ASERS

Theoretical and Practical Research in Economic Fields

Biannually
Volume II
Issue 2(4)
Winter 2011

ISSN 2068 – 7710

Summer 2011
Volume II, Issue 1(3)

ASERS Publishing
http://www.asers.eu/asers-publishing
ISSN 2068 – 7710

Contents:

Call for Papers
Summer_Issue 2012

Theoretical and Practical Research in Economic Fields

Many economists today are concerned by the proliferation of journals and the concomitant labyrinth of research to be conquered in order to reach the specific information they require. To combat this tendency, **Theoretical and Practical Research in Economic Fields** has been conceived and designed outside the realm of the traditional economics journal. It consists of concise communications that provide a means of rapid and efficient dissemination of new results, models and methods in all fields of economic research.

Theoretical and Practical Research in Economic Fields publishes original articles in all branches of economics – theoretical and empirical, abstract and applied, providing wide-ranging coverage across the subject area.

Journal promotes research that aim at the unification of the theoretical-quantitative and the empirical-quantitative approach to economic problems and that are penetrated by constructive and rigorous thinking. It explores a unique range of topics from the frontier of theoretical developments in many new and important areas, to research on current and applied economic problems, to methodologically innovative, theoretical and applied studies in economics. The interaction between empirical work and economic policy is an important feature of the journal.

Theoretical and Practical Research in Economic Fields is indexed in RePEC, IndexCopernicus, EBSCO, ProQuest and very soon in CEEOL databases.

The primary aim of the Journal has been and remains the provision of a forum for the dissemination of a variety of international issues, empirical research and other matters of interest to researchers and practitioners in a diversity of subject areas linked to the broad theme of economic sciences.

All the papers will be first considered by the Editors for general relevance, originality and significance. If accepted for review, papers will then be subject to double blind peer review.

Invited manuscripts will be due till May 25st, 2012, and shall go through the usual, albeit somewhat expedited, refereeing process.

Deadline for submission of proposals:	25st May 2012
Expected Publication Date:	1st June 2012
Web:	www.asers.eu/journals/tpref/
E-mail:	tpref@asers.eu

To prepare your paper for submission, please see full author guidelines in the following file:
TPREF_Full_Paper_Template.doc, then send it via email at tpref@asers.eu.

A MATHEMATICAL MODEL FOR A COMPANY'S ADVERTISING STRATEGY

Laura UNGUREANU
Spiru Haret University, **Romania**
laura.ungureanu@spiruharet.ro

Abstract:

Models are better means of approximating reality, suitable for most economic phenomena which are generally represented by dynamical processes. Economic mathematicians have begun their study of this type of processes and have reached so far that today they are able to elaborate dynamical bifurcation diagrams that include all mathematical phenomena and Hopf bifurcation, in particular. In this paper, we have explained the behavior of an advertising model that consists out of a Cauchy problem made for/from a system of ordinary differential equations.

An advertising model is written in the form a Cauchy problem for a system of two first order ordinary differential equations involving two real parameters. For two particular values of them it is shown that a degenerate Bogdanov-Takens bifurcation phenomenon occurs. This implies an extremely complex behavior of the economic model for advertising.

Keywords: dynamical model, advertising strategy, limit cycle, advertising spiral.

JEL Classification: C02, C32, M37.

1. Mathematic model

This advertising model firstly elaborated in 1950 has been the subject of a more rigorous analysis in 1992, by Feichtinger. It was afterwards associated with one of the epidemic theories - as advertising can be compared with "spreading germs". Potential buyers (*X*) are „infected" by these germs through advertising and thus they get in contact with brand consumers (*Y*).

$$\begin{cases} \dot{X} = k - aXY + \beta Y \\ \dot{Y} = aXY - \delta Y \end{cases}$$

where $a(t) = \alpha Y$ is the contact rate of advertising in a given time *t*, presumed to be proportional with the number of regular buyers; β represents the distribution rate of competing brands; ε stands as the rate of migration, mortality or oblivion; $\delta = \varepsilon + \beta$. Thus, the system becomes (Tu, 1994):

$$\begin{cases} \dot{X} = k - aXY^2 + \beta Y \\ \dot{Y} = aXY^2 - \delta Y \end{cases} \quad (1.1)$$

