Theoretical and Practical Research in Economic Fields

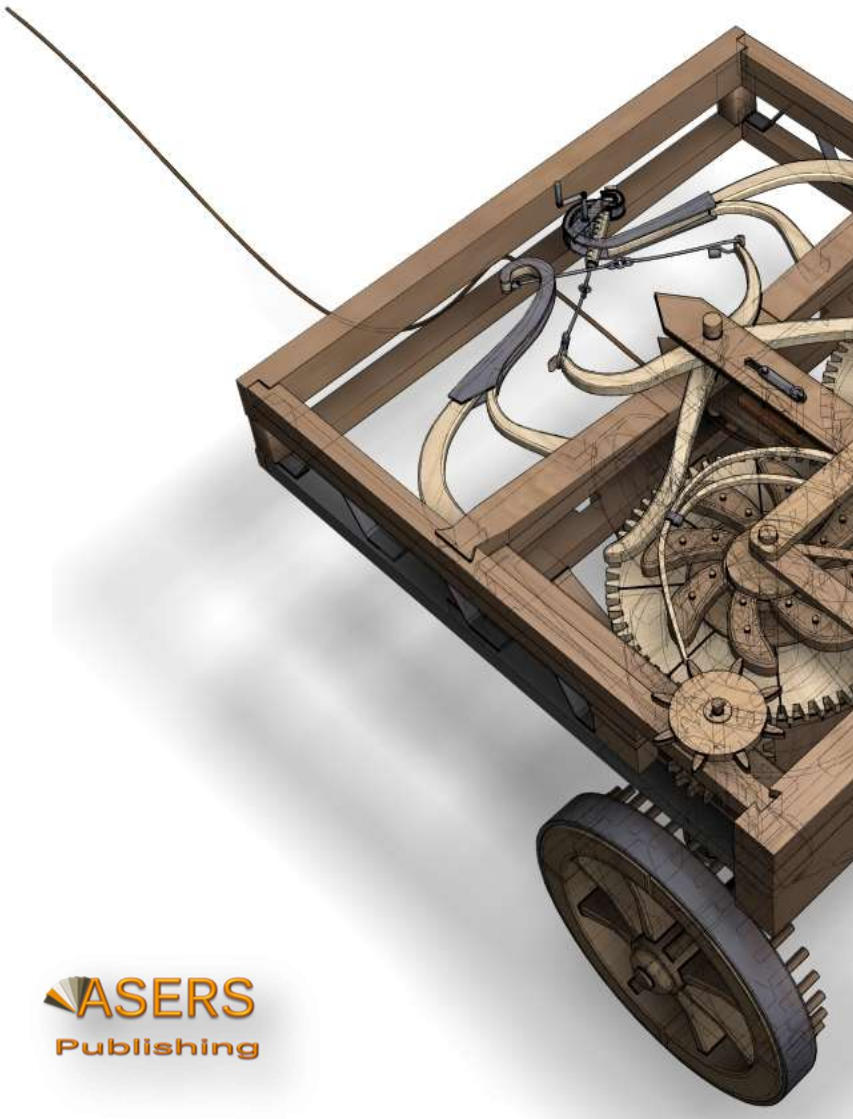

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Table of Contents:

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Theoretical and Practical Research in Economic Fields

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FOREIGN TRADE AND MACROECONOMIC EFFECTS OF EXPORTS

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Abstract:

We find that the IS-LM model and the national income model reinforce each other in the context of exports and the export multiplier. Using simple differentiation techniques, we derive several relationships with respect to foreign trade. As predicted by Keynes, exports have a favorable effect on national income and the interest rate. Exports also increase the average price level and the exchange rate in the country. Through the mechanism of the export multiplier exports increase imports, savings, and consumption. A higher propensity to import reduces national income and fosters a negative trade balance. Similar is the effect of a higher exchange rate which discourages exports and encourages imports.

***Keywords*:** export multiplier; propensity to import; interest rate; exchange rate; IS-LM model; national income model.

JEL Classification: E12; E21; E43; F40.

Introduction

According to King (1998) Giblin (1930) was the first to imply the export multiplier in discussing the effect of a decrease in export demand on Australia's national income. He had also estimated the Australian export multiplier as early as 1929. Harrod (1933) developed the idea that the level and growth of output given a balance of payment constraint depend on the foreign trade multiplier. The multiplier was later called an export multiplier. Harrod's idea of the foreign trade multiplier lies at the basis of the Keynesian investment multiplier which was the core of the *General Theory* advanced by Keynes (1936). Sordi and Vercelli (2012) maintain that the *General Theory* investment multiplier became the building block of the *IS-LM* model in what is a standard version of the Keynesian static model. This intertwined the *IS-LM* model with the national-income model in its various versions. Our paper delineates some of the main forms of both models illustrating that the effect of exports on different macroeconomic variables, but particularly on national income and equilibrium interest rate, is identical.

According to King (1998) Harrod's trade multiplier works in many countries and differences in export performance are an important source of international growth rate differences. Kaldor (1975) claims that Keynes's idea of unemployment in a recession diverted the attention from the foreign trade multiplier which in a dynamic setting is far more important in explaining the growth and pace of industrial development. Bairam (1993) suggests that Harrod's foreign trade multiplier is larger for developing countries than for developed ones and that the value of the multiplier is related to the stage of development of the respective country.

