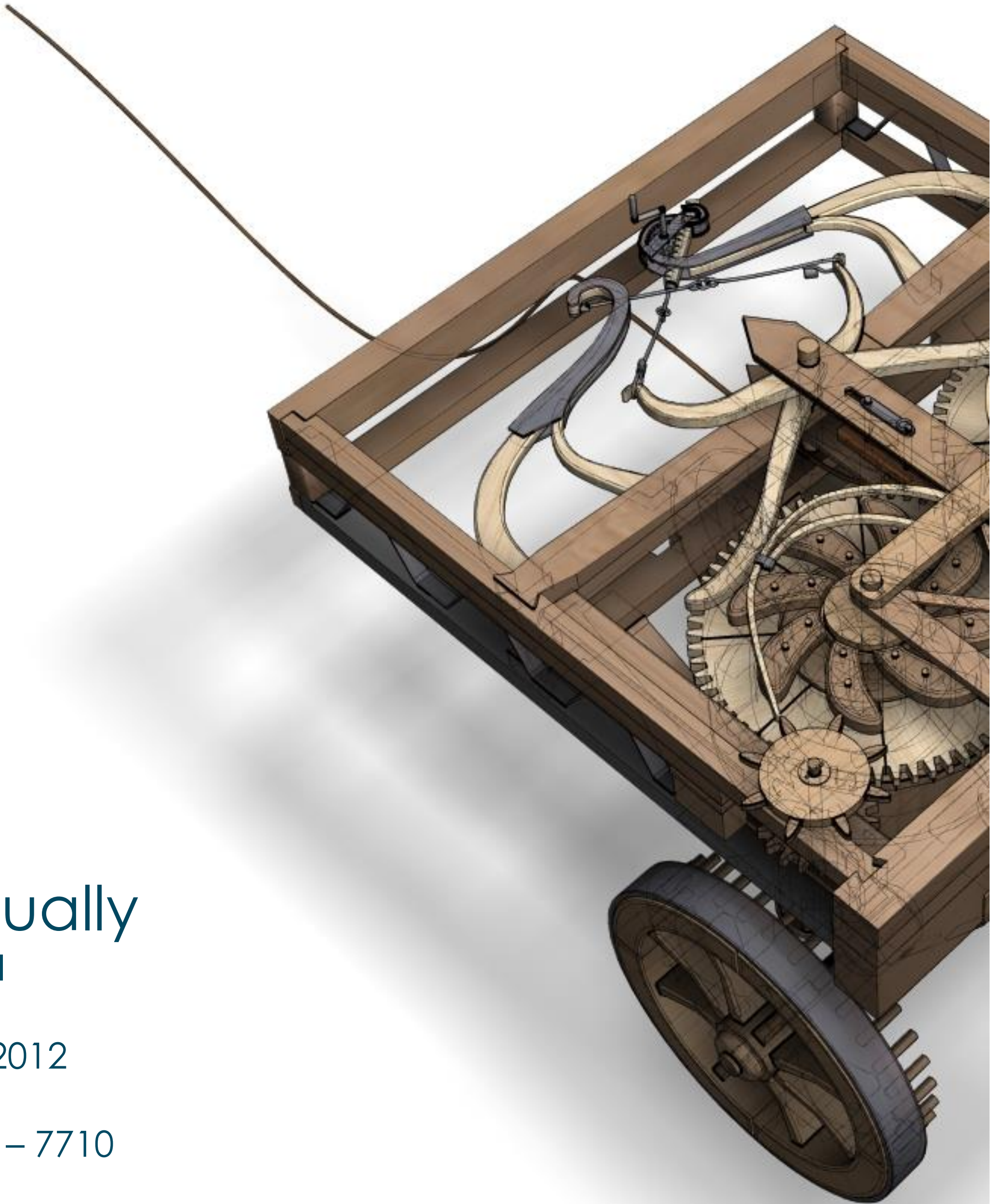
ASERS

Theoretical and Practical Research in Economic Fields



Biannually

Volume III
Issue 1(5)
Summer 2012

ISSN 2068 – 7710

Summer 2012
Volume III, Issue 1(5)

ASERS Publishing
http://www.asers.eu/asers-publishing
ISSN 2068 – 7710

Contents:

Call for Papers
Winter_Issue 2012

Theoretical and Practical Research in Economic Fields

Many economists today are concerned by the proliferation of journals and the concomitant labyrinth of research to be conquered in order to reach the specific information they require. To combat this tendency, **Theoretical and Practical Research in Economic Fields** has been conceived and designed outside the realm of the traditional economics journal. It consists of concise communications that provide a means of rapid and efficient dissemination of new results, models and methods in all fields of economic research.

Theoretical and Practical Research in Economic Fields publishes original articles in all branches of economics – theoretical and empirical, abstract and applied, providing wide-ranging coverage across the subject area.

Journal promotes research that aim at the unification of the theoretical-quantitative and the empirical-quantitative approach to economic problems and that are penetrated by constructive and rigorous thinking. It explores a unique range of topics from the frontier of theoretical developments in many new and important areas, to research on current and applied economic problems, to methodologically innovative, theoretical and applied studies in economics. The interaction between empirical work and economic policy is an important feature of the journal.

Theoretical and Practical Research in Economic Fields, starting with its first issue, it is indexed in RePEC, IndexCopernicus, EBSCO, ProQuest and CEEOL databases.

The primary aim of the Journal has been and remains the provision of a forum for the dissemination of a variety of international issues, empirical research and other matters of interest to researchers and practitioners in a diversity of subject areas linked to the broad theme of economic sciences.

All the papers will be first considered by the Editors for general relevance, originality and significance. If accepted for review, papers will then be subject to double blind peer review.

Deadline for submission of proposals:	15th November 2012
Expected Publication Date:	January 2013
Web:	www.asers.eu/journals/tpref/
E-mail:	tpref@asers.eu

To prepare your paper for submission, please see full author guidelines in the following file: TPREF_Full_Paper_Template.doc, then send it via email at tpref@asers.eu.

