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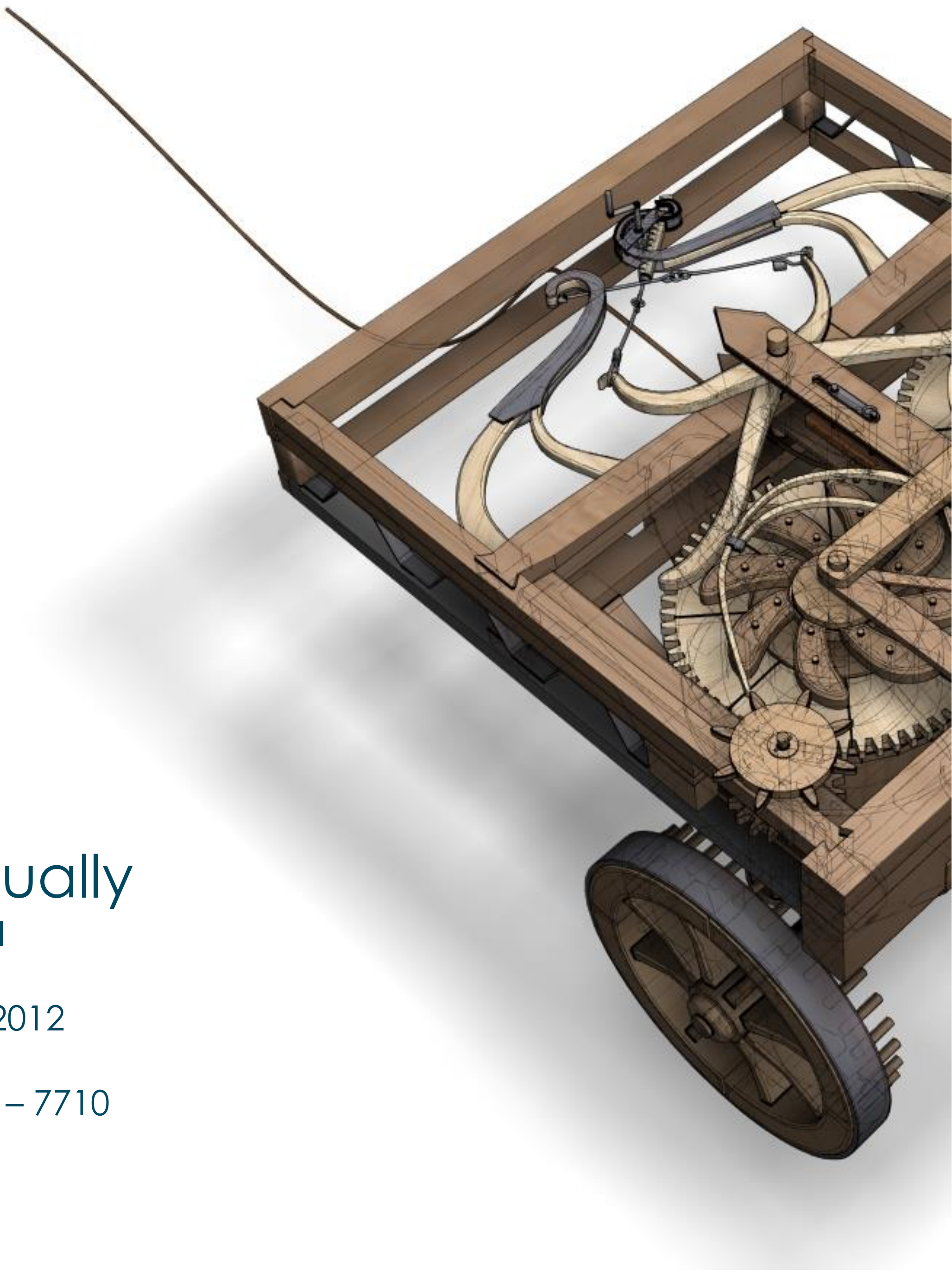

