), (Radier, *et al.* 2016), (Arce, *et al.* 2024). Our model addresses this gap by explicitly incorporating the risk-free rate.

Our model builds upon the foundation of existing credit spread estimation approaches, particularly those suitable for tractability and applicability in data-scarce environments (Pye 1974), (Resti and Sironi 2007), (Voloshyn 2014), (Roggi 2015). Similar to these works, we prioritize a parsimonious model structure. However, a key distinction lies in our explicit incorporation of the risk-free rate. While some models, like "p*LGD"[7] (Hull 2018) implicitly address the risk-free rate through its influence on default probabilities, we underline the direct effect as well. This aligns with the understanding of credit spreads as a reflection of both firm-specific factors and broader economic forces (Fabozzi and Mann 2005).

2. Contractual and Expected Cash Flows

Our analysis considers a corporate bond with an assumed zero liquidity spread. We compare it to a government or benchmark bond with the same maturity (n coupon periods) and coupon rate (c). Corporate bond valuation can be achieved through two main equivalent approaches (Bessis 2002):

1. Discounting Contractual Cash Flows with Yield (which contains credit spread): This method reflects the risk premium demanded by investors for holding the corporate bond. The contractual cash flows (principal and coupon payments) are discounted using a yield that incorporates risk-free rate and a credit spread ($r + s$). This discounted value represents the price of the corporate bond denoted as $P(r + s)$.

2. Discounting Expected Cash Flows with Risk-Free Rate: This method focuses on the expected (probability-weighted) cash flows. Each contractual cash flow is weighted by its probability of occurrence and then discounted by the risk-free rate. The sum of these discounted expected cash flows represents the price of the corporate bond denoted as $P(r, \pi)$, where r is the risk-free rate and π is the unconditional probability of default. In this approach, investors are assumed to be risk-neutral.

When $P(r, \pi)$ is used, it assumes different functional form compared to $P(r + s)$ or $P(r)$, however for simplicity, assume *P* to represent the pricing function of a bond in general.

Expected Loss and Price Relationship

Alternatively, the price difference between the risk-free bond and the identical corporate bond represents the expected loss (EL) associated with the corporate bond. This expected loss reflects the potential shortfall experienced by investors if the corporation defaults (Garp 2011), (Petitt, Pinto and Pirie 2015). The corporate bond price can also be derived as the difference between the risk-free bond price $P(r)$ and the expected loss EL (see Appendix 1 for details). This approach reinforces the equivalence between the two valuation methods discussed earlier.

This relation can be expressed as follows:

$$P(r + s) = P(r, \pi) = P(r) - EL \tag{1}$$

After some transformations (Melik-Parsadanyan and Galstyan 2017), the price of the corporate bond with n coupon payments and c coupon rate can be expressed as follows:

$$\begin{aligned} P(r + s) = {} & \frac{c \cdot \left(1 - \sum_{k=1}^{1} \pi_k\right) + c \cdot \sum_{k=1}^{1} \pi_k \cdot RR}{(1 + r)^1} + \dots \\ & + \frac{c \cdot \left(1 - \sum_{k=1}^{i} \pi_k\right) + c \cdot \sum_{k=1}^{i} \pi_k \cdot RR}{(1 + r)^i} + \dots \\ & + \frac{(c + 100) \cdot (1 - \sum_{k=1}^{n} \pi_k) + (c + 100) \cdot \sum_{k=1}^{n} \pi_k \cdot RR}{(1 + r)^n} \end{aligned} \tag{2}$$

where RR is the recovery rate and π_k is the unconditional default probability in the $(k - 1, k]$ –th interval.

[6] *i.e.* in that case the credit spreads must be constant in theory when the alternative of risk-free investing is not available for investors

[7] The product of default probability and loss given default

Equation (2) plays an important role in deriving the credit spread formula presented in our paper (details in Appendix 3). The next section delves into the specifications of default probabilities and recovery rates, integral components of these derivations.

3. Modeling Framework

For robust credit spread calculations, our model framework necessitates clear specifications regarding loss given default (LGD) and default probabilities (PD). To ensure clarity and avoid misinterpretations, we explicitly define these building blocks.

The literature offers two primary approaches to calculating recovery rates (Duffie and Singleton, 1999). These approaches determine the recovery rate as a proportion of either the market value prior to default (Allen 2013) or the nominal value of the bond (Chan-Lau 2006). Our analysis adopts the market-value-prior-to-default approach for consistency.

A critical aspect of our model involves establishing a connection between unconditional (π_k) and conditional (p_i) default probabilities. This link is particularly relevant when incorporating historical default rates published by rating agencies[8] as surrogates for modeled unconditional probabilities. In practical applications, historical data often provides a more readily available alternative to complex credit risk modeling. Appendix 2 details the mathematical relationship between conditional and unconditional probabilities and their approximation by an average default probability.

