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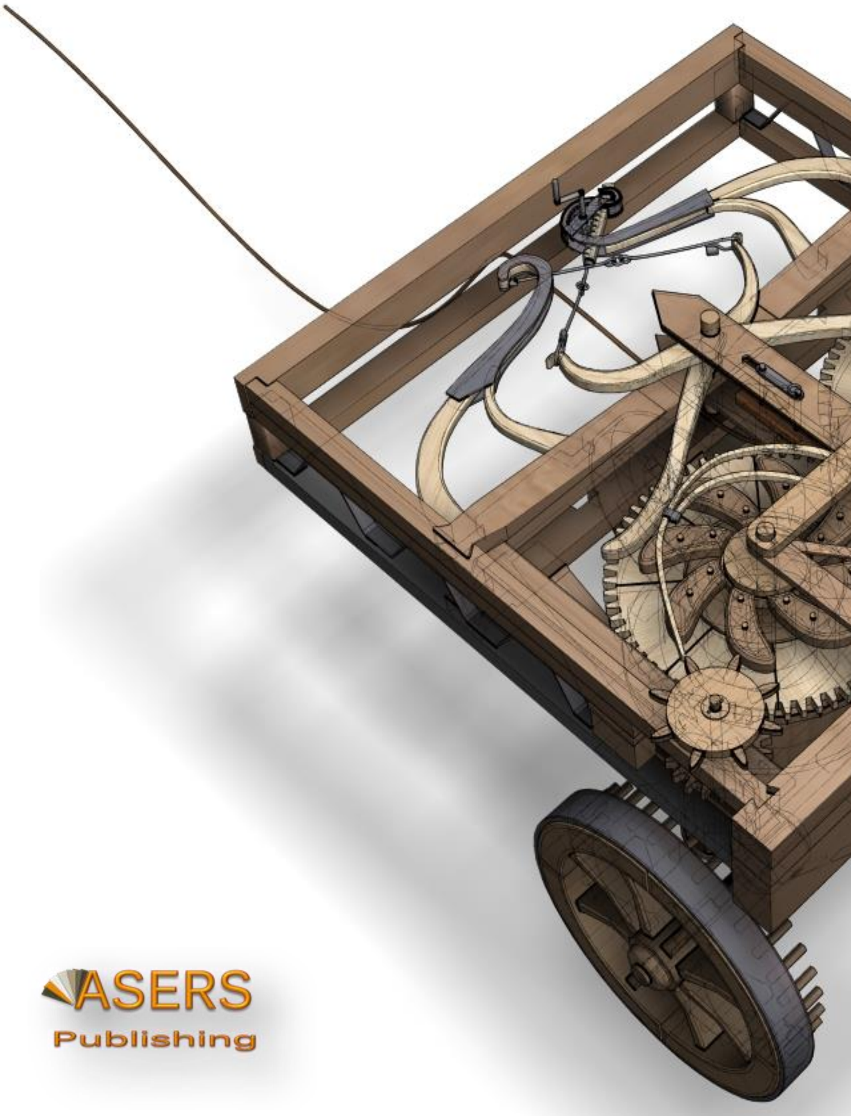

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Contents:

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THE EQUIVALENCE OF BERTRAND EQUILIBRIUM IN A DIFFERENTIATED DUOPOLY AND COURNOT EQUILIBRIUM IN A DIFFERENTIATED OLIGOPOLY

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Abstract

This paper examines whether Bertrand equilibrium in a differentiated duopoly can be duplicated with Cournot competition or not. We show that the degree of product differentiation plays an important role in the duality between those equilibria. Our main claims are two. One is that there exists a unique duality condition that satisfies the equivalence between Bertrand and Cournot equilibrium irrespectively of market structures. The other is that the number of firms of Cournot competition that satisfies Bertrand equilibrium increases with the degree of product differentiation.

