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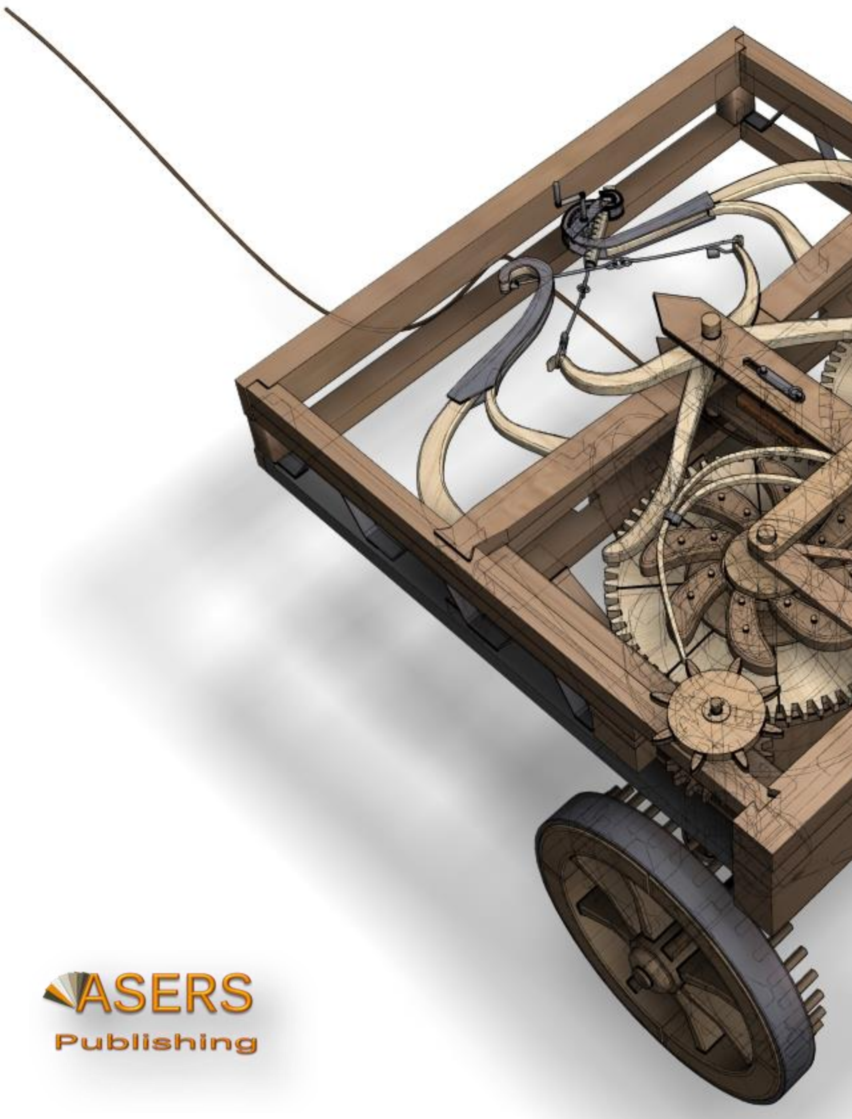

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AGGREGATION WITH TWO-MEMBER HOUSEHOLDS AND HOME PRODUCTION

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Abstract:

This note explores the problem of family labor supply decision in an economy with two-member households, joint home production, and fixed cost of joint labor supply. Even though the labor supply decisions are not indivisible per se, the presence of such fixed cost and partners with unequal labor productivity create non-convexities. The note shows how lotteries as in Rogerson (1988) can again be used to convexify consumption sets, and we perform aggregation over individual preferences. The main result demonstrated in the paper is that aggregate preferences of males do not differ from individual level ones. However, for females, the disutility of non-market work at the aggregate becomes separable from market work, but keeps its original (logarithmic) form, while the female labor elasticity of the market hours' supply increases from unity to infinity.