For an average p default probability, it can be established that

$$p = 1 - \sqrt[i]{1 - \sum_{k=1}^{i} \pi_k} = 1 - \sqrt[i]{\prod_{k=1}^{i} (1 - p_k)} \tag{3}$$

where π_k is the unconditional default probability in $k - th$ coupon period, with this meaning that $\sum_{k=1}^{i} \pi_k$ is the cumulative default probability, and p_i is the default probability in the i-th coupon period conditioned upon prior survival.

The approximation of conditional probabilities using average value of p allows us to obtain an analytical solution to the equation (2) for a corporate bond price.

Particularly, given the relation(3), the equation (2) can be expressed as follows:

$$P(r + s) = \sum_{i=1}^{n} \frac{c \cdot (1 - p)^i + c \cdot \left(1 - (1 - p)^i\right) \cdot RR}{(1 + r)^i} + \frac{100 \cdot (1 - p)^n + 100 \cdot (1 - (1 - p)^n) \cdot RR}{(1 + r)^n} \tag{4}$$

4. The Credit Spread

Using the equations in the previous parts and the properties of modified duration, the credit spread formula is derived. Particularly, equation (4) can be transformed as follows (details in Appendix 3):

$$P(r + s) - P(r) = LGD \cdot \left(P\left(r + p \cdot \frac{1 + r}{1 - p}\right) - P(r) \right) \tag{5}$$

Using the properties[9]of modified duration, we derive the credit spread of a corporate bond as the following:

$$s \approx LGD \cdot \frac{p}{1 - p} \cdot (1 + r) \tag{6}$$

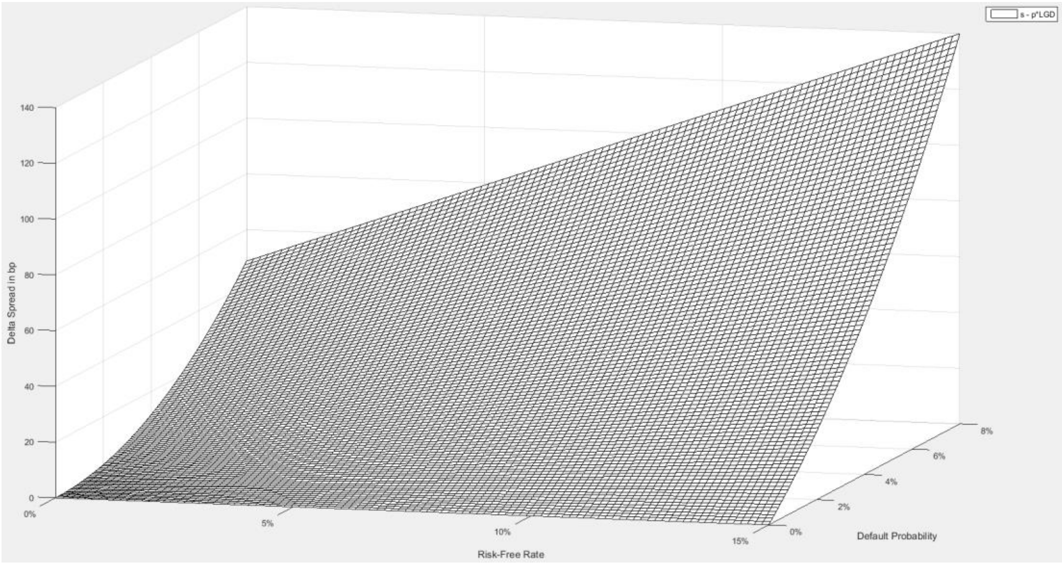
Equation (6) directly expresses the risk-free rate as an alternative investment, and shows an increased importance of the default probability. We compare our model with a model with no direct risk-free rate inclusion ("$p \cdot LGD$" framework) discussed in (Hull 2018).We show that the former approximation can underestimate

[8] See, for example, *Moody's Investors Service*, Corporates - Global: Annual default study: Defaults will rise modestly in 2019 amid higher volatility, *Exhibit 45*

[9]We consider the modified duration of the risk-free bond for a yield-to-maturity r as $\frac{P(r) - P(r + \Delta r)}{P(r)} = -MD \cdot \Delta r$. For the left-hand side of the equation(5) $\Delta r = s$ and for the right-hand side: $\Delta r = p \cdot \frac{1 + r}{1 - p}$

credit spreads compared to our model and the issue is of greater magnitude in the environments of high risk-free rates and default probabilities (Figure 1). The question is pertinent for the emerging market economies where the environment is described with high risk-free rates and high default probabilities.

Figure 1. Difference between the proposed spread and "$p \cdot LGD$" approach



Source: authors' calculations

5. Model Properties and Policy Implications

Our model offers valuable insights into the behavior of credit spreads under varying conditions. To isolate the effects of changes in default probability (p) and risk-free rate (r) we assume no causal relationship between them.

