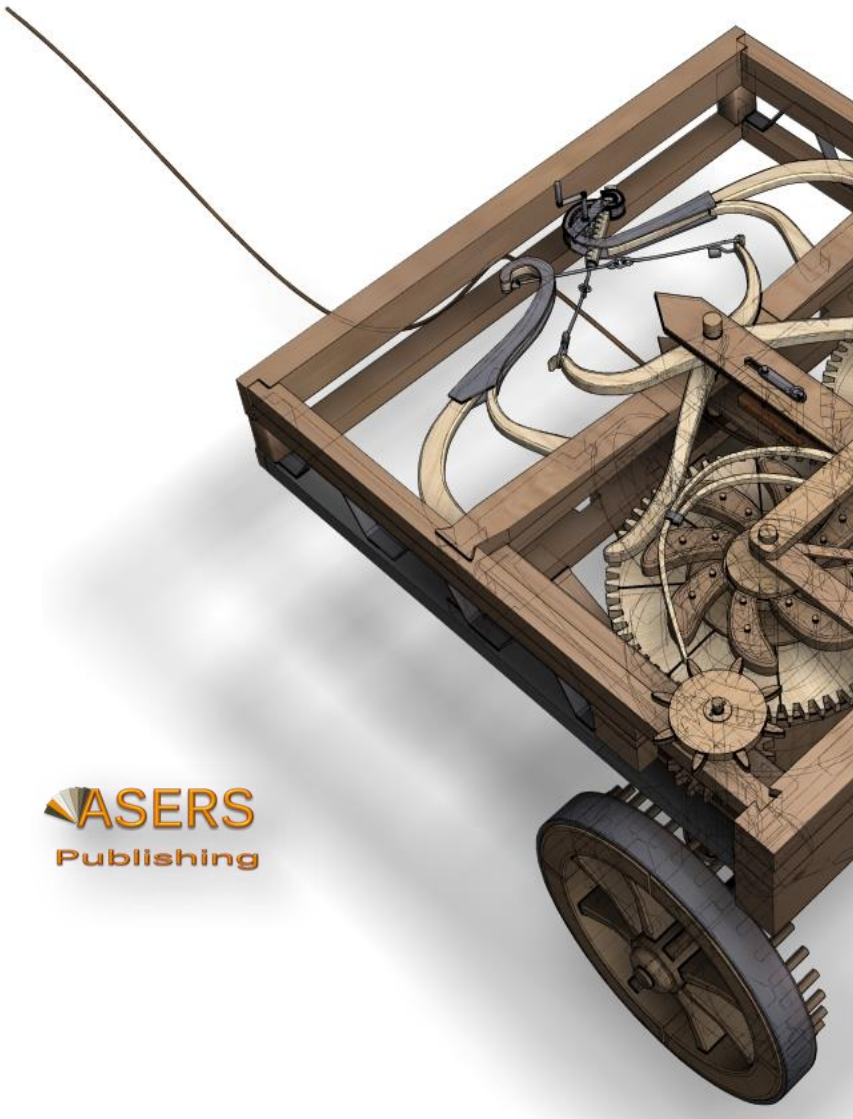
ASERS

Theoretical and Practical Research in Economic Fields



Biannually
Volume VIII
Issue 2(16)
Winter 2017

ISSN 2068 – 7710
Journal **DOI**
http://dx.doi.org/10.14505/tpref

ASERS Publishing

is an advanced e-publisher struggling to bring further worldwide learning, knowledge and research. This transformative mission is realized through our commitment to innovation and enterprise, placing us at the cutting-edge of electronic delivery in a world that increasingly considers the dominance of digital content and networked access not only to books and journals but to a whole range of other pedagogic services.

In both books and journals, ASERS Publishing is a hallmark of the finest scholarly publishing and cutting-edge research, maintained by our commitment to rigorous peer-review process.

Using pioneer developing technologies, ASERS Publishing keeps pace with the rapid changes in the e-publishing market.

ASERS Publishing is committed to providing customers with the information they want, when they want and how they want it. To serve this purpose, ASERS publishing offers digital Higher Education materials from its journals, courses and scientific books, in a proven way in order to engage the academic society from the entire world.

