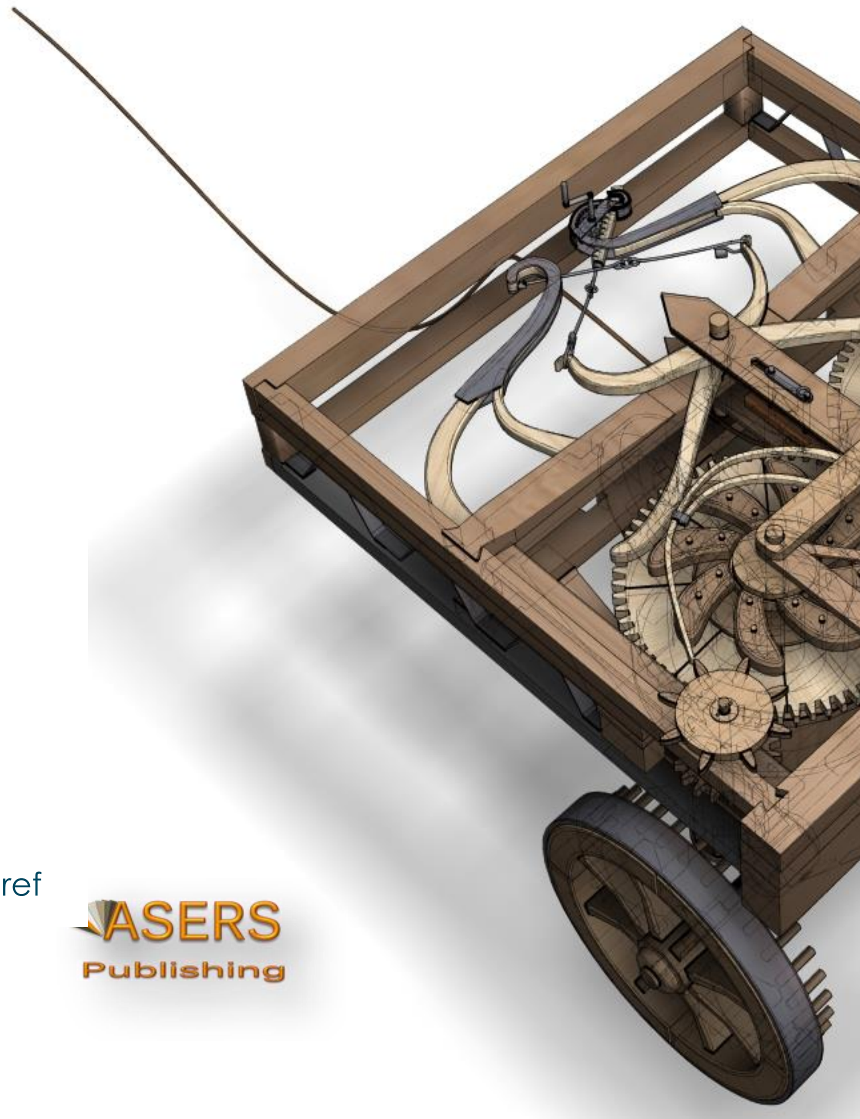
Theoretical and Practical Research in Economic Fields



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AGGREGATION WITH A NON-CONVEX LABOR SUPPLY DECISION, UNOBSERVABLE EFFORT, AND INCENTIVE ("FAIR") WAGES

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Abstract

The purpose of this note is to explore the problem of a non-convex labor supply decision in an economy with unobservable e_ort and incentive ("fair") wages a la Danthine and Kurmann (2004), and explicitly perform the aggregation presented there without a formal proof, and thus provide - starting from micro-foundations - the derivation of the expected utility functions used for the aggregate household. We show how lotteries as in Rogerson (1988) can be used to convexify consumption sets, and aggregate over individual preferences. With a discrete labor supply decisions, the elasticity of aggregate labor supply becomes a function of effort.

Keywords: Aggregation; Indivisible labour; Unobservable e_ort; Fair wages; Non-convexities.

JEL Classification: E1; J22; J41.