Keywords: Bertrand equilibrium, Cournot equilibrium, product differentiation, equivalence

JEL Classification: D21, D43, L13

1. Introduction

It is well known, from the viewpoint of social welfare, that price competition is more efficient than quantity competition. Suppose a duopoly situation where firms produce a homogeneous product and marginal costs are constant and equal for both firms. In a price competition, the price equals the marginal cost, while, in a quantity competition, the price is above it. Even though both products are differentiated, the results are the same: the equilibrium price under Cournot competition is higher than it under Bertrand competition except a perfect differentiation.

Since Dixit (1979) has first proposed a differentiated duopoly, many studies have been produced an array of extensions and generalization of Singh and Vives (1984)[1]. The literature on the relationship between Cournot and Bertrand competition has been concerned with two main issues. One stream focuses on the comparison between Bertrand and Cournot competition in both a horizontal and a vertical market. The other is to analyze it in a mixed oligopoly.

Even though many studies have focused on the comparison between competition modes, the duality between Cournot and Bertrand competition has received little attention. So, we consider whether Bertrand equilibrium can be duplicated by Cournot competition in a horizontal (or vertical) oligopoly or not. We show that the degree of product differentiation plays an important role in the duality between those models in a differentiated duopoly. Our main conclusions are two. First, we find a unique duality condition that satisfies the equivalence between Bertrand and Cournot equilibrium irrespectively of market structures. Second, the number of firms, in Cournot competition, that satisfies Bertrand equilibrium increases with the degree of product differentiation.

The paper is organized as follows. In section 2, we set up the model. Section 3 analyzes whether Bertrand equilibrium can be duplicated with Cournot competition in a horizontal market. Section 4 examines it in a vertical structure. Finally, we conclude the concluding remarks.

2. The Model

We consider an economy with a duopolistic sector, consisting of two firms produce a differentiated good. Both firms operate under constant returns to scale. Each firm's unit cost of production equals c exogenously. We analyze whether Bertrand equilibrium can be duplicated with Cournot competition in a differentiated industry. The demand structure of our model is adapted from Dixit (1979). A representative consumer maximizes $U(q_I, q_j) - \sum p_i q_i, i, j = 1,2 \; and \; i \neq j$, where q_i is the quantity, and p_i is its price. U is assumed to be quadratic and strictly concave $U(q_i, q_j) = a(q_i + q_j) - (q_i^2 + 2dq_iq_j + q_j^2)/2$, where $d \in [0,1], i, j = 1, 2$, and $\mathrm{i} \neq \mathrm{j}$. This utility function gives rise to a linear demand structure. The Inverse and direct demands are as follows:

$$p_i = a - q_i - dq_j \; and \; q_i = \frac{a(1-d) - p_i + dp_j}{1-d^2}; \; i, j = 1, 2, and \; i \neq j, \tag{1}$$

The parameter *d* of the demand function expresses the degree of product differentiation, ranging from zero when goods are independent to one when the goods are perfect substitutes.

3. Horizontal Oligopoly

In this section, we analyze whether Bertrand equilibrium can be duplicated with Cournot competition in a horizontal oligopolistic market. As a benchmark, we show that Bertrand equilibrium differs from Cournot equilibrium. In Cournot competition both firms choose quantities, in Bertrand competition, prices. In both cases, the equilibrium concept is non-cooperative Nash equilibrium. In Cournot competition, firm *i* chooses q_i so as to maximize its profit, taking as a given q_j, while, in Bertrand competition, it chooses p_i so as to maximize its profit, taking as a given p_j as follows:

$$\max_{q_i} \pi_i = (a - c - q_i - dq_j)q_i,$$

$$\max_{p_i} \pi_i = (p_i - c)\left(\frac{a(1-d) - p_i + dp_j}{1-d^2}\right).$$

It is straightforward to compute Bertrand and Cournot equilibrium. We obtain the equilibrium quantities and prices under Bertrand and Cournot competition, respectively, as follows:

[1]For a horizontal market, see Cheng (1985), Dastidar (1997), Qiu (1997), Hackner(2000), and Amir and Jin (2001), and for a vertical market, Correa-Lopez (2007), Arya et al. (2008), Mukherjee *et al.* (2012), Alipranti *et al.* (2014), and Lee and Choi (forthcoming).

$$q^B = \frac{a-c}{2+d-d^2}, \tag{2.1}$$

$$p^B = c + \frac{(1-d)(a-c)}{2-d}, \tag{2.2}$$

$$q^C = \frac{a-c}{2+d}, \tag{2.3}$$

$$p^C = c + \frac{a-c}{2+d}, \tag{2.4}$$

where the superscript '*B*' and '*C*' denotes Bertrand and Cournot competition, respectively.