Robinson (1952) presents the foreign trade multiplier graphically whereas Metzler (1942) and Machlup (1943) apply algebraic and arithmetic models to depict its effect. Similar is the treatment of the Harrod foreign trade multiplier provided by Kennedy and Thirlwall (1979) and Thirlwall (1982). Our

research gravitates to this second type of methods. It applies neither graphical, nor empirical testing of the export multiplier and the various effects of exports on the macroeconomy. We use a purely theoretical, mathematical approach applying implicit differentiation. The first part examines general functions and is, in this sense, an illustration of implicit differentiation and the implicit-function theorem. The part which follows uses simple definite functions seeking thus equilibrium values and tracing the effect of exports on those. In addition to exports, we investigate the role of imports, the exchange rate, the propensity to import, etc. The paper is theoretical, rather than empirical using a simple setting of the major macroeconomic functions and variables.

Some of our key findings are that exports affect national income and the interest rate favorably. We find this both in the context of the national-income model and the *IS-LM* model. Through the export multiplier exports affect positively imports, savings, and the average price level in the country. We also study the influence of the exchange rate on national income, interest rate and the average price level. We demonstrate mathematically how a higher exchange rate can discourage exports and encourage imports stimulating thus a negative trade balance. In the context of definite functions, we also find the effect of the marginal propensity to import on national income. In the context of the mutual trade between two countries we find their interdependence. The exports of each country are assumed to be a share of its aggregate investment where the exports of one country, Home, are imports for the other one, Foreign, and vice versa. We derive that the exports of Home are positively related to Foreign's investment and those of Foreign to Home's investment.

Part 1 is an introduction and a brief literature review. We do not aim a complete literature review of the history of the export multiplier. Part 2 discusses foreign trade in a general-function setting. Part 3 introduces some simple definite-form functions. The paper ends with conclusions.

1. Macroeconomic Effects of Exports in the Context of General Functions

In the standard *IS-LM* model we have equilibrium on two markets, the capital market and the money market. In the capital market aggregate investment equals savings while in the money market money demand equals money supply. This is reflected by the following equilibrium equations.

$$I(i) = S(Y, i) \qquad \frac{dI}{di} = I' < 0 \qquad 0 < S_Y < 1 \qquad S_i > 0$$

where investment is a decreasing function of the interest rate and savings increase with national income and the interest rate, with S_Y being the marginal propensity to save. On the money market money demand equals money supply, or

$$M_{so} = L(Y, i) \qquad L_Y > 0 \qquad L_i < 0$$

The liquidity function is positively related to national income (transaction demand for money) and negatively to the interest rate while money supply is assumed to be exogenously determined since it depends on the monetary policy of the central bank. Accounting for equilibrium in foreign trade,

$$X_o = M(Y) \qquad 0 < M' < 1$$

where $\frac{dM}{dY} = M'$ is the marginal propensity to import and exports are exogenously determined. Adding the last equation to the capital market identity gives the following form of the *IS-LM* model.

$$I(i) + X_o = S(Y, i) + M(Y)$$

$$L(Y, i) = M_{so}$$

Solving in the form of implicit functions,

$$I(i) + X_o - S(Y, i) - M(Y) = 0$$

$$L(Y,i) - M_{so} = 0$$

gives rise to the Jacobian

$$|J| = \begin{vmatrix} -S_Y - M' & I' - S_i \\ L_Y & L_i \end{vmatrix} = -L_i(S_Y + M') - L_Y(I' - S_i) > 0$$

Writing off the implicit-function theorem with respect to exogenous exports,

$$\begin{bmatrix} -S_Y - M' & I' - S_i \\ L_Y & L_i \end{bmatrix} \begin{bmatrix} \frac{dY}{dX_o} \\ \frac{di}{dX_o} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

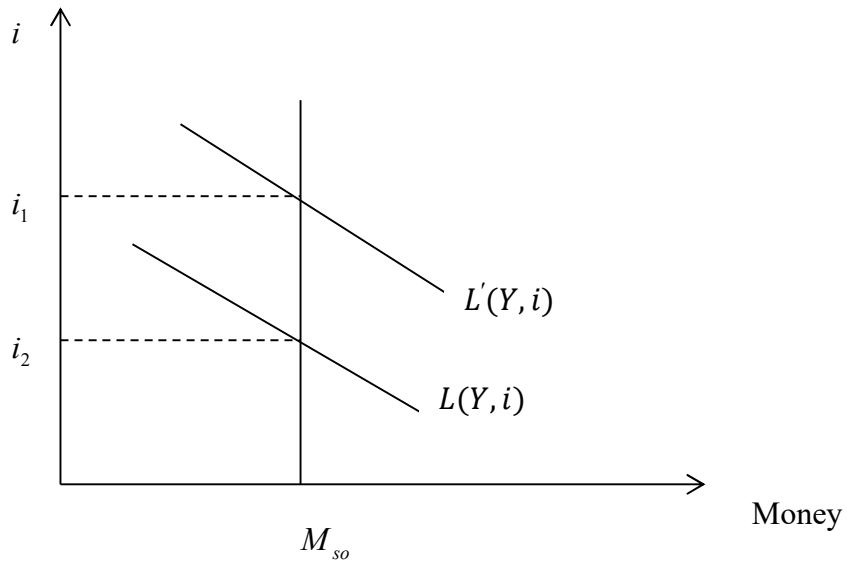
Solving for the two comparative-static derivatives,

$$\frac{dY}{dX_o} = \frac{\begin{vmatrix} -1 & I' - S_i \\ 0 & L_i \end{vmatrix}}{|J|} = \frac{L_i}{L_i(S_Y + M') + L_Y(I' - S_i)} > 0$$

$$\frac{di}{dX_o} = \frac{\begin{vmatrix} -S_Y - M' & -1 \\ L_Y & 0 \end{vmatrix}}{|J|} = \frac{L_Y}{L_Y(S_i - I') - L_i(S_Y + M')} > 0$$

The export multiplier $\frac{dY}{dX_o}$ is positive since exports increase national income. We can also see the positive effect of exports on the interest rate. This is achieved through the liquidity function where exports increase income and the transaction demand for money grows which ultimately leads to higher interest rates. Figure 1 demonstrates this graphically.

