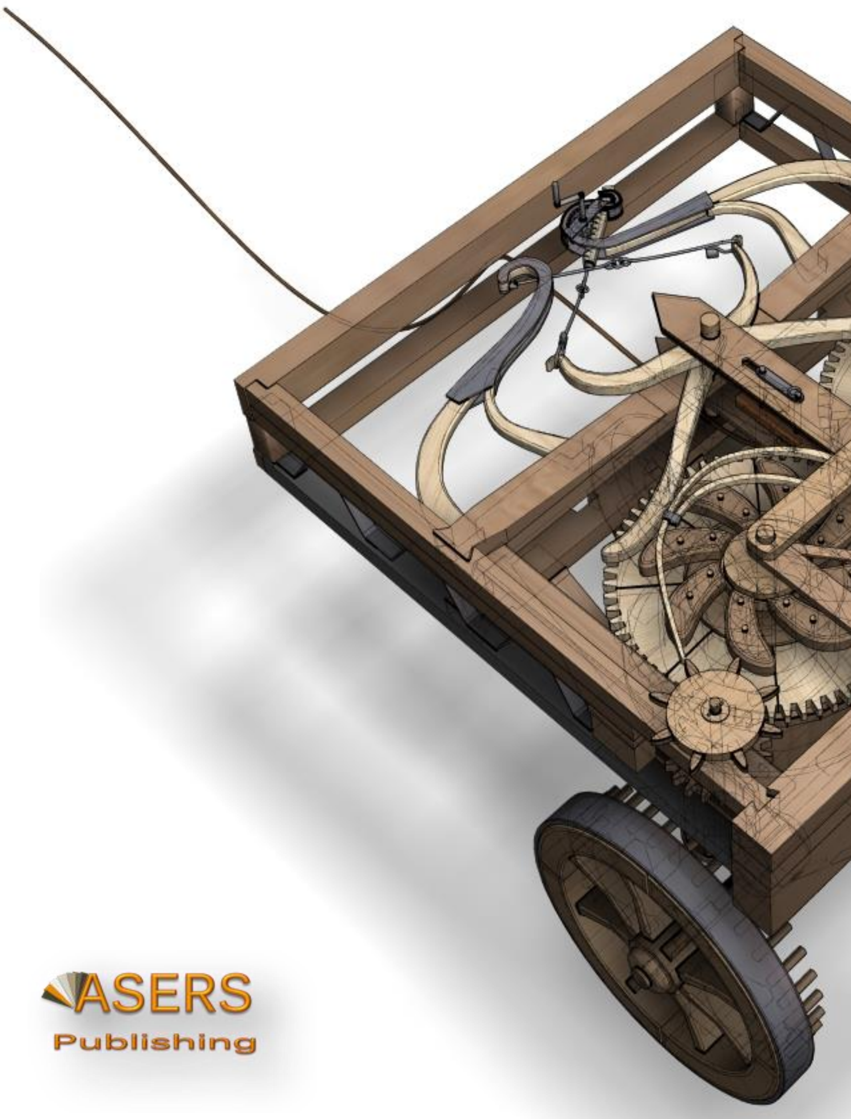
ASERS

Theoretical and Practical Research in Economic Fields



Biannually

Volume VII
Issue 2(14)
Winter 2016

ISSN 2068 – 7710
Journal **DOI**
http://dx.doi.org/10.14505/tpref

ASERS Publishing

ASERS Publishing is an advanced e-publisher struggling to bring further worldwide learning, knowledge and research. This transformative mission is realized through our commitment to innovation and enterprise, placing us at the cutting-edge of electronic delivery in a world that increasingly considers the dominance of digital content and networked access not only to books and journals but to a whole range of other pedagogic services.

In both books and journals, ASERS Publishing is a hallmark of the finest scholarly publishing and cutting-edge research, maintained by our commitment to rigorous peer-review process.

Using pioneer developing technologies, ASERS Publishing keeps pace with the rapid changes in the e-publishing market.

ASERS Publishing is committed to providing customers with the information they want, when they want and how they want it. To serve this purpose, ASERS publishing offers digital Higher Education materials from its journals, courses and scientific books, in a proven way in order to engage the academic society from the entire world.

