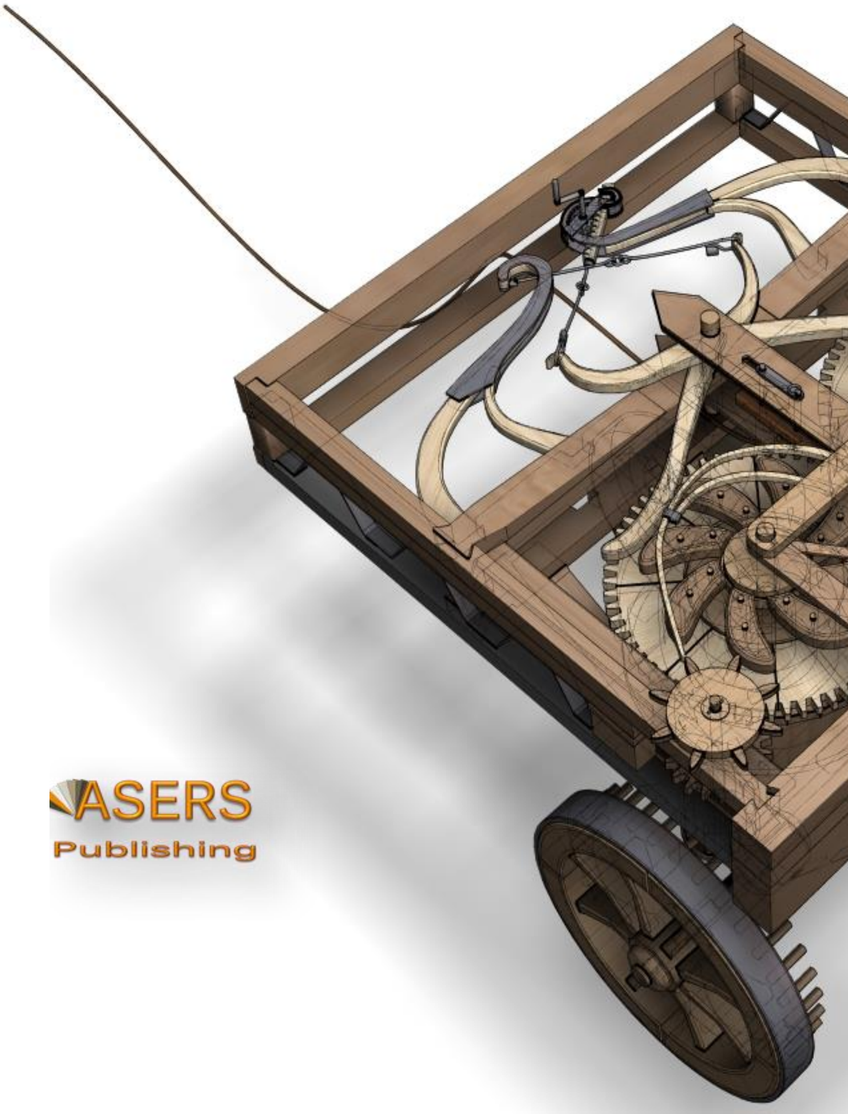
Theoretical and Practical Research in Economic Fields



Biannually
Volume XII
Issue 1(23)
Summer 2021

ISSN 2068 – 7710
Journal **DOI**
https://doi.org/10.14505/tpref

ASERS
Publishing

ASERS Publishing is an advanced e-publisher struggling to bring further worldwide learning, knowledge and research. This transformative mission is realized through our commitment to innovation and enterprise, placing us at the cutting-edge of electronic delivery in a world that increasingly considers the dominance of digital content and networked access not only to books and journals but to a whole range of other pedagogic services.

In both books and journals, **ASERS Publishing** is a hallmark of the finest scholarly publishing and cutting-edge research, maintained by our commitment to rigorous peer-review process.

Using pioneer developing technologies, **ASERS Publishing** keeps pace with the rapid changes in the e-publishing market.

ASERS Publishing is committed to providing customers with the information they want, when they want and how they want it. To serve this purpose, ASERS publishing offers digital Higher Education materials from its journals, courses and scientific books, in a proven way in order to engage the academic society from the entire world.

