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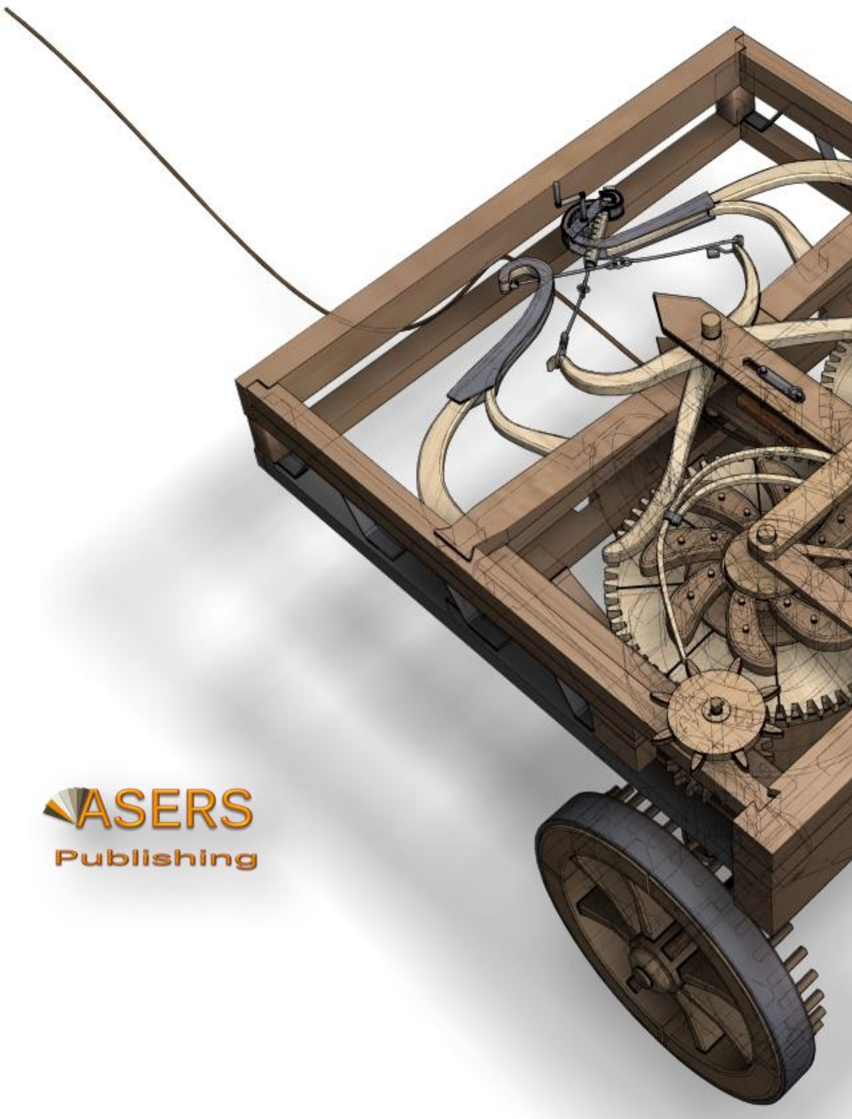

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# Contents:

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# OPINIONS ON THE THEORIES OF SAVAGE AND DE FINETTI

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**Abstract:**

*Feduzi, Runde and Zappia (2012, 2014, 2017) have claimed repeatedly that de Finetti and Savage formally allowed imprecise , indeterminate, non-additive probabilities to be used by decision makers in their normative theory of decision making .The only way that non additivity can be formally incorporated into a decision theory is by the use of a variable similar to Keynes's w or Ellsberg's ρ. Nowhere in anything written by de Finetti and/or Savage in their lifetimes does any such variable appear in their formal theoretical analysis or in any of their supporting axioms. Savage stated that he did not know how to integrate such a variable that would take account of vagueness, into the additive probability measure that represented his formal, normative theory. Feduzi, Runde and Zappia (2012, 2014, 2017) have misinterpreted the de Finetti and Savage concerns and sympathy for vacillating ,indecisive decision makers, who were confronted with vague and ambiguous evidence, with their recognition that defining one's personal probabilities can be a difficult task ,with the erroneous and mistaken belief that de Finetti and Savage incorporated a role for imprecise and indeterminate probabilities within their formal, subjectivist, normative theory of probability and their SEU decision theory ,which rests completely on additive probability.*