Volume VIII
Issue 2(16)
Winter, 2017

ASERS Publishing
http://www.asers.eu/asers-publishing
ISSN 2068 – 7710
Journal's Issue **DOI**:
http://dx.doi.org/10.14505/tpref.v8.2(16).00

Contents:

Many economists today are concerned by the proliferation of journals and the concomitant labyrinth of research to be conquered in order to reach the specific information they require. To combat this tendency, **Theoretical and Practical Research in Economic Fields** has been conceived and designed outside the realm of the traditional economics journal. It consists of concise communications that provide a means of rapid and efficient dissemination of new results, models and methods in all fields of economic research.

Theoretical and Practical Research in Economic Fields publishes original articles in all branches of economics – theoretical and empirical, abstract and applied, providing wide-ranging coverage across the subject area.

Journal promotes research that aim at the unification of the theoretical-quantitative and the empirical-quantitative approach to economic problems and that are penetrated by constructive and rigorous thinking. It explores a unique range of topics from the frontier of theoretical developments in many new and important areas, to research on current and applied economic problems, to methodologically innovative, theoretical and applied studies in economics. The interaction between empirical work and economic policy is an important feature of the journal.

Theoretical and Practical Research in Economic Fields, starting with its first issue, it is indexed in EconLit, RePEC, EBSCO, ProQuest, Cabell Directories and CEEOL databases.

The primary aim of the Journal has been and remains the provision of a forum for the dissemination of a variety of international issues, empirical research and other matters of interest to researchers and practitioners in a diversity of subject areas linked to the broad theme of economic sciences.

All the papers will be first considered by the Editors for general relevance, originality and significance. If accepted for review, papers will then be subject to double blind peer review.

Invited manuscripts will be due till May 10th, 2018, and shall go through the usual, albeit somewhat expedited, refereeing process.

Deadline for submission of proposals: 10th May 2018

Expected publication date: June 2018

Website: http://journals.aserspublishing.eu/tpref

E-mail: tpref@aserspublishing.eu, asers.tpref@gmail.com

To prepare your paper for submission, please see full author guidelines in the following file: TPREF_Full_Paper_Template.docx, on our site.

Volume VIII, Issue 2(16), Winter 2017

DOI: http://dx.doi.org/10.14505/tpref.v8.2(16).06

AGGREGATION WITH SEQUENTIAL INDIVISIBLE AND CONTINUOUS LABOR SUPPLY DECISIONS AND AN INFORMAL SECTOR

Aleksandar VASILEV
Independent Researcher, Bulgaria
alvasilev@yahoo.com

Suggested Citation:

Vasilev, A. (2017). Aggregation with Sequential Indivisible and Continuous Labor Supply Decisions and an Informal Sector, *Theoretical and Practical Research in Economic Field*, (Volume VIII, Winter), 2(16): 143-147. DOI:10.14505/tpref.v8.2(16).06. Available from: http://journals.aserspublishing.eu/tpref.

Article's History:

Received August 2017; *Revised* September 2017; *Accepted* October 2017.

Abstract:

The purpose of this paper is to explore the problem of non-convex labor supply decision in an economy with both discrete and continuous labor decisions. In contrast to the setup in Vasilev (2016a), here each household faces a sequential labor market choice - an indivisible labor supply choice in the market sector, and conditional on non-working in the official sector, a divisible hours choice in the informal sector. We show how lotteries as in Rogerson (1988) can again be used to convexify consumption sets, and aggregate over individual preferences. With a mix of sequential discrete and continuous labor supply decisions, aggregate disutility of non-market work becomes separable from market work, and the elasticity of the latter increases from unity to infinity.

Keywords: Aggregation; Indivisible labour; Sequential Lotteries; Discrete-continuous mix; Informal economy; Non-convexities

JEL Classification: E1; J22; J46

1. Introduction and Motivation

The purpose of this paper is to explore the problem of non-convex labor supply decision in an economy with both discrete and continuous labor decisions, and explicitly perform the aggregation presented in Vasilev (2015a) without a formal proof, and thus provide - starting from micro-foundations - the derivation of the expected utility functions used for the aggregate household. In contrast to the setup in Vasilev (2016a), here each household faces a sequential labor market choice - an indivisible labor supply choice in the market sector, and conditional on non-working in the official sector, a divisible hours choice in the informal sector. Such a modelling choice is along the lines of Vasilev (2016b, 2016c). The novelty relative to those studies is that the earlier setups were dealing with private-public sector and full-time work vs. overtime, respectively, while here the focus is on the market vs unofficial work margin. In this paper, we show how lotteries as in Rogerson (1988) can still be used to convexify consumption sets, and aggregate over individual preferences. With a mix of sequential discrete and continuous labor supply decisions, aggregate disutility from informal sector work becomes separable from market work, and the elasticity of the latter increases from unity to infinity