Volume XII
Issue 1(23)
Summer 2021

ASERS Publishing
http://www.asers.eu/asers-publishing
ISSN 2068 – 7710
Journal's Issue **DOI**:
https://doi.org/10.14505/tpref.v12.1(23).00

Table of Contents:

1 Is Slow Economic Growth Originating from the Total External Debt Stock in the Democratic Republic of Congo? Olivier Munene MUPENDA 5

2 Pluralism as a Recommended Research Practice for Central Banks in Addressing Welfare Concerns on the Experience of COVID-19 Emerson Abraham JACKSON 13

3 Estimating the Relationship between Governance, Economic Growth, Inequality and Poverty Mohammed TOUITOU 21

4 Improvement of Methodical Approaches to the Management of the System of Economic Security of Bakery Industry Enterprises Valeriia BONDARENKO-BEREHOVYCH 31

5 A Model with Knowledge Externalities and Educational Policy Aleksandar VASILEV 39

6 Age and Gender - Specific Excess Mortality during the COVID-19 Pandemic in Hungary in 2020 Csaba G. TÓTH 42

7 COVID-19: Behavior of Public Finances Tools in Democratic Republic of Congo. Economic Situation and Perspectives Yannick LUBONGO MBILU 47

8 A Note on *Gensys’* Minimality Alessandro SACCAL 57

Call for Papers

Volume XII, Issue 2(24), Winter 2021

Theoretical and Practical Research in Economic Fields

research to be conquered in order to reach the specific information they require. To combat this tendency, **Theoretical and Practical Research in Economic Fields** has been conceived and designed outside the realm of the traditional economics journal. It consists of concise communications that provide a means of rapid and efficient dissemination of new results, models and methods in all fields of economic research.

Theoretical and Practical Research in Economic Fields publishes original articles in all branches of economics – theoretical and empirical, abstract and applied, providing wide-ranging coverage across the subject area.

Journal promotes research that aim at the unification of the theoretical-quantitative and the empirical-quantitative approach to economic problems and that are penetrated by constructive and rigorous thinking. It explores a unique range of topics from the frontier of theoretical developments in many new and important areas, to research on current and applied economic problems, to methodologically innovative, theoretical and applied studies in economics. The interaction between empirical work and economic policy is an important feature of the journal.

Theoretical and Practical Research in Economic Fields, starting with its first issue, it is indexed in EconLit, RePEC, ProQuest, Cabell Directories and CEEOL databases.

The primary aim of the Journal has been and remains the provision of a forum for the dissemination of a variety of international issues, empirical research and other matters of interest to researchers and practitioners in a diversity of subject areas linked to the broad theme of economic sciences.

All the papers will be first considered by the Editors for general relevance, originality and significance. If accepted for review, papers will then be subject to double blind peer review.

Invited manuscripts will be due till November 10th, 2021, and shall go through the usual, albeit somewhat expedited, refereeing process.

Deadline for submission of proposals: 10th November 2021

Expected publication date: December 2021

Website: http://journals.aserspublishing.eu/tpref

E-mail: tpref@aserspublishing.eu

To prepare your paper for submission, please see full author guidelines in the following file: TPREF_Full_Paper_Template.docx, on our site.

Theoretical and Practical Research in Economic Fields

DOI: https://.doi.org/10.14505/tpref.v12.1(23).08

A NOTE ON *GENSYS'* MINIMALITY

Alessandro SACCAL
Independent Researcher, Italy
saccal.alessandro@gmail.com

Suggested Citation:
Saccal, A. (2021). A Note on *Gensys'* Minimality, *Theoretical and Practical Research in Economic Fields* (Volume XII, Summer 2021), 1(23): 57 - 60. DOI:10.14505/tpref.v12.1(23).08
Article's History:
Received 11nd of January 2021; *Revised* 9th of February 2021; *Accepted* 10th of March 2021; *Published* of 30th of June 2021.

Abstract:

Gensys'non-minimality is shown analytically and necessary and sufficient conditions for vector autoregression representations of states in outputs are presented.

Keywords: gensys; minimality; state space.

JEL Classification: C02; C32.

Introduction

Sims' (2001) Matlab solution algorithm to linear rational expectation models is called gensys. Does it deliver minimal linear time invariant state space representations? Namely, is gensys sufficient for minimal linear time invariant state space representations? The example produced by Komunjer and Ng (2011) shows that the answer is negative: $\mathrm{G} \not\longrightarrow MR,$ since $\exists x \in U$ such that $G \times \wedge \neg MR x$in which $G \equiv$gensys, $MR \equiv$Minimal representation, $X \equiv$counterexample and $U \equiv$universe (*i.e.* domain of discourse). This note shows such analytically, presenting necessary and sufficient conditions for vector autoregression representations of states in outputs.