Keywords: family labor supply; home production; aggregation

JEL Classification: E1; J22; J46

1. Introduction

The standard real business cycle model, *e.g.* Kydland and Prescott (1982), does not capture well labor market behavior. After looking at micro evidence and finding that males and females differ in terms of their labor supply elasticities, Cho and Rogerson (1988) demonstrated that the existence of families, which earlier representative-agent models ignores, might be important at the aggregate level. After all, a typical household is not a one-member entity, but mostly comprises two members. The household usually pools resources together (to diversify risk) and makes joint decisions, aiming to maximize the utility of the family. This note explores the problem of family labor supply as in Cho and Rogerson (1988), where the partners have different productivities, and in case both partners work in the market sector, the family faces a certain fixed cost (*e.g.*, increased inconvenience due to the tight working schedules of both partners). The novelty in this note is the introduction of home production technology as well, where the latter also provides consumption flows. The note explores how the availability of home production interacts with the families' structure and whether this matters for aggregate preferences. After all, non-market production is likely to be important particularly for families, who enjoy it jointly, *e.g.* house cleaning, cooking, etc. In addition, the availability of joint home production technology provides an alternative use of time for each of the members in the household. This note shows that despite the fact that all labor decisions are continuous, the presence of family labor supply fixed cost introduces a non-convexity. Therefore, lotteries as in Rogerson (1988) can again be used to convexify households' consumption sets, and aggregation over individual preferences. The main result demonstrated in the paper is that aggregate preferences of males do not differ from individual level ones. However, for females, the disutility of non-market work at the aggregate becomes separable from market work, but keeps its original (logarithmic) form, while the elasticity of the market hours supply increases from unity to infinity. This result is reminiscent to the one found in Vasilev (2016) in a model with home production and indivisible labor choice in the market sector, but without families.

2. Model Description

2.1. Household problem

The theoretical setup presented in this note is a one-period economy without physical capital. The representation is identical to an economy where the capital has been already chosen optimally. There is a unit measure of identical male individuals (indexed by subscript "1"), and a unit measure of identical female individuals (indexed by subscript "2"). In the paper, "households" and "families" will be used interchangeably. There is no formal "marriage" decision in the model, in the sense that each male is already matched with a female. For simplicity, it will be assumed there is no bargaining within the family, no possibility for divorce and thus no options outside marriage. The household's role is to pool income and to make joint decisions. The preferences of the partners in each family are also the same, and are defined over composite consumption (*c*) and leisure (*l*), and are represented by a separable utility function *u(c,l)* as follows

$$u = \ln(c) + \ln(l),$$

where

c =[a (c_m)^b+(1-a)(c_n)^b]^(1/b),

is, as in McGrattan, Rogerson and Wright (1997), a Constant Elasticity of Substitution (CES) aggregation of market- and non-market ("home") consumption, denoted by c_m and c_n, respectively. Parameters a and $1-a$, where $0 < a < 1$, denote the weights attached to different consumption categories in the aggregate consumption bundle, and parameter $b > 0$ measures the degree of substitutability between market and home production.

Each household member has a unit endowment of time, which can be either supplied in the market sector, used to produce non-market output, or enjoyed as leisure, hence

$$l = 1 - h_m - h_n.$$

Non-market output (y_n) is non-tradable and non-storable consumption good (c_n), and can be produced using labor as follows:

$$y_n = c_n = h_{1,n} + h_{2,n},$$

where each member of each household can supply any number of hours in the non-market sector. Home production is jointly enjoyed by the male and the female in the family, thus $c_n = c_{1,n} = c_{2,n}$. More specifically, non-market output is a public good from the family point of view, while market consumption is a private (and rivalrous) good.

The labor choice in the market sector is also continuous. The hourly wage rate in the market sector is w_1 for males, and w_2 for females, which are taken as given.[36] Finally, each household claims an equal share of the representative firm's profit, denoted by π.[37] The budget constraint that each household faces is then

$$c_{1,m} + c_{2,m} = w_1 h_{1,m} + w_2 h_{2,m} + \pi$$

Household's utility maximization problem is to

$$U = \ln c_1 + \ln c_2 + \ln(1 - h_{1,m} - h_{1,n}) + \ln(1 - h_{2,m} - h_{2,n}) - I(h_{1,m}, h_{2,m})\tau$$

where $I(h_{1,m}, h_{2,m})$ is an index functions, which takes a value of unity in case of a positive labor supply by both parents ($h_{1,m}h_{2,m} > 0$), and zero otherwise. $\tau > 0$ represents the size of the fixed cost incurred when both parents are working.

The problem is to max Eq. (6) subject to (2) - (5). The problem is thus one of choosing $\{(c_{1,m}, h_{1,m}, h_{1,n}), (c_{2,m}, h_{2,m}, h_{2,n})\}$ optimally by taking $\{w_1, w_2, \pi\}$ as given.[38]

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[36] In equilibrium, in order to be consistent with data, we will calibrate the model so that $w_1 > w_2$.

[37] This is a technical assumption, which guarantees positive consumption even for unemployed people.

[38] Note that by choosing $h_{1,n}$ and $h_{2,n}$, the household chooses optimally c_n as well.