THE GOVERNMENT-TAXPAYER GAME

David CARFI
IAMIS, Institute for the Applications of Mathematics and Integrated Sciences
Department of Mathematics, University of California at Riverside, **USA**
davidcarfi71@yahoo.it
Caterina FICI
Business Administrator at VF of V. Fici, **Italy**
caterinafici@libero.it

Abstract:

In this paper, we model - quantitatively – a possible realistic interaction between a taxpayer and his Government. We formalize, in a general setting, this strategic interaction. Moreover, we analyze completely a particular realistic sample of the general model. We determine the entire payoff space of the sample game; we find the unique Nash equilibrium of the interaction; we determine the payoff Pareto maximal boundary, the conservative payoff zone and the conservative core of the game (part of Pareto boundary greater than the conservative payoff of the game). Finally, we suggest possible compromise solutions between the two players. From an economic point of view, the sample chosen gives an example of normative settings, for which, there is no reason (convenience), for the taxpayer, to evade the taxes or to declare less than his real income, when his behavior is conservative (defensive, risk-averse). Moreover, the two proposed compromise solutions (which realize the maximum collective gain) could be significantly applied to distinguished taxpayer (big companies and so on).

Keywords: Government; taxpayer; tax; fiscal policy; tax evasion.

JEL Classification: E62; H21; E42; G38; G18; G28; H26.

1. The Model

We shall consider a two-player normal form gain game, G = (f, >), representing the rational interaction between the **Government (first player 1)** and a **Taxpayer (second player 2)**, in a country. The payoff function f from the product E × F into IR^2 of our game shall be defined upon the bi-strategy space of the game, Cartesian product of the respective strategy spaces of the two players, and with values into the payoff universe IR^2. The two components f_1 and f_2 of the function f are the respective payoff function of the two players.

1.1. Strategy spaces

Government**.** The unit interval **U** = [0,1] is the strategy set of the Government, a probabilistic interval: each element x of the unit interval **U** is the probability that the Government checks the real (true) income of the Taxpayer.

Taxpayer**.** The compact interval F = [0,V] is the range of possible income declarations of the Taxpayer, each element y in F is a possible income that the Taxpayer may decide to declare to the Government, V is the true income of the taxpayer.

1.2. Payoff function of the Government

We consider, firstly, the payoff function of the Government, in his interaction with the Taxpayer 2; it is, as usual, a real function f_1: [0,1] × [0,V] → **R**. To construct the payoff function f_1 (that is, to define its "correspondence law") we shall proceed step by step.

First step. Firstly, we consider the case in which the Government checks the Taxpayer declaration, that is, the strategy 1 of the Government. In this case, we have

$f_1(1, y) = tV + g(t)(V - y) - C$, for every strategy y in F, where:

a) the fixed real number t, belonging to IU, is the percentage (the unit interval U is now interpreted in a different way), due by the Taxpayer to the Government, upon his real income V. Hence, a first income of the Government, in this case, is the discounted cash flow tV;
b) the real number g(t) > t is the fixed percentage due, by the taxpayer, to the Government upon his own non-declared income (V – y); so that, the Government receives also the discounted cash flow g(t)(V - y);

c) at last, the real number C is the cost afforded by the Government to check the tax evasion.

Second step. Secondly, we consider the case in which the Government does not check (at all) the taxpayer declaration and accept the strategy y of the Tax-payer as his true income, so that, we have:

$f_1(0,y) = ty$,

for every strategy y of the second player.

Third step. To obtain the values of the function f_1, on the remaining part of the bi-strategy space, we shall use the von Neumann mixed-extension method, but only with respect to the first finite strategy space {0,1} of the Government. In other terms, we shall consider – for every strategy y of the Taxpayer - the mixed extension of the finite stochastic variable L(y): {0,1} → **R**, defined by

$L(y)(0) = f_1(0,y) = ty$ and $L(y)(1) = f_1(1, y) = tV + g(t)(V - y) - C$,

by using the probabilistic scenarios **only for the actions of the Government** (see later for a robust justification of this probabilistic choice and its applicability). We so have:

$$
\begin{aligned}
f_1(x, y) &= \mathbf{E}_{(1-x,\, x)}(L(y)) = \\
&= \mathbf{E}_{(1-x,\, x)}(ty,\ tV + g(t)(V - y) - C) = \\
&= (1- x)\, ty + x\,[tV+ g(t)(V- y) - xC] = \\
&= x\,[tV + g(t)(V- y) - C - ty\,]+ ty .
\end{aligned}
$$

1.3. Payoff function of the second player, the Taxpayer

In general, for the payoff function of the second player, we have:

$f_2(0,y) = (V- y) + (1 - t)y$, for every y in F; indeed, when the government does not check the possible evasion, the Taxpayer net income is:

1) the non-declared income (V- y) (considered as it is, since there are no taxes on it);
2) plus the declared income y minus the tax ty , which player 2 has to pay because of the declaration y.

When the Government decides to check the declaration of the Taxpayer, we obtain:

$f_2(1, y) =(1-t)V - g(t)(V- y)$,

for every y in F, indeed, when the Government checks the possible evasion, the Taxpayer net income is:

1) the non-declared income (V- y) minus a higher tax g(t)(V- y) - with respect to the usual taxation t(V- y) - because of the evasion;
2) plus the real income V minus the tax tV on the real income V, which player 2 has to pay because the Government, after the check, knows the real income V of the Taxpayer.

Mixed extension. By adopting the von Neumann mixed extension method, as before only on the Government strategies, we obtain:

$$
\begin{aligned}
f_2(x,y) &= \mathbf{E}_{(1-x,\, x)}((V - y)+(1 - t)y,\ (1- t)V - g(t)(V- y)) = \\
&= (1- x)\,((V - y) + (1 - t)y)+ x((1- t)V - g(t)(V- y)),
\end{aligned}
$$

for every pair (x,y) in the bi-strategy space.

Payoff Function of Government - Taxpayer Game. Resuming the above results, we can finally give the definition of the payoff vector function of our entire game G; it is defined by

$$f(x,y) = (x\,[tV + g(t)(V- y) - C - ty\,] + ty,\ (1- x)\,[(V - y)+ (1 - t)y] + x[(1- t)V - g(t)(V- y)]),$$

for every (x,y) in the strategic square S.