1.2. Case scenario - $k = 0$

In this particular situation, the system becomes $\begin{cases} \dot{X} = -aXY^2 + \beta Y \\ \dot{Y} = aXY^2 - \delta Y \end{cases}$ and allows an infinite number of equilibrium points of type $(X, 0)$. The linearized approximation of such a point is given by $\begin{cases} \dot{X} = \beta Y \\ \dot{Y} = -\delta Y \end{cases}$ and defined by the Jacobian matrix $\begin{pmatrix} 0 & \beta \\ 0 & -\delta \end{pmatrix}$. The equation that asserts this matrix is $s^2 + \delta s = 0$ and allows its own values, $s_1 = 0$ and $s_2 = -\delta$, which makes the equilibrium point non-hyperbolic. Its non-hyperbolicity makes it impossible to apply a Hartman-Grobman theorem. This is why we cannot link the portrait phase of the nonlinear dynamical system to the portrait phase of the corresponding linear dynamical system, in the sense of topological equivalence. The eigen vectors corresponding are $v_1(-\beta,\delta)$ and $v_2(1,0)$. The set of equilibrium points of the linear system are situated on the line $Y=0$.

1.3. Case scenario $k \neq 0$

In this case, the following transformations can be made $x=\frac{\alpha k}{\delta\varepsilon}X$, $y=\frac{\varepsilon}{k}Y$, $\gamma=\frac{\alpha k^2}{\delta\varepsilon^2}$, $\theta=\frac{\beta}{\delta}$, $u=x-1$, $v=y-1$ and changing the time variable $r=\delta t$, this is the result:

$$\begin{cases} \dot{u}=-\gamma\left(u+\psi v+2uv+v^2+uv^2\right) \\ \dot{v}=u+v+2uv+v^2+uv^2 \end{cases} \tag{1.2}$$

where $\psi=2-\phi$.

The above system (1.2) can also be written as:

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix}=\begin{pmatrix} -\gamma & -\gamma\psi \\ 1 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix}+g(u,v)\begin{pmatrix} -\gamma \\ 1 \end{pmatrix}$$

where $g(u,v)=2uv+v^2+uv^2$

2. Equilibrium points

Taking into account α 's economic significance, the result is generally $\gamma\neq 0$. Also, because $\varepsilon\neq 0$ then $\psi\neq 1$. Under these circumstances, the system allows only a single equilibrium point $r_0(0,0)$, the origin. The linearized approximation is the system:

$$\begin{cases} \dot{U}=-\gamma U-\gamma\psi V \\ \dot{V}=U+V \end{cases} \tag{2.1}$$

and the Jacobian matrix $A=\begin{pmatrix} -\gamma & -\gamma\psi \\ 1 & 1 \end{pmatrix}$ with the characteristic polynomial of $P(\lambda)=\lambda^2+(\gamma-1)\lambda+\gamma\psi-\gamma=0$ which has the discriminant $\Delta=(\gamma-1)^2-4\gamma(\psi-1)=\gamma^2-4\gamma\psi+2\gamma+1$ and values of $\lambda_{1,2}=\frac{1}{2}\left(1-\gamma\pm\sqrt{\Delta}\right)=\frac{1-\gamma}{2}\pm\frac{1}{2}\sqrt{(\gamma+1)^2-4\gamma\psi}$.

We notice that $\Delta=0$ represents – in the parameter space (γ,ψ) - a hyperbola whose center is given by $C\left(0,\frac{1}{2}\right)$ and whose asymptote equations are $\gamma=0$ and $\gamma-4\psi+2=0$. In the domain held between the bows of the hyperbola which are the origin of the landmark, we have $\Delta>0$, and beyond this point, the discriminant is negative. Concerning the values, the following situation arises (Figure 2.1):