Credit Spread Sensitivities

Equation (6) reveals that both p and r have positive sensitivities on credit spreads[10]. In other words, an increase (decrease) in either p or r leads to an increase (decrease) in credit spread. However, the model suggests a stronger sensitivity to changes in p compared to r[11].

Monetary Policy Implications

This sensitivity has significant implications for monetary policy. To maintain a constant yield level when p rises, all else equal, the risk-free rate needs to be lowered. Our analysis indicates that, on average, a 72 basis point (bp) decrease in r is needed to offset a one percentage point increase[12] in p. This effect is further amplified in environments with higher initial interest rates. Also, the model can be used to analyze the impact of risk-free rate on corporate bond credit spreads and yields. This in turn can serve as another tool to analyze the monetary policy transmission channel.

Indirect Effects and Real-World Considerations

While the direct influence of risk-free rate on credit spreads is captured by our model, there are also indirect effects to consider. Existing literature (Longstaff and Schwartz 1995), (Duffee 1998) highlights the negative relationship between risk-free rate and credit spread[13]. In alignment with this established relationship, a rising r can lead to a decrease in p, which would then cause a decrease in credit spread due to the higher p sensitivity.

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[10] $\frac{ds}{dp} = \frac{1}{(1-p)^2} \cdot LGD \cdot (1 + r) > 0; \frac{ds}{dr} = \frac{p}{(1-p)} \cdot LGD > 0$

[11] $\frac{ds}{dp} > \frac{ds}{dr}\ if\ r > p \cdot (1 - p) - 1, For\ the\ extreme\ case\ r > -0.75.$ In reality, risk-free rates are above the extreme value.

[12] The average change is calculated over probabilities ranging between 0.5-8.5 percent. Additional -0.72 b.p. is needed for 1% increased initial interest rates. For example, on average 75.8 b.p. decrease is needed for 5% initial interest rate environment to offset 1% increase in default probabilities.

[13] Risk-free rates decrease during economic downturn, which implies increased defaults.

Break-Even Default Probability

An interesting application of the model lies in break-even default probability analysis. Assuming only the risk-free rate and credit spread contribute to the corporate bond yield, the break-even default probability is the point at which the yield to maturity reaches zero[14]. This concept can be mathematically expressed using the equation (7).

$$p_{b/e} = \frac{r}{r - LGD \cdot (1 + r)} \tag{7}$$

For a given r and LGD, if the default probability is lower than $p_{b/e}$, the yield to maturity of that bond (without liquidity premium) is negative (Figure 2).

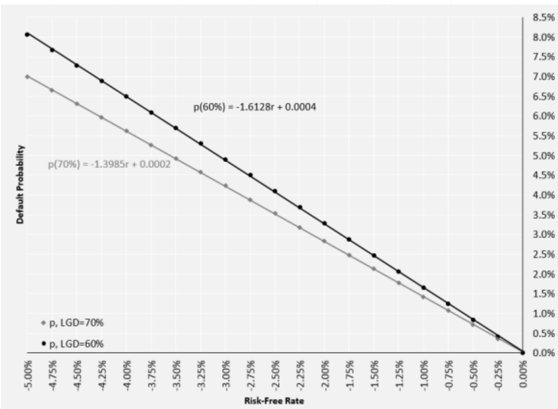
Yield Spreads

Another important point can be stressed using this analysis (Figure 3). The proposed framework allows to gain insight into liquidity and credit spread issues in negative yield spreads. An empirical analysis of this type can be found in (Bhanot and Guo 2011). For given rating grades, using the proposed model we can calculate risk-free rates at which the yields hold positive only due to liquidity premiums ($r + s = 0$).

Exchange Rate Risk and Local Currency Bonds

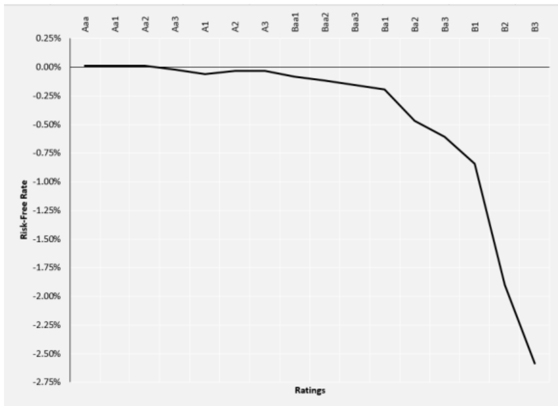
Moreover, taking into the consideration the link between risk–free rate and currency, the proposed framework enhances the analyses in the field of emerging market local currency bonds' valuation by constituting risk free rate as far as the exchange rate risk is priced in local currency bonds. In our opinion, the proposed framework can be used to derive exchange rate spreads of local currency bonds relative to foreign currency denominated bonds (for example, USD). Within this view, p is the probability of devaluation of local currency during the bonds' maturity, and LGD is the magnitude of the devaluation.