2. Numerical Example

To build up a computable and realistic example, we shall put:

$t = 25\% = ¼$, $g(t) = ½ = 50\%$, $C = ¼$ and $V = 1$.

Remark (on the strategy sets).

(1) In the above example, we are normalizing the real income V; so that, also the declaration strategy y belongs to the compact unit interval **U** = [0,1], 0 means total Tax Evasion (declaration 0), 1 means No Tax Evasion (the declaration y equals the real income V).

(2) Any strategy x of the first player belongs to [0,1], but the meaning is completely different, as it is emphasized in the following remark.

Remark (interpretation of strategy spaces). The interpretations of our strategy spaces are obvious and recalled below:

a) the strategy space E is a probabilistic strategy space;
b) the strategy space F (of the second player) is a "money" strategy space;
c) any strategy x of E has a probabilistic meaning: probability 0 means "No check the possible tax evasion of the Taxpayer"; probability 1 means "to check the possible tax evasion of the player 2";
d) from a *frequency* point of view, the probabilistic strategy x is realizable by checking n = x m taxpayer declarations, where m is the total number of taxpayers.

2.1. Payoff functions

Let us see the form of our particular payoff functions.

Payoff function of the Government

In our numerical example, we have:

$$\begin{aligned} f_1(x, y) &= x((1/4)V + (1/2)(V- y) - (1/4) - (1/4)y) + (1/4)y = \\ &= x((3/4) – (1/2)y – (1/4) – (1/4)y)) + (1/4)y = \\ &= x((1/2) – (3/4)y)) + (1/4)y = \\ &= x/2 - (3/4)xy + y/4, \end{aligned}$$

for every pair (x,y) in the square $\mathbf{U}^2$.

Payoff function of the taxpayer

In our numerical example, we have:

$$\begin{aligned} f_2(x, y) &= (1- x)(1- y + (3/4)y + x((3/4) – (1/2)(1- y)) = \\ &= (1- x)(1- (1/4)y) + x(1/4)+(y/2)) = \\ &= (1- (y/4)) - x + (x/4)y + (x/4) + (x/2)y = \\ &= 1- (3/4)x + (3/4)xy - y/4, \end{aligned}$$

for every pair (x,y) in $\mathbf{U}^2$.

Payoff function of the sample game

Concluding, in our numerical example, the payoff function of the entire game is defined by

$$\begin{aligned} f(x, y) &= (x/2 + y/4 – (3/4)xy, 1 - (3/4)x + (3/4)xy – y/4) = \\ &= (1/4)(2x + y - 3xy, -3x – y + 3xy) + (0,1), \end{aligned}$$

for every strategy profile (x,y) of the game G.

Tridimensional representation of the game (f,>). Here, we present a 3D representation of the game (f,>). This representation consists in the union of the graphs of the payoff functions. The mostly higher surface is the graph of the Government payoff function, the mostly lower surface is the graph of the Tax-payer payoff function.

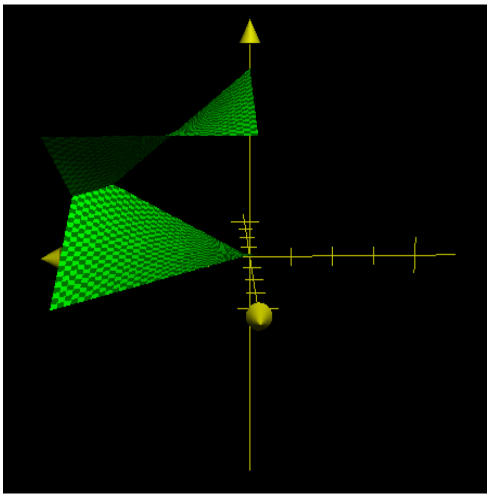


Figure 1. 3D representation of f

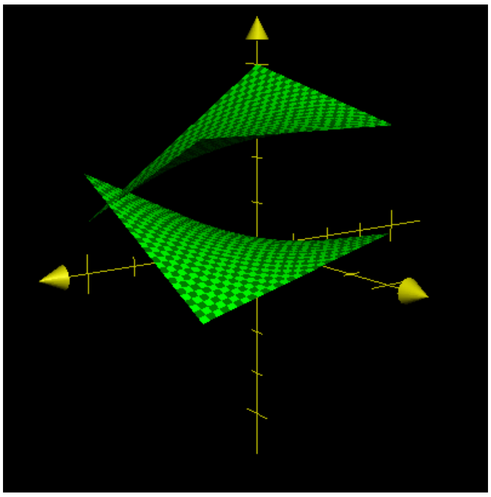


Figure 2. 3D representation of f

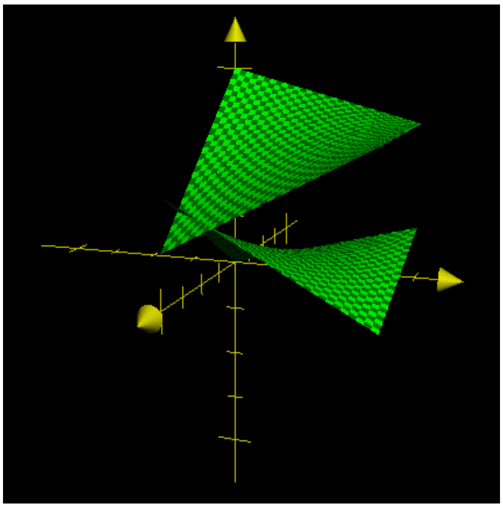


Figure 3. 3D representation of f

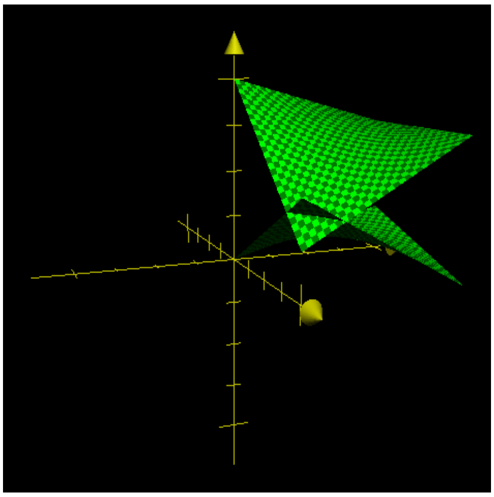


Figure 4. 3D representation of f

Note that, there is a connected part of the bi-strategy square on which the Tax-payer function is greater than the Government function. We represent it in the following figure.