Figure 1. Effect of increased money demand (due to increased exports) on equilibrium interest rate



As exports increase national income, the transaction demand for money shifts outwards from $L(Y,i)$ to $L'(Y,i)$. This increases the equilibrium interest rate from i_1 to i_2. Additionally, we can find a set of auxiliary derivatives.

$$\frac{dM}{dX_o} = M' \frac{dY}{dX_o} = \frac{M' L_i}{L_i(S_Y + M') + L_Y(I' - S_i)} > 0$$

$$\frac{dI}{dX_o} = I' \frac{di}{dX_o} = \frac{I' L_Y}{L_Y(S_i - I') - L_i(S_Y + M')} < 0$$

$$\frac{dS}{dX_o} = S_Y \frac{dY}{dX_o} + S_i \frac{di}{dX_o} = \frac{S_Y L_i - S_i L_Y}{L_i(S_Y + M') + L_Y(I' - S_i)} > 0$$

By increasing national income exports allow the nation to consume more foreign goods. Exports also have a favorable effect on aggregate savings since they increase both national income and the interest rate. The effect on aggregate investment is adverse since exports raise the interest rate.

We can check the validity of the *IS-LM* model and the effect of exports on national income, interest rate, investment, etc, using the standard national-income model. Comparing the results ensures their robustness. We assume the following national-income model.

$$Y = C(Y,i) + I(i) + X_o - M(Y) \quad 0 < C_Y, M' < 1 \qquad L_Y > 0 \quad I', C', L' < 0$$

$$kY + L(Y,i) = M_{so} \qquad k = \text{ positive constant}$$

where Y, C, I, X_o, M, L and M_{so} stand for national income, consumption, investment, exports, imports, money demand and money supply, respectively. All derivatives are assumed to be continuous. Solving,

$$Y - C(Y,i) - I(i) - X_o + M(Y) = 0$$
$$kY + L(Y,i) - M_{so} = 0$$

Writing off the implicit-function theorem with respect to exogenous exports,

$$\begin{bmatrix} 1 - C_Y + M' & -C' - I' \\ k + L_Y & L' \end{bmatrix} \begin{bmatrix} \frac{dY}{dX_o} \\ \frac{di}{dX_o} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|J| = L'(1 - C_Y + M') + (k + L_Y)(C' + I') < 0$$

The implicit function theorem holds, and we find the effect of exports on national income and the interest rate.

$$\frac{dY}{dX_o} = \frac{\begin{vmatrix} 1 & -C' - I' \\ 0 & L' \end{vmatrix}}{|J|} = \frac{L'}{L'(1 - C_Y + M') + (k + L_Y)(C' + I')} > 0$$

$$\frac{di}{dX_o} = \frac{\begin{vmatrix} 1 - C_Y + M' & 1 \\ k + L_Y & 0 \end{vmatrix}}{|J|} = -\frac{k + L_Y}{L'(1 - C_Y + M') + (k + L_Y)(C' + I')} > 0$$

Increased exports again increase national income and the interest rate. The results reaffirm the *IS-LM* model. It is interesting that exports again increase national income through speculative demand for money while the interest rate through the transactional demand for money.

$$\frac{dY}{dX_o} = \frac{\begin{vmatrix} -1 & I' - S_i \\ 0 & L_i \end{vmatrix}}{|J|} = \frac{L_i}{L_i(S_Y + M') + L_Y(I' - S_i)} > 0$$

$$\frac{di}{dX_o} = \frac{\begin{vmatrix} -S_Y - M' & -1 \\ L_Y & 0 \end{vmatrix}}{|J|} = \frac{L_Y}{L_Y(S_i - I') - L_i(S_Y + M')} > 0$$

An expanded national-income model is one which accounts for the price level where p is the level of domestic prices and net exports are the difference between exports and imports. We can find not only the effect of exports on national income and the interest rate but on the price level as well.

$$Y = C(Y,p) + I(Y,i) + G_o + X_o - M(Y,p) \qquad 0 < C_Y, M_Y < 1 \qquad C_p, I' < 0$$

$$\frac{M_{so}}{p} = L(Y,i) \qquad I_Y, M_p > 0 \qquad L_Y > 0 \qquad L_i < 0$$

$$p = p_o + g(Y) \qquad g_Y > 0$$

Rewriting the model in an implicit form,

$$Y - C(Y,p) - I(Y,i) - G_o - X_o + M(Y,p) = 0$$

$$L(Y,i) - \frac{M_{so}}{p} = 0$$

$$p - p_o - g(Y) = 0$$

$$\begin{bmatrix} 1 - C_Y - I_Y + M_Y & -I' & -C_p + M_p \\ L_Y & L_i & \frac{M_{so}}{p^2} \\ -g_Y & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{dY}{dX_o} \\ \frac{di}{dX_o} \\ \frac{dp}{dX_o} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$|J| = I'\left(L_Y + g_Y \frac{M_{so}}{p^2}\right) + L_i\left[1 - C_Y - I_Y + M_Y - g_Y(C_p - M_p)\right] < 0$$