2. Model Setup

The theoretical setup follows to a great extent Vasilev (2015a). The economy is static, there is no physical capital, and agents face a sequential convex decision, followed by a continuous labor supply decision in a two-sector economy. Since the focus is on a one-period world, the model abstracts away from technological progress, population growth and uncertainty. There is a large number of identical one-member households, indexed by *i* and

distributed uniformly on the [0,1] interval. In the exposition below, we will use small case letters to denote individual variables and suppress the index *i* to save on notation.

2.1. Description of the Model

Each household maximizes the following utility function:

$$U(c,l)=\ln c+\alpha\ln l, \quad (1)$$

where *c* denotes consumption of market output, *l* is the leisure enjoyed by each individual household, and $\alpha>0$ is the relative weight attached to utility of leisure. Each household is endowed with a time endowment of unity, which can be split between hours worked in either the official sector, h_m, hours worked in the informal economy ("black market"), h_b, and leisure *l*, so that

$$h_m+h_b+l=1 \quad (2)$$

The households make a sequential labor supply choice: The first is whether to work full-time in the market sector, or not at all. In other words, $h_m \in \{0;\bar{h}\}$. Conditional on not working in the market sector, a household may decide to go and work in the grey sector, where it can supply any number of hours, *i.e.* $h_b \in [0,1]$. That is, the first labor choice is indivisible, while the second is divisible. Also, it will be assumed that h_b=0 whenever $h_m=\bar{h}$, or a household employed full-time in the market sector would choose not to supply any hours in the grey economy. This assumption is put in place to guarantee that each worker can only participate in one of the production sectors. Next, the hourly wage rate in the official ("market") sector and the implicit rate in the informal economy ("black market") sectors are denoted by w^m and w^b, respectively. Finally, the households own the firm in the market economy, and are entitled an equal share of the profit (π).

The problem faced by a household that decides to work full-time in the market sector is then to set $h_m=\bar{h}$ and enjoy

$$U^m=\ln(w^m\bar{h}+\pi)+\alpha\ln(1-\bar{h}), \quad (3)$$

while a household that decides not to work in the market sector chooses $h_b \in [0,1]$ to maximize its utility function

$$\max_{h_b} U^b=\ln(w^b h_b+\pi)+\alpha\ln(1-h_b) \quad (4)$$

The optimal labor choice in the grey economy is then characterized by the following first-order condition:

$$\frac{w^b}{w^b h_b+\pi}=\frac{\alpha}{1-h_b}, \quad (5)$$

or

$$h_b=\frac{w^b-\alpha\pi}{(1+\alpha)w^b} \quad (6)$$

That is, optimal choice of hours worked in the informal economy is a function of both the wage and profit rate in the official sector, which the household takes as given (and which in equilibrium would depend on the aggregate hours supplied to the firm operating in the market sector; therefore, non-market hours are a function of total market hours).

2.2. Stand-in firm: market sector

There is a representative firm in the model economy, which operates in the market sector. It produces a homogeneous final product using a production function that requires labor H_m as the only input. For simplicity, output price will be normalized to unity. The production function $f(H_m)$ features decreasing returns to scale: $f''(H_m)<0$, $f'(H_m)>0$, $f'(0)=\infty$, $f'(\bar{h})=0$. The representative firm acts competitively by taking the wage rate *w* as given and chooses H_m to maximize profit:

$$\pi=f(H_m)-w^m H_m \qquad \text{s.t. } 0 \le H_m \le \bar{h}\,. \tag{7}$$

In equilibrium, there will be positive profit, which follows from the assumptions imposed on the production function.