Figure 2. Break-even default probabilities vs negative risk-free rates



Source: authors' calculations

Figure 3. Ratings vs break-even risk-free rates



Source: authors' calculations

Conclusions and Further Research

This paper presents the development of credit spread estimation model that directly incorporates the risk-free rate as a crucial element, acknowledging its dual role as both a macroeconomic indicator and a competing investment option. This aligns with established financial theory regarding the relationship between risk-free rates and credit spreads.

The model's novelty lies in its significant applicability to emerging markets characterized by high risk-free rates and default probabilities. By incorporating risk-free rate into the model, it provides a new alternative for corporate bonds valuation in emerging markets with limited data or depth. Our analysis highlights substantial discrepancies between our model and the benchmark "p*LGD" framework in such settings (refer to Figure 1). Empirical testing is recommended to further validate the model's effectiveness in real-world scenarios.

Furthermore, the model offers practical advantages, particularly for timely decision-making. It allows for the incorporation of historical default probabilities, a readily available alternative to complex credit risk modeling. A formula within the model facilitates this integration.

[14] $y = r + s = 0, where\ s = \frac{p_{b/e}}{1 - p_{b/e}} \cdot LGD \cdot (1 + r)$

Beyond credit spread estimation, the model extends to analyzing foreign exchange spreads for local currency bonds. Additionally, the break-even analyses presented offer valuable insights for investors, risk managers, and policymakers.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Declaration of Use of Generative AI and AI-assisted Technologies

The authors declare that they have not used generative AI and AI-assisted technologies during the preparation of this work.

References

[1] Allen, S. (2013). In *Financial Risk Management: A Practitioner's Guide to Managing Market and Credit Risk*, 468-471. Hoboken, New Jersey: John Wiley and Sons, Inc.

[2] Arce, M., *et al.* (2024). The Yields That Bind: Treasury Bill Yields and the Philippine. *DLSU Business and Economics Review* 34 (1). Available at: https://animorepository.dlsu.edu.ph/ber/vol34/iss1/30

[3] Bao, J., Pan, J. and Wang, J. (2011). The illiquidity of corporate bonds. *The Journal of Finance,* 66(3): 911-946. DOI: 10.1111/j.1540-6261.2011.01655.x

[4] Ben-Abdellatif, M., *et al.* (2024). A two-factor structural model for valuing corporate securities. *Rev Deriv Res* 27: 203-225. DOI: https://doi.org/10.1007/s11147-024-09203-2

[5] Bessis, J. (2002). *Risk management in banking*, 538-539. West Sussex, England: John Wiley and Sons Ltd.

[6] Bhanot, K., and Guo, L. (2011). Negative Credit Spreads: Liquidity and Limits to Arbitrage. *The Journal of Fixed Income,* 21 (1): 32-41. DOI: 10.3905/jfi.2011.21.1.032

[7] Black, F., and Cox, J. C. (1976). Valuing Corporate Securities: Some Effects Of Bond Indenture Provisions. *The Journal of Finance*, 31 (2): 351-367. DOI: 10.1111/j.1540-6261.1976.tb01891.x

[8] Boss, M., *et al.* (2009). Modeling Credit Risk through the Austrian Business Cycle: An Update of the OeNB Model. *Financial Stability Report, Oesterreichische Nationalbank (Austrian Central Bank)* 85-101.

[9] Castro, V. (2012). Macroeconomic determinants of the credit risk in the banking system: The case of the GIPSI. *NIPE Working Paper Series.* Available at: http://www2.eeg.uminho.pt/economia/nipe/docs/2012/NIPE_WP_11_2012.pdf

[10] Chan-Lau, J. A. (2006). Market-Based Estimation of Default Probabilities and Its Application to Financial Market Surveillance. *IMF Working Paper WP/06/104.*

[11] Chen, H., Cui, R., He, Z. and Milbradt, K. (2014). Quantifying Liquidity And Default Risks of Corporate Bonds. *NBER Working Paper Series Working Paper 20638.*

[12] Chen, L., Lesmond, D. A. and Wei, J. (2007). Corporate Yield Spreads and Bond Liquidity. *The Journal of Finance*, 62 (1): 119-149. DOI: 10.1111/j.1540-6261.2007.01203.x

[13] Driessen, J. (2005). Is Default Event Risk Priced in Corporate Bonds? *The Review of Financial Studies*, 18 (1): 165-195. DOI: doi.org/10.1093/rfs/hhi009

[14] Duffee, G. R. (1998). The Relation Between Treasury Yields and Corporate Bond Yield Spreads. *The Journal of Finance*, 53 (6): 2225-2241. DOI: 10.1111/0022-1082.00089

[15] Duffie, D., and Singleton, K. J. (1999). Modeling Term Structures of Defaultable Bonds. *The Review of Financial Studies Special*, 12 (4): 687-720. DOI: 10.1093/rfs/12.4.687

[16] Ericsson, J., Jacobs, K. and Oviedo, R. (2009). The Determinants of Credit Default Swap Premia. *The Journal of Financial and Quantitative Analysis,* 109-132. DOI: 10.1017/S0022109009090061

[17] Fabozzi, F. J., and Mann, S. V. (2005). *The Handbook of Fixed Income Securities.* The McGraw-Hill Companies.