1. Gensys State Space, Minimality and VAR

Gensys gives rise to the unique and stable solution $[x_{1t}\ x_{2t}]^\top = [(A_{11}0)(0\ 0)]^\top [x_{1t-1}\ x_{2t-1}]^\top + [B_{11}B_{21}]^\top u_t, \forall t \in \mathbb{Z}, x_{1t} \in \mathbb{R}^{n_{x_1}}, x_{2t} \in \mathbb{R}^{n_{x_2}}, u_t \in \mathbb{R}^{n_u}, A_{11} \in \mathbb{R}^{n_{x_1} \times n_{x_\mathrm{u}}}, B_{11} \in \mathbb{R}^{n_{x_1} \times n_{x_\mathrm{u}}}$ and $B_{21} \in \mathbb{R}^{n_{x_1} \times n_{x_\mathrm{u}}}$; x_{1t} is a vector of non-expectational variables, X_{2t} is a vector of expectational variables and is a vector of inputs (i.e.shocks). Such a solution is the transition equation of a linear time invariant state space representation in discrete time: $[x_{1t}\ x_{2t}]^\top = [(A_{11}\ 0)\ \ (0\ 0)]^\top [x_{1t-1}\ x_{2t-1}]^\top + [B_{11}\ B_{21}]^\top u_t \longleftrightarrow x_t = Ax_{t-1} + Bu_t,$ $\forall x_t \in \mathbb{R}^{n_x},\ A \in \mathbb{R}^{n_x \times n_x}$ and $\mathrm{B} \in \mathbb{R}^{n_x \times n_u};$ x_t is a vector states such that $\mathrm{n}_x = n_{x_1} + n_{x_2}.$ Let $\mathrm{M} \in \mathbb{R}^{n_y \times n_x}$ give rise to $\mathrm{Mx}_t = MAx_{t-1} + MBu_t \longleftrightarrow y_t = Cx_{t-1} + Du_t,\ \forall y_t \in \mathbb{R}^{n_y},\ C \in \mathbb{R}^{n_y \times n_x}$ and $\mathrm{D} \in \mathbb{R}^{n_y \times n_u}$. It is the measurement equation of a linear time invariant state space representation in discrete time, in which y_t is a vector of outputs; M is called measurement matrix.

Linear time invariant state space representations are minimal *if and only if* rank $\mathrm{r}_C = r_O = n_x$ for controllability matrix $\mathrm{C}=[\cdots\ A^{n_x-1}B]$ and observability matrix $\mathrm{O}=[\cdots\ CA^{n_x-1}]^\top.$ Non-minimal representations can be reduced to minimal ones by the Kalman decomposition: the economic interpretation is invariant (see Franchi (2013)). Assume that the representation be minimal: $\mathrm{x}_{mt} = A_m x_{mt-1} + B_m u_t \text{ and } y_t = C_m x_{mt-1} + Du_t.$

Assume that D be non-singular and thus square: $\mathrm{n}_y = n_u.$ Solve the measurement equation for u_t and plug it into the transition equation:

$x_{mt} = \left(A_m - B_m D^{-1} C_m\right) x_{mt-1} + B_m D^{-1} y_t = F_m x_{mt-1} + B_m D^{-1} y_t;$ notice that $F_m \equiv A_m - B_m D^{-1} C_m.$ Solve it backwards, satisfying causality: $x_{mt} = \sum_{j=0}^{\infty} F_m^j B_m D^{-1} y_{t-j}$ *if and only if* F_m is stable, namely, F_m's characteristic polynomial eigenvalues are less than one in modulus, $|\lambda_{F_m(\lambda)}| < 1$ for $F_m(\lambda) = F_m - \lambda I$ in $\det[F_m(\lambda)] = 0.$ Plug this into the measurement equation: $y_t = \sum_{j=0}^{\infty} F_m^j B_m D^{-1} y_{t-j-1} + D u_t.$

Thus: there exists a vector autoregression of infinite order $\mathrm{VAR}(\infty)$ *if and only if* F_m is stable; there exists a vector autoregression of finite order $\mathrm{VAR}(k)$ for $k<\infty$ *if and only if* F_m is nilpotent, namely, F_m's characteristic polynomial eigenvalues are zero, $\lambda_{F_m(\lambda)} = 0.$ See Franchi (2013), Franchi and Paruolo (2014), Fernández-Villaverde *et al.* (2007), Ravenna (2007) and Franchi and Vidotto (2013) for further detail.