2.3. Stand-in firm: unofficial sector

Each worker in the unofficial sector has access to an individual concave production function ("backyard technology") that uses only labor, $g(h_b)$, where $g'(h_b)>0$, $g''(h_b)<0$, $g'(0)=\infty$, $g'(1)=0$. Each firm in the unofficial sector will then hire labor h_b in every period to maximize static profit

$$\max_{h_b} g(h_b)-w^b h_b \qquad \text{s.t. } 0\le h_b\le 1. \tag{8}$$

With free entry, there are zero profits in the sector, hence the implicit wage w^b in the unofficial sector equals the average product of labor, *i.e.*

$$w^b=\frac{g(h^b)}{h_b}. \tag{9}$$

2.4. Decentralized competitive equilibrium (DCE): Definition

A DCE is defined by allocations $\{c_m,c_b,h_m,h_b\}$, wage rates $\{w,w^b\}$, and aggregate profit $(\Pi=\pi)$ s.t. (1) all households maximize utility; (2) the stand-in firm in the unofficial sector maximizes profit; (3) the implicit wage rate in the unofficial sector is such that profits in the grey economy are zero; (4) all markets clear.

3. Characterization the DCE and derivation of the aggregate utility function

It will be shown that in the DCE, if it exists, only some of the households will be employed in the official sector and work full-time, while the rest will fo to the unofficial sector and chose how many hours to work. Following the arguments in Rogerson (1988) and Hansen (1985), it can be easily shown that polar cases in which each household either works in the official, or in the unofficial sector, cannot not be equilibrium outcomes. Therefore, it must be the case that a proportion of the agents in the economy are working in the unofficial sector, while the rest will be supplying labor services in the official sector. Denote this mass of officially employed by λ, and the officially unemployed by $1-\lambda$. Workers in the official sector will receive consumption c_m, while those working in the unofficial sector will consume c_b. Note that λ can be interpreted also as the probability of being chosen to work in the unofficial sector: This probability is determined endogenously in the model, as workers would seek for the optimal balance between the net return from working across the sectors (at the margin). No matter of the employment outcome in the market sector, ex post every household enjoys the same utility level. Thus, in equilibrium $H_m=\lambda\bar{h}$. From the firm's point of view then the market wage is set equal to:

$$w^m = f'(\lambda\bar{h}) \tag{10}$$

Firm's profit is then

$$\pi=\Pi=f(\lambda\bar{h})-f'(\lambda\bar{h})\lambda\bar{h}>0\,, \tag{11}$$

which follows from the dreasing returns to scale featured by the production function. Next, to show that the DCE actually exists, it is sufficient to show the existence of a fixed point $\lambda\in(0,1)$ by analyzing a non-linear equation using the fact that in equilibrium utility is the same for all households. Note that from the result that grey-sector hours are a function of total market hours, it follows now that unofficial hours are a function of the proportion of households employed in the market sector (since $\bar{h}$ is now a parameter). Furthermore, using the Implicit Function Theorem on the first-order condition for h_n, we can show $dh_n/d\lambda>0$, which follows from the complementarity between the two types of income: from labor in the unofficial sector, and the capital income from the claim on profits. In other words, the more people work in the official sector, the lower the profits are; profit is zero if everyone works in the market sector. This in turn stimulates the household to work more in the unofficial

sector in order to increase total income and achieve higher consumption in the household's utility function (while also trying not to decrease utility from leisure that much).

It is trivial to show that everyone working in the market sector (λ=1) is not an equilibrium, since then w^m=0. From the ex-ante symmetry assumption for households, market consumption would be the same for both market workers and those not selected for work in the official sector, while the latter would enjoy higher utility out of leisure (holding h_m fixed), hence there is no benefit of working. Similarly, nobody working in the market sector (λ=0) is not an equilibrium outcome either, since the firm would then offer a very high wage for the first unit of labor, and by taking a full-time job a marginal worker could increase his/her utility a lot.

Thus, if there is a DCE, then it must be that not all households would receive the same consumption bundle. If $\lambda \in (0,1)$ is an equilibrium, then total utility for households that work in the market sector should equal to the utility of households that do not supply any hours in the market sector. This equation is monotone in λ, as the utility function is a sum of monotone functions, and the $h_b(\lambda)$ term is quantitatively small. Thus, we can explore the behavior of that function (the difference between the utility of working in the market sector and working in the grey economy) as we let λ vary in the (0,1) interval. As $\lambda \to 0$, the left-hand-side dominates (utility of working is higher), while when $\lambda \to 1$ the right-hand-side dominates (utility of not working is higher), where the results follow from the concavity of the utility functions and the production technologies. In addition, from the continuity of those functions, $\exists \lambda \in 0,1)$, which is consistent with equilibrium. The unique value of λ follows from the monotonicity of the utility and production functions. Let c_m^* and c_b^* denote equilibrium consumption allocations of individuals selected for work in the market sector, and those who will work in the informal sector.