[18] Garp, P. J. (2011). Vol. Sixth Edition, in *Financial Risk Manager Handbook Plus Test Bank*, 504-505. Hoboken, New Jersey: John Wiley and Sons, Inc.,.

[19] He, Z., and Wei Xiong. (2012). Rollover Risk and Credit Risk. *The Journal of Finance,* 67 (2): 391-429. DOI:10.1111/j.1540-6261.2012.01721.x

[20] Hull, J.C. (2018). *Risk Management and Financial Institutions.* Haboken, New Jersey: John Wiley and Sons Inc.

[21] Leland, H. E. (1994). Corporate Debt Value, Bond Covenants, and Optimal Capital Structure. *The Journal of Finance* 49: 1213-1252. DOI: 10.1111/j.1540-6261.1994.tb02452.x

[22] Lepone, A., and Wong, B. (2009). Determinants of Credit Spread Changes: Evidence from the Australian Bond Market. *The Australasian Accounting Business and Finance Journal.*

[23] Longstaff, F. A., and Schwartz, E. S. (1995). A Simple Approach to Valuing Risky Fixed and Floating Rate Debt. *The Journal of Finance,* 50 (3): 789-819. DOI:10.1111/j.1540-6261.1995.tb04037.x

[24] Longstaff, F.A., Mithal, S. and Neis, E. (2005). Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Credit Default Swap Market. *The Journal of Finance,* 60 (5): 2213-2253. DOI:10.1111/j.1540-6261.2005.00797.x

[25] Maglione, F. (2024). Introducing and testing the Carr model of default. *Quantitative Finance*, 1–22. DOI:https://doi.org/10.1080/14697688.2024.2368081

[26] Melik-Parsadanyan, V., and Galstyan, A. (2017). Price and Yield of a Bond with Credit Risk. *Alternative* 4: 537-544.

[27] Merton, R.C. (1974). On The Pricing Of Corporate Debt: The Risk Structure. *The Journal of Finance* Volume 29 (2): 447-470. DOI: 10.1111/j.1540-6261.1974.tb03058.x

[28] Ohyama, S., and Sugimoto, T. (2007). The determinants of credit spread changes in Japan. *Bank of Japan Working Paper Series.*

[29] Petitt, B., Pinto, J. and Pirie, W. (2015). Fixed Income Analysis. 212-213; 308-315. CFA Institute.

[30] Pye, G. (1974). Gauging the Default Premium. *Financial Analysts Journal,* 30 (1): 49-52. DOI:10.2469/faj.v30.n1.49

[31] Radier, G., Majoni, A., Njanike, K. and Kwaramba, M. (2016). Determinants of bond yield spread changes in South Africa. *African Review of Economics and Finance* 50-81.

[32] Resti, A., and Sironi, A. (2007). *Risk Management and Shareholders' Value in Banking. From Risk Measurement Models to Capital Allocation Policies.* West Sussex: John Wiley and Sons Ltd.

[33] Roggi, O. (2015). *Risk, Value And Default.* Singapore: World Scientific Publishing.

[34] Schwarz, K. (2019). Mind the Gap: Disentangling Credit and Liquidity in Risk Spreads. *Review of Finance,* 23 (3): 557-597. DOI:10.1093/rof/rfy034

[35] Specht, L. (2023). An Empirical Analysis of European Credit Default Swap Spread Dynamics. *Junior Management Science* 1-42.

[36] Voloshyn, I. (2014). How to Treat Interest Losses in Estimation of Credit Risk Spread. August. Accessed May 17, 2020. https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2475755

Appendix 1

In this section we provide the decomposition of corporate bond price into risk-free price and expected loss.

In the second method of valuation, the future cash flows are weighted by the corresponding probabilities of default and no-default. In the case of no default, the future cash flows comprise coupon payments (and the principal for the last period), and in the event of default the future cash flows are recovered values. Probability weighted cash flows are riskless (because in the scope of the probability the cash flows are considered certainly receivable) and are discounted at risk-free rates. Denote by RR and LGD the recovery rate and loss given default respectively ($0 \leq LGD \leq 1, RR = 1 - LGD$), by π_k the unconditional default probability in $(k-1, k]$ time interval, and the price of risk-free bond at i-th period by $P_i(r)$. The price of the bond $P(r,\pi)$ is expressed as follows:

$$P(r,\pi) = \sum_{i=1}^{N} \frac{c \cdot \left(1 - \sum_{k=1}^{i} \pi_k\right) + \pi_i \cdot RR \cdot P_i(r)}{(1 + r)^i} + \frac{100 \cdot \left(1 - \sum_{k=1}^{n} \pi_k\right) + \pi_n \cdot RR \cdot P_n(r)}{(1 + r)^n}$$