Since C_Y, I_Y are marginal propensities, respectively, to consume and to save where investment equals savings in equilibrium, their sum should not exceed 1. It follows that the second parenthesized term is positive, and the Jacobian is negative.

$$\frac{dY}{dX_o} = \frac{\begin{vmatrix} 1 & -I' & -C_p + M_p \\ 0 & L_i & \frac{M_{so}}{p^2} \\ 0 & 0 & 1 \end{vmatrix}}{|J|} = \frac{L_i}{I'\left(L_Y + g_Y \frac{M_{so}}{p^2}\right) + L_i\left[1 - C_Y - I_Y + M_Y - g_Y(C_p - M_p)\right]} > 0$$

$$\frac{di}{dX_o} = \frac{\begin{vmatrix} 1 - C_Y - I_Y + M_Y & 1 & -C_p + M_p \\ L_Y & 0 & \frac{M_{so}}{p^2} \\ -g_Y & 0 & 1 \end{vmatrix}}{|J|} =$$

$$= -\frac{L_Y + g_Y \frac{M_{so}}{p^2}}{I'\left(L_Y + g_Y \frac{M_{so}}{p^2}\right) + L_i\left[1 - C_Y - I_Y + M_Y - g_Y(C_p - M_p)\right]} > 0$$

$$\frac{dp}{dX_o} = \frac{\begin{vmatrix} 1 - C_Y - I_Y + M_Y & -I' & 1 \\ L_Y & L_i & 0 \\ -g_Y & 0 & 0 \end{vmatrix}}{|J|} =$$

$$= \frac{L_i g_Y}{I'\left(L_Y + g_Y \frac{M_{so}}{p^2}\right) + L_i\left[1 - C_Y - I_Y + M_Y - g_Y(C_p - M_p)\right]} > 0$$

As usual, exports stimulate national income through speculative demand and the interest rate through transactional demand. Furthermore, exports increase domestic prices by increasing national income. A country that exports heavily eventually achieves a higher standard of living. We can redefine the model to check the effect of national imports, assuming they are exogenous.

$$Y = C(Y,p) + I(Y,i) + G_o + X(Y,p) - M_o \qquad 0 < C_Y < 1 \qquad C_p, I', X_p < 0$$

$$\frac{M_{so}}{p} = L(Y,i) \qquad I_Y, X_Y > 0 \qquad L_Y > 0 \qquad L_i < 0$$

$$p = p_o + g(Y) \qquad g_Y > 0$$

This time we find the effect of imports on the endogenous variables. Rewriting in an implicit form,

$$Y - C(Y,p) - I(Y,i) - G_o - X(Y,p) + M_o = 0$$

$$L(Y,i) - \frac{M_{so}}{p} = 0$$

$$p - p_o - g(Y) = 0$$

With respect to exogenous imports,

$$\begin{bmatrix} 1 - C_Y - I_Y - X_Y & -I' & -C_p - X_p \\ L_Y & L_i & \frac{M_{so}}{p^2} \\ -g_Y & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{dY}{dM_o} \\ \frac{di}{dM_o} \\ \frac{dp}{dM_o} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

$$|J| = I'\left(L_Y + g_Y \frac{M_{so}}{p^2}\right) + L_i\left[1 - C_Y - I_Y - X_Y - g_Y(C_p + X_p)\right] < 0$$

Given that the sum of the propensities C_Y, I_Y is less than 1, the second parenthesized term is positive and so the Jacobian is negative. This allows solving for the following comparative-static derivatives.

$$\frac{dY}{dM_o} = \frac{\begin{vmatrix} -1 & -I' & -C_p - X_p \\ 0 & L_i & \frac{M_{so}}{p^2} \\ 0 & 0 & 1 \end{vmatrix}}{|J|} =$$

$$= -\frac{L_i}{I'\left(L_Y + g_Y \frac{M_{so}}{p^2}\right) + L_i\left[1 - C_Y - I_Y - X_Y - g_Y(C_p + X_p)\right]} < 0$$

$$\frac{di}{dM_o} = \frac{\begin{vmatrix} 1 - C_Y - I_Y - X_Y & -1 & -C_p - X_p \\ L_Y & 0 & \frac{M_{so}}{p^2} \\ -g_Y & 0 & 1 \end{vmatrix}}{|J|} =$$

$$= \frac{L_Y + g_Y \frac{M_{so}}{p^2}}{I'\left(L_Y + g_Y \frac{M_{so}}{p^2}\right) + L_i\left[1 - C_Y - I_Y - X_Y - g_Y(C_p + X_p)\right]} < 0$$

$$\frac{dp}{dM_o} = \frac{\begin{vmatrix} 1 - C_Y - I_Y - X_Y & -I' & -1 \\ L_Y & L_i & 0 \\ -g_Y & 0 & 0 \end{vmatrix}}{|J|} =$$

$$= -\frac{L_i g_Y}{I'\left(L_Y + g_Y \frac{M_{so}}{p^2}\right) + L_i\left[1 - C_Y - I_Y - X_Y - g_Y(C_p + X_p)\right]} < 0$$