Introduction

The purpose of this note is to explore the problem of a non-convex labor supply decision in an economy with unobservable effort and incentive ("fair") wages a la Danthine and Kurmann (2004), and explicitly perform the aggregation presented there without a formal proof, and thus provide - starting from micro-foundations - the derivation of the expected utility function used for the aggregate household. We show how lotteries as in Rogerson (1988) can be used to convexify consumption sets, and aggregate over individual preferences. With a discrete labor supply decisions, the elasticity of aggregate labor supply becomes a function of effort.

1. The Model

The theoretical setup follows to a great extent Vasilev (2017). To simplify the analysis, the model economy here is static, without physical capital, and agents will face a non-convex labor supply decision. Effort exerted by workers is a productive input in the final goods sector, but unobservable, and thus not directly contractible. However, producers understand that while workers do not like exerting effort, they derive utility from returning the gift of a generous wage by supplying a higher effort level even in an environment of costly monitoring. This leads to the firm paying an incentive wage. Since the focus is on a one-period world, the model abstracts away from technological progress, population growth and uncertainty. There is a large number of identical one-member households, indexed by *i* and distributed uniformly on the [0; 1] interval. In the exposition below, we will use small case letters to denote individual variables and suppress the index *i* to save on notation.

1.1. Application functionality

Each household maximizes the following utility function:

$$\ln c - hG(e) \tag{1.1}$$

where *c* denotes consumption of household *i, h* is the fraction of time available to household *i* that is spent working, and *e* is the level of effort exerted. The total time endowment available to each household *i* is normalized to unity, thus leisure, $l = 1 - h$, is implicitly expressed as time off-work. The novelty here is the G(e) utility term, which, as in Vasilev (2017), is included to capture that workers may derive additional dis-utility from exerting effort.

As in Hansen (1985) and Rogerson (1988) household i's labor supply is assumed to be indivisible, *i.e.* $h \in \{0; 1\}$. The problem faced by a household that decides to work full-time is then to set $h = 1$ and enjoy

$$U^w = lnc^w - G(e^w), \tag{1.2}$$

where $c^w = w + \pi$ and e^w are the consumption and effort levels when working. Note that the effort level will be determined implicitly from its optimality condition $G^{'}(e) = 0$, which does not depend on the other model variables. In contrast, a household that decides not to work chooses $h = 0$ and enjoys

$$U^u = lnc^u, \tag{1.3}$$

where $c^u = \pi$ is the consumption level when the household is not working.

1.2. Effort function

As in Danthine and Kurmann (2004), the effort is modelled as follows:

$$G(e) = (e - (\phi_0 + \phi_1 \ln w + \phi_2 \ln N))^2 \tag{1.4}$$

where $\phi_0, \phi_1 > 0, \phi_2 < 0$, and N denotes aggregate employment. Thus, in equilibrium,

$$e = \phi_0 + \phi_1 \ln w + \phi_2 \ln N \tag{1.5}$$

In other words, if the worker receives an incentive ("fair") wage, s/he would supply a higher level of effort. On the other hand, the higher the employment, or the tighter the labor market, the lower the incentive to exert effort.

1.3. Stand-in firm

There is a representative firm in the model economy. It produces a homogeneous final product using a production function that requires labor *H* as the only input. For simplicity, output price will be normalized to unity. The production function $f(eH)$ features decreasing returns to scale (for any effort level): $f^{'}(eH) > 0, f^{''}(eH) < 0, f^{'}(0) = 1, f^{'}(e) = 0$. The representative firm acts competitively by setting the wage rate *w* and choosing *H* to maximize profit by stimulating optimal effort:

$$\Pi = f(eH) - WH \text{ s.t. } 0 \leq H \leq 1 \tag{1.6}$$

and

$$e = \phi_0 + \phi_1 \ln w + \phi_2 \ln N. \tag{1.7}$$

In equilibrium, there will be positive profit, which follows from the assumptions imposed on the production function.

1.4. Decentralized Competitive Equilibrium (DCE): Definition

A DCE is defined by allocations $\{c^w; c^u; e; H\}$, wage rate *w*, and aggregate profit $\Pi = \pi$, s.t. (1) all households maximize utility; (2) the stand-in firm maximizes profit; (3) all markets clear.

