Claim 1.1 (Marshall Lerner condition for the nominal money supply) *The first derivative of the current account with respect to the nominal money supply is positive if and only if the sum of (i) the elasticity of real exports to the real exchange rate and (ii) the modulus elasticity of real imports to the real exchange rate is greater than one. Formally:*

$$ca_{M_S} > 0 \longleftrightarrow \eta_{ex_e} + |\eta_{im_e}| > 1.$$

Proof. Consider the current account equation expressed in real terms: $ca = ex - e \cdot im$.
Differentiate it with respect to the nominal money supply: $ca_{M_S} = ex_e e_{m_S} m_{S_{M_S}} - \left(e_{m_S} m_{S_{M_S}} im + e \cdot im_e e_{m_S} m_{S_{M_S}} \right)$.

Divide it by
$e_{m_S} m_{S_{M_S}}$: $\dfrac{ca_{M_S}}{e_{m_S} m_{S_{M_S}}} = ex_e - (im + e \cdot im_e)$.
Multiply it by
$\dfrac{e}{ex}$: $\dfrac{ca_{M_S} e}{e_{m_S} m_{S_{M_S}} ex} = \dfrac{e}{ex}(ex_e - im - e \cdot im_e)$.

Now, at efficiency the current account is null: $ca = 0 \longleftrightarrow ex = e \cdot im$.
Moreover, recall the definition of elasticities:
$\eta_q := \dfrac{q_p p}{q}$.

Therefrom, substitute $\dfrac{e}{ex}$ with $\dfrac{1}{im}$ and apply the definition of elasticities: $\dfrac{ca_{M_S}}{e_{m_S} m_{S_{M_S}} im} = \eta_{ex_e} - \dfrac{1}{im}(im + e \cdot im_e) \longrightarrow \dfrac{ca_{M_S}}{e_{m_S} m_{S_{M_S}} im} = \eta_{ex_e} - 1 - \eta_{im_e}$.

It follows that ca_{M_S} is positive if and only if $\eta_{ex_e} - 1 - \eta_{im_e}$ is positive: $ca_{M_S} > 0 \longleftrightarrow \eta_{ex_e} - 1 - \eta_{im_e} > 0$.
Finally, recall that price elasticities of demand are negative and that real exports are supplied, and real imports demanded; the elasticity of real imports to the real exchange rate is therefore negative: $\eta_{im_e} < 0$ and $|\eta_{im_e}| = -\eta_{im_e}$. Consequently, ca_{M_S} is positive if and only if $\eta_{ex_e} + |\eta_{im_e}|$ is greater than one: $ca_{M_S} > 0 \longleftrightarrow \eta_{ex_e} + |\eta_{im_e}| > 1. QED$

Such is a Marshall Lerner condition for an increase in the current account given an increase in the real exchange rate (sc. depreciation) due to an increase in the nominal money supply.

Claim 1.2 (Marshall Lerner condition for prices) *The first derivative of the current account with respect to prices is negative if and only if the sum of (i) the elasticity of real exports to the real exchange rate and (ii) the modulus elasticity of real imports to the real exchange rate is smaller than one. Formally:*

$$ca_p < 0 \longleftrightarrow \eta_{ex_e} + |\eta_{im_e}| < 1.$$

Proof. Consider the current account equation expressed in real terms: $ca = ex - e \cdot im$.
Differentiate it with respect to prices:
$ca_p = ex_e e_{m_S} m_{S_p} - \left(e_{m_S} m_{S_p} im + e \cdot im_e e_{m_S} m_{S_p} \right)$.
Divide it by
$e_{m_S} m_{S_p}$: $\dfrac{ca_p}{e_{m_S} m_{S_p}} = ex_e - (im + e \cdot im_e)$.
Multiply it by
$\dfrac{e}{ex}$: $\dfrac{ca_p e}{e_{m_S} m_{S_p} ex} = \dfrac{e}{ex}(ex_e - im - e \cdot im_e)$.

Substitute $\frac{e}{ex}$ with $\frac{1}{im}$ and apply the definition of elasticities: $\frac{ca_p}{e_{m_S} m_{S_p} im} = \eta_{ex_e} - \frac{1}{im}(im + e \cdot im_e) \longrightarrow \frac{ca_p}{e_{m_S} m_{S_p} im} = \eta_{ex_e} - 1 - \eta_{im_e}$.

It follows that ca_p is negative if and only if $\eta_{ex_e} - 1 - \eta_{im_e}$ is negative, namely, if and only if $\eta_{ex_e} + |\eta_{im_e}|$ is smaller than one: $ca_p < 0 \longleftrightarrow \eta_{ex_e} - 1 - \eta_{im_e} < 0 \longleftrightarrow \eta_{ex_e} + |\eta_{im_e}| < 1. QED$

Such is a Marshall Lerner condition for a decrease in the current account given a decrease in the real exchange rate (sc. appreciation) due to an increase in prices.