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A CLOSED FORM SOLUTION FOR A GROWTH MODEL WITH EXTERNALITIES AND PUBLIC SPENDING

Oliviero A. CARBONI
University of Sassari, **Italy**
Paolo RUSSU
University of Sassari, **Italy**

Abstract:

This paper studies the equilibrium dynamics of a growth model with public spending. The model considers negative production externalities by explicitly including them as unfavorable effects in the production function. Differently from conventional analysis here government spending along with private production, also generate negative externalities. The model simultaneously determines the optimal shares of consumption, capital accumulation, taxes and composition of the two different public allocations, which maximize the representative household's lifetime utilities in a centralized economy. Moreover, with one restriction on the parameters we fully determine the solutions path for all variables of the model and determine the conditions for balanced growth. Given the active role the government has in determining the level of production, the higher the externalities compared to the optimum, the lower the tax rate.

Keywords: growth models, fiscal policy, public spending composition, externalities.

JEL classification: O40, H50, E13, H20, D62.

1. Introduction

The relation between fiscal policy and economic growth is a central and controversial issue in the growth literature. Within this debate academics and policy makers have been particularly concerned about the potential negative effects deriving from industrialization. On the one hand it is stated that the economic benefits of improving production are related to more employment, more consumption and potential high tax revenue. On the other hand it is stressed that, while economic growth improves quality of life through produced goods, it also reduces welfare since it generates substantial negative externalities, such as congestion and environmental degradation.

The list of negative externalities associated with production may be considerably long. They mainly concern effects on residents' welfare such as: crowding and congestion cities, roads and public transportation and cities, noise, litter, air contamination and global warming effects, degradation of nature, increased urbanization, and increased crime rate. Congestion and environmental degradation caused by an expansion of manufacturing are essentially effects that are not taken into account by individual firms. Hence, production expansion is more desirable to an individual firm than it is to society. In this case competitive equilibria are not Pareto optimal. To correct this distortion, the government's fiscal policy should internalize such adverse effects. Among the public policies affecting the productive sector, taxation and government spending play an especially important role.

The logic behind the congestion effects of average capital stock is commonly recognized in the existing literature. Barro and Sala-i-Martin (1992), for instance, suggest that almost all public services are characterized by a certain degree of congestion and Eicher and Turnowski (2000) argue that congestion adversely affects the equilibrium growth rate. There are two main approaches to deal with environment externalities: government interventions (pigovian taxes, subsidies and direct regulation) and public abatement policies.

It is commonly recognized that tightening environmental policies generally has an adverse effect on economic growth because it crowds out private expenditure, including investment (Haung, and Cai, 1994; Ligthart, and van der Ploeg, 1994). Conversely, several studies propose that environmental tax policies may support economic growth. Ewijk, and Wijnbergen (1995), Bovenberg, and Smulders (1995), and Bovenberg, and de Mooij (1997), show that environmental taxes improve the quality of the environment, which in turn ameliorates the efficiency of other productive inputs, thereby promoting economic growth (see also Ricci, 2002 and Nakada, 2004 on similar positions).

Concerning the public abatement policy it should be mentioned that there are essentially two positions in the economics literature. The first assumes that private firms deal with abatement (Bovenberg, and de Moiij, 1997; Bovenberg, and Smulders, 1995). The second presumes that abatement spending is financed by the government. Within the latter, Gruver (1976) investigates the optimal division of investment between pollution control capital and directly productive capital. The model considers pollution as a flow positively related to aggregate output, negatively related to the stock of pollution control capital, and having a negative impact on utility. Greiner (2005) analyzes a growth model where pollution affects only the utility of a representative household but does not affect production possibilities directly by entering the aggregate production function. However, there is an indirect effect of pollution on output, because it is supposed that resources are used for abatement activities.