From Eq. (2.1), Eq. (2.2), Eq. (2.3), and Eq. (2.4), we obtain $p^C - p^B = d^2(a-c)/(4-d^2) \geq 0$ (and similary $q^B - q^C = d^2(a-c)/(4+4d-d^2-d^3) \geq 0$, which are nonnegative. Quantities are lower and prices are higher in Cournot than in Bertrand competition.

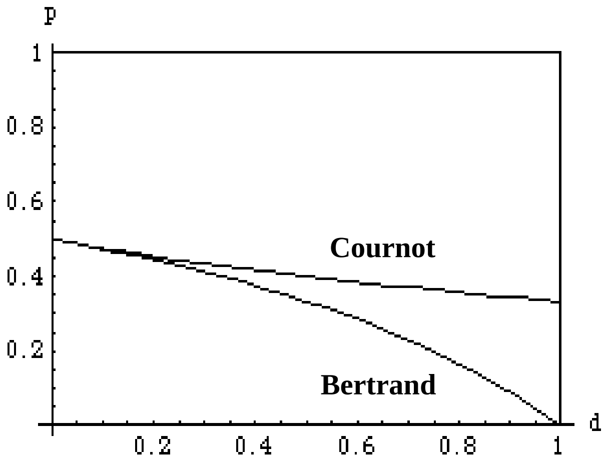

Figure 1. Equilibrium Prices between Bertrand and Cournot when $a - c = 1$ and $c = 0$

From now on, we show that Bertrand equilibrium can be duplicated with Cournot competition. Consider an economy in which two types of firm, $T \in \{i, j\}$, producing a differentiated good at a constant marginal (=average) cost *c*. Each type consists of *n* number of firms. The inverse demand is given by:

$$p^i = a - Q_K^i - dQ_K^j i,j = 1,2, i \neq j, K = 1, \dots, n, \tag{3}$$

where $Q_K^i = q_1^i + \cdots + q_n^i \text{and} Q_K^j = q_1^j + \cdots + q_n^j$.

Firm 1 in type *i* sets its quantity q_1^i so as to maximize its profit for given rival firms' outputs $(Q_K^i - 1 = q_2^i + \cdots + q_n^i \; and \; Q_K^j = q_1^j + \cdots + q_n^j)$ as follows:

$$\max_{q_1^i} \pi_1^i = (p^i - c)q_1^i = \left(a - c - Q_K^i - dQ_K^j\right)q_1^i.$$

Differentiating the maximization problem with respect to q_1^i, we obtain the reaction function as follows:

$$\frac{\partial \pi_1^i}{\partial q_1^i} = a - c - Q_K^i - dQ_K^j - q_1^i = 0. \tag{4}$$

On the other hand, firm 1 in type *j* sets its quantity q_1^j so as to maximize its profit for given rival firms' outputs $(Q_K^i = q_1^i + \cdots + q_n^i \; and \; Q_K^j - 1 = q_2^j + \cdots + q_n^j)$ as follows:

$$\max_{q_1^j} \pi_1^j = (p^j - c)q_1^j = \left(a - c - Q_K^j - dQ_K^i\right)q_1^j.$$

Differentiating the maximization problem with respect to q_1^j, we obtain the reaction function as follows:

$$\frac{\partial \pi_1^j}{\partial q_1^j} = a - c - Q_K^j - dQ_K^i - q_1^j = 0. \tag{5}$$

Summing Eq. (4) and Eq. (5) and solving them, we obtain the equilibrium quantity as follows:

$$q_K^i = q_K^j = \frac{a - c}{(1 + d)n + 1}, \tag{6.1}$$

$$Q_K^i = Q_K^j = \frac{(a - c)n}{(1 + d)n + 1}. \tag{6.2}$$

Comparing Eq. (2.1) and Eq. (2.3) to Eq. (6.2) and Eq. (3), we obtain the following result:

$$n = \frac{1}{1 - d^2}. \tag{7}$$

We summarized the results in Proposition 1.