Claim 1.3 (Marshall Lerner condition for export demand) *The first derivative of the current account with respect to export demand is positive if and only if the sum of (i) the elasticity of real exports to the real exchange rate, (ii) the modulus elasticity of real imports to the real exchange rate and (iii) the quotient of the elasticity of real exports to export demand, the elasticity of the real exchange rate to money demand and the elasticity of money demand to export demand is greater than one. Formally:*

$$ca_{ed} > 0 \longleftrightarrow \eta_{ex_e} + |\eta_{im_e}| + \frac{\eta_{ex_{ed}}}{\eta_{e_{m_D}} \eta_{m_{D_{ed}}}} > 1.$$

Proof. Consider the current account equation expressed in real terms: $ca = ex - e \cdot im$. Differentiate it with respect to export demand: $ca_{ed} = ex_e e_{m_D} m_{D_{ed}} + ex_{ed} - (e_{m_D} m_{D_{ed}} im + e \cdot im_e e_{m_D} m_{D_{ed}})$.

Divide it by $e_{m_D} m_{D_{ed}}$: $\frac{ca_{ed}}{e_{m_D} m_{D_{ed}}} = ex_e + \frac{ex_{ed}}{e_{m_D} m_{D_{ed}}} - (im + e \cdot im_e)$.

Multiply it by $\frac{e}{ex}$, substitute it with $\frac{1}{im}$ where needed and apply the definition of elasticities: $\frac{ca_{ed} e}{e_{m_D} m_{D_{ed}} ex} = \frac{ex_e e}{ex} + \frac{ex_{ed} e}{e_{m_D} m_{D_{ed}} ex} - \frac{e}{ex}(im + e \cdot im_e)$ $\longrightarrow \frac{ca_{ed}}{e_{m_D} m_{D_{ed}} im} = \eta_{ex_e} + \frac{ex_{ed} e}{e_{m_D} m_{D_{ed}} ex} - (1 + \eta_{im_e})$.

Multiply it by $\frac{m_D ed}{m_D ed}$ and apply the definition of elasticities: $\left(\frac{m_D ed}{m_D ed}\right) \frac{ca_{ed}}{e_{m_D} m_{D_{ed}} im} = \frac{m_D ed}{m_D ed}\left(\eta_{ex_e} + \frac{ex_{ed} e}{e_{m_D} m_{D_{ed}} ex} - 1 - \eta_{im_e}\right)$ $\longrightarrow \frac{ca_{ed}}{e_{m_D} m_{D_{ed}} im} = \eta_{ex_e} + \frac{\eta_{ex_{ed}}}{\eta_{e_{m_D}} \eta_{m_{D_{ed}}}} - 1 - \eta_{im_e}$.

It follows that ca_{ed} is positive if and only if $\eta_{ex_e} + \frac{\eta_{ex_{ed}}}{\eta_{e_{m_D}} \eta_{m_{D_{ed}}}} - 1 - \eta_{im_e}$ is positive, namely, if and only if $\eta_{ex_e} + |\eta_{im_e}| + \frac{\eta_{ex_{ed}}}{\eta_{e_{m_D}} \eta_{m_{D_{ed}}}}$ is greater than one: $ca_{ed} > 0 \longleftrightarrow \eta_{ex_e} + \frac{\eta_{ex_{ed}}}{\eta_{e_{m_D}} \eta_{m_{D_{ed}}}} - 1 - \eta_{im_e} > 0 \longrightarrow \eta_{ex_e} + |\eta_{im_e}| + \frac{\eta_{ex_{ed}}}{\eta_{e_{m_D}} \eta_{m_{D_{ed}}}} > 1. QED$

Such is a Marshall Lerner condition for an increase in the current account given a decrease in the real exchange rate (sc. appreciation) due to an increase in export demand.

Claim 1.4 (Marshall Lerner condition for import demand) *The first derivative of the current account with respect to import demand is negative if and only if the sum of (i) the elasticity of real exports to the real exchange rate, (ii) the modulus elasticity of real imports to the real exchange rate and (iii) the quotient of the elasticity of real imports to import demand, the modulus elasticity of the real exchange rate to money demand and the elasticity of money demand to import demand is smaller than one. Formally:*

$$ca_{id} < 0 \longrightarrow \eta_{ex_e} + |\eta_{im_e}| + \frac{\eta_{im_{id}}}{|\eta_{e_{m_D}}|\eta_{m_{D_{id}}}} < 1.$$

Proof. Consider the current account equation expressed in real terms:
$ca = ex - e \cdot im.$

Differentiate it with respect to import demand:
$ca_{id} = ex_e e_{m_D} m_{D_{id}} - \left[e_{m_D} m_{D_{id}} im + e\left(im_e e_{m_D} m_{D_{id}} + im_{id}\right)\right].$

Divide it by $e_{m_D} m_{D_{id}}$: $\dfrac{ca_{id}}{e_{m_D} m_{D_{id}}} = ex_e - \left[im + e\left(im_e + \dfrac{im_{id}}{e_{m_D} m_{D_{id}}}\right)\right].$