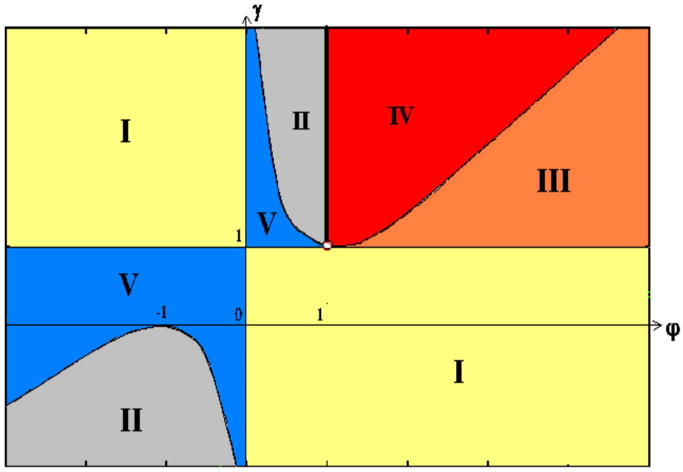

Figure 2.1. The dynamical bifurcation diagram

In the domain **I**, the origin is a saddle point; in domain **II** – it is a repulsive focus; in **III** – an attracting knot; in domain **IV –** an attracting focus; in domain **V –** a repulsive knot, on the semi-line $\gamma = 1$, where $\psi > 1$is the center, and **O** is a non-hyperbolic point (double zero).

The only points from the parameter space for which the equilibrium point isn't a hyperbolic point are those situated on the semi-line $\gamma = 1\ \psi \geq 1$. For all the other values taken by the γ and ψ parameters, the origin is a hyperbolic point and – based on the Hartman-Grobman theorem – the phase portraits from the nonlinear case are topological equivalents to those of the linearized. Hence, we deal with the non-hyperbolic case in which the information provided by the associated linearized system will have to be added. In the case $\gamma = 1$the initial system becomes:

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} -1 & -\psi \\ 1 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} + g(u,v)\begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{2.2}$$

The matrix of the linearized system is $A = \begin{pmatrix} -1 & -\psi \\ 1 & 1 \end{pmatrix}$ and its values are purely imaginary $\lambda_{1,2} = \pm i\omega$ cu $\omega = \sqrt{\psi - 1}$ and $\left.\frac{\partial \operatorname{Re} \lambda_1}{\partial \gamma}\right|_{\gamma=1} = -\frac{1}{2} \neq 0$. So, for $\psi > 1$ (invariant) and $\gamma = 1$, a Hopf bifurcation is revealed. For $\lambda_1 = i\omega$, the complex vector is:

$$v = \begin{pmatrix} i\omega - 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} + i\begin{pmatrix} \omega \\ 0 \end{pmatrix}.$$

The real vector subspace of (1.1) for a base of $B = \{\bar{v}_1(-1,1), v_2(\omega,0)\}$; the crossing matrix from the initial base to base B is $P = \begin{pmatrix} -1 & \omega \\ 1 & 0 \end{pmatrix}$ and so the formula that allows the changing of the coordinates of a vector is $\begin{pmatrix} u \\ v \end{pmatrix} = P\begin{pmatrix} x \\ y \end{pmatrix}$. In base B, the A matrix assumes the canonical shape $A' = P^{-1}AP = \begin{pmatrix} 0 & -\omega \\ \omega & 0 \end{pmatrix}$ and it yields:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 0 & -\omega \\ \omega & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} h(x,y) \\ k(x,y) \end{pmatrix} \quad (2.3)$$

where $h(x,y) = 2(-x+\omega y)x + x^2 + (-x+\omega y)x^2$

$k(x,y) = 0$

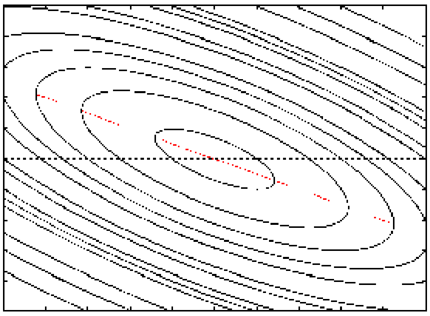

Figure 2.2a. Linear case (center)

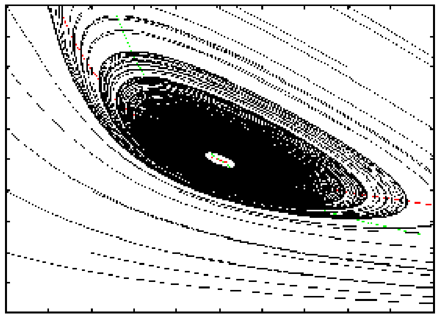

Figure 2.2b. Non-linear case (Hopf bifurcation point)