Volume VII
Issue 2(14)
Winter, 2016

ASERS Publishing
http://www.asers.eu/asers-publishing
ISSN 2068 – 7710
Journal's Issue **DOI**:
http://dx.doi.org/10.14505/tpref.v7.2(14).00

Contents:

Many economists today are concerned by the proliferation of journals and the concomitant labyrinth of research to be conquered in order to reach the specific information they require. To combat this tendency, **Theoretical and Practical Research in Economic Fields** has been conceived and designed outside the realm of the traditional economics journal. It consists of concise communications that provide a means of rapid and efficient dissemination of new results, models and methods in all fields of economic research.

Theoretical and Practical Research in Economic Fields publishes original articles in all branches of economics – theoretical and empirical, abstract and applied, providing wide-ranging coverage across the subject area.

Journal promotes research that aim at the unification of the theoretical-quantitative and the empirical-quantitative approach to economic problems and that are penetrated by constructive and rigorous thinking. It explores a unique range of topics from the frontier of theoretical developments in many new and important areas, to research on current and applied economic problems, to methodologically innovative, theoretical and applied studies in economics. The interaction between empirical work and economic policy is an important feature of the journal.

Theoretical and Practical Research in Economic Fields, starting with its first issue, it is indexed in EconLit, RePEC, EBSCO, ProQuest, Cabell Directories and CEEOL databases.

The primary aim of the Journal has been and remains the provision of a forum for the dissemination of a variety of international issues, empirical research and other matters of interest to researchers and practitioners in a diversity of subject areas linked to the broad theme of economic sciences.

All the papers will be first considered by the Editors for general relevance, originality and significance. If accepted for review, papers will then be subject to double blind peer review.

Invited manuscripts will be due till May 15th, 2017, and shall go through the usual, albeit somewhat expedited, refereeing process.

Deadline for submission of proposals:	15th May, 2017
Expected publication date:	June, 2017
Website:	http://www.asers.eu/publishing/index.php/tpref
E-mail:	tpref@asers.eu, asers.tpref@gmail.com

To prepare your paper for submission, please see full author guidelines in the following file: TPREF_Full_Paper_Template.doc, then send it via email at tpref@asers.eu.

DOI: http://dx.doi.org/10.14505/tpref.v7.2(14).04

DISCRETE TIME OR CONTINUOUS TIME, THAT IS THE QUESTION: THE CASE OF SAMUELSON'S MULTIPLIER-ACCELERATOR MODEL

Yinghao LUO
Independent researcher, P.R.China,
luoyh5000@163.com
Mingmin LUO
Zhejiang University of Finance and Economics, P.R.China
luomm2004@163.com

Suggested Citation:

Luo, Y., Luo, M. (2016). Discrete time or continuous time, that is the question: the case of Samuelson's Multiplier-Accelerator model, *Theoretical and Practical Research in Economic Field*, (Volume VII, Winter 2016), 2(14):155-159. DOI:10.14505/tpref.v7.2(14).04. Available from: http://www.asers.eu/publishing/index.php/tpref.

Article's History:

Received November, 2016; *Revised* November, 2016; *Accepted* December, 2016.

Abstract:

A basic problem in economic dynamics is the choice of continuous-time or discrete-time in mathematical modeling. In this paper, we study the continuous-time Samuelson's multiplier-accelerator model and compare this continuous-time model with its classical discrete-time model. We find that although time scales do not affect the probability of return to equilibrium solution in Samuelson's multiplier-accelerator model, but time scales have an influence on the perfect symmetry of periodic motion.

Keywords: Samuelson's multiplier-accelerator model, time scales, perfect symmetry, probability, return to equilibrium

JEL Classification: B41, C02, E1, E2, E32, H5

1. Introduction

Measurement cannot be separated from theory. Theoretical thinking is better through a mathematical representation. The choice of mathematical representations depends on the essential features in empirical observation and theoretical perspective. A mathematical representation should be powerful enough to display stylized features to be explained and simple enough to manage its mathematical solution to be solved.

The choice of the kind of 'time' (continuous or discrete) to be used in the construction of dynamic models is a moot question. We know that such a choice implies the use of different analytical tools: differential equations in continuous time and difference equations in discrete time. In mathematical economics and econometrics, many models are in the forms of difference equations in discrete-time without much justification. There were only a few economists using continuous-time models.

Economists like use discrete-time models more than continuous-time model in economic modeling because, on the one hand, economic data are reported in terms of discrete-time such as annual data、seasonal data and monthly data, on the other hand, discrete-time model is easy to run regression. However, compared with discrete-time model, continuous-time models have different behavioral solutions and different stability conditions in nature.