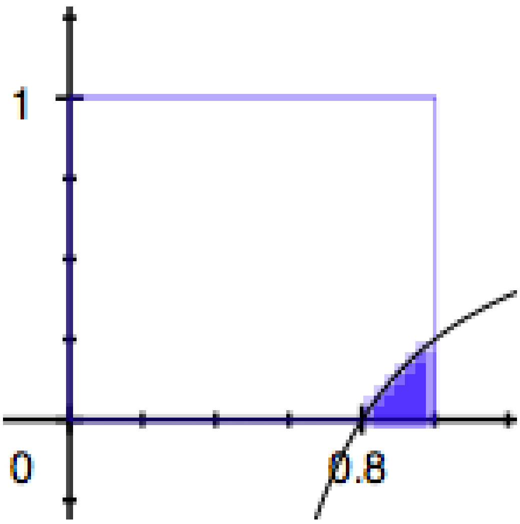


Figure 5. Tax-payer function is greater than the Government function

3. Digression: Why the taxpayer should pay the taxes?

Note that our game (f,>) is a non-zero and non-constant sum game: the aggregate payoff function s of our game G is the real function defined on the bi-strategy space by

$s(x, y) = -\frac{1}{4}x + 1$, for every probability strategy x of the Government. The interpretation is quite clear:

a) 1 is the total income of the tax payer, which is in this context the only effective income of the game;
b) and ¼ x is the expense of the Government for checking the evasion, when it decides to employ the strategy x.

Observe, moreover, that the maximum, on the strategy space **U,** of this social sum s is attained at any bi-strategy point (0,y), with y in **U**, and this maximum is 1 (the Taxpayer real income). So that, the maximum collective gain $1 = \max_U(s)$, corresponds to the situation in which the Government does not check the declaration of the Taxpayer (0 expenses for checking it). Of course, in this case, the Taxpayer (if he is aware of the Government strategy) will choose to declare nothing (his strategy 0) and *the total collective payoff remains only in the Taxpayer's hands*. Obviously this last situation is a selfish bad scenario for the human society: but this is also the reason at the root of the tax evasion.

The *key-solution for the collectivity* is that the tax tV (or ty or g(t)(V-y), and so on…) should be used by the Government "much better" than how much the taxpayer itself can do! In other terms, *to convince the Taxpayer to pay the taxes*, the Government should employ the capitals deriving from taxes at an income rate i_G such that, for any reasonable tax payer individual income rate i_T, one has (for instance in the case of truthful declaration)

$$(1+ i_G)\, tV + (1+ i_T)(1- t)V > (1+ i_T)V,$$

i.e., a rate of income that makes the social sum $(1+ i_G)\, tV + (1+ i_T)(1- t)V$ greater than the potential future income $(1+ i_T)V$, of the Taxpayer.

4. The Complete Analysis of Our Sample Game

In this section we conduct the complete analysis of the game (f, >). At this aim, we observe that the payoff function f, of our numerical example, is viewable as 0.25 g + (0,1), where we have considered the "payoff kernel" g, defined by

$$g(x,y)= (2x + y - 3xy, -3x – y + 3xy),$$

for every bi-strategy (x,y). We shall study only this kernel g, since any information on the game (f, >) is deductible from the game (g, >), by a 0.25 rigid contraction and then by a (0, 1) translation.

4.1. The bi-strategy space

As we already saw, the bi-strategy space of our game is the square $\mathbf{U}^2$ it is represented in the following figure.

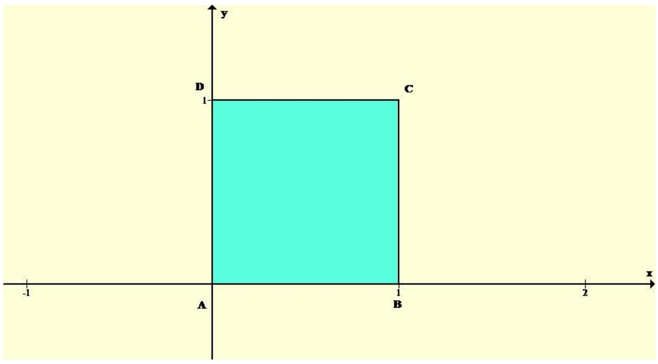


Figure 6. The bi-strategy space (square) S

4.2. The Payoff space

Our first significant aim is to find the payoff space of the game (g, >), that is the image g(S) of the vector payoff function g (the image of the bi-strategy square S under g). At this aim, we transform the topological boundary fr(S) of the square S, that is the union of the 4 edges of the square; and moreover, we have also to transform the critical zone cr(g) of the vector function g (set of bi-strategies at which the Jacobian matrix of g is not-invertible): the boundary of the image of g, is contained into the union of the two images g(fr(S)) and g(cr(g)):

$fr(g(S)) \subseteq g(fr(S)) \cup g(cr(g))$.

So let's go to study the Critical zone of g.

Critical zone. The critical zone of the function g is the set of all bi-strategies (x,y) at which the Jacobian matrix (family) is not invertible. The Jacobian family of the function g - defined by

$g(x,y)= (2x + y - 3xy, -3x – y + 3xy)$,

for every bi-strategy (x,y) - is the pair of gradients

$J(g)(x,y) = (grad(g_1), grad(g_2)) =$

$= ((2 - 3y, 1 - 3x), (3y - 3, 3x - 1))$.

The Jacobian determinant of the function g at the point (x, y) is the determinant of the vector family J(g)(x, y), that is the real number:

$$\det J(g)(x,y) = (2 - 3y)(3x - 1) + (3y - 3)(3x - 1) =$$
$$= 6x - 2 - 9xy + 3y + 9xy - 3y - 9x + 3 =$$
$$= -3x + 1.$$

So the Jacobian family is not invertible at the point (x, y) if and only the abscissa x is equal to 1/3. Concluding the critical zone of the function g is the segment cr(g) = **U**(0,1) + (1/3,0), or, if you prefer, the segment cr(g) = [(1/3,0), (1/3,1)].