**Keywords:** indeterminate; imprecise and precise probabilities; interval valued probabilities

**JEL Classification:** B10; B12; B16; B20; B22

## 1. Introduction

This paper is organized in the following manner. Section Two presents Savage's formal, normative theory of subjective probability that underlies his normative theory to maximize subjective expected utility (SEU). This formal, normative theory is based on several axioms. A brief summary of these axioms and the normative rule to maximize SEU is covered. There is no role in the formal, normative theory for imprecise, and certainly not indeterminate, probabilities that are interval valued. Savage's normative theory is representative of de Finetti 's normative theory and of any developments of the normative theory carried out by means of co-authorship between themselves. Section Three looks at Keynes's mathematical representation and incorporation of sub and non-additivity in his conventional coefficient of weight and risk, c, and his corresponding, explicit introduction of non-additivity into Part II of the A Treatise on Probability. Section Four demonstrates that NOWHERE in any of the articles published by Feduzi, Runde and Zappia (2012, 2014, 2017) do they ever show where Savage, de Finetti, and Savage and de Finetti, supposedly introduced into their formal, normative theory ANY role for sub or non-additive probabilities.

## 2. Savage's Formal, Normative Theory of Maximizing SEU

Savage's formal, normative theory, to maximize SEU using additive probabilities, is based on the following seven axioms that are presented in summary form below. All seven of these axioms were accepted by de Finetti.

*Axiom A.1* states that "the preference relation is transitive and that all acts are comparable." (Karni 2005, 5; Savage 1954).

*Axiom A.2* states that "the preference between acts depend solely on the consequences in states in which the payoffs of the two acts being compared are distinct." (Karni 2005, 5; Savage 1954).

*Axiom A.3* states that" the ordinal ranking of consequences is independent of the event and the act that yield them." (Karni 2005, 5; Savage 1954).

*Axiom A.4* states that "the betting preferences be independent of the specific consequences that define the bets." (Karni 2005, 5-6; Savage 1954)

*Axiom A.5* states that "the decision-making problem and the qualitative probabilities (are) nontrivial by ruling out that the decision maker is indifferent among all acts." (Karni 2005, 6; Savage 1954).

*Axiom A.6* states that "no consequence is either infinitely better or infinitely worse than any other consequence". It "implies that there are infinitely many states of the world and that if there exists a probability measure representing the decision maker's beliefs, it must be nonatomic. Moreover, the probability measure is defined on the set of all events, hence it is finitely additive (that is, for every event $E$, $0 \leq \pi(\mathrm{E}) \leq 1, \pi(\mathrm{S}) = 1$ and for any two disjoint events, $E$ and $E_0$, $\pi(\mathrm{E} \cup E_0) = \pi(\mathrm{E}) + \pi(E_0)$)." (Karni 2005, 6; Savage 1954).

*Axiom A.7* states that there is a "is a monotonicity requirement asserting that if the decision maker considers an act strictly better (worse) than each of the payoffs of another act on a given non-null event, then the former act is conditionally strictly preferred (less preferred) than the latter." (Karni 2005, 6; Savage 1954).

Savage's formal normative theory, Subjective Expected Utility (SEU) states that "Let < be a preference relation on F. Then the following two conditions are equivalent:

(i) "<" satisfies axioms A.1-A.7.

(ii) There exists a unique, non-atomic, finitely additive, probability measure π on S such that $\pi(\mathrm{E}) = 0$ if and only if E is null, and a bounded, unique up to a positive affine transformation, real-valued function u on C …" (Karni 2005, 7-8; Savage 1954)

Thus, the formal, normative goal of the decision maker is to Maximize SEU.

There is absolutely no role in this formal, normative theory for non-additive or sub additive probabilities (imprecise or indeterminate interval valued probabilities) whatsoever.

## 3. Keynes's normative theory of indeterminate and imprecise probability based on Boole's upper and lower probabilities.