In this paper, we study the continuous-time Samuelson's multiplier-accelerator model and compare this continuous-time model with its classical discrete-time model. We find that although time scales do not affect the probability of return to equilibrium solution in Samuelson's multiplier-accelerator model, but time scales have an influence on the perfect symmetry of periodic motion.

The structure of the rest of the paper is as follows. In part 2, we give discrete-time Samuelson's multiplier-accelerator model. In part 3, we give continuous-time Samuelson's multiplier-accelerator model. In part 4 we give the probability of return to equilibrium solution in Samuelson's multiplier-accelerator model. In part 5 we give a conclusion.

2. Discrete time Samuelson's multiplier-accelerator model

The original version of the Samuelson's multiplier-accelerator is in discrete time (Samuelson 1939):

$$C_t = aY_{t-1} \tag{1}$$

$$I_t = b\left(C_t - C_{t-1}\right) \tag{2}$$

$$Y_t = C_t + I_t + G \tag{3}$$

where C is consumption, I is investment, G is government expenditure, Y is income, and 0<a<1, b>0.

We have a second-order difference equation

$$Y_t - a\left(1+b\right)Y_{t-1} + abY_{t-2} = G \tag{4}$$

The model has five types of solutions:

(1) Monotonically converging regime A and its borderline;
(2) Damped oscillation regime B;
(3) Explosive oscillation regime C;
(4) Monotonically diverging regime D and its borderline;
(5) Periodic oscillation curve PQ'.

We should notice that the periodic oscillation occurs only at the borderline between B and C regime, *i.e.* curve PQ'. Patterns in the parameter space are shown in Figure 1.

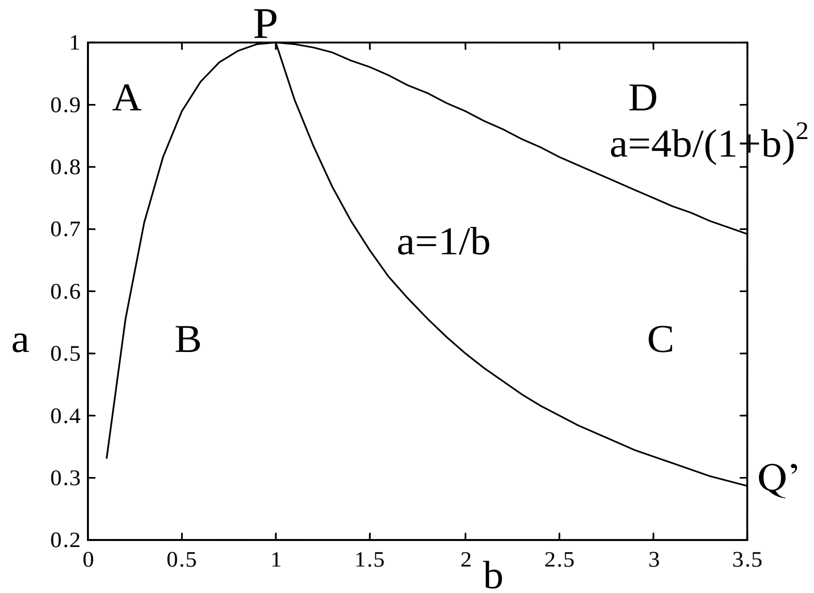


Figure 1. Discrete-time Samuelson's multiplier-accelerator diagram.
A and its borderline stands for monotonically converging; B, damped oscillation; PQ', periodic oscillation; C, explosive oscillation; D and its borderline, monotonically diverging.

3. Continuous time Samuelson's multiplier-accelerator model

We discuss the continuous time version of the above Samuelson's model to demonstrate the relation between discrete time and continuous time linear models.

We simply replace the difference by the derivative in the above Samuelson's model. We have

$$C(t) = aY(t-1) \;\; = a\left[Y(t) - \left(Y(t) - Y(t-1)\right)\right] \;\; = a[Y(t) - Y'(t)] \tag{5}$$

$$I(t)=b[C(t)-C(t-1)]=ba\left[\left(Y(t)-Y'(t)\right)\ -\left(Y(t-1)-Y'(t-1)\right)\right]\ =ba\left[Y'(t)-Y''\ (t)\right]\quad (6)$$

In (5), Y(t)-Y'(t) stands for the correction of current income, so the consumption at time t, C(t), can be regarded as linear function of the correction of the income at time t, Y(t). In (6), Y'(t)-Y"(t) stands for the correction of the change of current income, so the investment at time t, I(t), can be regard as linear function of the correction of the change of the income at time t, Y(t).