This is the normal shape and the a coefficient, introduced by Guckenheimer, gives stability to the orbit; evaluated by $x = y = 0$ it yields:

$$a = \frac{1}{16}\left(h_{xxx} + h_{xyy}\right) + \frac{1}{16\omega} h_{xy}\left(h_{xx} + h_{yy}\right) = \frac{1}{16}(-6-4) = -\frac{5}{8} < 0$$

Because $a < 0$ - according to the Hopf bifurcation theorem – the result is that the system has a stable periodic orbit (Figure 2.2). In this case, the complex conjugate values are purely imaginary; hence, the origin is a repulsive subcritical Hopf bifurcation point (Figure 2.3). The equilibrium point is non-hyperbolic, defined as a condition of involution ($trA = 0$), so that it is situated in co-dimension one.

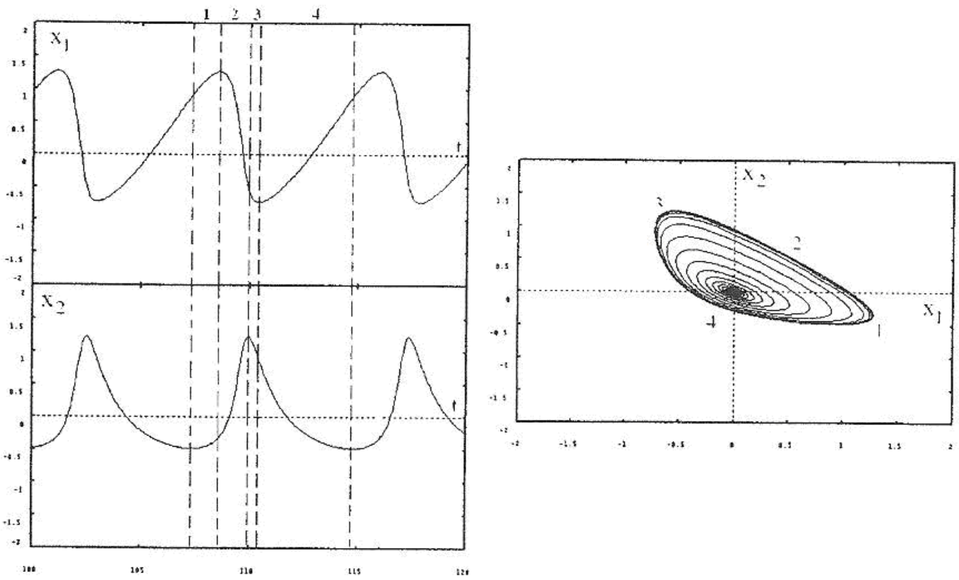

Figure 2.3. Limit cycle, an inside trajectory and the oscillations for $\gamma = 0.9$ and $\psi = 2$

3. The case of $\gamma = \psi = 1$

In the particular case that $\gamma = \psi = 1$, the system becomes $\begin{cases} \dot{u} = -\left(u + v + 2uv + v^2 + uv^2\right) \\ \dot{v} = u + v + 2uv + v^2 + uv^2 \end{cases}$. Its equilibrium points are situated on the equation curve of $u + v + 2uv + v^2 + uv^2 = 0$, which is the equivalent of the algebraic system $\begin{cases} v+1=0 \\ u+v+uv=0 \end{cases}$. So, the equilibrium points describe the line $v=-1$ and the hyperbola $u+v+uv=0$. Furthermore, there is also the fixed variety given by $v=-u$, tangent at the origin of the hyperbola $u+v+uv=0$ (Figure 3.1). The sense of this variety is given by the sign of the derivative $\dot{u}=v^2(1+v)$.