2. Symmetric Case

Let x_{1t} be symmetrically semi-measurable, namely, let half of its rows be measurable: $x_t = [x_{M1t}\ x_{N1t}\ x_{2t}]^\top$ such that $n_{x_{M1}} = n_{x_{N1}},\ A = [(A_{11_{11}}\ A_{11_{12}}\ 0)\ (A_{11_{21}}\ A_{11_{22}}\ 0)\ (0\ 0\ 0)]^\top,\ B = [B_{11_{11}}\ B_{11_{21}}\ B_{21}]^\top,$ $M=[1\ 0\ 0],\ y_t = x_{m1t},\ C = [A_{11_{11}}\ A_{11_{12}}\ 0]$ and $D=B_{11_{11}}.$ Record $r_{\mathcal{C}}$ for $\mathcal{C}$ and $r_{\mathcal{O}}$ for $\mathcal{O}$: $n_x = r_{\mathcal{C}} = 3 > r_{\mathcal{O}} = 2,$ thus, the representation is controllable, non-observable and therefrom non-minimal.

Reduce the representation to minimality by the Kalman decomposition: construct similarity transformation matrix $\mathcal{T}=[\mathcal{O}_{r_{\mathcal{O}}}\ v_{n_x - r_{\mathcal{O}}}]^\top$ such that $\bar{x}_{co\bar{o}t} = \mathcal{T}^{-1} x_t,\ \bar{A}_{co\bar{o}} = \mathcal{T}^{-1} A \mathcal{T},\ \bar{B}_{co\bar{o}} = \mathcal{T}^{-1} B,\ \bar{C}_{co\bar{o}} = C\mathcal{T},\ \bar{\mathcal{C}}_{co\bar{o}} = \mathcal{T}^{-1}\mathcal{C}$ and $\bar{\mathcal{O}}_{co\bar{o}} = \mathcal{O}\mathcal{T};$ select the first $r_{\mathcal{O}} = 2$ states such that $\bar{x}_{cot} = x_{mt},\ \bar{A}_{co} = A_m,\ \bar{B}_{co} = B_m,\ \bar{C}_{co} = C_m,\ \bar{\mathcal{C}}_{co} = \mathcal{C}_m$ and $\bar{\mathcal{O}}_{co} = \mathcal{O}_m.$

Computing $F_m,\ F_m(\lambda)$ and $|\lambda_{F_m(\lambda)}|,$ F_m first eigenvalue matrix $\Lambda_1 \equiv \lambda_{1Fm(\lambda)} = -[A_{11_{12}} B_{11_{21}} - A_{11_{22}} B_{11_{11}}] B_{11_{11}}^{-1}$ and F_m second eigenvalue matrix $\Lambda_2 \equiv \lambda_{2Fm(\lambda)} = \mathbf{0};$ notice that $A_{11_{12}} \in \mathbb{R}^{n_{x_{M1}} \times n_{x_{N1}}},\ B_{11_{21}} \in \mathbb{R}^{n_{x_{N1}} \times n_u},\ A_{11_{22}} \in \mathbb{R}^{n_{x_{N1}} \times n_{x_{N1}}},\ B_{11_{11}} \in \mathbb{R}^{n_{x_{M1}} \times n_u}.$ Thus, there exists a $\mathrm{VAR}(k),\ \forall k \leq \infty,$ of x_t in y_t *if and only if* $|\lambda_{\Lambda_1(\lambda)}| \in [0,\ 1)$ for $\Lambda_1(\lambda) = \Lambda_1 - \lambda I$ in $\det[\Lambda_1(\lambda)] = 0.$

Such a **gensys** condition is necessary and sufficient for a vector autoregression representation of the states in the outputs in the symmetric case, furthering $|\lambda_{F_m(\lambda)}| \in [0,\ 1)$ and acting as the analytical *counterexample* to the syntactic implication 'Minimal linear time invariant state space representations *if* **gensys'.**

3. Complete and Asymmetric Case

Let x_{1t} be fully measurable, namely, let all of its rows be measurable: $M=[1\ 0],\ y_t = x_{1t},\ C = [A_{11}\ 0]$ and $D=B_{11}.$ Record $r_{\mathcal{C}}$ for $\mathcal{C}$ and $r_{\mathcal{O}}$ for $\mathcal{O}$: $n_x = r_{\mathcal{C}} = 2 > r_{\mathcal{O}} = 1,$ thus, the representation is controllable, non-observable and therefrom non-minimal.