Recapitulating: the critical zone is the subset of points (x,y) of the plane with x = 1/3 and y in **U**, it is the segment [P, Q] represented in the following figure 2. Note that the payoff critical zone is the unique point P' = (2/3,-1) (see Appendix 3).

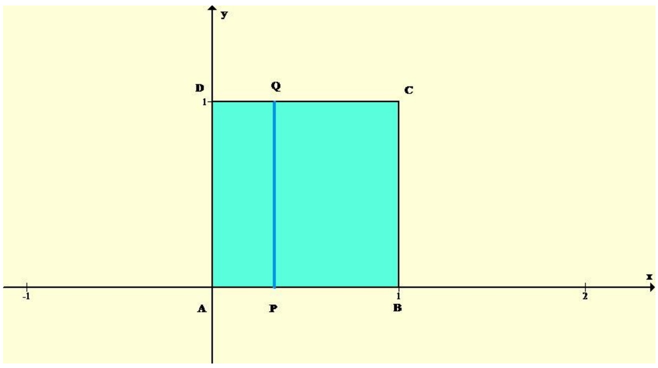


Figure 7. The Critical zone [P,Q] in bi-strategy space

Payoff space. Since the boundary of the payoff space is contained into the union of the transformation of the boundary of the square S and since the critical zone in the payoff space in reduced to the single point P' = (2/3, -1), we deduce that the boundary of the payoff space is indeed the transformation of the boundary of the square S. So we have the following Figure 3 (for the calculations see Appendix 3). In Figure 3, it is evident the geometric importance of the knot P'.

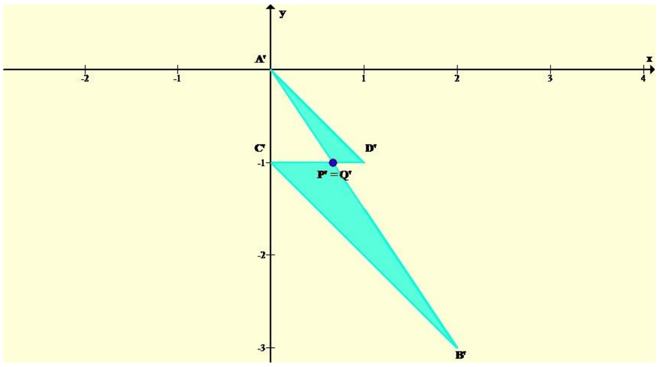


Figure 8. The payoff space of the game

4.3. Nash equilibrium

The **unique Nash equilibrium** is the pair N = (1/3, 2/3), the corresponding Nash payoff N' belongs to the payoff critical zone, so it is the payoff P' = (2/3, -1): this is simple to check and we give the calculations in Appendix 1.

The **Nash payoff of the original game** (f, >) is the payoff (1/6, 3/4). As we'll see, the Nash payoff of the taxpayer is his conservative one.

4.4. Conservative phase

Also the conservative bi-value of the game belongs to the payoff critical zone {P'}, as shown in the following figure 4. We give all the calculations in Appendix 2. The conservative strategies are $x^{\#}$ = 1/3 and $y^{\#}$ =1, respectively. Note that *the unique conservative strategy of the taxpayer is the truthful declaration* 1.

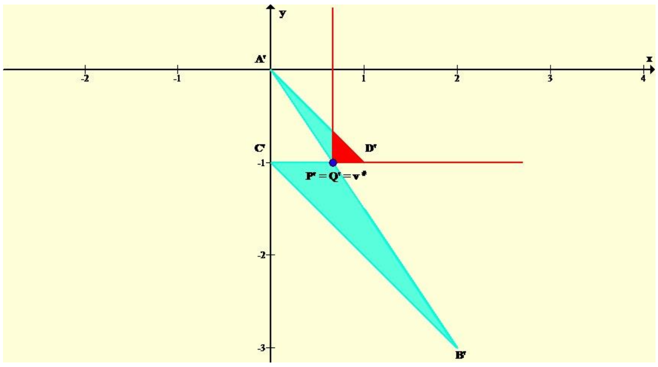


Figure 9. The conservative zone of the game (g, >) in its payoff space

4.5. Pareto boundary

The Pareto maximal boundary is (straightforwardly) the union of segments [A', D'] and [H', B'], shown in the below figure (the point H' - which is not Pareto efficient - is denoted by a little triangle). Note that it is bounded but not compact and (evidently) not connected.

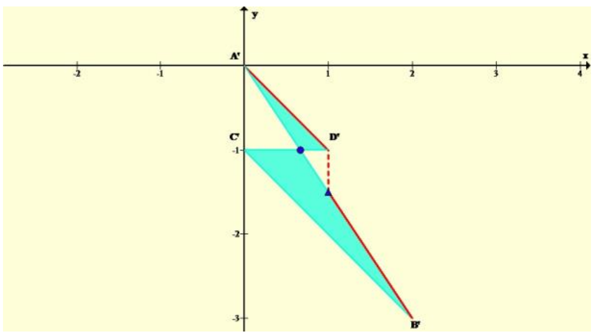


Figure 10. The Pareto boundary in the payoff space (bounded but not closed)

4.6. The conservative core

The core is simply a compact segment, as it is shown in the figure below. It has the important characteristic to be entirely with the maximum collective (aggregate) value of the game.