From the equation above, follows:

$$P(r,\pi) = \sum_{i=1}^{n} \frac{c}{(1 + r)^i} + \frac{100}{(1 + r)^n} - \sum_{i=1}^{n} \frac{c \cdot \sum_{k=1}^{i} \pi_k}{(1 + r)^i} - \frac{100 \cdot \sum_{k=1}^{n} \pi_k}{(1 + r)^n} +$$

$$+ \sum_{i=1}^{n} \frac{P_i(r) \cdot \pi_i}{(1 + r)^i} - \sum_{i=1}^{n} \frac{P_i(r) \cdot \pi_i \cdot LGD}{(1 + r)^i} = P(r) + \sum_{i=1}^{n} \frac{P_i(r) \cdot \pi_i - c \cdot \sum_{k=1}^{i} \pi_k}{(1 + r)^i} -$$

$$- \frac{100 \cdot \sum_{k=1}^{n} \pi_k}{(1 + r)^n} - EL = P(r) - EL$$

In the equation above, $P(r)$ denotes the price of the risk-free bond, and EL denotes expected loss.

The equation above holds, because:

$$\sum_{i=1}^{n} \frac{P_i(r) \cdot \pi_i - c \cdot \sum_{k=1}^{i} \pi_k}{(1 + r)^i} - \frac{100 \cdot \sum_{k=1}^{n} \pi_k}{(1 + r)^n}$$

$$= \left[\begin{array}{c} \frac{\pi_1}{1 + r} \cdot \left(c + \frac{c}{1 + r} + \dots + \frac{c}{(1 + r)^{n-1}}\right) + \\ \frac{\pi_2}{(1 + r)^2} \cdot \left(c + \frac{c}{1 + r} + \dots + \frac{c}{(1 + r)^{n-2}}\right) + \dots + \frac{\pi_n}{(1 + r)^n} \cdot (c + 100) \end{array} \right] -$$

$$- \left[\frac{c}{1 + r} \cdot \pi_1 + \frac{c}{(1 + r)^2} \cdot (\pi_1 + \pi_2) + \dots + \frac{c + 100}{(1 + r)^n} \cdot (\pi_1 + \pi_2 + \dots + \pi_n)\right] =$$

$$= \frac{\pi_1}{1 + r} \cdot \left(c + \frac{c}{1 + r} + \dots + \frac{c}{(1 + r)^{n-1}} - c - \frac{c}{(1 + r)} - \dots - \frac{c}{(1 + r)^{n-1}}\right) +$$

$$+ \frac{\pi_2}{(1 + r)^2} \cdot \left(c + \frac{c}{1 + r} + \dots + \frac{c}{(1 + r)^{n-2}} - c - \frac{c}{1 + r} - \dots - \frac{c}{(1 + r)^{n-2}}\right) +$$

$$+ \dots + \frac{\pi_n}{(1 + r)^n} \cdot (c + 100 - c - 100) = 0$$

Appendix 2

In this section we provide information regarding the link between conditional and unconditional default probabilities.

Denote by i_+ and i_- correspondingly the events of default and no-default in the $(i-1,i]$ time interval, the conditional probability of default in the i-th time period by p_i, where the event of default is conditioned upon the events of no prior default. These relationships mathematically are expressed as follows:

$$P\left(i_+ | \bigcap_{k=1}^{i-1} k_-\right) = p_i \qquad P\left(i_- | \bigcap_{k=1}^{i-1} k_-\right) = 1 - p_i$$

Consider particular cases of survival until i-th time period for $k = \overline{1,3}$.

- $k = 1 \Longrightarrow P(1_-) = 1 - p_1$;
- $k = 2 \Longrightarrow P(2_- \cap 1_-) = P(2_- | 1_-) \cdot P(1_-) = (1 - p_2) \cdot (1 - p_1)$
- $k = 3 \Longrightarrow P(3_- \cap 2_- \cap 1_-) = P(3_- | 2_- \cap 1_-) \cdot P(2_- \cap 1_-) =$
  $= (1 - p_3) \cdot (1 - p_2) \cdot (1 - p_1)$
- $k = 3 \Longrightarrow P(3_+ \cap 2_- \cap 1_-) = P(3_+ | 2_- \cap 1_-) \cdot P(2_- \cap 1_-) = p_3 \cdot (1 - p_2) \cdot (1 - p_1)$

Generally: $P\left(\bigcap_{k=1}^{i} k_-\right) = \prod_{k=1}^{i}(1 - p_k)$, $P\left(i_+ \cap \bigcap_{k=1}^{i-1} k_-\right) = p_k \cdot \prod_{k=1}^{i-1}(1 - p_k)$

Consider the sum of probabilities of default and no default in the i-th time period conditioned upon no default in prior two periods:

$$\begin{aligned} P(3_+ \cap 2_- \cap 1_-) + P(3_- \cap 2_- \cap 1_-) \\ = (1 - p_3) \cdot (1 - p_2) \cdot (1 - p_1) + p_3 \cdot (1 - p_2) \cdot (1 - p_1) \\ = (1 - p_2) \cdot (1 - p_1) = P(2_- \cap 1_-) \end{aligned}$$