1.5. Characterization of the DCE and derivation of the aggregate utility function

It will be shown that in the DCE, if it exists, only some of the households will be employed and work full-time, while the rest will be unemployed. Following the arguments in Rogerson (1988) and Hansen (1985), it can be easily shown that polar cases in which each household either, or a case in which nobody works, cannot not be equilibrium outcomes. Therefore, it must be the case that a only proportion of the agents in the economy are working. Denote this mass of employed by *n*. Workers will receive consumption nc^w, while those who are unemployed will consume c^u. Note that *n* can be interpreted also as the probability of being chosen to work: This probability is determined endogenously in the model, as workers would seek for the optimal balance between the net return from working in terms of increased utility of consumption, which, however, comes at the expense of lower utility out of leisure. Note that no matter of the employment outcome, ex post every household enjoys the same utility level. Thus, in equilibrium $H = n = N$. As derived earlier, the wage is set equal to:

$$\ln w = \frac{\phi_1 - \phi_0}{\phi_1} - \frac{\phi_2}{\phi_1} \ln n \tag{1.8}$$

Firm's profit is then

$$\pi = f(ne) - wn > 0 \tag{1.9}$$

which follows from the decreasing returns to scale featured by the production function. Next, to show that the DCE actually exists, it is sufficient to show the existence of a fixed point $n \in (0,1)$ by analyzing a non-linear equation using the fact that in equilibrium utility is the same for all households.

It is trivial to show that everyone working in the market sector (n = 1) is not an equilibrium, since then w = 0. From the ex ante symmetry assumption for households, market consumption would be the same for workers and those not selected for work, while the latter would enjoy higher utility out of leisure, hence there is no benefit of working. Similarly, nobody working (n = 0) is not an equilibrium outcome either, since the firm would then offer a very high wage for the first unit of labor, and by taking a full-time job a marginal worker could increase his/her utility a lot.

Thus, if there is a DCE, then it must be that not all households would receive the same consumption bundle. If $n \in (0,1)$ is an equilibrium, then total utility for households that work should equal to the utility of households that do not supply any hours. This equation is monotone in *n*, as the utility function is a sum of monotone functions. Thus we can explore the behavior of that function (the difference between the utility of working and not working) as we let *n* vary in the (0, 1) interval. As $n \to 0$, the left-hand-side dominates (utility of working is higher), while when $n \to 1$, the right-hand-side dominates (utility of not working is higher), where the results follow from the concavity of the utility functions and the production technology. In addition, from the continuity of those functions, $\exists!\, n \in (0,1)$, which is consistent with equilibrium. The unique value of *n* follows from the monotonicity of the utility and production functions. Let c^w and c^u denote equilibrium consumption allocations of individuals selected for work, and those unemployed.

Given the indivisibility of the labor supply, the equilibrium allocation obtained above is not Pareto optimal, as demonstrated in Rogerson (1988). More specifically, a social planner could make everyone better-off by using an employment lottery in the first stage and choosing the fraction *n* of individual households to work and give everyone consumption $nc^w + (1-n)c^u$, In order to show this, we need to check that such an allocation is feasible, and that it provides a higher level of total utility. Showing feasibility is trivial as total market labor input and total consumption are identical to the corresponding individual equilibrium values.

Next, we will show that the new allocation, which is independent of the household's employment status, makes households better-off. This is indeed the case, where the strict inequality follows from the concavity of the logarithmic function. Thus, the SP is indeed giving in expected utility terms an allocation that is an improvement over the initial equilibrium allocation. If households can pool income together and doing so, they will be able to equalize consumption across states, i.e.,$c = c^w = c^u$, so aggregate utility becomes

$$\ln c - G(e)n \tag{1.10}$$

which is the representation in Danthine and Kurmann (2004). On the aggregate, when each household faces an indivisible labor choice, the representative agent obtained from the aggregation features different preferences of work, as the disutility of work is now a function of effort.