Economides and Philippopoulos (2008) investigate Ramsey second-best optimal policy in a general equilibrium model of growth augmented with renewable natural resources. Natural resources are depleted by private economic activity but they can also be maintained by public policy. The government uses tax revenues to finance infrastructure services and cleanup policy. Policy instruments (the tax rate on polluting activities and the allocation of tax revenue between infrastructure and cleanup policy) are chosen optimally. Gupta and Barman (2009) analyze the properties of optimal fiscal policy in the presence of productive public expenditure and environmental degradation. They consider the level of consumption as the source of pollution. Government allocates its tax revenue between pollution abatement expenditure and productive public expenditure. Degradation of environmental quality reduces the effective benefit of public investment expenditure. One common feature of all these models is that they treat externalities as being generated by private production while the government acts as a “cleaner up” and subtracts the required resources from the private sector.

Finally, a more recent line of research in economic growth models, has focused on the role played by negative externalities and has established the existence of indeterminate equilibrium paths. Chen and Lee (2007) consider a social constant returns economy where a congestion effect generates negative aggregate externalities. Itaya (2008) shows how pollution may affect indeterminacy results in a one-sector growth model with social increasing returns. In Meng, and Yip (2008) indeterminacy derives from negative capital externalities. In Antoci *et al.* (2005) and in Antoci, and Sodini (2009) negative externalities may generate indeterminacy in an economy where private goods can be consumed as substitutes for free access environmental goods.

Starting from the above discussion, the aim of this work is to study the equilibrium dynamics of a growth model with public spending where negative aggregate production externalities are explicitly included in the production function as unfavorable effects. The model simultaneously determines the optimal shares of consumption, capital accumulation, taxes and composition of the two different public expenditures which maximize a representative household’s lifetime utilities in a centralized economy. Optimality is determined by deriving the first best optimum of the social planner in the presence of adverse effects deriving from aggregate production. Moreover, under the condition $\sigma=\alpha$ (Uzawa, 1965; Smith, 2006; Chilarescu, 2008; Hiraguchi, 2009) the model supplies a closed form solution and determines the conditions for a balanced growth (Carboni, and Russu, 2012).

The paper has the following structure: section 2 contains the model background, section 3 outlines the analytical model, section 4 describes the dynamics and section 5 concludes.

2. Model Background

In the last decades, a vast literature has emerged on the relationship between fiscal policy and long-run economic growth, and the composition of public expenditure has become a central question in growth studies. For instance, Lee (1992), Devarajan *et al.* (1996) expand on Barro’s model, allowing different kinds of government expenditure to have different impacts on growth. Devarajan *et al.* (1996) consider two productive services (expressed as flow variables) with two different productivities and derive the conditions under which a change in the composition of expenditure leads to a higher steady-state growth rate of the decentralized economy. By using the distinction between productive and non-productive spending (Glomm and Ravikumar, 1997; Kneller *et al.*, 1999), they are able to determine the optimal composition of different kinds of expenditures, based on their relative elasticities. Following a similar line, Chen (2006) investigates the optimal composition of public spending in an endogenous growth model with a benevolent government. He establishes the optimal productive public service share of the total government budget and the optimal public consumption share, determined by policy and structural parameters.

Also within an endogenous growth framework, Ghosh, and Roy (2004) introduce public capital and public services as inputs in the production of the final good. They show that optimal fiscal policy in an economy depends on the tax rate and on the share of spending for the accumulation of public capital and the provision of public services. Finally, employing a neoclassical framework Carboni, and Medda (2011, a, b) consider two different kinds of public capital through accumulation and determine the government size and the mix of government expenditures which maximize the rate of growth and the long-run level of per capita income.

Following this strand of literature this paper analyzes the equilibrium dynamics of a growth model with public finance, where two different allocations of public resources are considered. We consider the fiscal policy as a part of the aggregate economy by explicitly including the public sector in the production function. This generates a potential relationship between government and production.