Proposition 1. If the number of firms in each type is equivalent to $1/(1 - d^2)$, Bertrand equilibrium can be perfectly duplicated with Cournot competition.

By differentiating Eq. (7) with respect to d, we have the following result.

Lemma 1. The number of firms, in Cournot competition, that satisfies Bertrand equilibrium increases with the degree of product differentiation.

4. Vertical Oligopoly

We extend our model to vertical structures (monopolistic and bilateral duopoly). The timing of the games is as follows. At stage one, each upstream firm sets its input price (w in a monopolistic duopoly and (w_i, w_j) in a bilateral duopoly). At stage two, each downstream firm sets the quantity.

As a benchmark, we first consider a monopolistic duopoly in which an upstream firm produces an input and sells it at one price to two downstream firms. In Cournot competition, at stage two, downstream firm *i* sets its quantity q_i so as to maximize its profit for given rival's quantity q_j and input price w and in Bertrand competition, downstream firm *i* sets its price p_i so as to maximize its profit for a given rival's output price p_j and input price w. Therefore, its maximization problems are as follows:

$$\max_{q_i} \pi_i = (p_i - w)q_i = \left(a - w - q_i - dq_j\right)q_i,$$

$$\max_{p_i} \pi_i = (p_i - w)\left(\frac{a(1 - d) - p_i + dp_j}{1 - d^2}\right).$$

We obtain the equilibrium quantities and prices under Bertrand and Cournot competition, respectively, as follows:

$$q_1^B = \frac{a-w}{2+d-d^2}, \quad (8.1)$$

$$p_1^B = \frac{(1-d)a \mp w}{2-d}, \quad (8.2)$$

$$q_1^C = \frac{a-w}{2+d}, \quad (8.3)$$

$$p_1^C = \frac{a+(1+d)w}{2+d}. \quad (8.4)$$

At stage one, the upstream firm sets the input price w so as to maximize its profit. Its maximization problems under Cournot and Bertrand competition are, respectively, as follow:

$$\max_w \Pi = (w-c)(q_1+q_2) = \frac{2(w-c)(a-w)}{2+d},$$

$$\max_w \Pi = (w-c)(q_1+q_2) = \frac{2(w-c)(a-w)}{2+d-d^2}.$$

We obtain the equilibrium input prices, quantities, and prices under Bertrand and Cournot competition, respectively, as follows:

$$w^B = w^C = \frac{a+c}{2}, \quad (9.1)$$

$$q_i^B = \frac{a-c}{2(2+d-d^2)}, \quad (9.2)$$

$$p_i^B = c + \frac{(a-c)(3-2d)}{2(2-d)}, \quad (9.3)$$

$$q_i^C = \frac{a-c}{2(2+d)}, \quad (9.4)$$

$$p_i^C = c + \frac{(a-c)(3+d)}{2(2+d)}. \quad (9.5)$$

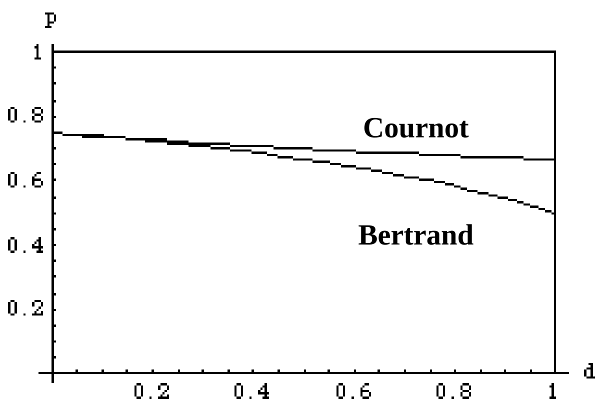

Figure 2. Equilibrium Prices in a Monopolistic Duopoly when $a - c = 1$ and $c = 0$