Generally:

$$P\left(i_+ \cap \bigcap_{k=1}^{i-1} k_-\right) + P\left(\bigcap_{k=1}^{i} k_-\right) = p_k \cdot \prod_{k=1}^{i-1}(1 - p_k) + \prod_{k=1}^{i}(1 - p_k) = \prod_{k=1}^{i-1}(1 - p_k) = P\left(\bigcap_{k=1}^{i-1} k_-\right)$$

Therefore:

$$P\left(\bigcap_{k=1}^{i} k_-\right) + P\left(i_+ \cap \bigcap_{k=1}^{i-1} k_-\right) + P\left((i-1)_+ \cap \bigcap_{k=1}^{i-2} k_-\right) + \dots + P(2_+ \cap 1_-) =$$

$$= P(1_-) = 1 - p_1 \Longrightarrow P\left(\bigcap_{k=1}^{i} k_-\right) + P\left(i_+ \cap \bigcap_{k=1}^{i-1} k_-\right) + \dots + P(1_+) = 1 \Longrightarrow$$

$$P\left(\bigcap_{k=1}^{i} k_-\right) = 1 - \sum_{k=1}^{i} P\left(k_+ \cap \bigcap_{j=1}^{k-1} j_-\right) \qquad (1')$$

Equation $(1')$ means that the event of no default until i-th time interval is the supplement of the events of no default in every prior interval, moreover the event of default is considered with the intersection of events of prior no default, e.g. $i_+ \cap \bigcap_{j=1}^{i-1} j_-$.

Denote by π_i the unconditional probability of default in the i –th interval with no prior default:

$$\pi_i = P\left(i_+ \cap \bigcap_{k=1}^{i-1} k_-\right)$$

Equation (1′) can be expressed as $P\left(\bigcap_{k=1}^{i} k_{-}\right) = 1 - \sum_{k=1}^{i} \pi_k$, and as far as $P\left(\bigcap_{k=1}^{i} k_{-}\right)$=$\prod_{k=1}^{i}(1 - p_k)$, then

$$1 - \sum_{k=1}^{i} \pi_k = \prod_{k=1}^{i}(1 - p_k)$$

Assume that conditional probabilities of default are equally likely for every time interval. If the conditional probabilities of default are approximated by p, the relationship above can be expressed (Melik-Parsadanyan and Galstyan 2017) as follows

$$1 - \sum_{k=1}^{i} \pi_k = (1 - p)^i$$

Appendix 3

In this section we provide information regarding the derivations of the credit spread estimation model.

Equation 4 can be expressed as follows:

$$P(r + s) = \sum_{i=1}^{n} \frac{c \cdot (1 - p)^i \cdot (1 - RR) + c \cdot RR}{(1 + r)^i} + \frac{100 \cdot (1 - p)^n \cdot (1 - RR) + 100 \cdot RR}{(1 + r)^n}$$

From the equation above follows:

$$P(r + s) = \left[\sum_{i=1}^{n} \frac{c \cdot (1 - p)^i \cdot (1 - RR)}{(1 + r)^i} + \frac{100 \cdot (1 - p)^n \cdot (1 - RR)}{(1 + r)^n}\right] + + \left[\sum_{i=1}^{n} \frac{\mathrm{c} \cdot \mathrm{RR}}{(1 + \mathrm{r})^{\mathrm{i}}} + \frac{100 \cdot \mathrm{RR}}{(1 + \mathrm{r})^{\mathrm{n}}}\right] = (1 - RR) \cdot \left[\sum_{i=1}^{n} \frac{c}{\left(\frac{1 + r}{1 - p}\right)^i} + \frac{100}{\left(\frac{1 + r}{1 - p}\right)^n}\right] + + \mathrm{RR} \cdot \left[\sum_{i=1}^{n} \frac{\mathrm{c}}{(1 + \mathrm{r})^{\mathrm{i}}} + \frac{100}{(1 + \mathrm{r})^{\mathrm{n}}}\right] = (1 - RR) \cdot \left[\sum_{i=1}^{n} \frac{c}{\left(1 + \frac{r + p}{1 - p}\right)^i} + \frac{100}{\left(1 + \frac{r + p}{1 - p}\right)^n}\right] + + \mathrm{RR} \cdot \left[\sum_{i=1}^{n} \frac{\mathrm{c}}{(1 + \mathrm{r})^{\mathrm{i}}} + \frac{100}{(1 + \mathrm{r})^{\mathrm{n}}}\right]$$

It follows that the price of the bond can be decomposed into the following

$$P(r + s) = (1 - RR) \cdot P\left(\frac{r + p}{1 - p}\right) + RR \cdot P(r)$$

Where $P\left(\frac{r + p}{1 - p}\right)$ is the price of the bond with (c) coupon and (n) coupon periods discounted at the $\frac{r + p}{1 - p}$ rate, and $P(r)$ is the price of the same bond discounted at $r$rate.