In line with Devarajan *et al.* (1996) and Ghosh, and Gregoriu (2008) we consider the two types of public expenditure entering as flows in the production function. All government activities are considered to be production-enhancing according to their respective elasticities. Hence, the government can influence private production through spending for different types of public investment such as roads and highways, telecommunication systems, RandD capital stock, other infrastructures (Aschauer, 1989; Kneller *et al.*, 1999) or for simple services, such as the maintenance of infrastructure networks and keeping law and order. The different impact of each type of government spending on production makes is all the more necessary to disaggregate the public budget into its various components.

The model considers negative production externalities by explicitly including them as unfavorable effects in the production function. Unlike conventional analysis, here government spending, along with private production, is also assumed to generate negative externalities. This is supposed to have an effect on optimal public spending and tax decision. We analyze the case where the social planner internalizes the externalities in Ghosh, and Gregoriu (2008). Differently from their work which considers four control variables (c, τ, g_1, g_2 in their terminology), we endogenize y so that the social planner directly accounts for, τ and φ in the maximization decision while taking into consideration the negative aggregate externalities coming from production. Moreover, employing a Cobb-Douglas production function our model ends up with three equations. Hence, the complexity of the dynamic system is reduced.

It worth highlighting that Zhang (2011) supplies an analytical expression of the balanced growth solution in a multi-sector model. He calculates the optimal distribution coefficient of fixed capital investment and of labor hour, the proportion of production, the economic growth rate, the rate of change of the price index, and rental rates of different fixed capital. However, differently from our work his analysis does not consider optimal fiscal policy.

3. Model Set Up

In this section we model the government expenditure composition as a part of the aggregate economy. We explicitly include the public sector in the production function as a distinct input based on the rationale that government services are not a substitute for private factors, and resources cannot be easily transferred from one sector to another. Public capital provides flows of rival, non-excludable public services, which would not be provided by the market. Flows are proportional to the relative stocks and enter the production function together with private capital.

The model considers two different categories of public spending. The first (G_1) is traditional core productive spending. The second (G_2) is a broad concept of capital, namely "institutional" spending embracing all the activities which are designed to improve the environment in which firms can effectively operate (Glaeser *et al*, 2004). Both components of government expenditure are complementary with private production (e.g. private vehicles can be used more productively when the quality of the road network increases). Following Barro (1990) and most of the recent work in growth studies, in our specification productive government expenditure is introduced as a flow (Glomm, and Ravikumar, 1994; Turnowski, and Fischer, 1995; Devarajan *et al.*, 1996; Eicher, and Turnovsky, 2000; Ghosh, and Gregoriu, 2008).[1]

[1] An alternative method is to allow the government also to accumulate stocks of durable consumption goods and physical infrastructure capital (Arrow, and Kurz, 1969; Futagami *et al*. (1993); Fisher, and Turnovsky, 1998; Carboni, and Medda, 2011a, b, among others). Although attractive in terms of realism, this approach would substantially increase the dimensionality of the dynamic system. The introduction of two public capital stocks along with private capital would imply a macro dynamic equilibrium with three state variables which considerably complicates the formal analysis (Turnovsky, and Fisher, 1995).

We assume that there is a large number of infinitely lived households and firms that is normalized to one, that population growth is zero and that there is no entry or exit of firms. The representative firm produces a single composite good using private capital (k) which is broadly defined to encompass physical and human capital, and two public inputs, G_1 and G_2, based on CES technology:[2]

$$y = k^{\varepsilon} G_1^{\beta_1} G_2^{\beta_2} \overline{y}^{-\psi} \tag{1}$$

where $\varepsilon = 1 - \beta_1 - \beta_2$, $\overline{y}^{-\psi}$ represents the negative externalities deriving from aggregate production ($\overline{y}$) (Chang *et al.*, 2011).