Conclusions

The purpose of this note is to explore the problem of a non-convex labor supply decision in an economy with unobservable effort and incentive ("fair") wages a la Danthine and Kurmann (2004), and explicitly perform the aggregation presented there without a formal proof, and thus provide - starting from micro-foundations - the derivation of the expected utility function used for the aggregate household. We show how lotteries as in Rogerson (1988) can be used to convexify consumption sets, and aggre- gate over individual preferences. With a discrete labor supply decisions, the elasticity of aggregate labor supply is a function of effort.

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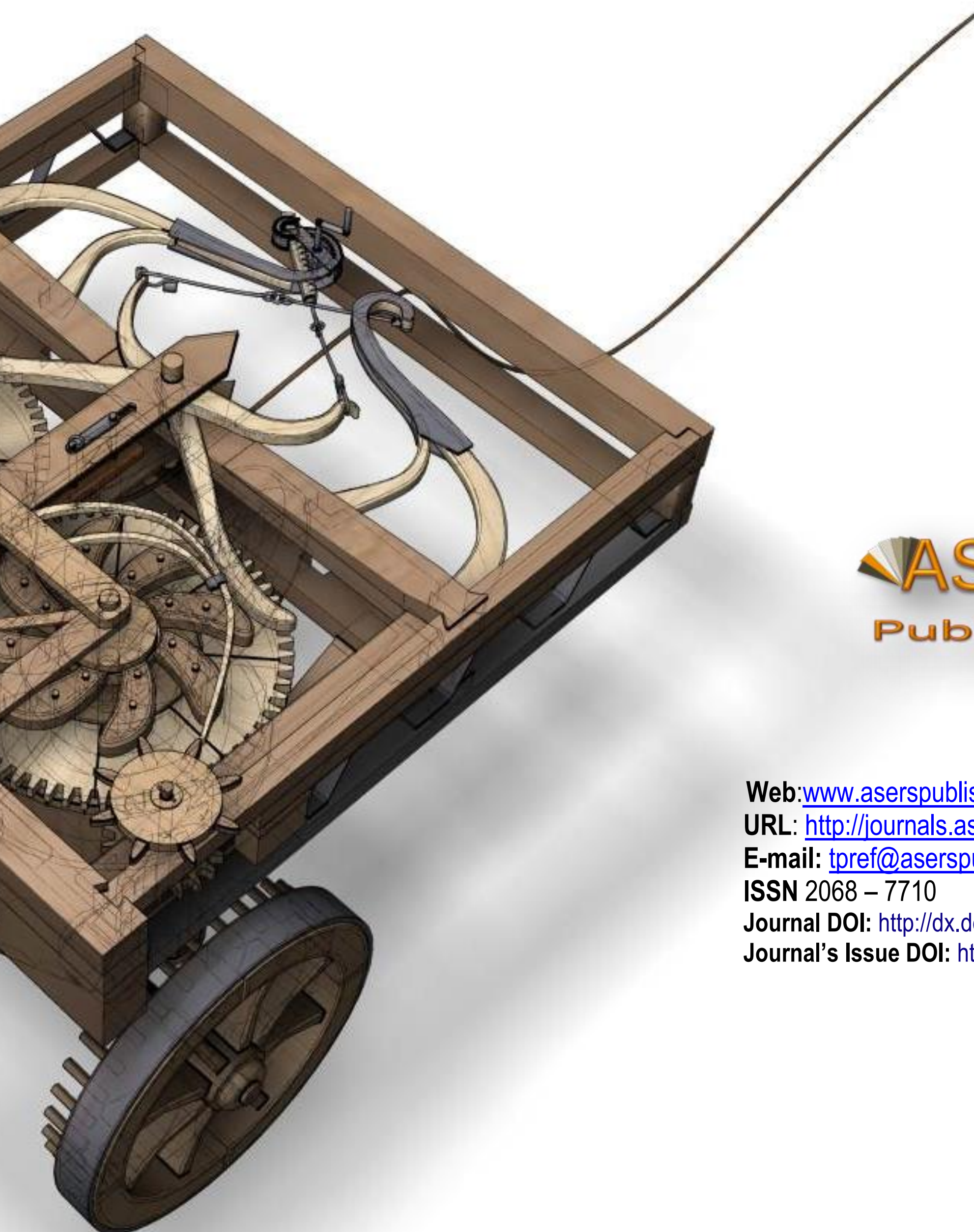

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