Moreover, as far as $\frac{r + p}{1 - p} = r + \frac{p}{1-p}(1 + r)$, it follows that

$$P(r + s) = (1 - RR) \cdot P\left(r + \frac{p}{1 - p} \cdot (1 + r)\right) + RR \cdot P(r)$$

Subtracting $P(r)$ from both sides gives Equation (5)

$$P(r + s) - P(r) = (1 - RR) \cdot P\left(r + \frac{p}{1 - p} \cdot (1 + r)\right) + (RR - 1) \cdot P(r) = = (1 - RR) \cdot \left[P\left(r + \frac{p}{1 - p} \cdot (1 + r)\right) - P(r)\right] = = LGD \cdot \left[P\left(r + \frac{p}{1 - p} \cdot (1 + r)\right) - P(r)\right]$$

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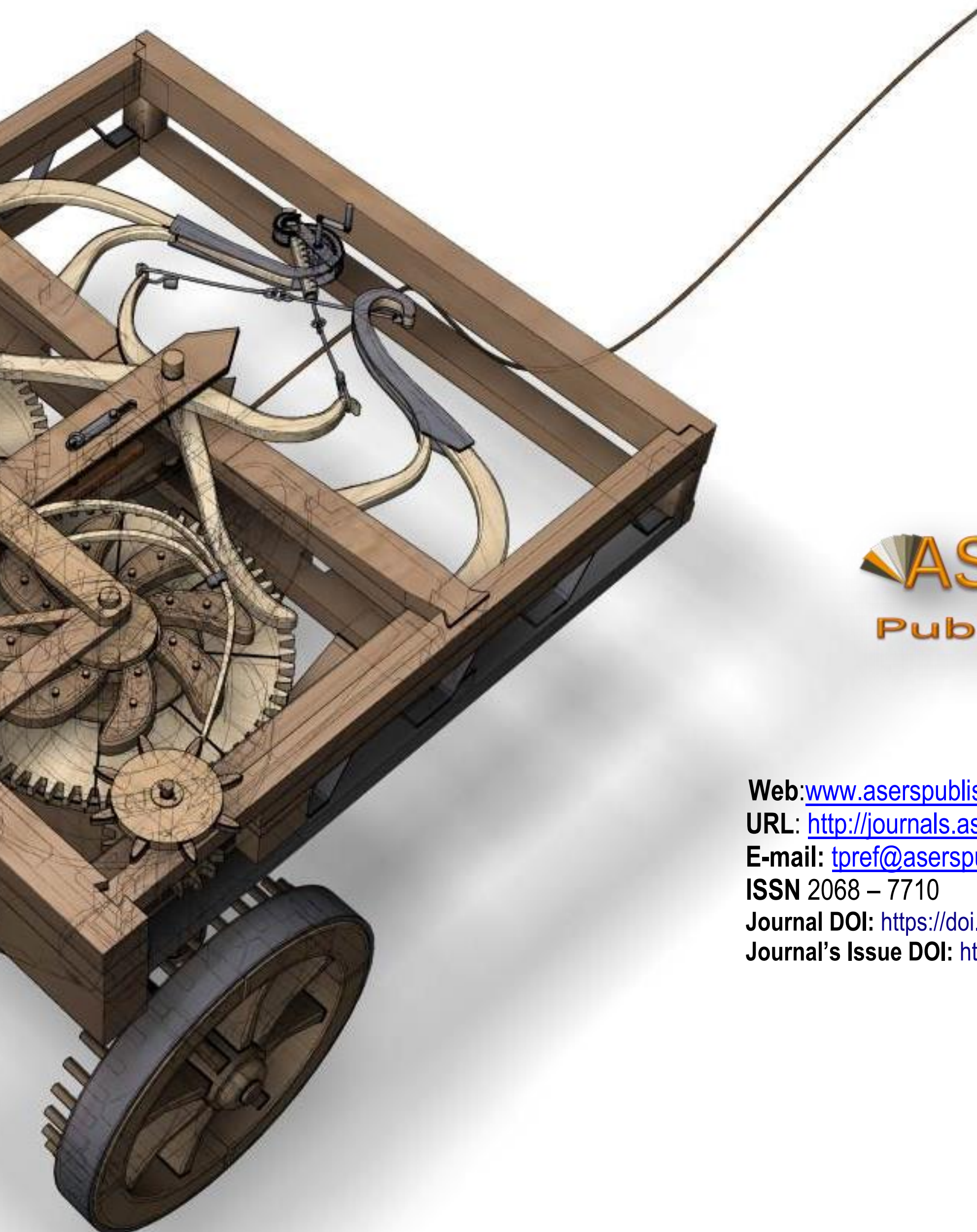

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