As a benchmark, we also consider a bilateral duopoly in which each upstream firm produces an input and sells it to its own downstream firm. In Cournot competition, at stage two, downstream firm *i* sets its quantity

q_i so as to maximize its profit for given rival's quantity q_j and input price w_i. On the other hand, in Bertrand competition, downstream firm *i* sets its price p_i so as to maximize its profit for given rival's output price p_j and input price w_i. Therefore, its maximization problems are, respectively, as follows:

$$\max_{q_i} \pi_i = (p_i - w_i)q_i = (a - w_i - q_i - dq_j),$$

$$\max_{p_i} \pi_i = (p_i - w_i)q_i = (p_i - w_i)\left(\frac{a(1-d) - p_i + dp_j}{1-d^2}\right).$$

We obtain the equilibrium price and quantity under Bertrand and Cournot competition, respectively, as follows:

$$q_i^B = \frac{a(2-d-d^2) - (2-d^2)w_i + dw_j}{4-5d^2+d^4}, \tag{10.1}$$

$$p_i^B = \frac{a(2-d-d^2) + 2w_i + dw_j}{4-5d^2+d^4}, \tag{10.2}$$

$$q_i^C = \frac{a(2-d) - 2w_i + dw_j}{4-d^2}, \tag{10.3}$$

$$p_i^C = \frac{a(2-d) + (2-d^2)w_i + dw_j}{4-d^2}. \tag{10.4}$$

At stage one, each upstream firm sets the input prices (w_i, w_j) so as to maximize its profit. Upstream firm *i*'s maximization problems are, respectively, as follow:

$$\max_{w_i} \Pi_i = (w_i - c)q_i = (w_i - c)\left(\frac{a(2-d) - 2w_i + dw_j}{4-d^2}\right),$$

$$\max_{w_i} \Pi_i = (w_i - c)(q_1 + q_2) = (w_i - c)\left(\frac{a(2-d-d^2) - (2-d^2)w_i + dw_j}{4-5d^2+d^4}\right).$$

Finally, we obtain the equilibrium input prices, quantities, and prices under Bertrand and Cournot competition, respectively, as follows:

$$w_i^B = w_j^B = c + \frac{(2-d-d^2)(a-c)}{4-d-2d^2}, \tag{11.1}$$

$$w_i^C = w_j^C = c + \frac{(2-d)(a-c)}{4-d}, \tag{11.2}$$

$$q_i^B = q_j^B = \frac{(2-d^2)(a-c)}{(2+d-d^2)(4-d-2d^2)}, \tag{11.3}$$

$$p_i^B = p_j^B = c + \frac{2(1-d)(3-d^2)(a-c)}{(2-d)(4-d-d^2)}, \tag{11.4}$$

$$q_i^C = q_j^C = \frac{2(a-c)}{(2+d)(4-d)}, \tag{11.5}$$

$$p_i^C = p_j^C = c + \frac{(6-d^2)(a-c)}{(2+d)(4-d)}. \tag{11.6}$$

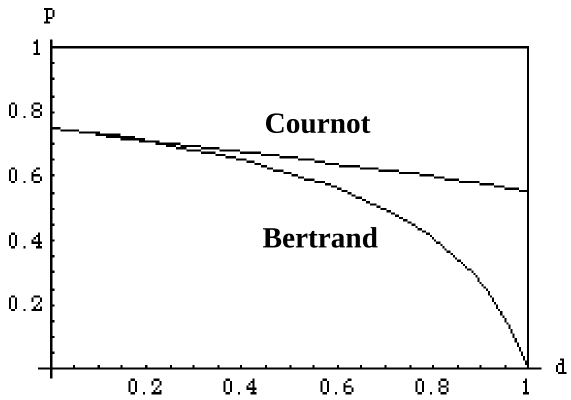

Figure 3. Equilibrium Prices in a Bilateral Duopoly when $a - c = 1$ and $c = 0$

Next, we analyze the equivalence between Bertrand and Cournot equilibrium in vertical structures. We first consider a monopolistic duopoly in which an upstream firm produces an input and sells it at one price to two types of downstream firms, $M \in \{i, j\}$, producing a differentiated good. Each type consists of *n* number of downstream firms. The upstream firm has a constant marginal (=average) cost *c*. For simplicity, one unit of the final product needs exactly one unit of the input and the cost of transforming the input into the final product is normalized to zero. The timing of the games is as follows. At stage one, the upstream firm sets the input price. At stage two, each downstream firm chooses the output.

At stage 2, under Eq. (3), downstream firm 1 in type *i* sets its quantity q_1^i so as to maximize its profit for given rival firms' quantities ($Q_{K-1}^i = q_2^i + \cdots + q_n^i \; and \; Q_K^j = q_1^j + \cdots + q_n^j$) and input price w as follows:

$$\max_{q_1^i} \pi_1^i = (p^i - w)q_1^i = (a - w - Q_K^i - dQ_K^j)q_1^i.$$

Differentiating the maximization problem with respect to q_1^i, we obtain the reaction function as follows:

$$\frac{\partial \pi_1^i}{\partial q_1^i} = a - w - Q_K^i - dQ_K^j - q_1^i = 0. \tag{12}$$

On the other hand, downstream firm 1 in type *j* sets its output q_1^j so as to maximize its profit for given rival firms' outputs ($Q_K^i = q_1^i + \cdots + q_n^i \; and \; Q_{K-1}^j = q_2^j + \cdots + q_n^j$) as follows:

$$\max_{q_1^j} \pi_1^j = (p^j - w)q_1^j = (a - w - Q_K^j - dQ_K^i)q_1^j.$$

Differentiating the maximization with respect to q_1^j, we obtain the reaction function as follows:

$$\frac{\partial \pi_1^j}{\partial q_1^j} = a - w - Q_K^j - dQ_K^i - q_1^j = 0. \tag{13}$$

Summing (12) and Eq. (13) and solving them, we obtain the equilibrium quantities as follows:

$$q_K^i = q_K^j = \frac{a - w}{n + dn + 1}, \tag{14.1}$$

$$Q_K^i = Q_K^j = \frac{(a - w)n}{n + dn + 1}. \tag{14.2}$$

At stage one, the upstream firm sets the input price w so as to maximize its profit. Its maximization problem is as follows:

$$\max_{w} \Pi = (w-c)\left(Q_K^i + Q_K^j\right) = 2(w-c)\left(\frac{(2-w)n}{n+dn+1}\right).$$

Differentiating the maximization with respect to w, we obtain the reaction function as follows:

$$\frac{\partial \Pi}{\partial w} = \frac{2n(a+c-2w)}{n+dn+1} = 0. \tag{15}$$

Solving (15), we obtain the equilibrium input price as follows:

$$w = \frac{a+c}{2}. \tag{16.1}$$

Substituting (16.1) into (14.1) and (14.2), we obtain the equilibrium quantity for each downstream firm and for industry, respectively, as follows:

$$q_K^i = q_K^j = \frac{a-c}{2(n+dn+1)}, \tag{16.2}$$

$$Q_K^i = Q_K^j = \frac{(a+c)n}{2(n+dn+1)}. \tag{16.3}$$

Comparing (9.3) to (16.3), we obtain the following result:

$$n = \frac{1}{1-d^2}. \tag{17}$$

Proposition 2. If the number of downstream firms of each type is equivalent to $1/(1-d^2)$, If the number of firms in each type is equivalent to $1/(1-d^2)$, Bertrand equilibrium in a monopolistic duopoly can be perfectly duplicated with Cournot competition.

Finally, we turn to a bilateral duopolistic market in which each upstream firm, $U \in \{i, j\}$, produces an input at a constant marginal (=average) cost *c*. Each upstream firm sells its input to its *n* number of downstream firms.