2.1 Firms

There is a representative firm producing a homogeneous final consumption good using male and female labor. For simplicity, its price is normalized to unity. As in Cho and Rogerson (1988), it will be assumed that female hours are only an imperfect substitute to male hours, with degree of proportionality $0 < \nu < 1$. This modeling choice reflects difference in productivities as proxied by wages in data, e.g. Reder (1962). Finally, the production function features decreasing returns to scale and is given by

$$Y = F(H_{1,m} + \nu H_{2,m}), F'(.) > 0, F''(.) < 0, F'(0) = \infty, F'(\bar{h}_{1,m} + \nu \bar{h}_{2,m}) = 0,$$

where, as in Cho and Rogerson (1988), $\bar{h}_{1,m}$ and $\bar{h}_{2,m}$ are the optimal market labor supply decisions made by individual males and females, respectively. The last constraint means that the firm faces a capacity constraint: If the male and female in each household work full-time in the market sector, the marginal product of an hour worked is zero. Together with the decreasing returns to scale that would produce positive economic profit (and thus guarantee a positive level of market consumption) in equilibrium.

The firm acts competitively by taking the wage rates $\{w_1, w_2\}$ as given, and chooses hours $\{H_{1,m}, H_{2,m}\}$ employed to maximize profit.

3. Decentralized Competitive Equilibrium (DCE)

A Decentralized Competitive Equilibrium (DCE) is defined by consumption and labor allocations for males and females, wage rates, and aggregate profit $\Pi(= \pi)$ such that (i) all households maximize utility; (ii) the stand-in firm maximizes profit; (iii) all markets clear.

4. Characterizing the DCE

It will be shown that in the DCE, if it exists, only some of the females in the households will be employed, but everyone enjoys the same utility level of consumption.[39] From the firm's problem, the wage rates are

$$w_1 = F'(H_{1,m} + \nu H_{2,m}),$$
$$w_2 = \nu F'(H_{1,m} + \nu H_{2,m}).$$

Thus, $w_1 > w_2$ follows directly from the assumption that female hours are imperfect substitutes for male hours ($\nu \in (0,1)$) in the production of market consumption. Moreover, if there is a DCE in which not all females work, then it must be the case that not all females in the households would receive the same consumption bundle. Denote this mass of employed females by ϕ, and the proportion of unemployed females by $1 - \phi$. Then the total fixed cost resulting from family labor supply equals $\phi\tau$. Note that this setup is very similar to a setup with non-convexities, as in Rogerson (1988). More specifically, even though there is no indivisibility in the individual female labor supply decision sets, due to the presence of fixed costs when both partners work, the setup is isomorphic to a model where the females can only supply $h_{2,m} \in [0, \bar{h}_2]$.

Thus, in equilibrium, aggregate male and female hours are $H_{1,m} = \bar{h}_{1,m}$ and $H_{2,m} = \phi\bar{h}_{2,m}$, respectively, and using those expressions, we can compute equilibrium profit. Next, if ϕ is the equilibrium proportion of females employed, then total utility for females that work in the market sector should equal to the utility of females that do not supply any hours in the market sector. In particular, the females that work will have higher utility of consumption, while those who do not work enjoy higher utility of leisure. Let $c^e_{2,m}$ denote the market consumption of working females, and $c^u_{2,m}$ denote the market consumption of non-working females. Similarly, let c^e_2 denote the composite consumption of working females, and c^u_2 denote the composite consumption of non-working females, where $c^e_{2,m} > c^u_{2,m}$, and $c^e_2 > c^u_2$. Females that work, supply $\bar{h}_{2,m}$ in the market, and females that do not work, supply zero hours in the market.

Next, from the symmetry of the production of the home production output, it follows that the choice for home hours will be symmetric, with $h_n = h_{1,n} = h_{2,n}$. Furthermore, $h_n = h_n(\phi)$, *i.e.*, non-market hours are a function of the proportion of females employed in the market sector. Using the Implicit Function Theorem on the optimality condition for home hours, we can show $\frac{dh_n}{d\phi} > 0$, which follows from the complementarity between the two types of consumption in the household's utility function. To show that the DCE actually exists, it is sufficient to

[39]As demonstrated below, this follows from the difference in productivity (wages). In a way, it can be also inferred from the model setup that the females also bear the fixed cost in case both partners in the family work in the market sector.