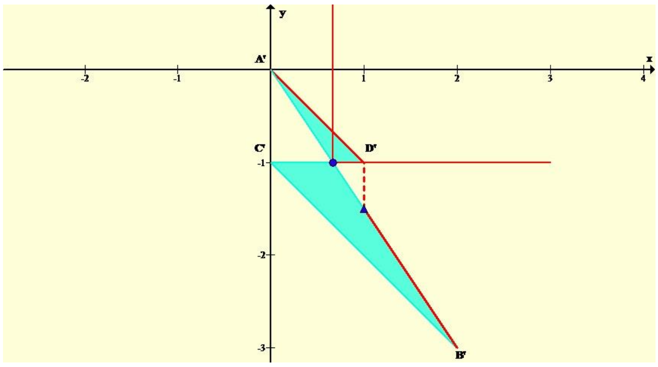


Figure 11. The conservative core (which coincides with the Nash core) in the payoff space

4.7. Compromise solution

We propose two compromise solutions, maximizing the collective payoff of the game. They are almost coincident. We propose Kalai-Smorodisky solutions with threat point the Nash equilibrium and utopia points, the sup of the game or the sup of the core. They are both win-win solutions with respect to the Nash payoff.

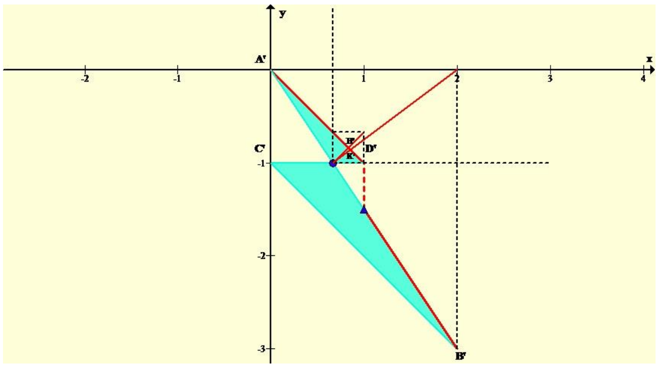


Figure 12. The Kalai-Pareto compromise solution and the core compromise in the payoff space

Important remark 2 (risk neutral solution of the game). To stress more the question, let au study the payoff function of the taxpayer at its Nash equilibrium strategy y = 2/3. We have

$h(x) := g_2(x, 2/3) = - 3x - 2/3 + 2x = - 2/3 - x,$

for every Government strategy x. Assume the taxpayer be risk neutral and without any information about the Government strategy, then we have to consider the mean value of the above function h, it is

$\mu = (½) (- 2/3 - 5/3) = - 7/6 < -1 = v_2^{\#},$

so that, even a risk neutral taxpayer, will decide to defend himself by the conservative strategy 1.

Important remark 3 (Stackelberg behavior). Assume that the taxpayer is aware that the Government is being to use his Nash equilibrium strategy 1/3. Then, the payoff function to be considered is the section g_2 (1/3,), given by $g_2(1/3, y) = -1$, for every y in E: also in this case, we have the conservative payoff -1.

Conclusions

The goals of our paper are resumed in what follows.

- We model, in a *very general and applicable framework*, the interaction between a Government of a country and any possible tax-payer of the country itself, by using a *realistic probability (frequency) approach for the checking evasion strategy* of the Government;
- We propose a *realistic and realizable, by normative actions and laws*, particular sample of the general model. In this sample, the *Nash equilibrium* shows a situation in which the payoff of the taxpayer equals

the *conservative payoff* of the taxpayer himself. Not only this circumstance is, by itself, the worst possible one for a tax evasion, but moreover, *the taxpayer could attain this Nash/conservative payoff by declaring truthfully his real income*, not incurring in any punishment.

- Furthermore, since at least the conservative payoff (and in this case also Nash payoff) of the tax payer is certainly reached by the adoption of his unique conservative strategy 1, for *a risk averse or risk neutral tax payer, there is no reason to declare less than his real income.*
- In our realistic particular example, the *conservative strategy of the tax payer (the most likely one*) shows a situation in which *a tax payer has no convenience to declare an income inferior than the real one;*
- We propose, by the way, a measure of the *collective loss in the above Nash equilibrium*, and we show its position by the *total knowledge of the entire payoff space* of the sample-interaction.
- We show *two (quantitatively close) compromise solutions*, applicable (by binding contracts) in the case of *great distinguished tax-payers*, maximizing the collective gain of the society.

References

[1] Carfi, D., and Schilirò, D. 2011. Crisis in the Euro Area. Coopetitive Game Solutions as New Policy Tools, *Theoretical and Practical Research in Economic Fields* 2(1): 23-36.

[2] Carfi, D., and Perrone, E. 2011. Game Complete Analysis of Bertrand Duopoly, *Theoretical and Practical Research in Economic Fields* , Volume II, Issue 1(3), pp: 5-22.

[3] Carfi, D., and Musolino, F. 2011. Fair Redistribution in Financial Markets: a Game Theory Complete Analysis, *Journal of Advanced Studies in Finance* 2(2): 74-100.

[4] Carfi, D. 2009. Payoff space in C1-games, *Applied Sciences* (APPS) 11: 1-16.

[5] Carfi, D., and Ricciardello, A. 2010. An algorithm for payoff space in C1-games. *AAPP*: 88(1).

[6] Carfi, D., and Ricciardello, A. 2012. Topics in Game Theory. AAPP Monograph 9.

[7] Ehrlich, I. 1973. Participation in Illegitimate Activities: A Theoretical and Empirical Investigation, *The Journal of Political Economy* 81(3): 521-526.

[8] Carfi, D., and Perrone, E. 2012. “Asymmetric Cournot duopoly: game complete analysis“, MPRA Paper 37093, University Library of Munich, Germany.

[9] Carfì, D. and Fici, C. 2012. "The government-taxpayer game“, MPRA Paper 38506, University Library of Munich, Germany.

[10] Carfì, D. and Schilirò, D. 2012. "Global green economy and environmental sustainability: a coopetitive model“, MPRA Paper 37018, University Library of Munich, Germany.

[11] Carfì, D. and Perrone, E. 2012. "Game complete analysis of symmetric Cournot duopoly“, MPRA Paper 35930, University Library of Munich, Germany.

[12] Carfì, D. Bagileri, D. and Dagnino, G. 2012. "Asymmetric RandD alliances and coopetitive games“, MPRA Paper 37095, University Library of Munich, Germany.

[13] Carfì, D. and Pintaudi, A. 2012. "Optimal participation in illegitimate market activities: complete analysis of 2-dimensional cases“, MPRA Paper 37822, University Library of Munich, Germany.