Interestingly, the effect of imports is opposite to that of exports. Imports reduce national income through speculative demand, the interest rate through transaction demand and domestic prices through the effect on national income. A country whose imports grow disproportionately with exports would eventually experience a declining living standard and falling prices. This is also consistent with the price-specie flow mechanism by which, as the country consumes more foreign goods and becomes indebted, it experiences lower prices and interest rates which eventually can stimulate exports. In this context, it is good to analyze the exchange rate in relation to exports. If the exchange rate e is considered, the national-income model could take the form

$$Y = C(Y) + I(Y) + G_o + X_o - M(Y,e) \quad 0 < C_Y, M_Y < 1 \qquad I_Y, L_Y > 0$$

$$M_s(e) = L(Y) \qquad M_e = \frac{\partial M}{\partial e} > 0 \qquad M_s' = \frac{\partial M_s}{\partial e} > 0$$

The country's imports increase with an increase in the exchange rate. Solving by implicit differentiation,

$$Y - C(Y) - I(Y) - G_o - X_o + M(Y,e) = 0$$

$$M_s(e) - L(Y) = 0$$

$$\begin{bmatrix} 1 - C_Y - I_Y + M_Y & M_e \\ -L_Y & M_s' \end{bmatrix} \begin{bmatrix} \frac{dY}{dX_o} \\ \frac{de}{dX_o} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|J| = M_s'(1 - C_Y - I_Y + M_Y) + L_Y M_e > 0$$

We find the effect of exports on Y and e as

$$\frac{dY}{dX_o} = \frac{\begin{vmatrix} 1 & M_e \\ 0 & M_s' \end{vmatrix}}{|J|} = \frac{M_s'}{M_s'(1 - C_Y - I_Y + M_Y) + L_Y M_e} > 0$$

$$\frac{de}{dX_o} = \frac{\begin{vmatrix} 1 - C_Y - I_Y + M_Y & 1 \\ -L_Y & 0 \end{vmatrix}}{|J|} = \frac{L_Y}{M_s^{'}(1 - C_Y - I_Y + M_Y) + L_Y M_e} > 0$$

National income increases as the nation exports more. The exchange rate also increases with exports. As the nation exports more, its currency appreciates relative to other currencies. As a secondary effect this increase in the exchange rate can stimulate imports and discourage exports. To demonstrate this, we modify the national-income model.

$$Y = C(Y) + I(Y,i) + G_o + X(e_o) - M(Y,e_o) \qquad 0 < C_Y, M_Y < 1 \qquad I_Y, L_Y, M_e > 0$$

$$M_{so} = L(Y,i) \qquad X_e, I', L_i < 0$$

Rearranging,

$$Y - C(Y) - I(Y,i) - G_o - X(e_o) + M(Y,e_o) = 0$$

$$L(Y,i) - M_{so} = 0$$

$$\begin{bmatrix} 1 - C_Y - I_Y + M_Y & -I^{'} \\ L_Y & L_i \end{bmatrix} \begin{bmatrix} \frac{dY}{de_o} \\ \frac{di}{de_o} \end{bmatrix} = \begin{bmatrix} X_e - M_e \\ 0 \end{bmatrix}$$

$$|J| = L_i(1 - C_Y - I_Y + M_Y) + L_Y I^{'} < 0$$

$$\frac{dY}{de_o} = \frac{\begin{vmatrix} X_e - M_e & -I^{'} \\ 0 & L_i \end{vmatrix}}{|J|} = \frac{L_i(X_e - M_e)}{L_i(1 - C_Y - I_Y + M_Y) + L_Y I^{'}} < 0$$

$$\frac{di}{de_o} = \frac{\begin{vmatrix} 1 - C_Y - I_Y + M_Y & X_e - M_e \\ L_Y & 0 \end{vmatrix}}{|J|} = -\frac{L_Y(X_e - M_e)}{L_i(1 - C_Y - I_Y + M_Y) + L_Y I^{'}} < 0$$

The exchange rate would lower national income. If the exchange rate is maintained continuously high, this will affect national income adversely since exports would be reduced and the nation will continue to import, which will most likely lead to a negative trade balance. National income and the interest rate fall. Since a high exchange rate stimulates imports and discourages exports, the effect on net exports and the trade balance is negative.

$$NX = X(e_o) - M(Y,e_o)$$

$$\frac{dNX}{de_o} = \frac{d\left[X(e_o) - M(Y,e_o)\right]}{de_o} = \frac{dX}{de_o} - \frac{dM}{de_o} < 0$$

The national income-model can further be expanded by the level of domestic prices p.

$$Y = C(Y,p) + I(Y,i) + G_o + X(e_o) - M(Y,e_o) \qquad 0 < C_Y, M_Y < 1 \qquad C_p, I^{'}, X_e < 0$$
$$I_Y, M_e > 0$$

$$M_s(p) = L(Y,i) \qquad L_Y > 0 \qquad M_{sp}, L_i < 0$$

$$p = p_o + g(Y) \qquad g_Y > 0$$

Rearranging the system,

$$Y - C(Y,p) - I(Y,i) - G_o - X(e_o) + M(Y,e_o) = 0$$

$$M_s(p) - L(Y,i) = 0$$

$$p - p_o - g(Y) = 0$$

Solving for the effect of the exchange rate on the endogenous variables Y, p, i,

$$\begin{bmatrix} 1 - C_Y - I_Y + M_Y & -C_p & -I' \\ -L_Y & M_{sp} & -L_i \\ -g_Y & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{dY}{de_o} \\ \frac{dp}{de_o} \\ \frac{di}{de_o} \end{bmatrix} = \begin{bmatrix} X_e - M_e \\ 0 \\ 0 \end{bmatrix}$$