show the existence of a fixed point $\phi \in (0,1)$ by analyzing a non-linear equation using that in equilibrium utility has to be the same for all households. First, it is trivial to show that an equilibrium where all females working ($\phi = 1$) is not possible, since by assumption then it follows that $w_1 = F'(\bar{h}_{1,m} + \nu\bar{h}_{2,m}) = 0$. Similarly, no females working ($\phi = 0$) is not an equilibrium outcome either, since the firm would offer a very high wage for the first unit of female labor, and by taking a full-time job a worker could increase her utility. Finally, since the equation equalizing female utility independent of employment status is monotone in ϕ, as the utility function is a sum of monotone continuous functions, and from the assumptions imposed on the production function. Then from the intermediate value theorem it directly follows that there exists a unique $\phi \in (0,1)$ that is consistent with equilibrium.[40] Given the indivisibility of the female labor supply in the market sector, the equilibrium allocation obtained above contains non-convexity, and is therefore not Pareto optimal, as demonstrated in Rogerson (1988). More specifically, a Social Planner (SP) could make everyone better off by using employment lottery and choosing the fraction ϕ of females to work and give every female the same consumption $c_2 = \phi c^{e}_{2,m} + (1-\phi)c^{u}_{2,m}$, independent of the employment status. Note that such an allocation is both feasible, and provides a higher level of total utility. Showing feasibility is trivial, it follows directly from the presence of unit mass of the females in the model.

Next, it will be shown that the new allocation, which is independent of household's employment status, makes households better off since it generates higher utility on average.[41] This follows from the convexity of the CES aggregation and the concavity of the logarithmic function. Thus, the SP is indeed giving in expected utility terms an allocation that is an improvement over the initial equilibrium allocation.

Then, as a result of pooling resources within the family, another round of Pareto-improving redistribution can be implemented by giving everyone (male or female) the same composite consumption[42]

$$\hat{c} = \frac{1}{2}[c_1 + c_2]$$

In other words, the family in the model acts as an insurance mechanism that can achieve complete insurance across the members of the family. Such redistribution is again feasible and makes everyone better off.[43] Aggregate preferences now look as follows

$$U = 2\ln[\hat{c}] + \ln[1 - \bar{h}_{1,m} - h_n(phi)] + \phi\ln[1 - \bar{h}_{2,m} - h_n(\phi)] + (1-\phi)\ln[1 - h_n] - \phi\tau$$

Note that since the fraction of females employed is

$\phi = H_{2,m}/\bar{h}_{2,m}$

By letting

$$A = \frac{\tau}{\bar{h}_{2,m}} > 0,$$

the aggregate utility function simplifies to

$$U = 2\ln[\hat{c}] + \ln[1 - \bar{h}_{1,m} - h_n(\phi)] + \ln[1 - h_n(\phi)] - AH_{2,m} + H_{2,m}\frac{1}{\ln[1 - h_n(\phi)]}$$

Finally, letting

$$B = A + \frac{1}{\ln[1 - h_n(\phi)]} > 0,$$

and also using that aggregate hours supplied by males

[40]More precisely, there are a lot of equilibria: in each equilibrium the names of the females working are different, but in every equilibrium the same fraction of females ϕ that works is the same.

[41]For now, the fixed cost can be ignored, as it is present in both cases.

[42]This means giving everyone the same market consumption, as the non-market consumption is the same from the very beginning.

[43]Note that the redistribution is identical to the one where we equalize consumption within each family, and then equalize consumption across families.

$$H_{1,m} = \bar{h}_{1,m},$$

results in an aggregate utility of the form

$$U = 2\ln C + \ln[1 - H_{1,m} - h_n(\phi)] + \ln[1 - h_n(\phi)] - BH_{2,m}.$$

The aggregate utility function above is of an interesting and novel form: On the aggregate, when there is a continuum of a two-member household with fixed cost of family labor supply and home production, the representative agent obtained from the aggregation procedure features different preferences of work. More specifically, the disutility of labor when it comes to male hours is still logarithmic (so there is no difference between the individual labor supply elasticity for males and the same elasticity for males at the aggregate level), but in the female market sector is now linear, while the disutility of home labor is logarithmic as in the individual utility function. The split of the two types of labor (market and non-market) for females, but not males, is a novel result that is driven by the fact that market labor for females behaves as if it were indivisible, while non-market hours were divisible

Conclusion

Our model with two-member households and home production can generate large aggregate labor supply elasticities while being consistent with the micro evidence (see Heckman and MaCurdy 1980, MaCurdy 1981, Killingsworth 1983, Kydland 1984, and the references therein.) that most individuals have a very low labor supply elasticity. In addition, the interaction with home production produces different elasticity of market vs. non-market labor supply for females only. This heterogeneity in aggregate preferences for labor between males and females is an interesting finding as well, and due to the fixed cost of joint labor supply, despite the homogeneity of preferences at the individual level. Lastly, given that the population in the model has a combined mass of two, the scaling factor in front of the utility from aggregate consumption will affect the degree of substitution between consumption and hours.

As a possible venue for future research, we plan to extend the model to a dynamic and stochastic setting, and feed the derived aggregate utility function above in a sophisticated Real-Business-Cycle model with physical capital, distortionary taxation, government spending, and home production to investigate the effect of those preferences for the transmission of technology and fiscal shocks. However, such investigations are beyond the scope of this note.

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