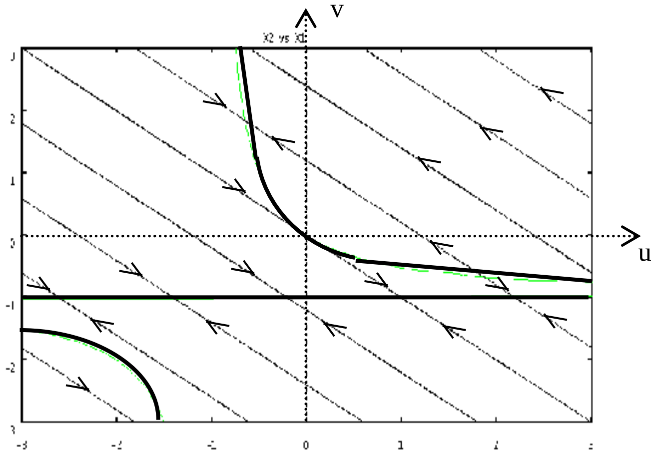

Figure 3.1. The set of equilibrium points

At the origin, the Jacobian matrix resembles $A=\begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix}$ and we obtain $\det A=0$, $trA=0$, $A\neq O_2$, being situated in the conditions of the Bogdanov-Takens bifurcation, which means co-dimension two, with two involution conditions.

For a given equilibrium point M_0 on the hyperbola $H: u+v+uv-0$, $M_0(u_0,v_0)$, the Jacobian matrix of the linearized system is $B=\begin{pmatrix} -1-2v_0-v_0^2 & -1 \\ 1+2v_0+v_0^2 & 1 \end{pmatrix}$ and the corresponding equation has its roots in $\lambda_1=0$, $\lambda_2=-2v_0-v_0^2$. For $\lambda_1=0$, the main direction is given by the vector $\overline{w_1}(1+u_0,-1-v_0)$, and for $\lambda_2=-2v_0-v_0^2$ the main direction is given by $\overline{w_2}(1,-1)$.

The tangent at M_0 on H is given by the right side of the equation $(v_0+1)u+(u_0+1)v+u_0+v_0=0$. And what is interesting enough to observe is that $\overline{w_1}$ is a direction vector for this tangent.

3.1. The analysis around the equilibrium $(0,0)$

In order to have this equilibrium for the zero values of the parameters, we performed the transformation $\gamma=1+\alpha_1$, $\psi=1+\alpha_2$. Thus (1.2) becomes:

$$\begin{cases} \dot{u}=-(1+\alpha_1)[u+(1+\alpha_2)v+v^2+uv^2+2uv] \\ \dot{v}=u+v+2uv+v^2+uv^2 \end{cases} \tag{3.1}$$

corresponding to the matrix

$$\overline{A}(\alpha)=\begin{pmatrix} -(1+\alpha_1) & -(1+\alpha_1)(1+\alpha_2) \\ 1 & 1 \end{pmatrix} \tag{3.2}$$

For $\overline{\alpha} = 0$, it has the eigenvalues $\lambda_1(0) = \lambda_2(0) = 0$. The corresponding eigenvectors are $\overline{v_0} = (-1.1)^T$, $\overline{v_1} = (0.1)^T$ and the generalized eigenvectors are $\overline{\omega_0} = (-1.0)^T$, $\overline{\omega_1} = (1.1)^T$.
Let us use the linear transformation

$$\begin{cases} u = -u_1 \\ v = u_1 + v_1 \end{cases} \tag{3.3}$$

to turn (3.1) into:

$$\begin{cases} \dot{u}_1 = (1+u_1)(\alpha_2 u_1 + v_1 + \alpha_2 v_1 + u_1^2 - v_1^2 - u_1^3 - 2u_1^2 v_1 - u_1 v_1^2) \\ \dot{v}_1 = -\alpha_2(1+\alpha_1)u_1 - (\alpha_1 + \alpha_2 + \alpha_1\alpha_2)v_1 - \alpha_1 u_1^2 + \alpha_1 v_1^2 + \alpha_1 u_1^3 + \\ \qquad + 2\alpha_1 u_1^2 v_1 + \alpha_1 u_1 v_1^2 \end{cases} \tag{3.4}$$