Given the indivisibility of the labor supply in the market sector, the equilibrium allocation obtained above is not Pareto optimal, as demonstrated in Rogerson (1988). More specifically, a social planner (SP) could make everyone better off by using an employment lottery in the first stage and choosing the fraction λ of individual households to work in the market sector and give everyone consumption $\lambda c_m^* + (1-\lambda)c_b^*$. In order to show this, we need to check that such an allocation is feasible, and that it provides a higher level of total utility. Showing feasibility is trivial as total market labor input and total consumption are identical to the corresponding individual equilibrium values.

Next, we will show that the new allocation, which is independent of household's employment status in the market sector, makes households better off since it generates higher utility on average. This is indeed the case, where the strict inequality follows from the convexity of the CES aggregation and the concavity of the logarithmic function. Thus, the SP is indeed giving in expected utility terms an allocation that is an improvement over the initial equilibrium allocation. If households can pool income together and doing so, they will be able to equalize consumption across states, *i.e,* $c = c_m^* = c_b^*$. Substituting the expressions for consumption and total hours in the aggregate utility and rearranging terms yields

$$\ln c + AH_m + B\ln(1-h_b). \qquad (12)$$

where A >0 and B>0 are functions of model parameters. The resulting aggregate utility function is of an interesting and novel form. On the aggregate, when each household faces a sequence of indivisible and divisible labor choices, the representative agent obtained from the aggregation features different preferences of work: as in Vasilev (2016a) with home production, the disutility of work in the market sector is now linear, while the disutility of labor in the informal sector h_b is logarithmic as in the individual utility function, built separated from market hours. The split of the two types of labor is driven by the fact that market labor was indivisible, while hours supplied in the informal sector were divisible. In addition, the sequential discrete-continuous labor supply decision could be quantitatively important for the transmission of business cycle shocks and welfare effects of different government policies. However, such investigations are left beyond the scope of this note. In general, in terms of moment matching at business-cycle frequencies, models with indivisible market hours, *e.g.* the original Hansen (1985) and Vasilev (2015c) with home production sector, are an improvement over identical models with divisible labor supply in the official sector. Therefore, the model could be useful to policymakers interested in implementing labor policies in countries with large informal sectors.

Conclusions

This paper explored the problem of non-convex labor supply decision in an economy with both discrete and continuous labor decisions. In contrast to the setup in Vasilev (2016a), here each household faces a sequential labor market choice - an indivisible labor supply choice in the market sector, and a divisible hours choice in the informal sector. The novelty was that the aggregated utility function produced interesting non-linearities that were not present at individual level. With a mix of sequential discrete and continuous labor supply decisions, the disutility from informal sector hours work becomes separable from official sector hours, and the elasticity of the latter increases from unity to infinity.

References

[1] Hansen, G. 1985. Indivisible labor and the business cycle. *Journal of Monetary Economics* 16: 309–327.

[2] Rogerson, R. 1988. Indivisible labor, lotteries and equilibrium. *Journal of Monetary Economics* 21: 3–16.

[3] Vasilev, A. 2015a. Welfare gains from the adoption of proportional taxation in a general-equilibrium model with a grey economy: the case of Bulgaria's 2008 flat tax reform. *Economic Change and Restructuring*, 48: 169-185. DOI: 10.1007/s10644-015-9159-5

[4] Vasilev, A. 2015b. Welfare Effect of Flat Income Tax Reform: The Case of Bulgaria, *Eastern European Economics*, 53: 205-220. DOI: 10.1080/00128775.2015.1033364