As a result, we have continuous time Samuelson's multiplier-accelerator model

$$C(t)=\ a[Y(t)-Y'(t)] \quad (5)$$
$$I(t)=\ ba[Y'(t)-Y''(t)] \quad (6)$$
$$Y(t)=C(t)+I(t)+G \quad (7)$$

We have a second-order differential equation

$$Y''(t)+\frac{1-b}{b}Y'(t)+\frac{1-a}{a*b}Y(t)=G/(a*b) \quad (8)$$

In (8), taking $Y''(t)=Y'(t)=\ 0$ we have equilibrium solution

$$Y(t)=\ G/(1-a) \quad (9)$$

The correspondent homogeneous equation of (8) is

$$Y''(t)+\frac{1-b}{b}Y'(t)+\frac{1-a}{a*b}Y(t)=0 \quad (10)$$

The characteristic equation is

$$\lambda^2+\frac{1-b}{b}\lambda+\frac{1-a}{a*b}=0 \quad (11)$$

two roots of the characteristic equation are

$$\lambda_1=\left(-b_1+\sqrt{\Delta}\right)/2$$

$$\lambda_2=\left(-b_1-\sqrt{\Delta}\right)/2$$

where $b_1=\frac{1-b}{b}$ b1, $\Delta=-b_1^2-4(1-a)/a*b$

A. When Δ>0, equation (11) have two different real roots and the general solutions of (8) are

$$Y(t)=C_1exp(\lambda_1 t)\ +C_2exp(\lambda_2 t)+G/(1-a)$$

If $b_1>0$, *i.e.*1> b, λ_1, λ_2 are all negative roots and the solutions are monotonically converging.

If $b_1<0$, *i.e.*1< b, λ_1, λ_2 are all positive roots and the solutions are monotonically diverging.

If $b_1=0$, Δ= -4(1-a)/(ba)<0 (for 0<a<1, b>0), so $b_1=0$ is impossible.

B. When Δ=0, equation (11) have two same real roots and the general solutions of (8) are

$$Y(t)=(C_1+C_2t\ )exp(-t\ b_1/2)+G/(1-a)$$

If $b_1>0$, *i.e.* 1> b, the solutions are monotonically converging.

If $b_1<0$, *i.e.* 1< b, the solutions are monotonically diverging.

If $b_1=0$, *i.e.* 1= b, a= 1, but 0<a<1, so $b_1=0$ is impossible.

When $\Delta<0$, $\lambda_1, \lambda_2=\alpha \pm i\beta$, Where $a=-b_1/2, b=\sqrt{-\Delta}/2$.

The general solutions of (8) are

$$Y(t)=(C_1cos(\beta t)\ +C_2sin(\beta t))exp(\alpha t)+G/(1-a)$$

If $\alpha>0$, *i.e.* $b_1<0$, *i.e.* 1 < b, the solutions are explosive oscillation

If $\alpha<0$, *i.e.* $b_1>0$, *i.e.* 1> b, the solutions are damped oscillation,

We have a periodic solution only when b = 1.

Similarly, this continuous-time model also has four dynamic regimes. Its pattern regimes are shown in Figure 2. Compared with the discrete-time Samuelson's model, the only difference is the changing of the periodic border.

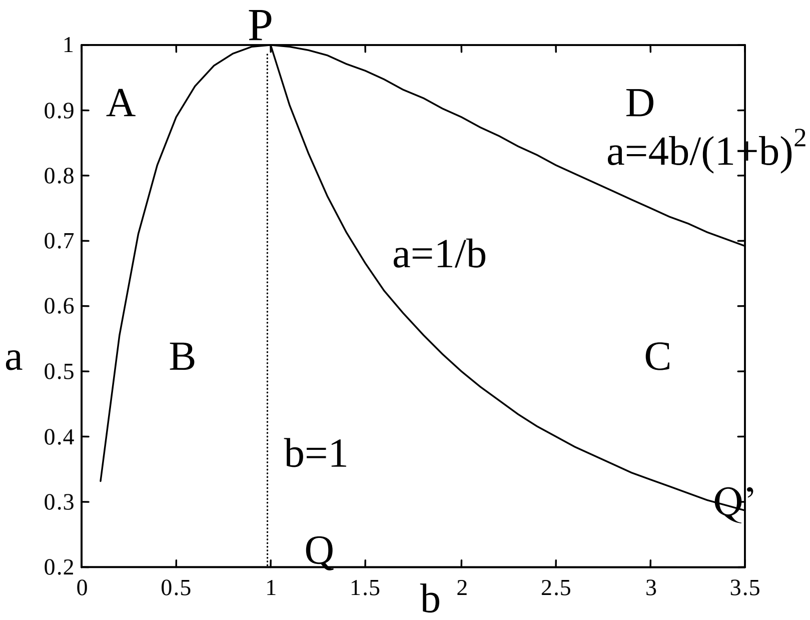


Figure 2. Continuous-time Samuelson's multiplier-accelerator diagram
The periodic boundary shifts from PQ' to PQ