Reduce the representation to minimality by the Kalman decomposition: construct $\mathcal{T}=[\mathcal{O}_{r_{\mathcal{O}}}\ v_{n_x - r_{\mathcal{O}}}]^\top = [(A_{11}\ 0)\ (0\ 1)]^\top$ and proceed as before, selecting the first $r_{\mathcal{O}} = 1$ states, so that

$$[x_{mt}\ y_t]^\top = [A_m\ C_m]^\top x_{mt-1} + [B_m\ D]^\top u_t \longleftrightarrow [A_{11}^{-1} x_{1t}\ x_{1t}]^\top = [A_{11}\ A_{11}^2]^\top A_{11}^{-1} x_{1t-1} + [A_{11}^{-1} B_{11}\ B_{11}]^\top u_t.$$

Computing $F_m,\ F_m(\lambda)$ and $|\lambda_{F_m(\lambda)}|,\ \lambda_{F_m(\lambda)} = F_m = A_{11} - A_{11}^{-1} B_{11} B_{11}^{-1} A_{11}^2 = \mathbf{0}.$ Thus, there exists a $\mathrm{VAR}(k),\ \forall k < \infty,$ of x_t in $y_t.$

The scenario of x_{1t} asymmetric semi-measurability, namely, $n_{x_{M1}} \neq n_{x_{N1}},$ is best studied case by case.

Conclusion

This note’s conclusion prescribes the reduction of **gensys’**representation to minimality as hereby shown.

References

[1] Fernández-Villaverde J., Rubio-Ramírez J., Sargent T., and Watson M. 2007. *ABCs (and Ds) of Understanding VARs.*American Economic Review 97, 3: 1021-1026. DOI:https://doi.org/10.1257/aer.97.3.1021

[2] Franchi M. 2013. *Comment on: Ravenna F. 2007. Vector Autoregressions and Reduced Form Representations of DSGE Models. Journal of Monetary Economics 54, 2048-2064.*Dipartimento di Scienze Statistiche Empirical Economics and Econometrics Working Papers Series, DSS-E3 WP 2013/2. https://www.dss.uniroma1.it/RePec/sas/wpaper/20132_Franchi.pdf.

[3] Franchi M., and Paruolo P. 2014. *Minimality of State Space Solutions of DSGE Models and Existence Conditions for Their VAR Representation.*Computational Economics 46, 4: 613–62. DOI:https://doi.org/10.1007/s10614-014-9465-4

[4] Franchi M., and Vidotto A. 2013. *A Check for Finite Order VAR Representations of DSGE Models.*Economics Letters 120, 1: 100-103. DOI: https://doi.org/10.1016/j.econlet.2013.04.013

[5] Komunjer I., and Ng S. 2011. *Dynamic Identification of Dynamic Stochastic General Equilibrium Models.*Econometrica 79, 6: 1995-2032. DOI:https://doi.org/10.3982/ECTA8916

[6] Ravenna F. 2007. *Vector Autoregressions and Reduced Form Representations of DSGE Models.*Journal of Monetary Economics 54, 7: 2048-2064. DOI:https://doi.org/10.1016/j.jmoneco.2006.09.002