[14] Carfì, D. and Musolino, F. 2012. "A coopetitive approach to financial markets stabilization and risk management“, MPRA Paper 37098, University Library of Munich, Germany.

[15] Carfì, D. and Schilirò, D. 2012. "A framework of coopetitive games:applications to the Greek crisis“, MPRA Paper 37855, University Library of Munich, Germany.

[16] Carfì, D. and Schilirò, D. 2012. "A model of coopetitive game for the environmental sustainability of a global green economy“, MPRA Paper 38508, University Library of Munich, Germany.

[17] Carfì, D., Patané, G. and Pellegrino, S. 2011. "Coopetitive games and sustainability in project financing“, MPRA Paper 32039, University Library of Munich, Germany.

[18] Carfì, D., Gambarelli, G. and Uristani, A. 2011. "Balancing pairs of interfering elements“, MPRA Paper 35335, University Library of Munich, Germany.

[19] Carfì, D. and Musolino, F. 2011. "Game complete analysis for financial markets stabilization", MPRA Paper 34901, University Library of Munich, Germany.

[20] Carfì, D., Ricciardello, A., and Agreste, S. 2011. "An Algorithm for payoff space in C1 parametric games", MPRA Paper 32099, University Library of Munich, Germany.

[21] Carfì, D., and Perrone, E. 2011. "Asymmetric Bertrand duopoly: game complete analysis by algebra system Maxima", MPRA Paper 35417, University Library of Munich, Germany.

[22] Carfì, D. and Schilirò, D. 2011. "A model of coopetitive games and the Greek crisis", MPRA Paper 32098, University Library of Munich, Germany.

[23] Carfì, D. 2011. "Financial Lie groups", MPRA Paper 31303, University Library of Munich, Germany.

[24] Carfì, D. and Schilirò, D. 2011. "Coopetitive games and global green economy", MPRA Paper 32035, University Library of Munich, Germany.

[25] Carfì, D. and Perrone, E. 2011. "Game complete analysis of Bertrand Duopoly", MPRA Paper 31302, University Library of Munich, Germany.

[26] Carfì, D. and Ricciardello, A. 2011. "Mixed extensions of decision-form games", MPRA Paper 28971, University Library of Munich, Germany.

[27] Carfì, D. and Schilirò, D. 2011. "A coopetitive model for the green economy", MPRA Paper 35245, University Library of Munich, Germany.

[28] Carfì, D. and Trunfio, A. 2011. "A non-linear coopetitive game for global green economy", MPRA Paper 32036, University Library of Munich, Germany.

[29] Carfì, D. and Schilirò, D. 2010. "Crisis in the Euro area: coopetitive game solutions as new policy tools", MPRA Paper 27138, University Library of Munich, Germany.

[30] Carfì, D. and Schilirò, D. 2010. "Crisis in the Euro area: coopetitive game solutions as new policy tools", MPRA Paper 31891, University Library of Munich, Germany, revised 26 Jun 2011.

[31] Carfì, D., Magaudda, M. and Schilirò, D. 2010. "Coopetitive game solutions for the Eurozone economy", MPRA Paper 26541, University Library of Munich, Germany.

[32] Carfì, D. 2009. "Differentiable game complete analysis for tourism firm decisions", MPRA Paper 29193, University Library of Munich, Germany.

[33] Carfì, D. and Ricciardello, A. 2009. "Non-reactive strategies in decision-form games", MPRA Paper 29262, University Library of Munich, Germany.

[34] Carfì, D. 2009. "Fibrations of financial events", MPRA Paper 31307, University Library of Munich, Germany.

[35] Carfì, D. 2009. "Decision-form games", MPRA Paper 29009, University Library of Munich, Germany.

[36] Carfì, D. 2009. "Reactivity in decision-form games", MPRA Paper 29001, University Library of Munich, Germany.

[37] Carfì, D. 2008. "Optimal boundaries for decisions", MPRA Paper 29243, University Library of Munich, Germany.

APPENDIX 1

Nash Equilibrium

We recall that the kernel function g is defined by
$g(x,y)= (2x + y - 3xy, -3x – y + 3xy)$,
for every x,y in the strategy square **U**. Then, for what concern the partial derivative of g_2 in the second argument, note that
$g_2 (x,)'(y) = \partial_2 g_2 (x, y) = -1 + 3 x \geq 0$,
so that the tax-payer section $g_2 (x,.)$ is strictly increasing if x > 1/3, it is constant if x = 1/3 and it is strictly decreasing when x < 1/3. Hence, the reaction correspondence B_2 (from E into F) is defined by $B_2(x) = \{0\}$ if x < 1/3, $B_2(x) = F$ if x = 1/3 and $B_2(x) = \{1\}$ if x > 1/3. Recapitulating, we have

$$B_2 (x) = \begin{cases} \{0\} & \text{if } x \in [0, 1/3[\\ [0,1] & \text{if } x = 1/3. \\ \{1\} & \text{if } x \in]1/3, 1] \end{cases}$$

On the other hand, we have $g_1 (x,y) = 2x + y – xy$, for any x, y in [0,1]. Since we have
$g_1 (.,y)' (x) = \partial_1 g_1 (x,y) = 2 – 3y \geq 0$,
then the section $g_1 (.,y)$ is strictly increasing if y < 2/3, it is constant if y = 2/3, and it is strictly decreasing if y > 2/3. Hence, the reaction correspondence B_1 (of F into E) is defined by $B_1(y) = \{1\}$ if y < 2/3, $B_1(y) = E$ if y = 2/3 and $B_1(y) = \{0\}$, when y > 2/3:

$$B_1 (y) = \begin{cases} \{1\} & \text{if } y \in [0, 2/3[\\ [0,1] & \text{if } y = 2/3. \\ \{0\} & \text{if } y \in]2/3, 1] \end{cases}$$

Nash Equilibria. We have (by intersecting the graph of B_2 with the inverse graph of the correspondence B_1) one unique Nash equilibrium of the game (g,>): the point N = (1/3, 2/3). This Nash equilibrium is also the Nash equilibrium of the original game (f,>). The **Nash payoff of the kernel game** (g,>) is the critical payoff point N' = g(N) = (2/3, – 1), this is obvious also because the entire critical zone (1/3,0) + **U**(0,1) is transformed into the single point P' = (2/3,-1).