4. The probability of return to equilibrium solution in Samuelson's multiplier-accelerator model

In the above analysis, we can find that the equilibrium solution of Samuelson's multiplier-accelerator model is the same as that of G/(1-a) for both discrete time and continuous time.

In the following, we discuss the probability of return to equilibrium solution in Samuelson's multiplier-accelerator model when the multiplier a and the accelerator b are in the range 0 <a <1 and 0 <b <= T. This probability is equal to the ratio of the area of the region returning to equilibrium solution to that of the whole region (0 <a <1 and 0 <b <= T).

From Figure 1 and Figure 2, we can find that the probability of return to equilibrium solution in Samuelson's multiplier-accelerator model is the same for both discrete time and continuous time, which is as follows.

When 0 <T <= 1, Prob = 100%.

When T>1, $Prob = \{4\left[ln(T+1) + 1/(T+1)\right] - 4ln2 - 1\}/T.$

When T→+∞, Prob = 0.

Note that when T>1, 1/T<Prob<1.

It can be seen that the probability of return to equilibrium solution in Samuelson's multiplier-accelerator model is a continuous function of T and monotonically decreasing (see Table 1).

Table 1. The probability of return to equilibrium solution in Samuelson's multiplier-accelerator model

T	1	1.7	15	2785	2860
Prob	100.00%	98.94%	50.45%	1.00%	0.98%

Where T is the length of the range of the accelerator b, and Prob is the probability of return to equilibrium solution. When 0 <T <= 1, Prob = 100%. When T>1, Prob = {4[ln(T+1) +1/(T+1)] - 4ln2 -1} / T. When T→+∞, Prob = 0.

Conclusion

In this paper, we study the continuous-time Samuelson's multiplier-accelerator model and compare this continuous-time model with its classical discrete-time model. We find that although time scales do not affect the probability of return to equilibrium solution in Samuelson's multiplier-accelerator model, but time scales have an influence on the perfect symmetry of periodic motion.

The probability of return to equilibrium solution in Samuelson's multiplier-accelerator model is getting smaller with the increase of the range of the accelerator b (the length of the range is equal to T) when the multiplier a is greater than 0 and less than 1, regardless of discrete time or continuous time.

When 0<T<=1, the probability of return to equilibrium solution in Samuelson's multiplier-accelerator model is 100%. The perfect symmetry of periodic motion of the continuous-time Samuelson's multiplier-accelerator model occurs at b = 1 (of course T = 1 at this time), while the perfect symmetry of the periodic motion of the discrete-time Samuelson's multiplier-accelerator model does not appear.

When T>1, the probability of return to equilibrium solution in Samuelson's multiplier-accelerator model is greater than 1/T and less than 1. The perfect symmetry of periodic motion of the continuous-time Samuelson's multiplier-accelerator model occurs at b = 1 and the perfect symmetry of the periodic motion of the discrete-time Samuelson's multiplier-accelerator model occurs at a = 1/b.

When T tends to +∞, the probability of return to equilibrium solution in Samuelson's multiplier-accelerator model is astonishing zero.

Therefore, although the difference equations have large similarity with the differential equations, when abstracting economic phenomenon into mathematical models, the choice between the difference equation and the differential equation is not just a matter of convenience. We should carefully examine its empirical and theoretical foundation.

Ackowledgement

We would like to thank Ping Chen and Laura Ungureanu for helpful suggestions.

References

[1] Samuelson, P.A. 1939. Interactions between the Multiplier Analysis and the Principle of Acceleration. *Review of Economic Statistics*, 4: 75-78, http://dx.doi.org/10.2307/1927758

ASERS

ASERS
Publishing

Web:www.asers.eu
URL: http://www.asers.eu/publishing/index.php/tpref
E-mail: asers@asers.eu tpref@asers.eu
ISSN 2068 – 7710
Journal DOI: http://dx.doi.org/10.14505/tpref
Journal'sIssue DOI: http://dx.doi.org/10.14505/tpref.v7.2(14).00