Multiply it by $\frac{e}{ex}$, substitute it with $\frac{1}{im}$ where needed and apply the definition of elasticities:

$$\frac{ca_{id} e}{e_{m_D} m_{D_{id}} ex} = \frac{ex_e e}{ex} - \frac{e}{ex}\left[im + e\left(im_e + \frac{im_{id}}{e_{m_D} m_{D_{id}}}\right)\right]$$

$$\longrightarrow \frac{ca_{id}}{e_{m_D} m_{D_{id}} im} = \eta_{ex_e} - \left(1 + \eta_{im_e} + \frac{e \cdot im_{id}}{e_{m_D} m_{D_{id}} im}\right).$$

Multiply it by $\frac{m_D id}{m_D id}$ and apply the definition of elasticities:

$$\left(\frac{m_D id}{m_D id}\right)\frac{ca_{id}}{e_{m_D} m_{D_{id}} im} = \frac{m_D id}{m_D id}\left[\eta_{ex_e} - \left(1 + \eta_{im_e} + \frac{e \cdot im_{id}}{e_{m_D} m_{D_{id}} im}\right)\right]$$

$$\longrightarrow \frac{ca_{id}}{e_{m_D} m_{D_{id}} im} = \eta_{ex_e} - 1 - \eta_{im_e} - \frac{\eta_{im_{id}}}{\eta_{e_{m_D}} \eta_{m_{D_{id}}}}.$$

Notice that the elasticity of the real exchange rate to money demand is negative: $\eta_{e_{m_D}} < 0$ and $|\eta_{e_{m_D}}| = -\eta_{e_{m_D}}$. It follows that ca_{id} is negative if and only if $\eta_{ex_e} - 1 - \eta_{im_e} - \frac{\eta_{im_{id}}}{\eta_{e_{m_D}} \eta_{m_{D_{id}}}}$ is negative, namely, if and only if $\eta_{ex_e} + |\eta_{im_e}| + \frac{\eta_{im_{id}}}{|\eta_{e_{m_D}}|\eta_{m_{D_{id}}}}$ is smaller than one:
$ca_{id} < 0 \longleftrightarrow \eta_{ex_e} - 1 - \eta_{im_e} - \dfrac{\eta_{im_{id}}}{\eta_{e_{m_D}} \eta_{m_{D_{id}}}} < 0 \longrightarrow \eta_{ex_e} + |\eta_{im_e}| + \dfrac{\eta_{im_{id}}}{|\eta_{e_{m_D}}|\eta_{m_{D_{id}}}} < 1$ or
$\eta_{ex_e} + |\eta_{im_e}| + \dfrac{\eta_{im_{id}}}{\eta_{e_{m_D}}|\eta_{m_{D_{id}}}|} < 1. QED$

Such is a Marshall Lerner condition for a decrease in the current account given an increase in the real exchange rate (sc. depreciation) due to an increase in import demand.

Since money demand is complex to measure the elasticity of the real exchange rate to money demand, $\eta_{e_{m_D}}$, that is to say, is not easily calculable, thus, for empirical testing one can specifically express the real exchange rate in terms of the real money supply and of export and import demand:

$e = f(\overset{+}{m_S}, \overset{-}{ed}, \overset{+}{id})$, *ceteris paribus.*

Export demand can be proxied via tariffs, quotas or even confidence and import demand is again foreign export demand: $id = ed^*$. It follows that the twofold Marshall Lerner condition for money demand is accordingly simplified:

(i)

$$ca_{ed} > 0 \longleftrightarrow \eta_{ex_e} + |\eta_{im_e}| + \frac{\eta_{ex_{ed}}}{\eta_{e_{ed}}} > 1;$$

(ii)

$$ca_{id} < 0 \longleftrightarrow \eta_{ex_e} + |\eta_{im_e}| - \frac{\eta_{im_{id}}}{\eta_{e_{id}}} < 1.$$

Nevertheless, misspecification problems suggest the regression of the real exchange rate on all independent variables of money demand. Identical issues in fact suggest the same for the calculation of the first derivative of money demand with respect to (i) export demand and to (ii) import demand within the orbit of the attendant elasticity measures, that is, of $m_{D_{ed}}$ in $\eta_{m_{D_{ed}}}$ and of $m_{D_{id}}$ in $\eta_{m_{D_{id}}}$.

Conclusion

What are the respective effects of a unit increase in money demand on the real exchange rate and on the current account, all else equal? The real exchange rate is known to appreciate, but the current account need not deteriorate, as the canonical Marshall Lerner condition instead seems to suggest. Indeed, an outward application thereof dictates a current account deterioration by virtue of a real exchange rate appreciation. A noumenal application thereof, which this work has presented, by contrast clarifies that thecurrent account deteriorates by virtue of a real exchange appreciation due to a fall in the real money supply, all else equal, and *vice versa;* it further specifies that the current account improves by virtue of a real exchange rate appreciation due to a rise in money demand, all else equal, and *vice versa.*

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