$$|J| = -g_Y(C_p L_i + M_{sp} I') + L_i(1 - C_Y - I_Y + M_Y) + L_Y I' < 0$$

$$\frac{dY}{de_o} = \frac{\begin{vmatrix} X_e - M_e & -C_p & -I' \\ 0 & M_{sp} & -L_i \\ 0 & 1 & 0 \end{vmatrix}}{|J|} = \frac{L_i(X_e - M_{se})}{-g_Y\left(C_p L_i + M_{sp} I'\right) + L_i(1 - C_Y - I_Y + M_Y) + L_Y I'} < 0$$

$$\frac{dp}{de_o} = \frac{\begin{vmatrix} 1 - C_Y - I_Y + M_Y & X_e - M_e & -I' \\ -L_Y & 0 & -L_i \\ -g_Y & 0 & 0 \end{vmatrix}}{|J|} =$$

$$= \frac{g_Y L_i(X_e - M_e)}{-g_Y\left(C_p L_i + M_{sp} I'\right) + L_i(1 - C_Y - I_Y + M_Y) + L_Y I'} < 0$$

$$\frac{di}{de_o} = \frac{\begin{vmatrix} 1 - C_Y - I_Y + M_Y & -C_p & X_e - M_e \\ -L_Y & M_{sp} & 0 \\ -g_Y & 1 & 0 \end{vmatrix}}{|J|} =$$

$$= \frac{(X_e - M_e)(g_Y M_{sp} - L_Y)}{-g_Y\left(C_p L_i + M_{sp} I'\right) + L_i(1 - C_Y - I_Y + M_Y) + L_Y I'} < 0$$

This expanded national-income model augmented by the price level reaffirms the results of the effect of the exchange rate on the macroeconomy. A higher exchange rate reduces national income, the average price level, and the equilibrium interest rate. A negative trade balance stimulated by a higher exchange rate reduces national income and lowers the standard of living in the country – both the interest rate and the price level fall.

3. Foreign Trade in the Context of Definite Functions

A standard national-income model includes the size of trade occurring between the country and the world. The model can be presented using definite, rather than general, functions. Net exports are the difference between the total exports and imports of the country. The simplest possible export multiplier can be expressed as follows.

$$Y = C + I_o + G_o + X_o - M$$

$$C = \alpha + \beta Y \qquad \alpha > 0$$

$$M = M_o + \mu Y \qquad 0 < \beta, \mu < 0$$

For equilibrium national income we have

$$\bar{Y} = \frac{I_o + G_o + X_o + \alpha - M_o}{1 - \beta + \mu}$$

Hence, the export multiplier is

$$\frac{d\bar{Y}}{dX_o} = \frac{1}{1 - \beta + \mu} > 0$$

Since $1 - \beta > 0$, the multiplier is positive. At the same time, this multiplier is smaller than the one without imports, i.e., if the nation follows protectionism and only exports without importing.

$$\frac{d\bar{Y}}{dX_o} = \frac{1}{1 - \beta} > 1$$

$$\frac{d\bar{Y}}{d\mu} = -\frac{I_o + G_o + X_o + \alpha - M_o}{(1 - \beta + \mu)^2} = -\frac{\bar{Y}}{1 - \beta + \mu} < 0$$

The marginal propensity to import μ has a negative effect on equilibrium national income. The more inclined the nation is to import, the lower its national income will be. A more complex export multiplier is one incorporating a tax rate τ.

$$Y = C + I_o + G_o + X_o - M$$

$$C = \alpha + \beta(1 - \tau)Y \qquad \alpha > 0 \qquad \beta, \mu, \tau \in (0,1)$$

$$M = \mu(1 - \tau)Y$$

X_o are the exogenous exports of the country and M are its imports. Consumption and imports are both considered functions of disposable income that is left after a tax rate of τ is imposed. Again, the marginal propensity to consume is β and μ denotes the marginal propensity to import. The export multiplier is easy to find. Substituting consumption and imports into the national-income equation,

$$\bar{Y} = \frac{\alpha + I_o + G_o + X_o}{1 - (\beta - \mu)(1 - \tau)}$$

We expect national income to be positive, thus, $1 - (\beta - \mu)(1 - \tau) > 0$.

$$\frac{dY}{dX_o} = \frac{1}{1 - (\beta - \mu)(1 - \tau)} > 0$$

The export multiplier is positive, *i.e.,* exports increase national income. Since it is expected that $\beta > \mu$, that is, the nation has a greater propensity to consume domestic, rather than foreign goods, the value of the export multiplier exceeds 1 and exports, indeed, have a multiplying effect on national income. At the same time, a higher tendency to consume foreign goods given by the marginal propensity to import has a negative effect on national income.

$$\frac{dY}{d\mu} = \frac{-(\alpha + I_o + G_o + X_o)(1 - \tau)}{[1 - (\beta - \mu)(1 - \tau)]^2} = \frac{-\bar{Y}(1 - \tau)}{1 - (\beta - \mu)(1 - \tau)} < 0$$

A slightly modified national-income model accounting for both trade and government produces similar results.

$$Y = C + I_o + G + X_o - M$$

$$C = \alpha + \beta(1 - \tau)Y \qquad \alpha > 0 \qquad \beta, \gamma, \mu, \tau \in (0,1)$$

$$G = \gamma Y$$

$$M = \mu(1-\tau)Y$$

This time the value of equilibrium national income is

$$\overline{Y} = \frac{\alpha + I_o + X_o}{1-\gamma+(\mu-\beta)(1-\tau)}$$

We expect national income to be positive. This gives the value of the export multiplier.

$$\frac{dY}{dX_o} = \frac{1}{1-\gamma+(\mu-\beta)(1-\tau)} > 0$$

The export multiplier is positive for positive national income. Furthermore, the effect of exports on imports is given by a chain derivative.