Keynes's logical theory of probability was developed technically in Part II of the *A Treatise on Probability* (TP 1921). In Chapters 10-14 of the TP, Keynes provides the first formal foundation in mathematical logic for the probability calculus based on additivity. However, at the same time, he introduces non-additivity explicitly into the discussion. In chapters 15-17 of the TP, Keynes provides the reader with 17 worked out problems, based on Boole's lower-upper interval valued probability approach presented on pp.265-268 of *The Laws of Thought* in 1854, that show how an interval valued approach to probability works. Many of the probability intervals are indeterminate, meaning that the additional, missing evidence is not going to become available to the decision maker in the future. Boole was the first to introduce indeterminate probability. In chapter 29 of the TP, Keynes introduces imprecise probability based on the application of Chebyshev's Inequality as a lower bound.

The introduction of non-additivity by Keynes is explicit. Apparently, no economist read this material in the 20th century. Let us start by providing Bertrand Russell's assessment of Part II of the TP. (see Russell 1931, 1932, 1959, 1996):

"Part II. "Fundamental Theorems", gives the definitions and axioms upon which the formal reasoning of the rest of the book is based, together with some propositions readily derivable from the definitions and axioms. It is impossible to summarize this Part, since it is already as condensed as possible. I cite only, as essential to the whole formal structure, the definitions of addition and multiplication: Addition: $\mathrm{ab/h} + \mathrm{a}\bar{b}\mathrm{/h} = \mathrm{a/h}$ (where $\bar{b}$ stands for not $-b$). Multiplication: $\mathrm{ab/h} = \mathrm{a/bh} \cdot \mathrm{b/h} = \mathrm{b/a\,h} \cdot \mathrm{a/h}$. Thus, addition and multiplication are not defined for any pair of probabilities, but only for such as have certain forms. This, of course, is connected with Mr. Keynes's view that probabilities are in general non-numerical. It is surprising how successful he is in raising his mathematical superstructure upon foundations which might have been thought inadequate to support" (Russell 1922, 121; boldface added).

Thus, Russell immediately understood that Keynes was introducing non-additivity explicitly. Note Russell's correct assessment above that Keynes's probabilities are generally non-additive. The probabilities of Savage and de Finetti are always additive (see Vicig and Seidenfeld 2012).

Keynes repeats his concerns about the general non-additivity of probability all through Part II of the TP.

*Keynes's First statement* concerning non-additivity is that

"IX. The Definition of Addition: $\mathrm{ab/h} + \mathrm{a}\bar{b}/\mathrm{h} = \mathrm{a/h}$, where $\bar{b}$ stands for the contradictory of b.

X. The Definition of Multiplication: ab/h = a/bh·b/h = b/ah·a/h. The symbolical development of the subject largely proceeds out of these definitions of Addition and Multiplication. It is to be observed that they give a meaning, not to the addition and multiplication of any pairs of probabilities, but only to pairs which satisfy a certain form. The definition of Multiplication may be read: 'the probability of both a and b given h is equal to the probability of a given bh, multiplied by the probability of b given h. (Keynes 1921, 120-121).

*Keynes's Second statement*:

"6. Addition and Multiplication. If we were to assume that probabilities are numbers or ratios, these operations could be given their usual arithmetical signification. In adding or multiplying probabilities we should be simply adding or multiplying numbers. But in the absence of such an assumption, it is necessary to give a meaning by definition to these processes. I shall define the addition and multiplication of relations of probabilities only for certain types of such relations. But it will be shown later that the limitation thus placed on our operations is not of practical importance. We define the sum of the probable relations $\mathrm{ab/h}$ and $\mathrm{a}\bar{b}/\mathrm{h}$ as being the probable relation a/h; and the product of the probable relations a/bh and b/h as being the probable relation ab/h. That is to say:

IX. $\mathrm{ab/h} + \mathrm{a}\bar{b}/\mathrm{h} = \mathrm{a/h}$ Def.

X. ab/h = a/bh · b/h = b/ah · a/h Def.

Before we proceed to the axioms which will make these symbols operative, the definitions may be restated in more familiar language. IX. may be read: "The sum of the probabilities of "both a and b" and of "a but not b", relative to the same hypothesis, is equal to the probability of *a* relative to this hypothesis." X. may be read: "The probability of "both a and b", assuming h, is equal to the product of the probability of b, assuming h, and the probability of a, assuming both b and h." Or in the current terminology we should have: "The probability that both of two events will occur is equal to the probability of the first multiplied by the probability of the second, assuming the occurrence of the first." It is, in fact, the ordinary rule for the multiplication of the probabilities of events which are not 'independent'. It has, however, a much more central position in the development of the theory than has been usually recognized." (Keynes 1921, 134-135)

*Keynes's Third statement:*

"Thus, we have to introduce as definitions what would be axioms if the meaning of addition and multiplication were already defined. In this latter case, we should have been able to apply the ordinary processes of addition and multiplication without any further axioms. As it is, we need axioms in order to make these symbols, to which we have given our own meaning, operative. When certain properties are associated, it is often more or less arbitrary which we take as defining properties and which we associate with these by means of axioms. In this case I have found it more convenient, for the purposes of formal development, to reverse the arrangement which would come most natural to common sense, full of preconceptions as to the meaning of addition and multiplication. I define these processes, for the theory of probability, by reference to a comparatively unfamiliar property, and associate the more familiar properties with this one by means of axioms. These axioms are as follows:

(iv.) If P, Q, R are relations of probability such that the products PQ, PR and the sums P + Q, P + R exist, then:

(iv.a) If PQ exists, QP exists, and PQ = QP.

If P + Q exists, Q + P exists and P + Q = Q + P." (Keynes 1921, 136-137)

*Keynes's Fourth statement:*

"A meaning has not been given, it is important to notice, to the signs of addition and multiplication between probabilities in all cases. According to the definitions we have given, P + Q and PQ have not an interpretation whenever P and Q are relations of probability, but in certain conditions only. Furthermore, if P+Q = R and Q = S+T, it does not follow that P + S + T = R, since no meaning has been assigned to such an expression as P + S + T. The equation must be written P + (S + T) = R, and we cannot infer from the foregoing axioms that (P + S) + T = R. The following axioms allow us to make this and other inferences in cases in which the sum P + S exists, i.e. when P + S = A and A is a relation of probability…

in every case in which the probabilities ….

exist, *i.e.*, in which these sums satisfy the conditions necessary in order that a meaning may be given to them in the terms of our definition.

….. if the sum R + S and the products

PR and PS exist as probabilities.

"7. From these axioms, it is possible to derive a number of propositions respecting the addition and multiplication of probabilities. They enable us to prove, for instance, that if P + Q = R + S then P-R = S-Q, provided that the differences P-R and S-Q exist; and that (P+Q)(R+S) = (P+Q)R+(P+Q)S = [PR+QR]+[PS+QS] = [PR+QS]+[QR+PS], provided that the sums and products in question exist. In general, any rearrangement which would be legitimate in an equation between arithmetic quantities is also legitimate in an equation between probabilities, provided that our initial equation and the equation which finally results from our symbolic operations can both be expressed in a form which contains only products and sums which have an interpretation as probabilities in accordance with the definitions. If, therefore, this condition is observed, we need not complicate our operations by the insertion of brackets at every stage, and no result can be obtained as a result of leaving them out, if it is of the form prescribed above, which could not be obtained if they had been rigorously inserted throughout. We can only be interested in our final results when they deal with actually existent and intelligible probabilities." (Keynes 1921, 137)

*Keynes's Fifth statement:*

"1. The possibility of numerical measurement, mentioned at the close of Chapter III., arises out of the Addition Theorem (24.1). In introducing the definitions and the axiom, which are required in order to make the convention of numerical measurement operative, we may appear, as in the case of the original definitions of Addition and Multiplication, to be arguing in an artificial way. This appearance is due, here as in Chapter XII., to our having given the names of addition and multiplication to certain processes of compounding probabilities in advance of postulating that the processes in question have the properties commonly associated with these names. As common sense is hasty to impute the properties as soon as it hears the names, it may overlook the necessity of formally introducing them. "(Keynes 1921, 158)

*Keynes's Sixth statement:*

"3. From these axioms and definitions combined with those of Chapter XII., it is easy to show (certainty being represented by unity and impossibility by zero) that we can manipulate according to the ordinary laws of arithmetic the "numbers" which by means of a special convention we have thus introduced to represent probabilities." (Keynes 1921, 159).