The government finances total public expenditure, $G_1 + G_2$, by levying a flat tax, τ, on income. In line with the main literature, we assume a permanent balanced government budget and rule out debt-financing of government spending (Barro, 1990; Futugami, Morita, and Shibata, 1993; Fisher, and Turnovsky, 1998). Public spending is financed by levying an average flat-rate tax on incomeτ $(0 < \tau < 1)$:

$$G_1 + G_2 = \tau y \tag{2}$$

$\phi(\text{or } 1 - \phi)$ denotes the share of public revenue allocated to $G_1(\text{or } G_2)$ so that

$$G_1 = \phi \tau y \tag{3}$$
$$G_2 = (1 - \phi) \tau y \tag{4}$$

The households own the firms and therefore receive all their output net of taxation which they either reinvest in the firms to increase their capital stock or use for consumption, depending on their preferences and the returns on private capital. Private investment by the representative household equals

$$\dot{k} = (1 - \tau) y - c \tag{5}$$

The central planner maximizes lifetime utility U given by

$$U(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma} \tag{6}$$

where c represents per capita consumption, and σ is the inter-temporal elasticity of substitution. Replacing (3) and (4) in (1), we obtain

$$y = k^{\alpha} \Lambda(\tau, \phi) \tag{7}$$

where $\Lambda(\tau, \phi) := (\tau\phi)^{\gamma_1} (\tau(1 - \phi))^{\gamma_2}$ and

$$\alpha = \frac{\varepsilon}{1 - \beta_1 - \beta_2 + \psi}, \gamma_1 = \frac{\beta_1}{1 - \beta_1 - \beta_2 + \psi}, \gamma_2 = \frac{\beta_2}{1 - \beta_1 - \beta_2 + \psi}.$$

We assume that the central planner chooses the functions$c$, τ and ϕ in order to solve the following problem

$$\underset{c,\tau,\phi}{MAX} \int_0^{\infty} \frac{c^{1-\sigma} - 1}{1 - \sigma} e^{-rt} dt \tag{8}$$

[2] This specific production function exhibits constant returns to scale at a disaggregate level because each firm takes $\overline{y}$ as given. On the contrary, the social planner can internalize the externality, thus obtaining decreasing returns.

subject to

$$\dot{k} = (1-\tau)k^{\alpha}(\tau\phi)^{\gamma_1}(\tau(1-\phi))^{\gamma_2} - c$$
$$k(0) > 0; \quad t \in 0, +\infty)$$

where $r > 0$ is the discount rate.

4. Dynamics

The current value of the Hamiltonian function associated to problem (8) is

$$H = \frac{c^{1-\sigma}-1}{1-\sigma} + \lambda((1-\tau)k^{\alpha}\Lambda(\tau,\phi) - c) \tag{9}$$

where λ is the co-state variable associated to k. By applying the Maximum Principle, the dynamics of the economy is described by the system

$$\dot{k} = \frac{\partial H}{\partial \lambda} = (1-\tau)k^{\alpha}\Lambda(\tau,\phi) - c \tag{10}$$
$$\dot{\lambda} = r\lambda - \frac{\partial H}{\partial k} = \lambda(r - \alpha(1-\tau)k^{\alpha-1}\Lambda(\tau,\phi)) \tag{11}$$

with the constraint

$$H_c = c^{-\sigma} - \lambda = 0 \tag{12}$$
$$H_{\tau} = (-k^{\alpha}\Lambda + k^{\alpha}(1-\tau)\Lambda\Lambda_{\tau})\lambda = 0 \tag{13}$$
$$H_{\phi} = (1-\tau)k^{\alpha}\Lambda_{\phi}\lambda = 0 \tag{14}$$

with $\Lambda_{\tau} = \frac{\partial \Lambda}{\partial \tau} = \frac{\Lambda}{\tau}(\gamma_1 + \gamma_2)$ and $\Lambda_{\phi} = \frac{\partial \Lambda}{\partial \phi} = \frac{\Lambda}{\phi}(\gamma_1 - \gamma_2)$.