$$\frac{dM}{dX_o} = \frac{dM}{dY}\frac{dY}{dX_o} = \frac{\mu(1-\tau)}{1-\gamma+(\mu-\beta)(1-\tau)} > 0$$

By exporting more, the country accumulates more income and can afford to import more. At the same time, a higher propensity to import and a stronger inclination to consume foreign goods reduces national income.

$$\frac{dY}{d\mu} = \frac{-(\alpha + I_o + X_o)(1-\tau)}{[1-\gamma+(\mu-\beta)(1-\tau)]^2} = -\frac{\overline{Y}(1-\tau)}{1-\gamma+(\mu-\beta)(1-\tau)} < 0$$

The greater the propensity to import, the smaller the national income. Using the chain rule again, we find the effect of exogenous exports on aggregate consumption. Since consumption is a share of national income, exports stimulate both national income and consumption.

$$\frac{dC}{dX_o} = \frac{dC}{dY}\frac{dY}{dX_o} = \frac{\beta(1-\tau)}{1-\gamma+(\mu-\beta)(1-\tau)} > 0$$

When the nation exports more, it can afford to consume more of both foreign and its own goods. It could be considered that the value of exports depends on aggregate investment. When firms invest more, they are likely to produce more and consequently export more abroad. This can be presented as

$$Y = C + I_o + G_o + X - M$$

$$C = \alpha + \beta(1-\tau)Y \qquad \alpha > 0 \qquad \beta, \mu, \tau, x \in (0,1)$$

$$X = xI_o$$

$$M = \mu(1-\tau)Y$$

where exports X depend on the level of investment of the country and M are its imports. Consumption and imports are both considered functions of disposable income that is left after a tax rate of τ is imposed. This gives

$$\bar{Y} = \frac{\alpha + (1+x)I_o + G_o}{1-(\beta-\mu)(1-\tau)}$$

The value of the investment multiplier is greater than 1 consistent with Keynesian theory.

$$\frac{dY}{dI_o} = \frac{(1+x)}{1-(\beta-\mu)(1-\tau)} > 1$$

Furthermore, we see that national income is positively related to the likelihood of firms to invest and the share of exports x in investment, since

$$\frac{dY}{dx} = \frac{I_o}{1-(\beta-\mu)(1-\tau)} > 0$$

Furthermore,

$$\frac{dY}{d\mu} = \frac{-(1-\tau)[\alpha+(1+x)I_o+G_o]}{[1-(\beta-\mu)(1-\tau)]^2} = \frac{-(1-\tau)\bar{Y}}{1-(\beta-\mu)(1-\tau)} < 0$$

The more likely the nation is to import, the lower the national income would be. Again, in the context of investment we can demonstrate the interplay between exports (imports) of one country and investment in another country. This time we introduce two countries trading with each other. Their economies are described by the following two national-income models.

$$Y_1 = C_1 + I_{1o} + X_1 - M_1 \qquad Y_2 = C_2 + I_{2o} + X_2 - M_2$$
$$C_1 = \alpha + \beta Y_1 \qquad C_2 = \gamma + \delta Y_2 \qquad \alpha, \gamma > 0$$
$$M_1 = \mu Y_1 \qquad M_2 = \lambda Y_2 \qquad 0 < \beta, \delta, \lambda, \mu < 1$$

Home's exogenous investment is I_{1o}, and that of Foreign is I_{2o}. With mutual trade the exports of Home are imports for Foreign and vice versa. Since the exports of Home are imports for Foreign and vice versa, we have $X_1 = M_2$ and $X_2 = M_1$. Substituting in the two national-income identities,

$$Y_1 = \alpha + \beta Y_1 + I_{1o} + \lambda Y_2 - \mu Y_1$$

$$Y_2 = \gamma + \delta Y_2 + I_{2o} + \mu Y_1 - \lambda Y_2$$

Rearranging and solving in a matrix form,

$$(1-\beta+\mu)Y_1 - \lambda Y_2 - \alpha - I_{1o} = 0$$

$$-\mu Y_1 + (1-\delta+\lambda)Y_2 - \gamma - I_{2o} = 0$$

Written in a matrix form,

$$\begin{bmatrix} 1-\beta+\mu & -\lambda \\ -\mu & 1-\delta+\lambda \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} \alpha + I_{1o} \\ \gamma + I_{2o} \end{bmatrix}$$

$$|J| = (1-\beta+\mu)(1-\delta+\lambda) - \lambda\mu = (1-\beta)(1-\delta+\lambda) + \mu(1-\delta) > 0$$

$$\bar{Y}_1 = \frac{(\alpha+I_{1o})(1-\delta+\lambda)+\lambda(\gamma+I_{2o})}{(1-\beta)(1-\delta+\lambda)+\mu(1-\delta)} \qquad \bar{Y}_2 = \frac{(\gamma+I_{2o})(1-\beta+\mu)+\mu(\alpha+I_{1o})}{(1-\beta)(1-\delta+\lambda)+\mu(1-\delta)}$$

The two national outputs are assumed to be positive where $0 < \beta, \delta, \lambda, \mu < 1$. Home's investment has a multiplying effect on the exports of Foreign and vice versa.

$$X_1 = M_2 = \lambda\bar{Y}_2 = \frac{\lambda[(\gamma+I_{2o})(1-\beta+\mu)+\mu(\alpha+I_{1o})]}{(1-\beta)(1-\delta+\lambda)+\mu(1-\delta)}$$

$$X_2 = M_1 = \mu\bar{Y}_1 = \frac{\mu[(\alpha+I_{1o})(1-\delta+\lambda)+\lambda(\gamma+I_{2o})]}{(1-\beta)(1-\delta+\lambda)+\mu(1-\delta)}$$

It is easy to see the interplay between the two economies – the exports of Home depend on the investment in Foreign and, vice versa, the exports of Foreign are positively related to Home's investment. The exports of each country are also a function of its own aggregate investment.