*Keynes's Seventh statement:*

"Many probabilities—in fact all those which are equal to the probability of some other argument which has the same premiss and of which the conclusion is incompatible with that of the original argument—are numerically measurable in the sense that there is some other probability with which they are comparable in the manner described above. But they are not numerically measurable in the most usual sense, unless the probability with which they are thus comparable is the relation of certainty. The conditions under which a probability a/h is numerically measurable and equal to q/r are easily

seen. It is necessary that there should exist probabilities $a_1/h_1$,

$a_2/h_2 \ldots, a_q/h_q \ldots, a_r/h_r$, such that

$a_1/h_1 = a_2/h_2 = \ldots = a_q/h_q = \ldots = a_r = h_r$,

$a/h = \sum a_s/h_s$, and $\sum a_s/h_s = 1$." (Keynes 1921, 159-160.)

Thus, exact numerical measurability requires additivity. Inexact numerical measurability will be non additive because, in general, you can't add intervals together so that the interval probabilities sum or equal to 1 .

Keynes provides the reader in chapter 26 of the TP with a formal, mathematical theory that explicitly integrates his weight of the evidence variable, w, into an explicit decision theory. Note that if w=0, then no probability exists.

Keynes's technical analysis is presented on p.315 and ft. 2 on p. 315 of the TP. The heart of Keynes's theory is that decision makers use decision weights that are non-additive or sub or super additive (sub-proportional or super-proportional), as opposed to the additive probability concept that assumes linearity. Keynes called his decision weight a "conventional coefficient of risk and weight, c" and presented it as

$$c=2pw/ [(1+q)\ (1+w)], \tag{1}$$

where p is the probability of success, q is the probability of failure, p+q =1, and w represents the weight of the relevant evidence, defined on the interval [0, 1]. That is, w measures the completeness of the relevant evidence upon which the probability estimates for p and q are based. Keynes defined w in the first paragraph of Chapter 6 on p. 71 of the TP to be a measure based on the absolute amount of relevant knowledge and relevant ignorance just as a probability measure was based on the total amount of favorable and unfavorable evidence. Note that w is not logical relation or entity V.V is the logical entity or relation w is a mathematical variable.

The conventional coefficient of risk and weight is easily rewritten as

$$c=p\ [1/(1+q)][2w/(1+w)] \tag{2}$$

In this rewritten version, c consists of the usual linear, additive p multiplied by two weights-the first weight,

$$[1/(1+q)], \tag{3}$$

deals with the problem of nonlinear probability preferences, while the second weight,

$$[2w/(1+w)], \tag{4}$$

deals with the problem of sub and non-additivity. The conventional coefficient, c, is directly connected to Keynes's interval valued probability discussed in Parts I, II, and III of the TP. If w=1 and we ignore nonlinear probability preferences, then the probabilities become additive. If w<1, any conventional coefficient can be translated into an interval valued probability.

Thus, Keynes argues that a decision maker should normatively take into consideration w. Define A to be the outcome and U(A) to be a utility function of A.

Then Keynes's formal, normative theory in the TP is to

Maximize either

$$cA \text{ or } cU(A) \tag{5}$$

where, given Keynes's discussions of the St. Petersburg Paradox in chapter 26 of the TP, U(A) is thrice differentiable.

Savage' s formal normative theory is to

$$\text{Maximize } pU(A), \tag{6}$$

where U(A) is a von Neumann-Morgenstern utility function and p is an additive subjective probability.