By straight calculation, we can write the values of the control variablesτ, ϕ which

$$\phi^{*} = \frac{\gamma_1}{\gamma_1+\gamma_2} \tag{15}$$
$$\tau^{*} = \frac{\gamma_1+\gamma_2}{1+\gamma_1+\gamma_2} \tag{16}$$

which in terms of the initial exponents become

$$\phi^{*} = \frac{\beta_1}{\beta_1+\beta_2} \tag{17}$$
$$\tau^{*} = \frac{\beta_1+\beta_2}{1+\psi} \tag{18}$$

It is worth noting that ψ (the unfavorable effect) has only effects on τ^{*} and that the optimal rate of taxation will be smaller than the case with no externalities ($\psi = 0$). This comes from the very assumption of the model which assumes government spending, along with private production, to be a source of negative undesired effects. This clearly affects optimal public spending and tax decision. More in detail, in order to reduce (increase) the externalities to the optimal level the central planner ought to reduce (increase) taxes and public spending.

By replacing equations (17) and (18) in (8) and noting that from equation (12)$\frac{\dot{c}}{c} = -\frac{1}{\sigma}\frac{\dot{\lambda}}{\lambda}$, one can write the following system, equivalent to (10)-(11)

$$\dot{k} = \Lambda^{\mathring{a}} k^{\alpha} - c \tag{19}$$

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}(\alpha \Lambda^{\mathring{a}} k^{\alpha - 1} - r) \tag{20}$$

where

$$\Lambda^{\mathring{a}} := \frac{\Lambda(\tau^{\mathring{a}}, \phi^{\mathring{a}})}{1+\gamma_1+\gamma_2} = \beta_1^{\gamma_1} \gamma_2^{\gamma_2} (1 + \gamma_1 + \gamma_2)^{1-\gamma_1-\gamma_2} \tag{21}$$

This condition is required in order to obtain a closed form solution and has been applied in Uzawa (1965) two-sector growth model, Smith (2006) while describing the Ramsey model, Chilarescu (2008) and Hiraguchi (2009) while describing the Lucas (1988) model.

Lemma 1 *If $\alpha = \sigma$ then the solution of equation (20) is given by*

$$c(t) = \frac{\omega c_0}{c_0 + (\omega k_0 - c_0) e^{rt}} k(t) \tag{22}$$

Proof. If we consider the variable defined as $x = \frac{c}{k}$, we can write the following differential equation

$$\frac{\dot{x}}{x} = \frac{\dot{c}}{c} - \frac{\dot{k}}{k},$$

replacing (19) and (20), we obtain

$$\frac{\dot{x}}{x} = \frac{\alpha}{\sigma} \Lambda^{\mathring{a}} k^{\alpha-1} - \frac{r}{\sigma} - \Lambda^{\mathring{a}} k^{\alpha-1} + \frac{c}{k} \tag{23}$$

under the hypothesis $\frac{\alpha}{\sigma} = 1$, we get $\frac{\dot{x}}{x} = -\frac{r}{\sigma} + x$, where for some $x(0) = x_0$ the solution is $x(t) = \frac{\omega}{1 + (\frac{\omega}{x_0} - 1) e^{\omega t}}$, where $\omega := \frac{r}{\sigma}$. But for some $x_0 = \frac{c_0}{k_0}$ the solution is given by (22).

Theorem 1 *Under the assumptions of the above lemma, the following statements are valid:*

- If $\omega k_0 - c_0 = 0$, then consumption per labor unit is always proportional to the capital per labor unit

$$c(t) = \omega k(t) \tag{24}$$

- If $\omega k_0 - c_0 > 0$, then

$$\frac{\dot{k}(t)}{k(t)} > \frac{\dot{c}(t)}{c(t)}, \qquad \forall t \tag{25}$$

- If $\omega k_0 - c_0 < 0$, then

$$\begin{cases} \frac{\dot{k}(t)}{k(t)} < \frac{\dot{c}(t)}{c(t)}, & \forall t \in (0, \bar{t}) \\ \frac{\dot{k}(t)}{k(t)} > \frac{\dot{c}(t)}{c(t)}, & \forall t > \bar{t} \end{cases} \tag{26}$$

where $\bar{t} := \frac{1}{\omega} ln(\frac{c_0}{|\omega k_0 - c_0|})$

For $c_0 \neq \omega k_0$

$$\lim_{t\to\infty}\left(\frac{\dot c}{c}-\frac{\dot k}{k}\right) = -\omega \tag{27}$$

that is, there exists a t^*, such that $\frac{\dot k}{k} \approx \frac{\dot c}{c} + \omega \Leftrightarrow c(t) = \omega k(t) e^{-\omega(t-t^*)}, \forall\, t > t^*$.