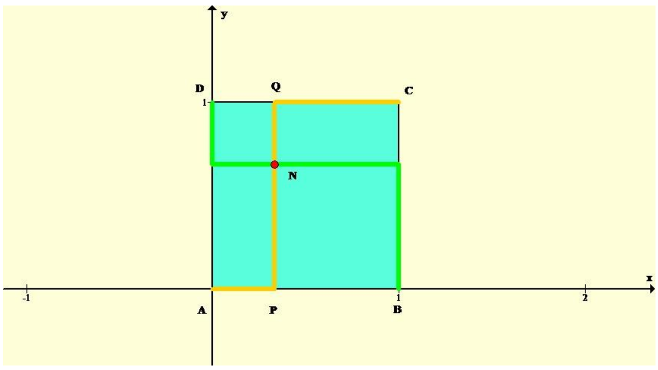


Figure A1. The best reply graphs with Nash equilibrium

The **Nash payoff of the original game** (f,>) is the critical payoff point
$(1/4)N' + (0, 1) = (1/4) (2/3, – 1) + (0, 1) = (1/6, ¾)$.
Note that this point guarantees a collective gain equal to 11/12.

Important remark 1 (risk averse solution of the game). Note that the Nash payoff of the taxpayer (in the original game (f,>)) is ¾, exactly the conservative payoff of the taxpayer in the game (f,>). So that, a rational risk averse taxpayer should prefer the conservative strategy $y^\# = 1$ (which guarantees at least this payoff ¾, independently on the Government strategy) to the uncertain scenario shown by the Nash equilibrium strategy 2/3: **a risk averse taxpayer has no reasons to choose other strategies than the conservative truthful declaration** $y^\# = 1$, in the conditions of our sample game (f,>).

APPENDIX 2

Conservative phase

The conservative value for the first player is

$v_1^\# = \sup_{x \in E} \inf_{y \in F} (2x + y - 3xy) = \sup_{x \in E} g_1^\# (x)$,

where $g_1^\# (x) = \inf_{y \in F} (2x + y - 3xy)$ is the *worst gain function for the first player*. Now, fixed $x \in [0, 1]$, we have

$g_1(x, .)'(y) = 1 - 3x$,

so the section $g_1 (x,.)$ is strictly increasing if $x < 1/3$, it is constant if $x = 1/3$ and it is strictly decreasing if $x > 1/3$. Hence, **the worst offensive correspondence** O_2 of the second player versus the first one is defined, from E into F, by $O_2(x) = \{0\}$ if $x < 1/3$, $O_2(x) = F$ if $x = 1/3$ and $O_2(x) = \{1\}$ if $x > 1/3$.

So that **the worst gain function for the Government** is defined by

$$g_1^\# (x) = \begin{cases} g_1 (x, 0) = 2x & \text{if } x \leq 1/3 \\ g_1 (x, 1) = 1 - x & \text{if } x > 1/3 \end{cases}$$

The unique maximum point of the above function is $x^\# = 1/3$, this is the **unique conservative strategy of the first player**. **The conservative value of the first player** is $v_1^\# = g_1^\# (x^\#) = 2/3$.

The conservative value for the second player is

$v_2^\# = \sup_{y \in F} \inf_{x \in E} (-3x - y + 3xy) = \sup_{y \in F} g_2^\# (y)$,

where

$g_2^\# (y) = \inf_{x \in E} (-3x - y + 3xy)$

is the **worst gain function for the second player**. Now, fixed $y \in [0, 1]$, we have

$g_2 (.,y)'(x) = -3 + 3y$,

so the section $g_2 (., y)$ is strictly decreasing if $y < 1$, it is constant if $y = 1$. Hence, the worst offensive correspondence O_1, form F into E, is defined by $O_1(y) = \{1\}$, if $y < 1$ and by $O_1(y) = E$, if $y = 1$.

So that, **the worst gain function for the Taxpayer** is defined by

$g_2^\# (y) = g_2 (1, y) = -3 + 2y$,

for every y in F. The unique maximum point of the above function (in F) is the strategy $y^\# = 1$, this is the **unique conservative strategy of the second player**.

The conservative value of the second player is $v_2^\# = g_2^\# (y^\#) = -1$.

The payoff at the conservative cross is the Nash Payoff, $g (1/3, 1) = (2/3, - 1)$.

APPENDIX 3

Payoff space

We study the image of the function g - defined by
$g(x, y) = (2x + y - 3xy, -3x – y + 3xy)$,
for every bi-strategy (x, y).

Image of side [A, B]. We have
$G(x, 0) = (2x, -3x)$,
for every strategy x in [0, 1], so that the image of the segment [A, B] is the straight line segment [A', B'], *where* A' = (0, 0) and B' = (2,-3).

Image of side [B, C]. We have
$g(1,y) = (2 - 2y, - 3 + 2y)$,
for every strategy y in [0, 1], so that the image of the segment [B, C] is the straight line segment [B', C'], *where* B' = (2,-3) and C' = (0, -1).

Image of side [C, D]. We have
$G(x,1) = (1 - x, – 1)$,
for every strategy x in [0, 1], so that the image of the segment [C, D] is the straight line segment [C', D'], *where* D' = (1, -1) and C' = (0, -1).

Image of side [A, D]. We have
$G(0, y) = (y, – y)$,
for every strategy y in [0, 1], so that the image of the segment [A, D] is the straight line segment [C', D'], *where* D' = (1, -1) and A' = (0, 0).

Image of the critical zone [P,Q]. We have
$g(1/3,y) = (2/3 , -1)$,
for every y in [0, 1], so that the image of the critical strategy segment [P, Q] is the degenerate straight line segment [P',Q'] (it is a unique point), where P' = Q' = (2/3, -1).

ASERS Publishing

ASERS

ASERS Publishing
Web: www.asers.eu
URL: http://www.asers.eu/asers-publishing
ISSN 2068 – 7710