The reader should note that Keynes 's theory (see Arthmar and Brady, 2012, 2016 and 2017; Brady, 1993, 1994, 2004a, 2004b) built on Boole (1854) and incorporated Savage's theory as a special case. Savage's theory, as well as de Finetti's, and Savage and de Finetti, assumes additivity and linear probability preferences. Keynes's theory allows for additivity or non-additivity, as well as linear probability preferences or nonlinear probability preferences

## 4. Feduzi, Runde and Zappia (2012, 2014, 2017)

Feduzi, Runde, and Zappia (2012, 2014, 2017) claim that Savage (see 1954, 1962, 1967, 1971), de Finetti (1930, 1931, 1955, 1964, 1967a, 1967b, 1967c), and/or Savage and de Finetti (1962) incorporated into their theoretical, formal, normative theory an explicit role for interval valued probability that was both imprecise and indeterminate:

"Viewed in this light, the possibility that there may be room for imprecision in probabilistic reasoning after all in de Finetti's *theoretical construct*, and in particular in its development and defense pursued jointly with Savage, is therefore an intriguing one." (Feduzi, Runde and Zappia 2017, 2; boldface and underline by the author)

The theoretical construct developed by Savage, de Finetti, and Savage and de Finetti as their normative theory was TO ALWAYS IN THEIR LIFETIMES maximize SEU. It is impossible to find any "…room for imprecision in probabilistic reasoning after all in de Finetti's *theoretical construct*."

Feduzi, Runde and Zappia (2012, 2014) put forth the claim that Savage, de Finetti, or Savage and de Finetti supported the use of interval valued probability in their theoretical, formal, normative theory:

"Relying on usually overlooked excerpts of de Finetti's works commenting on Keynes, Knight and interval valued probabilities, we argue that de Finetti suggested ***a relevant theoretical case for uncertainty to hold even when individuals are endowed with subjective probabilities***". (Feduzi, Runde and Zappia 2014, 1; boldface added by the author) and

"The purpose of this article is to show that Bruno de Finetti, famous as one of the three founding fathers of the subjective approach to probability assumed by the standard model, actually made ***a theoretical case for uncertainty within the subjectivist approach".*** *(*Feduzi, Runde and Zappia 2012, 329; boldface added by the author).

Of course, this is impossible to show because nowhere in any published work done by Savage, or de Finetti, or Savage and de Finetti, did they ever change their formal, theoretical, normative rule to maximize SEU. Feduzi, Runde and Zappia (2012, 2014, 2017) do not provide one piece of evidence that they did.

Feduzi, Runde and Zappia (2012, 2014, 2017) have confused and conflated the separate prescriptive, normative and descriptive roles that are used in decision theory .de Finetti, de Finetti and Savage, and Savage made it clear that there would and could be many descriptive violations of their normative theory. However, this was due to problems of ignorance and error on the part of the decision maker in failing to deal properly with vagueness or uncertainty.

Feduzi, Runde and Zappia (2012, 2014, 2017) provide evidence that Savage or de Finetti or Savage and de Finetti acknowledged that there would be significant descriptive violations of their theory. They prescribed their formal, theoretical normative theory for all decision makers who accepted their axioms. They never stated that all decision makers must accept their axioms. However, their formal theory to maximize SEU using additive probability never changed in their lifetimes. If Feduzi, Runde and Zappia continue to dispute this, then it is time for them to show exactly where in the published literature in the 20th century that changes to their formal, theoretical construct took place. If they can nor do so, they must withdraw their claims.

## Conclusions

Feduzi, Runde and Zappia (2012, 2014, 2017) provide no evidence whatsoever that Savage, or de Finetti, or Savage and de Finetti ever altered their theoretical formal, SEU construct. That construct was to Maximize SEU. The probabilities were at all times additive probabilities. These probabilities were obtained through proper scoring rules and elicitation techniques that aimed to explicitly eliminate vagueness, uncertainty, and ambiguity, and not deal with such issues. Their remedy for vagueness, uncertainty, and ambiguity on the part of decision makers was to provide additional training, practice, instruction, exercises and help to vacillating and unsure decision makers in order to help them in eliminating vagueness, uncertainty, and ambiguity. The remedy explicitly excluded any attempt to incorporate vagueness, uncertainty, and ambiguity into any formal, theoretical, normative approach to decision making.