Writing (3.4) in the form:

$$\begin{cases} \dot{u}_1 = v_1 + a_{00}(\alpha) + a_{10}(\alpha)u_1 + a_{01}(\alpha)v_1 + \frac{1}{2}a_{20}(\alpha)u_1^2 + a_{11}(\alpha)u_1 v_1 + \\ \qquad + \frac{1}{2}a_{02}(\alpha)v_1^2 + P_1(u_1, v_1, \alpha) \\ \dot{v}_1 = b_{00}(\alpha) + b_{10}(\alpha)u_1 + b_{01}(\alpha)v_1 + \frac{1}{2}b_{20}(\alpha)u_1^2 + b_{11}(\alpha)u_1 v_1 + \\ \qquad + \frac{1}{2}b_{02}(\alpha)v_1^2 + P_2(u_1, v_1, \alpha) \end{cases} \tag{3.5}$$

where $a_{kl}(\alpha)$ and $P_{1,2}(u_1, v_1, \alpha)$ are smooth functions of their arguments. We have $a_{00} = 0$; $a_{10} = \alpha_2(1+\alpha_1)$; $a_{01} = (1+\alpha_1)(1+\alpha_2) - 1$; $a_{20} = 2(1+\alpha_1)$; $a_{02} = -2(1+\alpha_1)$; $a_{11} = 0$ and $b_{00} = 0$; $b_{10} = -\alpha_2(1+\alpha_1)$; $b_{01} = -(\alpha_1 + \alpha_2 + \alpha_1\alpha_2)$; $b_{20} = -2\alpha_1$; $b_{02} = 2\alpha_1$; $b_{11} = 0$.

With the transformation:

$$\begin{cases} y_1 = u_1 \\ y_2 = v_1 + a_{00} + a_{10}u_1 + a_{01}v_1 + \frac{1}{2}a_{20}u_1^2 + a_{11}u_1 v_1 + \frac{1}{2}a_{02}v_1^2 + P(u, \cdot) \end{cases}$$

(3.4) becomes:

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = g_{00}(\alpha) + g_{11}(\alpha)y_1 + g_{01}(\alpha)y_2 + \frac{1}{2}g_{20}(\alpha)y_1^2 + g_{11}(\alpha)y_1 y_2 + \\ \qquad + \frac{1}{2}g_{02}(\alpha)y_2^2 + Q(y, \alpha) \end{cases} \tag{3.6}$$

for certain smooth functions $g_{kl}(\alpha)$, $g_{00}(0) = g_{10}(0) = g_{01}(0) = 0$ and a smooth functions $Q(y, \alpha) = O(\|y\|^3)$. One can verify that $g_{20}(0) = b_{20}(0)$, $g_{11}(0) = a_{20}(0) + b_{11}(0)$, $g_{02}(0) = b_{02}(0) + 2a_{11}(0)$.
With the transformation:

$$\begin{cases} y_1 = z_1 + \delta(\alpha) \\ y_2 = z_2 \end{cases}$$

(3.5) becomes (3.6):

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = g_{00} + g_{10}\delta + O(\delta^2) + (g_{10} + g_{20}\delta + O(\delta^2))z_1 + (g_{01} + g_{11}\delta + O(\delta^2))z_2 + \\ \qquad + \frac{1}{2}(g_{20} + O(\delta))z_1^2 + (g_{11} + O(\delta))z_1 z_2 + \frac{1}{2}(g_{02} + O(\delta))z_2^2 + O(\|z^3\|) \end{cases}$$

where $g_{11}(0) = a_{20}(0) + b_{11}(0) = 2 \neq 0$, there $\delta(\alpha) = -\frac{g_{01}(\alpha)}{g_{11}(0)}$ is completed by annihilating the coefficient of the term in z_2, in the equation (3.6). There (3.6) becomes:

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = h_{00}(\alpha) + h_{10}(\alpha)z_1 + \frac{1}{2}h_{20}(\alpha)z_1^2 + h_{11}(\alpha)z_1 z_2 + \frac{1}{2}h_{02}(\alpha)z_2^2 + R(z,\alpha) \end{cases} \quad (3.7)$$

Where:

$h_{00}(\alpha) = g_{00}(\alpha) + \dots$, $h_{10}(\alpha) = g_{10}(\alpha) - \frac{g_{20}(0)}{g_{11}(0)}g_{01}(\alpha) + \dots$, $h_{20}(0) = g_{20}(0)$, $h_{11}(0) = g_{11}(0)$, $h_{02}(0) = g_{02}(0)$.