Conclusion

Using the tools of implicit differentiation, we derive a set of relationships with respect to foreign trade. The national-income model and the *IS-LM* model advanced by Keynes reinforce each other since exports have a favorable effect on national income and the interest rate. Exports also increase the average price level and the exchange rate – as the nation exports more, it tends to become richer but also enjoys a higher standard of living. Through the export multiplier exports increase imports, savings, and the average price level. As the nation exports more, it can afford to consume more foreign goods and save more. On the other hand, if the nation has a stronger propensity to consume foreign, rather than domestic, goods, it tends to lose national income and incur a negative trade balance. Such a higher inclination for consuming foreign goods affects the trade balance adversely, as it discourages exports, while encouraging imports. The same effect is achieved by a higher exchange rate. We derive that a higher exchange rate reduces national income, the equilibrium interest rate, and the average price level in the country. In the context of definite functions, we also find the effect of the marginal propensity to import on national income. With the mutual trade between two countries, we find their interdependence. The exports of each country are assumed to be a share of its aggregate investment where the exports of one country, Home, are imports for the other one, Foreign, and vice versa. We derive that the exports of Home are positively related to Foreign's investment and those of Foreign to Home's investment.

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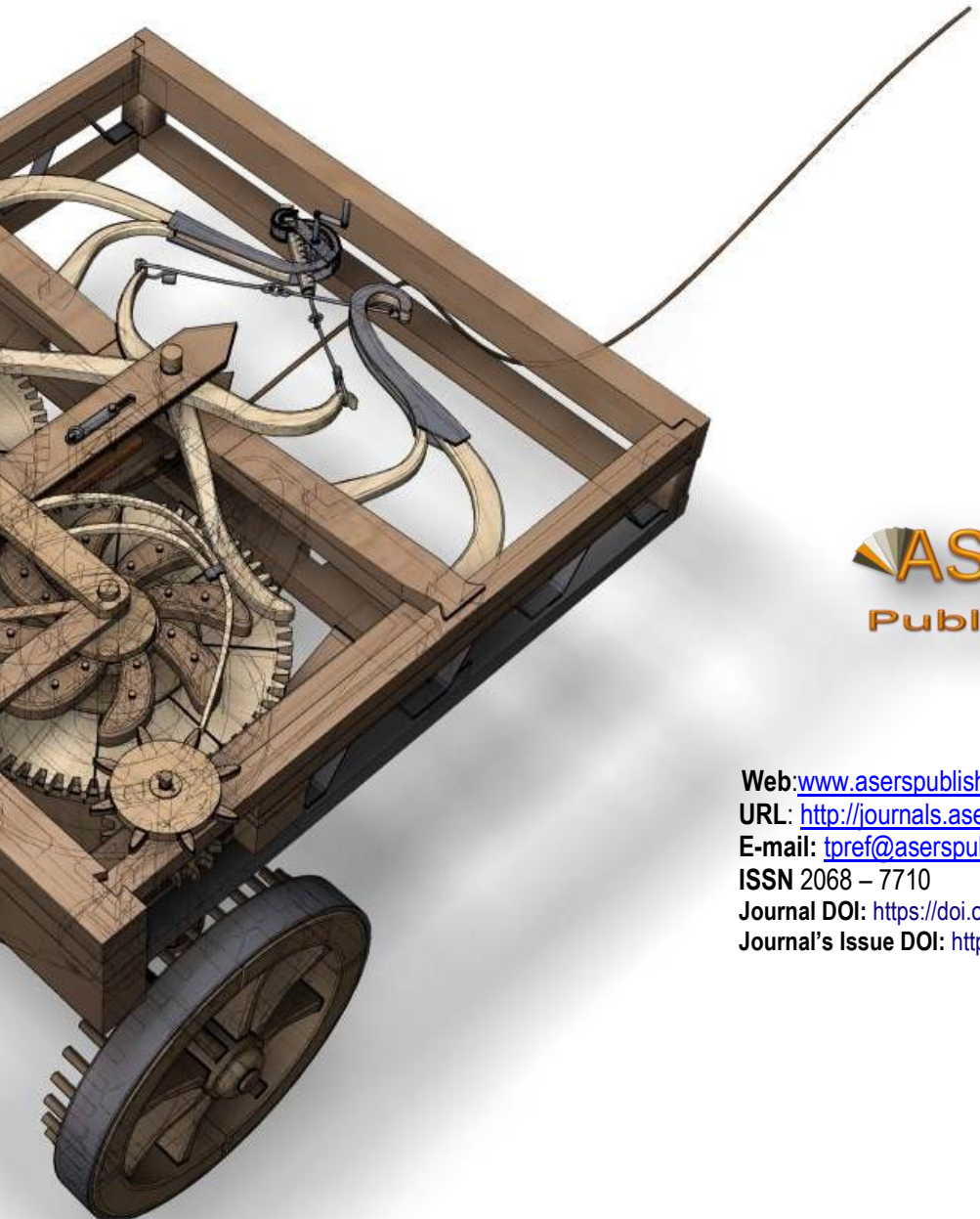

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