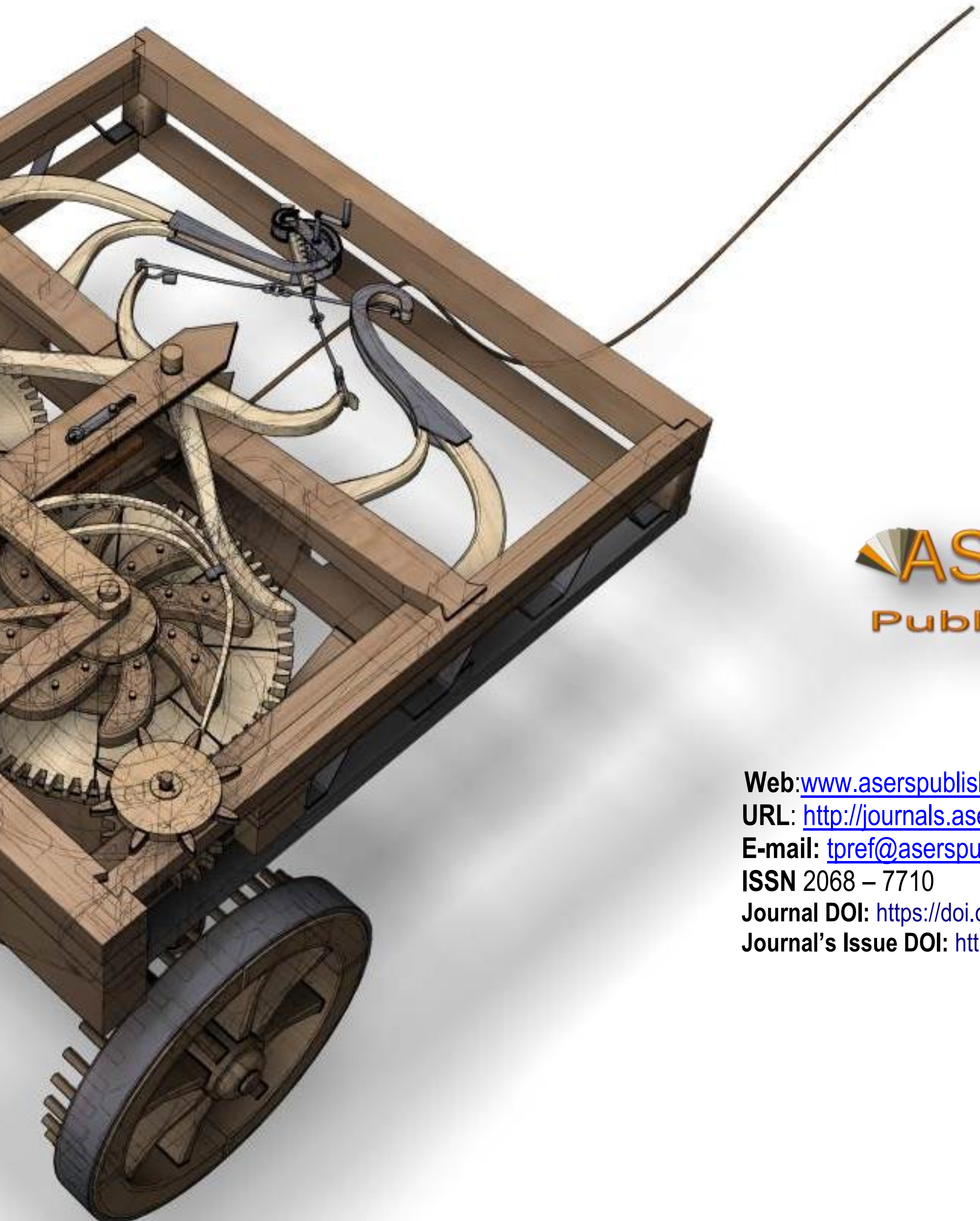
[7] Sims C. 2001. Solving Linear Rational Expectations Models. Computational Economics 20, 1-2: 1-20. DOI:https://doi.org/10.1023/A:1020517101123

Appendix

*Matlab*commands for symmetric case.

```
% gensys state space (symmetric case)
syms a1111 a1112 a1121 a1122 b1111 b1121 b21
A=[a1111 a1112 0; a1121 a1122 0; zeros(1, 3)];
B=[b1111; b1121; b21];
M=[1 0 0]; C=M*A; D=M*B;
% Controllability and observability
Con=[B A*B A*A*B];
fprintf('Controllability matrix rank')
rc=rank(Con)
Obs=[C; C*A; C*A*A];
fprintf('Observability matrix rank')
ro=rank(Obs)
% Similarity transformation
v=[0 0 1];
T=[Obs(1:2, 1:3); v];
invT=inv(T);
% Canonical and minimal decomposition
Ad = invT*A*T;
Bd = invT*B;
Cd = C*T;
Am = [Ad(1:2, 1:2)];
Bm = [Bd(1:2, 1:1)];
Cm = Cd(1:1, 1:2);
% Minimal controllability and observability
Conm=[Bm Am*Bm];
fprintf('Minimal controllability matrix rank')
rcm=rank(Conm)
Obsm=[Cm; Cm*Am];
fprintf('Minimal observability matrix rank')
rom=rank(Obsm)
% Minimal VAR representation
Fm=Am-Bm*inv(D)*Cm;
fprintf('Minimal VAR representation condition eigenvalues')
lambdas_Fm=eig(Fm)
```

ASERS



ASERS
Publishing

Web:www.aserspublishing.eu
URL: http://journals.aserspublishing.eu/tpref
E-mail: tpref@aserspublishing.eu
ISSN 2068 – 7710
Journal DOI: https://doi.org/10.14505/tpref
Journal's Issue DOI: https://doi.org/10.14505/tpref.v12.1(23).00