At stage 2, under (3), downstream firm 1 of type *i* sets its quantity q_1^i so as to maximize its profit for given rival firms' quantities ($Q_K^i - 1 = q_2^i + \cdots + q_n^i \; and \; Q_K^j = q_1^j + \cdots + q_n^j$) and input price w_i as follows:

$$\max_{q_1^i} \pi_1^i = \left(p^i - w_i\right)q_1^i = \left(a - w_i - Q_K^i - dQ_K^j\right)q_1^i.$$

Differentiating the maximization problem with respect to q_1^i, we obtain the reaction function as follows:

$$\frac{\partial \pi_1^i}{\partial q_1^i} = a - w_i - Q_K^i - dQ_K^j - q_1^i = 0. \tag{18}$$

On the other hand, downstream firm 1 of type *j* sets its quantity q_1^j so as to maximize its profit for given rival firms' quantitiess ($Q_K^i = q_1^i + \cdots + q_n^i \; and \; Q_{K-1}^j = q_2^j + \cdots + q_n^j$) and input price w_j as follows:

$$\max_{q_1^j} \pi_1^j = (p^j - w_j)q_1^j = (a - w_j - Q_K^j - dQ_K^i)q_1^j.$$

Differentiating the maximization problem with respect to q_1^j, we obtain the reaction function as follows:

$$\frac{\partial \pi_1^j}{\partial q_1^j} = a - w_j - Q_K^j - dQ_K^i - q_1^j = 0. \tag{19}$$

Summing (18) and (19) and solving them, we obtain the equilibrium quantities for each downstream firm and for industry, respectively, as follows:

$$q_K^i = q_K^j = \frac{(n(1-d)+1)a - (n+1)w_i - dnw_j}{n^2(1-d^2)+2n+1}, \tag{20.1}$$

$$Q_K^i = Q_K^j = \frac{n(n(1-d)+1)a - (n+1)w_i - dnw_j}{n^2(1-d^2)+2n+1}. \tag{20.2}$$

At stage one, the upstream firm *i* sets the input price w_i so as to maximize its profit for a given rival firm's input price w_j. Its maximization problem is as follows:

$$\max_{w_i} \Pi_i = (w_i - c)Q_K^i = (w_i - c)\left(\frac{n(n(1-d)+1)a - (n+1)w_i - dnw_j}{n^2(1-d^2)+2n+1}\right).$$

Differentiating the maximization with respect to w_i, we obtain the reaction function as follows:

$$\frac{\partial \Pi_i}{\partial w_i} = \frac{n(a+c) - dna + dnw_j}{2(n+1)} = 0. \tag{21}$$

Solving (21), we obtain the following result:

$$w_i = w_j = \frac{(n+1)(a+c) - dna}{2(n+1) - dn}. \tag{22.1}$$

Finally, we obtain the equilibrium quantities for each downstream firm and for industry, respectively, as follows:

$$q_K^i = q_K^j = \frac{(n+1)(a-c)}{(2(n+1)-dn)(n+dn+1)}, \tag{22.2}$$

$$Q_K^i = Q_K^j = \frac{n(n+1)(a-c)}{(2(n+1)-dn)(n+dn+1)}. \tag{22.3}$$

Comparing (22.3) to (11.4), we obtain the following result:

$$n = \frac{1}{1-d^2}. \tag{23}$$

Proposition 3. If the number of downstream firms in each type is equivalent to $1/(1-d^2)$, Bertrand equilibrium in a bilateral duopoly can be perfectly duplicated with Cournot competition.

Conclusion

This paper examines the relation between Bertrand equilibrium and Cournot equilibrium in a differentiated oligopoly. We consider whether Bertrand equilibrium can be duplicated by Cournot competition in a (or vertical) duopoly or not. We show that the degree of product differentiation plays an important role in the equivalence between Bertrand and Cournot equilibria in differentiated duopoly. Our main conclusions are two. First, we find a unique duality condition that satisfies the equivalence between Bertrand and Cournot equilibrium irrespectively of market structures. Second, the number of firms, in Cournot competition, that satisfies Bertrand equilibrium increases with the degree of product differentiation.

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