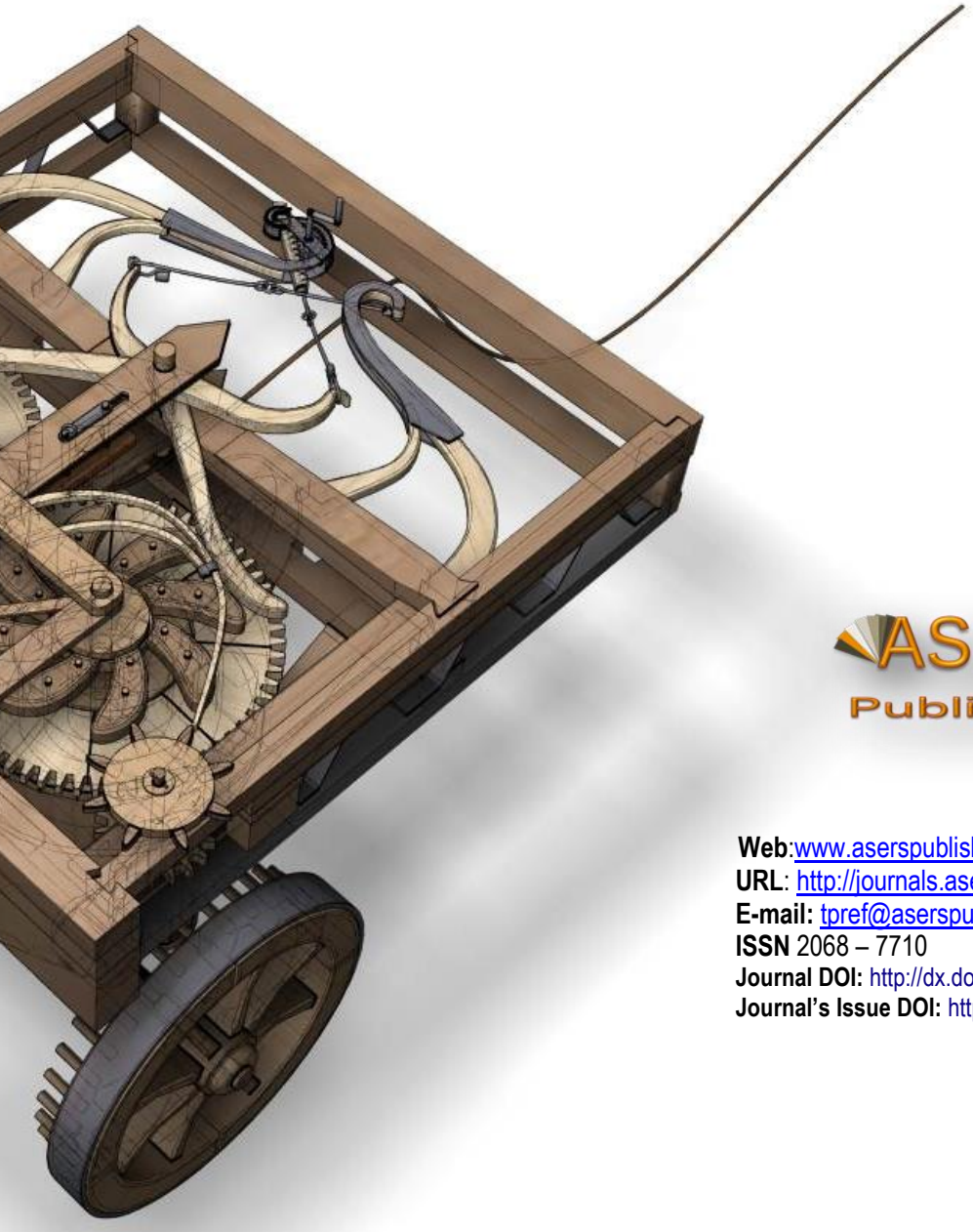
[5] Vasilev, A. 2015c. RBC models and the hours-wages puzzle: puzzle solved! *Economia Journal* 41: 117-130.

[6] Vasilev, A. 2016a. Aggregation with a mix of indivisible and continuous labor supply decisions: The case of home production, *International Journal of Social Economics* 43: 1507-1512. Available at: http://dx.doi.org/10.1108/IJSE-04-2015-0098

[7] Vasilev, A. 2016b. Aggregation with sequential non-convex public- and private-sector labor supply decisions, *Theoretical and Practical Research in Economics ASERS* 14: 173-178. Available at: http://dx.doi.org/10.14505/tpref

[8] Vasilev, A. 2016c. Straight time and Overtime: A Sequential-Lotteries Approach, *Theoretical and Practical Research in Economics ASERS* 13: 81-84. Available at: http://dx.doi.org/10.14505/tpref

ASERS



ASERS Publishing

Web:www.aserspublishing.eu
URL: http://journals.aserspublishing.eu/tpref
E-mail: tpref@aserspublishing.eu
ISSN 2068 – 7710
Journal DOI: http://dx.doi.org/10.14505/tpref
Journal's Issue DOI: http://dx.doi.org/10.14505/tpref.v8.2(16).00