By the time transformation $dt = (1 + \theta z_1)d\tau$, (3.7) reads:

$$\begin{cases} \dot{\xi}_1 = z_2 + \theta z_1 z_2 \\ \dot{\xi}_2 = h_{00} + (h_{10} + h_{00}\theta)z_1 + \frac{1}{2}(h_{20} + 2h_{10}\theta)z_1^2 + h_{11}z_1 z_2 + \frac{1}{2}h_{02}z_2^2 + O(\|z^3\|) \end{cases} \quad (3.8)$$

and by a coordinate transformation $\eta_1 = \xi_1$, $\eta_2 = \xi_2 + \theta\xi_1\xi_2$ (3.8) becomes:

$$\begin{cases} \dot{\eta}_1 = \eta_2 \\ \dot{\eta}_2 = f_{00}(\alpha) + f_{10}(\alpha)\eta_1 + \frac{1}{2}f_{20}(\alpha)\eta_1^2 + f_{11}(\alpha)\eta_1\eta_2 + \frac{1}{2}f_{02}(\alpha)\eta_2^2 + O(\|\eta^3\|) \end{cases} \quad (3.9)$$

where $f_{00}(\alpha) = h_{00}(\alpha)$, $f_{10}(\alpha) = h_{10}(\alpha) + h_{00}(\alpha)\theta(\alpha)$, $f_{20}(\alpha) = h_{20}(\alpha) + 2h_{10}(\alpha)\theta(\alpha)$, $f_{11}(\alpha) = h_{11}(\alpha)$, $f_{02}(\alpha) = h_{02}(\alpha) + 2\theta(\alpha)$

we choose θ such that the coefficient of η_2^2 vanishing, $\theta(\alpha) = -\frac{h_{02}(\alpha)}{2}$.

So far, we have:

$$\begin{cases} \dot{\eta}_1 = \eta_2 \\ \dot{\eta}_2 = \mu_1(\alpha) + \mu_2(\alpha)\eta_1 + A(\alpha)\eta_1^2 + B(\alpha)\eta_1\eta_2 + O(\|\eta^3\|) \end{cases} \quad (3.10)$$

where $\mu_1(\alpha)=h_{00}(\alpha)$, $\mu_2(\alpha)=h_{10}(\alpha)-\frac{1}{2}h_{00}(\alpha)h_{02}(\alpha)$ and $A(\alpha)=\frac{1}{2}\left(h_{20}(\alpha)-h_{10}(\alpha)h_{02}(\alpha)\right)$, $B(\alpha)=h_{11}(\alpha)$.

Now, in the method from Kuznetsov (Kuznetsov,1995) the next time transformation is $t=\left[\frac{B(\bar{\alpha})}{A(\bar{\alpha})}\right]\tau$. Due to the fact that $b_{20}=0$, it follows $A(\bar{0})=0$ and, so, we cannot continue the chain of transformations leading to the Bogdanov-Takens normal form. Hence, the point $(\gamma,\psi)=(1,1)$ corresponds to a degenerated Bogdanov-Takens bifurcation point. In order to obtain the corresponding normal form it necessary to apply other methods.

4. Economic interpretation of the results

From the economic point of view, and taking into account the significance of α, one can conclude that normally $\gamma\neq 0$. In the same manner, because $\varepsilon\neq 0$, then $\psi\neq 1$. In the equilibrium points that are situated on $\gamma=1$, $\psi>0$, the system allows a stable periodic orbit (with a stable limit cycle) consisting out of four phases (Figure 4.1):

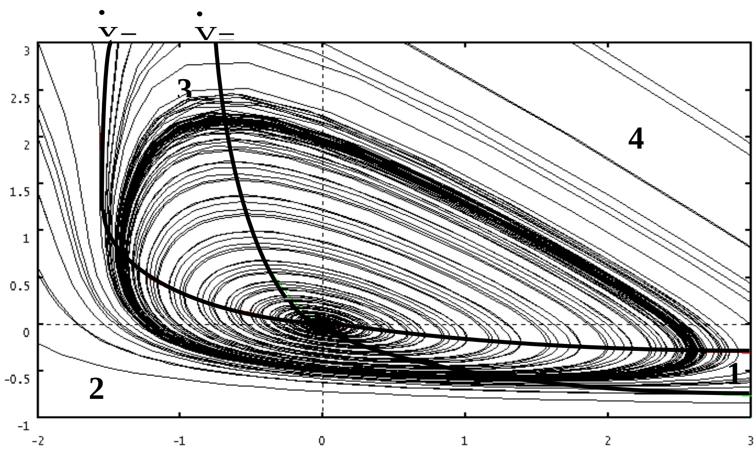

Figure 4.1. The stable periodic orbit, with four phases

1) prosperity - when X increases, Y increases; so, the number of potential buyers and the number of users both increase, hence it indicates a time of prosperity, when the product fills a gap in the market;