Feduzi, Runde and Zappia (2012, 2014, 2017) are extremely confused. Their confusion results from their severe misinterpretation of discussions made by Savage, or de Finetti, or Savage and de Finetti that deal with descriptive or prescriptive issues. None of their articles provide a single instance where Savage, or de Finetti, or Savage and de Finetti EVER change their formal, theoretical, normative theory to include variables representing imprecise or indeterminate interval valued probability.

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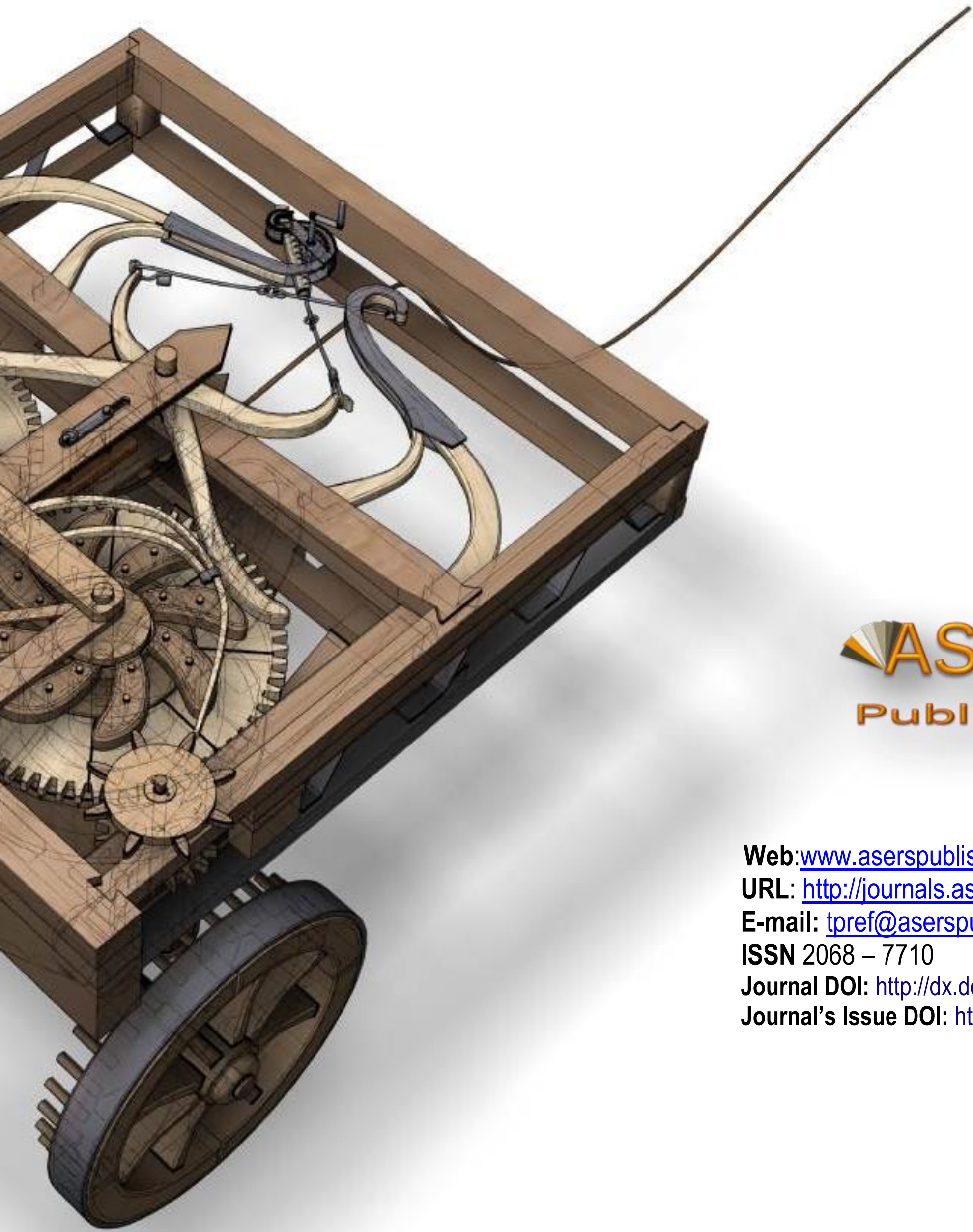

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