Proof. From 22, the first statement is obviously true. Differentiating $x(t)$, we obtain $\frac{\dot x}{x} = \frac{\dot c}{c} - \frac{\dot k}{k} = -\frac{r(\omega k_0 - c_0)}{c_0 e^{-\omega t} + (\omega k_0 - c_0)}$. Thus the next three statements follow as consequence.

As it is well known, a macroeconomics model exhibits balanced growth if consumption and capital grow at a constant rate while hours of work per time period stay constant that is if and only if $c_t = \omega k_t$.

Theorem 2 *If model exhibits balanced growth, the dynamic of the state variable $k(t)$ is given by*

$$k(t) = \left(\frac{\omega}{\Lambda^* + e^{\omega(\alpha-1)t}(k_0^{\alpha-1}\omega - \Lambda^*)}\right)^{\frac{1}{\alpha-1}} \tag{28}$$

Proof. To prove the theorem, observe that, in the case $c_0 = \omega k_0$, $\dot k(t) = \Lambda^* k^\alpha - \omega k$ is a Bernoulli differential equation.

Theorem 1 shows the relation between growth and the variables c and k when varying the initial conditions (c_0, k_0).

- *Case 1* realizes balanced growth.
- *Case 2* tells us that if the ratio between initial conditions $\left(\frac{c_0}{k_0}\right)$ is smaller than $\omega = \frac{r}{\sigma}$ (i.e. constant rate of time preference and constant elasticity of intertemporal substitution ratio) then the capital stock growth ratio $\left(\frac{\dot k}{k}\right)$ is greater than the growth rate of consumption $\left(\frac{\dot c}{c}\right)$ at any point in time.
- *Case 3* implies that if the ratio between initial conditions $\left(\frac{c_0}{k_0}\right)$ is larger than $\omega = \frac{r}{\sigma}$ then for a given initial period $(0;\bar t)$ the growth rate of capital stock is larger than that of consumption, while for the remaining time the opposite occurs.
- *Case 4* if $c_0 \neq \omega k_0$ then for a significantly large period of time ($t \to \infty$) consumption goes to zero given $c(t) = \omega k(t) e^{-\omega(t-t^*)}$.

Concluding Remarks

This work addresses the issue of optimal policy with externalities caused by aggregate production and analyzes the equilibrium dynamics of a growth model with public finance and two different allocations of public spending. Fiscal policy is part of the aggregate economy. This generates a potential relationship between government and production. In order to better capture reality, the model considers negative production externalities by explicitly including them as unfavorable effects in the production function. However, differently from the conventional paradigm, government spending, along with private production, also generates undesired effect.

In the ideal world represented in economic models, agents interact exclusively through the market as a function of their choices and actions. In the real world, the actions of an agent may alter the environment where other economic agents operate, regardless of, and in addition to, the effects generated by the price system. Hence, we emphasize the role played by taxation and public expenditures in the internalization of external effects aiming at achieving a socially desirable outcome. This is an important issue, because if such public intervention is optimally done, overall welfare in the economy improves.

The model analyzes the equilibrium dynamics and derives a closed form solution for the optimal shares of consumption, capital accumulation, taxes and composition of the two different public expenditures which maximize a representative household's lifetime utilities for a centralized economy. With one restriction on the parameters $(\alpha = \sigma)$ we fully determine the solutions path for all variables of the model and determine the conditions for a balanced growth. Finally, given the active role the government has in determining the level of aggregate production, the model suggests that the higher (lower) the externalities, the lower (higher) the tax rate and public spending ought to be.

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