2) saturation - when X decreases, Y increases; this reveals the time when the product saturates the market, it is well-known and the main concern of the buyer is "which product should I buy?";

3) free-fall - when X, Y both decrease; at this phase, the number of users decreases; it's a time of decrease, when the product is at a certain point of acceptance, its utility known but its place on the market is only due to its past reputation

4) come-back, recovery when X increases and Y decreases. In the recovery phase the buyer is reminded why the product exists, as it is relaunched by a new advertising campaign.

The purpose of advertising is as much to inform consumers regarding the existence of a new product (in terms of its functionality, usage, advantages compared with other similar products), as it is to orient them towards purchasing these products. Advertising campaigns must be correlated with the activities conducted in the market and with the actions of the launching – on a large scale – of the product.

The equations of the mathematical model proposed in this article can be extended taking into account that the estimation of parameters in such functions use statistic data that come from surveys, selective market research, family budget planning, research studies regarding the launch of a new product, studies concerning the life cycle of a certain long-term use product, yearly statistics, the degree of substitution and complementarily of certain products or the buyers' behavior.

5. Conclusion and prospects for further research

The apparition of the nonlinear dynamics had permitted the understanding and the development of some processes and methods that draw near the phenomenon to reality. The development of the theory of singularities and the theory of bifurcations had completed the multitude of instruments of analyzing and represents dynamics more and more complex giving the possibility of analyzing systems that were hard even impossible to use them from the traditional point of view. The study of nonlinear dynamics is very important for us because the economic systems are defined as nonlinear. Much of them contains multiple discontinuities and incorporates an inherent instability being permanently submitted to the actions of shocks and external and internal perturbations.

Generally, with the activities being more complex, with the need of planning, search for strategies and formal actions grows. The economic domain is a domain in which the uncertain grade and risk is very high and in which the planning plays an important role in trying to reduce this incertitude. In essence, the elaboration of strategies in this domain purposes a clear and systematic structure of the modulations in which the followed objectives can be touched by a judicious allocation of the resources by long or short term.

This study is only a starting point of economic dynamics. We are confronted with more difficult analytic problems: economic systems are described by unstable nonlinear dynamic equations of high dimensions with different adjustment speeds. The existence nonlinear theories have significant implications for economic forecasting, methodologies and so on.

References

[1] Arrowsmith, D. K., and Place, C. M. 1990. *An Introduction to Dynamical Systems*. Cambridge University Press.

[2] Beltrami, E. 1990. *Mathematics for Dynamic Modeling*. Academic Press, New York

[3] Braun, M. 1983. *Differential Equations and their Applications*. Third Edition, Springer, New York.

[4] Gandolfo, G. 1996. *Economic dynamics*, Springer, Berlin.

[5] Georgescu, A., Moroianu, M. and Oprea, I. 1999. *Teoria bifurcaţiei.* Piteşti University Publishing House, *in Romanian.*

[6] Hirsch, M. W. and Smale, S. 1996. *Differential Equations, Dynamical Systems and Linear Algebra.* Academic Press, New York.

[7] Kuznetsov, Y. A. 1995. *Elements of applied bifurcation theory.* Springer, New York.

[8] Tu, P.N.V. 1994. *Dynamical Systems*, Springer.

[9] Ungureanu, L. 2004. *Structural Stability and Bifurcation in two Models of Economic Dynamics*, Piteşti University Press, Piteşti (in Romanian).

[10] Zhang, W.B. 1990. *Economic Dynamics*. Springer-Verlag, Berlin.

ASERS Publishing
ASERS

ASERS Publishing
Web: www.asers.eu
URL: http://www.asers.eu/asers-publishing
ISSN 2068 – 7710